# Understanding Power System Harmonics 

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## 1. Introduction

Power systems are designed to operate at frequencies of 50 or 60 Hz . However, certain types of loads produce currents and voltages with frequencies that are integer multiples of the 50 or 60 Hz fundamental frequency. These higher frequencies are a form of electrical pollution known as power system harmonics.

Power system harmonics are not a new phenomenon. In fact, a text published by Steinmetz in 1916 devotes considerable attention to the study of harmonics in three-phase power systems. In Steinmetz's day, the main concern was third harmonic currents caused by saturated iron in transformers and machines. He was the first to propose delta connections for blocking third harmonic currents.

After Steinmetz's important discovery, and as improvements were made in transformer and machine design, the harmonics problem was largely solved until the 1930s and 40s. Then, with the advent of rural electrification and telephones, power and telephone circuits were placed on common rights-of-way. Transformers and rectifiers in power systems produced harmonic currents that inductively coupled into adjacent open-wire telephone circuits and produced audible telephone interference. These problems were gradually alleviated by filtering and by
minimizing transformer core magnetizing currents. Isolated telephone interference problems still occur, but these problems are infrequent because open-wire telephone circuits have been replaced with twisted pair, buried cables, and fiber optics.

Today, the most common sources of harmonics are power electronic loads such as adjustablespeed drives (ASDs) and switching power supplies. Electronic loads use diodes, siliconcontrolled rectifiers (SCRs), power transistors, and other electronic switches to either chop waveforms to control power, or to convert $50 / 60 \mathrm{~Hz}$ AC to DC. In the case of ASDs, DC is then converted to variable-frequency AC to control motor speed. Example uses of ASDs include chillers and pumps.

A single-phase power electronic load that you are familiar with is the single-phase light dimmer shown in Figure 1.1. By adjusting the potentiometer, the current and power to the light bulb are controlled, as shown in Figures 1.2 and 1.3.



Before firing, the triac is an open switch, so that practically no voltage is applied across the light bulb. The small current through the $3.3 \mathrm{k} \Omega$ resistor is ignored in this diagram.


After firing, the triac is a closed switch, so that practically all of $\mathrm{V}_{\text {an }}$ is applied across the light bulb.

Figure 1.1. Triac light dimmer circuit


Figure 1.2. Light dimmer current waveforms for firing angles

$$
\alpha=30^{\circ}, 90^{\circ}, \text { and } 150^{\circ}
$$



Figure 1.3. Normalized power delivered to light bulb versus $\alpha$

The light dimmer is a simple example, but it represents two major benefits of power electronic loads - controllability and efficiency. The "tradeoff" is that power electronic loads draw nonsinusoidal currents from AC power systems, and these currents react with system impedances to create voltage harmonics and, in some cases, resonance. Studies show that harmonic distortion levels in distribution feeders are rising as power electronic loads continue to proliferate and as shunt capacitors are employed in greater numbers to improve power factor closer to unity.

Unlike transient events such as lightning that last for a few microseconds, or voltage sags that last from a few milliseconds to several cycles, harmonics are steady-state, periodic phenomena that produce continuous distortion of voltage and current waveforms. These periodic nonsinusoidal waveforms are described in terms of their harmonics, whose magnitudes and phase angles are computed using Fourier analysis.

Fourier analysis permits a periodic distorted waveform to be decomposed into a series containing dc, fundamental frequency (e.g. 60 Hz ), second harmonic (e.g. 120 Hz ), third harmonic (e.g. 180 Hz ), and so on. The individual harmonics add to reproduce the original waveform. The highest harmonic of interest in power systems is usually the $25^{\text {th }}(1500 \mathrm{~Hz})$, which is in the low audible range. Because of their relatively low frequencies, harmonics should not be confused with radio-frequency interference (RFI) or electromagnetic interference (EMI).

Ordinarily, the DC term is not present in power systems because most loads do not produce DC and because transformers block the flow of DC. Even-ordered harmonics are generally much smaller than odd-ordered harmonics because most electronic loads have the property of halfwave symmetry, and half-wave symmetric waveforms have no even-ordered harmonics.

The current drawn by electronic loads can be made distortion-free (i.e., perfectly sinusoidal), but the cost of doing this is significant and is the subject of ongoing debate between equipment manufacturers and electric utility companies in standard-making activities. Two main concerns are

1. What are the acceptable levels of current distortion?
2. Should harmonics be controlled at the source, or within the power system?

## 2. Fourier Series

### 2.1. General Discussion

Any physically realizable periodic waveform can be decomposed into a Fourier series of DC, fundamental frequency, and harmonic terms. In sine form, the Fourier series is

$$
\begin{equation*}
i(t)=I_{a v g}+\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}\right) \tag{2.1}
\end{equation*}
$$

and if converted to cosine form, 2.1 becomes

$$
i(t)=I_{a v g}+\sum_{k=1}^{\infty} I_{k} \cos \left(k \omega_{1} t+\theta_{k}-90^{\circ}\right)
$$

$I_{\text {avg }}$ is the average (often referred to as the "DC" value $I_{d c}$ ). $I_{k}$ are peak magnitudes of the individual harmonics, $\omega_{o}$ is the fundamental frequency (in radians per second), and $\theta_{k}$ are the harmonic phase angles. The time period of the waveform is

$$
T=\frac{2 \pi}{\omega_{1}}=\frac{2 \pi}{2 \pi f_{1}}=\frac{1}{f_{1}} .
$$

The formulas for computing $I_{d c}, I_{k}, \theta_{k}$ are well known and can be found in any undergraduate electrical engineering textbook on circuit analysis. These are described in Section 2.2.

Figure 2.1 shows a desktop computer (i.e., PC) current waveform. The corresponding spectrum is given in the Appendix. The figure illustrates how the actual waveform can be approximated by summing only the fundamental, $3^{\text {rd }}$, and $5^{\text {th }}$ harmonic components. If higher-order terms are included (i.e., $7^{\text {th }}, 9^{\text {th }}, 11^{\text {th }}$, and so on), then the PC current waveform will be perfectly reconstructed. A truncated Fourier series is actually a least-squared error curve fit. As higher frequency terms are added, the error is reduced.

Fortunately, a special property known as half-wave symmetry exists for most power electronic loads. Have-wave symmetry exists when the positive and negative halves of a waveform are identical but opposite, i.e.,

$$
i(t)=-i\left(t \pm \frac{T}{2}\right)
$$

where $T$ is the period. Waveforms with half-wave symmetry have no even-ordered harmonics. It is obvious that the television current waveform is half-wave symmetric.


Figure 2.1. PC Current Waveform, and its $1^{\text {st }}, 3^{\text {rd }}$, and $5^{\text {th }}$ Harmonic Components
(Note - in this waveform, the harmonics are peaking at the same time as the fundamental. Most waveforms do not have this property. In fact, in many cases (e.g. a square wave), the peak of the fundamental component is actually greater than the peak of the composite wave.)

### 2.2 Fourier Coefficients

If function $i(t)$ is periodic with an identifiable period T (i.e., $i(t)=i(t \pm N T)$ ), then $i(t)$ can be written in rectangular form as

$$
\begin{equation*}
i(t)=I_{a v g}+\sum_{k=1}^{\infty}\left[a_{k} \cos \left(k \omega_{1} t\right)+b_{k} \sin \left(k \omega_{1} t\right)\right], \omega_{1}=\frac{2 \pi}{T}, \tag{2.2}
\end{equation*}
$$

where

$$
\begin{aligned}
& I_{a v g}=\frac{1}{T} \int_{t_{o}}^{t_{O}+T} i(t) d t \\
& a_{k}=\frac{2}{T} \int_{t_{o}}^{t_{o}+T} i(t) \cos \left(k \omega_{1} t\right) d t \\
& b_{k}=\frac{2}{T} \int_{t_{o}}^{t_{o}+T} i(t) \sin \left(k \omega_{1} t\right) d t
\end{aligned}
$$

The sine and cosine terms in (2.2) can be converted to the convenient polar form of (2.1) by using trigonometry as follows:

$$
\begin{align*}
& a_{k} \cos \left(k \omega_{1} t\right)+b_{k} \sin \left(k \omega_{1} t\right) \\
& \quad=\sqrt{a_{k}^{2}+b_{k}^{2}} \bullet \frac{a_{k} \cos \left(k \omega_{1} t\right)+b_{k} \sin \left(k \omega_{1} t\right)}{\sqrt{a_{k}^{2}+b_{k}^{2}}} \\
& \\
& =\sqrt{a_{k}^{2}+b_{k}^{2}} \bullet\left[\frac{a_{k}}{\sqrt{a_{k}^{2}+b_{k}^{2}}} \cos \left(k \omega_{1} t\right)+\frac{b_{k}}{\sqrt{a_{k}^{2}+b_{k}^{2}}} \sin \left(k \omega_{1} t\right)\right]  \tag{2.3}\\
& \quad=\sqrt{a_{k}^{2}+b_{k}^{2}} \bullet\left[\sin \left(\theta_{k}\right) \cos \left(k \omega_{1} t\right)+\cos \left(\theta_{k}\right) \sin \left(k \omega_{1} t\right)\right]
\end{align*}
$$

where

$$
\sin \left(\theta_{k}\right)=\frac{a_{k}}{\sqrt{a_{k}^{2}+b_{k}^{2}}}, \cos \left(\theta_{k}\right)=\frac{b_{k}}{\sqrt{a_{k}^{2}+b_{k}^{2}}}
$$

Applying trigonometric identity

$$
\sin (A+B)=\sin (A) \cos (B)+\cos (A) \sin (B)
$$


yields polar form

$$
\begin{equation*}
\sqrt{a_{k}^{2}+b_{k}^{2}} \bullet \sin \left(k \omega_{1} t+\theta_{k}\right) \tag{2.4}
\end{equation*}
$$

where

$$
\begin{equation*}
\tan \left(\theta_{k}\right)=\frac{\sin \left(\theta_{k}\right)}{\cos \left(\theta_{k}\right)}=\frac{a_{k}}{b_{k}} \tag{2.5}
\end{equation*}
$$

### 2.3 Phase Shift

There are two types of phase shifts pertinent to harmonics. The first is a shift in time, e.g. the $\pm T / 3$ among balanced a-b-c currents. If the PC waveform in Figure 2.2 is delayed by $\Delta T$ seconds, the modified current is


Figure 2.2. PC Current Waveform Delayed in Time

$$
\begin{gather*}
i(t-\Delta T)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1}(t-\Delta T)+\theta_{k}\right)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t-k \omega_{1} \Delta T+\theta_{k}\right) \\
=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\left(\theta_{k}-k \omega_{1} \Delta T\right)\right)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}-k \theta_{1}\right) \tag{2.6}
\end{gather*}
$$

where $\theta_{1}$ is the phase lag of the fundamental current corresponding to $\Delta T$. The last term in (2.6) shows that the individual harmonic phase angles are delayed by $k \theta_{1}$ of their own degrees.

The second type of phase shift is in phase angle, which occurs in wye-delta transformers. Wyedelta transformers shift voltages and currents by $\pm 30^{\circ}$, depending on phase sequence. ANSI standards require that, regardless of which side is delta or wye, the a-b-c phases must be marked so that the high-voltage side voltages and currents lead those on the low-voltage side by $30^{\circ}$ for
positive-sequence (and thus lag by $30^{\circ}$ for negative sequence). Zero sequences are blocked by the three-wire connection so that their phase shift is not meaningful.

### 2.4 Symmetry Simplifications

Waveform symmetry greatly simplifies the integration effort required to develop Fourier coefficients. Symmetry arguments should be applied to the waveform after the average (i.e., DC) value has been removed. The most important cases are

- Odd Symmetry, i.e., $i(t)=-i(-t)$,
then the corresponding Fourier series has no cosine terms,

$$
a_{k}=0,
$$

and $b_{k}$ can be found by integrating over the first half-period and doubling the results,

$$
b_{k}=\frac{4}{T} \int_{0}^{T / 2} i(t) \sin \left(k \omega_{1} t\right) d t
$$

- Even Symmetry, i.e., $i(t)=i(-t)$,
then the corresponding Fourier series has no sine terms,

$$
b_{k}=0,
$$

and $a_{k}$ can be found by integrating over the first half-period and doubling the results,

$$
a_{k}=\frac{4}{T} \int_{0}^{T / 2} i(t) \cos \left(k \omega_{1} t\right) d t
$$

Important note - even and odd symmetry can sometimes be obtained by time-shifting the waveform. In this case, solve for the Fourier coefficients of the time-shifted waveform, and then phase-shift the Fourier coefficient angles according to (A.6).

- Half-Wave Symmetry, i.e., $i\left(t \pm \frac{T}{2}\right)=-i(t)$,
then the corresponding Fourier series has no even harmonics, and $a_{k}$ and $b_{k}$ can be found by integrating over any half-period and doubling the results,

$$
a_{k}=\frac{4}{T} \int_{t_{o}}^{t_{o}+T / 2} i(t) \cos \left(k \omega_{1} t\right) d t, \quad k \text { odd }
$$

$$
b_{k}=\frac{4}{T} \int_{t_{o}}^{t_{o}+T / 2} i(t) \sin \left(k \omega_{1} t\right) d t, \quad k \text { odd. }
$$

Half-wave symmetry is common in power systems.

### 2.5 Examples

## - Square Wave

By inspection, the average value is zero, and the waveform has both odd symmetry and half-wave symmetry. Thus, $a_{k}=0$, and

$$
b_{k}=\frac{4}{T} \int_{t_{o}}^{t_{o}+T / 2} v(t) \sin \left(k \omega_{1} t\right) d t, k \text { odd }
$$



Solving for $b_{k}$,

$$
b_{k}=\frac{4}{T} \int_{0}^{T / 2} V \sin \left(k \omega_{1} t\right) d t=\frac{-4 V}{k \omega_{o} T} \cos \left(k \omega_{1} t\right)_{t=0}^{t=T / 2}=\frac{-4 V}{k \omega_{1} T}\left(\cos \left(\frac{k \omega_{1} T}{2}\right)-\cos (0)\right) .
$$

Since $\omega_{1}=\frac{2 \pi}{T}$, then

$$
\begin{aligned}
& b_{k}=\frac{-4 V}{2 k \pi}(\cos (k \pi)-1)=\frac{2 V}{k \pi}(1-\cos (k \pi)), \text { yielding } \\
& b_{k}=\frac{4 V}{k \pi}, k \text { odd. }
\end{aligned}
$$

The Fourier series is then

$$
\begin{equation*}
v(t)=\frac{4 V}{\pi} \sum_{k=1, k \text { odd }}^{\infty} \frac{1}{k} \sin \left(\mathrm{k} \omega_{1} t\right)=\frac{4 V}{\pi}\left[\sin \left(1 \omega_{1} t\right)+\frac{1}{3} \sin \left(3 \omega_{1} t\right)+\frac{1}{5} \sin \left(5 \omega_{1} t\right)+\cdots\right] . \tag{2.7}
\end{equation*}
$$

Note that the harmonic magnitudes decrease according to $\frac{1}{k}$.

## - Triangle Wave

By inspection, the average value is zero, and the waveform has both even symmetry and half-wave symmetry. Thus, $b_{k}=0$, and

$$
a_{k}=\frac{4}{T} \int_{t_{o}}^{t_{o}+T / 2} v(t) \cos \left(k \omega_{1} t\right) d t, k \text { odd. }
$$



Solving for $a_{k}$,

$$
\begin{aligned}
a_{k} & =\frac{4}{T} \int_{0}^{T / 2} V\left(1-\frac{4 t}{T}\right) \cos \left(k \omega_{1} t\right) d t=\frac{4 V}{T} \int_{0}^{T / 2} \cos \left(k \omega_{1} t\right) d t-\frac{16 V}{T^{2}} \int_{0}^{T / 2} t \cos \left(k \omega_{1} t\right) d t \\
& =\frac{4 V}{k \omega_{1} T}\left(\sin \left(\frac{k \omega_{1} T}{2}\right)-\sin (0)\right)-\left.\frac{16 V}{T^{2}} \frac{t \sin \left(k \omega_{1} t\right)}{k \omega_{1}}\right|_{t=0} ^{t=T / 2}+\frac{16 V}{T^{2}} \int_{0}^{T / 2} \frac{\sin \left(k \omega_{1} t\right)}{k \omega_{1}} d t \\
& =\frac{2 V}{k \pi} \sin (k \pi)-\frac{4 V}{k \pi} \sin (k \pi)+\frac{4 V}{k^{2} \pi^{2}}(1-\cos (k \pi)), k \text { odd. }
\end{aligned}
$$

Continuing,

$$
a_{k}=\frac{8 V}{k^{2} \pi^{2}}, k \text { odd. }
$$

The Fourier series is then

$$
\begin{align*}
v(t) & =\frac{8 V}{\pi^{2}} \sum_{k=1, k \text { odd }}^{\infty} \frac{1}{k^{2}} \cos \left(\mathrm{k} \omega_{1} t\right) \\
& =\frac{8 V}{\pi^{2}}\left[\cos \left(1 \omega_{1} t\right)+\frac{1}{9} \cos \left(3 \omega_{1} t\right)+\frac{1}{25} \cos \left(5 \omega_{1} t\right)+\cdots\right], \tag{2.8}
\end{align*}
$$

where it is seen that the harmonic magnitudes decrease according to $\frac{1}{k^{2}}$.

To convert to a sine series, recall that $\cos (\theta)=\sin \left(\theta+90^{\circ}\right)$, so that the series becomes

$$
\begin{equation*}
v(t)=\frac{8 V}{\pi^{2}}\left[\sin \left(1 \omega_{1} t+90^{\circ}\right)+\frac{1}{9} \sin \left(3 \omega_{1} t+90^{\circ}\right)+\frac{1}{25} \sin \left(5 \omega_{1} t+90^{\circ}\right)+\cdots\right] \tag{2.9}
\end{equation*}
$$

To time delay the waveform by $\frac{T}{4}$ (i.e., move to the right by $90^{\circ}$ of fundamental), subtract $\left(k \bullet 90^{\circ}\right)$ from each harmonic angle. Then, (2.9) becomes

$$
\begin{aligned}
v(t) & =\frac{8 V}{\pi^{2}}\left[\sin \left(1 \omega_{1} t+90^{\circ}-1 \bullet 90^{\circ}\right)+\frac{1}{9} \sin \left(3 \omega_{1} t+90^{\circ}-3 \bullet 90^{\circ}\right)\right) \\
& \left.+\frac{1}{25} \sin \left(5 \omega_{1} t+90^{\circ}-5 \bullet 90^{\circ}\right)+\cdots\right],
\end{aligned}
$$

or

$$
\begin{equation*}
v(t)=\frac{8 V}{\pi^{2}}\left[\sin \left(1 \omega_{1} t\right)-\frac{1}{9} \sin \left(3 \omega_{1} t\right)+\frac{1}{25} \sin \left(5 \omega_{1} t\right)-\frac{1}{49} \sin \left(7 \omega_{1} t\right) \cdots\right] . \tag{2.10}
\end{equation*}
$$

## - Half-Wave Rectified Cosine Wave

The waveform has an average value and even symmetry. Thus, $b_{k}=0$, and

$$
a_{k}=\frac{4}{T} \int_{0}^{T / 2} i(t) \cos \left(k \omega_{o} t\right) d t, k \text { odd. }
$$



Solving for the average value,

$$
\begin{align*}
& I_{\text {avg }}=\frac{1}{T} \int_{t_{O}}^{t_{o}+T} i(t) d t=\frac{1}{T} \int_{-T / 4}^{T / 4} I \cos \left(\omega_{1} t\right) d t=\left.\frac{I}{\omega_{o} T} \sin \left(\omega_{1} t\right)\right|_{t=-T / 4} ^{t=T / 4} \\
& \quad=\frac{I}{2 \pi}\left(\sin \frac{\omega_{1} T}{4}-\sin \frac{-\omega_{1} T}{4}\right)=\frac{I}{\pi} \sin \frac{\omega_{1} T}{4}=\frac{1}{\pi} \sin \frac{\pi}{2} . \\
& I_{\text {avg }}=\frac{I}{\pi} . \tag{2.11}
\end{align*}
$$

Solving for $a_{k}$,

$$
a_{k}=\frac{4}{T} \int_{0}^{T / 4} I \cos \left(\omega_{1} t\right) \cos \left(k \omega_{1} t\right) d t=\frac{2 I}{T} \int_{0}^{T / 4}\left(\cos (1-k) \omega_{1} t+\cos (1+k) \omega_{1} t\right) d t
$$

$$
=\left.\frac{2 I}{T}\left(\frac{\sin \left((1-k) \omega_{1} t\right)}{(1-k) \omega_{1}}+\frac{\sin \left((1+k) \omega_{1} t\right)}{(1+k) \omega_{1}}\right)\right|_{t=0} ^{t=T / 4}
$$

For $k=1$, taking the limits of the above expression when needed yields

$$
\begin{align*}
a_{1} & =\frac{2 I}{T} \bullet \lim _{(1-k) \omega_{1} \rightarrow 0}\left(\frac{\sin \left((1-k) \omega_{1} \frac{T}{4}\right)}{(1-k) \omega_{1}}\right)+\frac{I \sin (\pi)}{2 \pi} \\
& -\frac{2 I}{T} \bullet \lim _{(1-k) \omega_{1} \rightarrow 0} \frac{\sin (1-k) \omega_{1} \bullet 0}{(1-k) \omega_{1}}-\frac{I \sin (0)}{2 \pi} .  \tag{2.12}\\
a_{1} & =\frac{2 I}{T} \frac{T}{4}+0-0-0=\frac{I}{2}
\end{align*}
$$

For $k>1$,

$$
\begin{equation*}
a_{k}=\frac{I}{\pi}\left(\frac{\sin (1-k) \frac{\pi}{2}}{(1-k)}+\frac{\sin (1+k) \frac{\pi}{2}}{(1+k)}\right) \tag{2.13}
\end{equation*}
$$

All odd $k$ terms in (2.13) are zero. For the even terms, it is helpful to find a common denominator and write (2.13) as

$$
a_{k}=\frac{I}{\pi}\left(\frac{(1+k) \sin (1-k) \frac{\pi}{2}+(1-k) \sin (1+k) \frac{\pi}{2}}{1-k^{2}}\right), k>1, k \text { even. }
$$

Evaluating the above equation shows an alternating sign pattern that can be expressed as

$$
a_{k}=\frac{2 I}{\pi} \sum_{k=2,4,6, \cdots}^{\infty}(-1)^{\frac{k+2}{2}} \frac{1}{k^{2}-1}, k>1, k \text { even. }
$$

The final expression becomes

$$
i(t)=\frac{I}{\pi}+\frac{I}{2} \cos \left(\omega_{1} t\right)+\frac{2 I}{\pi} \sum_{k=2,4,6, \cdots}^{\infty}(-1)^{k / 2+1} \frac{1}{k^{2}-1} \cos \left(k \omega_{1} t\right)
$$

$$
\begin{equation*}
=\frac{I}{\pi}+\frac{I}{2} \cos \left(\omega_{1} t\right)+\frac{2 I}{\pi}\left[\frac{1}{3} \cos \left(2 \omega_{1} t\right)-\frac{1}{15} \cos \left(4 \omega_{1} t\right)+\frac{1}{35} \cos \left(6 \omega_{1} t\right)-\cdots\right] . \tag{2.14}
\end{equation*}
$$

## - Light Dimmer Current

The Fourier coefficients of the current waveform shown in Figure 1.2 can be shown to be the following:

For the fundamental,

$$
\begin{equation*}
a_{1}=\frac{-I_{p}}{\pi} \sin ^{2} \alpha, b_{1}=I_{p}\left[1-\frac{\alpha}{\pi}+\frac{1}{2 \pi} \sin 2 \alpha\right], \tag{2.15}
\end{equation*}
$$

where firing angle $\alpha$ is in radians, and $I_{p}$ is the peak value of the fundamental current when $\alpha=0^{\circ}$.

For harmonic multiples above the fundamental (i.e., $k=3,5,7, \ldots$ ),

$$
\begin{align*}
& a_{k}=\frac{I_{p}}{\pi}\left[\frac{1}{1-k}(\cos (1-k) \alpha-\cos (1-k) \pi)+\frac{1}{1+k}(\cos (1+k) \alpha-\cos (1+k) \pi)\right],  \tag{2.16}\\
& b_{k}=\frac{I_{p}}{\pi}\left[\frac{1}{1-k}(\sin (1-k) \pi-\sin (1-k) \alpha)+\frac{1}{1+k}(\sin (1+k) \alpha-\sin (1+k) \pi)\right] . \tag{2.17}
\end{align*}
$$

The waveform has zero average, and it has no even harmonics because of half-wave symmetry.

The magnitude of any harmonic $k$, including $k=1$, is $I_{k}=\sqrt{a_{k}^{2}+b_{k}^{2}}$. Performing the calculations for the special case of $\alpha=\frac{\pi}{2}$ radians (i.e., $90^{\circ}$ ) yields

$$
\begin{aligned}
& I_{1}=\frac{I_{p}}{\pi} \sqrt{1+\frac{\pi^{2}}{4}}=0.593 I_{p}, \text { and } I_{3}=\frac{I_{p}}{\pi}=0.318 I_{p} \\
& \frac{I_{3}}{I_{1}}=\frac{1}{\sqrt{1+\frac{\pi^{2}}{4}}}=0.537
\end{aligned}
$$

The above case is illustrated in the following Excel spreadsheet.

## Light_Dimmer_Fourier_Waveform.xls



Note - in the highlighted cells, the magnitude of $\mathrm{I}_{1}$ is computed to be 0.593 times the peak value of the fundamental current for the $\alpha=0^{\circ}$ case. The ratio of $\mathrm{I}_{3}$ to $\mathrm{I}_{1}$ is computed to be 0.537 .

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## 3. Definitions

### 3.1. RMS

The squared rms value of a periodic current (or voltage) waveform is defined as

$$
\begin{equation*}
I_{r m s}^{2}=\frac{1}{T} \int_{t_{o}}^{t_{o}+T} i(t)^{2} d t \tag{3.1}
\end{equation*}
$$

It is clear in (3.1) that the squared rms value of a periodic waveform is the average value of the squared waveform.

If the current is sinusoidal, the rms value is simply the peak value divided by $\sqrt{2}$. However, if the waveform has Fourier series

$$
i(t)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}\right)
$$

then substituting into (3.1) yields

$$
\begin{align*}
& I_{r m s}^{2}=\frac{1}{T} \int_{t_{0}}^{t_{o}+T}\left(\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}\right)\right)^{2} d t \\
& I_{r m s}^{2}=\frac{1}{T} \int_{t_{0}}^{t_{o}+T}\left(\sum_{k=1}^{\infty} I_{k}^{2} \sin ^{2}\left(k \omega_{1} t+\theta_{k}\right)+2 \sum_{m=1, n=1, m \neq n}^{\infty} \sum_{n}^{\infty} I_{m} I_{n} \sin \left(m \omega_{1} t+\theta_{m}\right) \sin \left(n \omega_{1} t+\theta_{n}\right)\right) d t \\
& I_{r m s}^{2}=\frac{1}{T} \int_{t_{0}}^{t_{o}+T}\left(\sum_{k=1}^{\infty} I_{k}^{2}\left(\frac{1-\cos 2\left(k \omega_{1} t+\theta_{k}\right)}{2}\right)\right. \\
& \left.+\sum_{m=1, n=1, m \neq n}^{\infty} I_{m}^{\infty} I_{n}\left(\frac{\cos \left((m-n) \omega_{1} t+\theta_{m}-\theta_{n}\right)}{2}+\frac{\cos \left((m+n) \omega_{1} t+\theta_{m}+\theta_{n}\right)}{2}\right)\right) d t \tag{3.2}
\end{align*}
$$

Equation (3.2) is complicated, but most of its terms contribute nothing to the rms value if one thinks of (3.2) as being the average value for one fundamental period. The average value of each $\cos 2\left(k \omega_{1} t+\theta_{k}\right)$ term is zero because the average value of a cosine is zero for one or more integer periods. Likewise, the average value of each $\cos \left((m \pm n) \omega_{1} t+\theta_{m} \pm \theta_{n}\right)$ term is also zero because $m$ and $n$ are both harmonics of the fundamental. Thus, (3.2) reduces to

$$
\begin{equation*}
I_{r m s}^{2}=\frac{1}{T} \int_{t_{o}}^{t_{o}^{+}+T}\left(\sum_{k=1}^{\infty} I_{k}^{2} \bullet \frac{1}{2}\right) d t=\frac{1}{2 T} \sum_{k=1}^{\infty} I_{k}^{2} \bullet\left(t_{o}+T-t_{o}\right)=\frac{1}{2} \sum_{k=1}^{\infty} I_{k}^{2}=\sum_{k=1}^{\infty}\left(\frac{I_{k}}{\sqrt{2}}\right)^{2}, \tag{3.3}
\end{equation*}
$$

where $I_{k}$ are peak values of the harmonic components. Factoring out the $\sqrt{2}$ yields

$$
\begin{equation*}
I_{r m s}^{2}=I_{1, r m s}^{2}+I_{2, r m s}^{2}+I_{3, r m s}^{2}+\cdots \tag{3.4}
\end{equation*}
$$

Equations (3.3) and (3.4) ignore any DC that may be present. The effect of DC is to add the term $I_{D C}^{2}$ to (3.3) and (3.4).

The cross products of unlike frequencies contribute nothing to the rms value of the total waveform. The same statement can be made for average power, as will be shown later. Furthermore, since the contribution of harmonics to rms add in squares, and their magnitudes are often much smaller than the fundamental, the impact of harmonics on rms is usually not great.

### 3.2. THD

The most commonly-used measure for harmonics is total harmonic distortion (THD), also known as distortion factor. It is applied to both voltage and current. THD is defined as the rms value of the harmonics above fundamental, divided by the rms value of the fundamental. DC is ignored. Thus, for current,

$$
\begin{equation*}
T H D_{I}=\frac{\sqrt{\sum_{k=2}^{\infty}\left(\frac{I_{k}}{\sqrt{2}}\right)^{2}}}{\frac{I_{1}}{\sqrt{2}}}=\frac{\sqrt{\frac{1}{2} \sum_{k=2}^{\infty} I_{k}^{2}}}{\frac{I_{1}}{\sqrt{2}}} \tag{3.5}
\end{equation*}
$$

The same equation form applies to voltage $T H D_{V}$.
THD and rms are directly linked. Note that since

$$
I_{r m s}^{2}=\frac{1}{2} \sum_{k=1}^{\infty} I_{k}^{2}
$$

and since

$$
T H D_{I}^{2}=\frac{\frac{1}{2} \sum_{k=2}^{\infty} I_{k}^{2}}{\frac{I_{1}^{2}}{2}}=\frac{\left(\sum_{k=1}^{\infty} I_{k}^{2}\right)-I_{1}^{2}}{I_{1}^{2}}
$$

then rewriting yields

$$
\sum_{k=1}^{\infty} I_{k}^{2}=I_{1}^{2}\left(1+T H D_{I}^{2}\right)
$$

so that

$$
\frac{1}{2} \sum_{k=1}^{\infty} I_{k}^{2}=\frac{I_{1}^{2}}{2}\left(1+T H D_{I}^{2}\right)
$$

Comparing to (3.3) yields

$$
I_{r m s}^{2}=\frac{1}{2} \sum_{k=1}^{\infty} I_{k}^{2}=\frac{I_{1}^{2}}{2}\left(1+T H D_{I}^{2}\right)=I_{1, r m s}^{2}\left(1+T H D_{I}^{2}\right)
$$

Thus the equation linking THD and rms is

$$
\begin{equation*}
I_{r m s}=I_{1, r m s} \sqrt{1+T H D_{I}^{2}} \tag{3.6}
\end{equation*}
$$

Because 1. line losses are proportional to the square of rms current (and sometimes increase more rapidly due to the resistive skin effect), and 2 . rms increases with harmonics, then line losses always increase when harmonics are present. For example, many PCs have a current distortion near 1.0 (i.e., $100 \%$ ). Thus, the wiring losses incurred while supplying a PC are twice what they would be in the sinusoidal case.

Current distortion in loads varies from a few percent to more than $100 \%$, but voltage distortion is generally less than $5 \%$. Voltage THDs below 0.05 , i.e. $5 \%$, are considered acceptable, and those greater than $10 \%$ are definitely unacceptable and will cause problems for sensitive equipment and loads.

### 3.3. Average Power

Harmonic powers (including the fundamental) add and subtract independently to produce total average power. Average power is defined as

$$
\begin{equation*}
P_{a v g}=\frac{1}{T} \int_{t_{0}}^{t_{0}+T} p(t) d t=\frac{1}{T} \int_{t_{0}}^{t_{0}+T} v(t) i(t) d t \tag{3.7}
\end{equation*}
$$

Substituting in the Fourier series of voltage and current yields

$$
P_{\text {avg }}=\frac{1}{T} \int_{t_{o}}^{t_{o}}\left(\sum_{k=1}^{\infty} V_{k} \sin \left(k \omega_{1} t+\delta_{k}\right) \bullet \sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}\right)\right) d t
$$

and expanding yields

$$
\begin{aligned}
P_{\text {avg }} & =\frac{1}{T} \int_{t_{0}}^{t_{o}+T}\left(\sum_{k=1}^{\infty} V_{k} I_{k} \sin \left(k \omega_{1} t+\phi_{k}\right) \bullet \sin \left(k \omega_{1} t+\theta_{k}\right)\right. \\
& \left.+\sum_{m=1, n=1, m \neq n}^{\infty} \sum_{n=1}^{\infty} V_{m} I_{n} \sin \left(m \omega_{1} t+\phi_{m}\right) \sin \left(n \omega_{1} t+\theta_{n}\right)\right) d t, \\
P_{\text {avg }} & =\frac{1}{T} \int_{t_{o}}^{t_{o}+T}\left(\sum_{k=1}^{\infty} V_{k} I_{k}\left(\frac{\cos \left(\phi_{k}-\theta_{k}\right)-\cos \left(2 k \omega_{1} t+\phi_{k}+\theta_{k}\right)}{2}\right)\right. \\
& \left.+\sum_{m=1, n=1, m \neq n}^{\infty} \sum_{m}^{\infty} I_{n}\left(\frac{\cos \left((m-n) \omega_{1} t+\theta_{m}-\theta_{n}\right)}{2}+\frac{\cos \left((m+n) \omega_{1} t+\theta_{m}+\theta_{n}\right)}{2}\right)\right) d t
\end{aligned}
$$

As observed in the rms case, the average value of all the sinusoidal terms is zero, leaving only the time invariant terms in the summation, or

$$
\begin{equation*}
P_{a v g}=\sum_{k=1}^{\infty} \frac{V_{k} I_{k}}{2} \cos \left(\phi_{k}-\theta_{k}\right)=\sum_{k=1}^{\infty} V_{k, r m s} \bullet I_{k, r m s} \bullet d p f_{k}=P_{1, a v g}+P_{2, a v g}+P_{3, a v g}+\cdots, \tag{3.8}
\end{equation*}
$$

where $d p f_{k}$ is the displacement power factor for harmonic $k$.

The harmonic power terms $P_{2, a v g}, P_{3, \text { avg }}, \cdots$ are mostly losses and are usually small in relation to total power. However, harmonic losses may be a substantial part of total losses.

Equation (3.8) is important in explaining who is responsible for harmonic power. Electric utility generating plants produce sinusoidal terminal voltages. According to (3.8), if there is no harmonic voltage at the terminals of a generator, then the generator produces no harmonic power. However, due to nonlinear loads, harmonic power does indeed exist in power systems and causes additional losses. Thus, it is accurate to say that

- Harmonic power is parasitic and is due to nonlinear equipment and loads.
- The source of most harmonic power is power electronic loads.
- By chopping the 60 Hz current waveform and producing harmonic voltages and currents, power electronic loads convert some of the " 60 Hz " power into harmonic power, which in turn propagates back into the power system, increasing system losses and impacting sensitive loads.

For a thought provoking question related to harmonic power, consider the case shown in Figure 3.1 where a perfect $120 \mathrm{Vac}(\mathrm{rms})$ power system with $1 \Omega$ internal resistance supplies a triac-
controlled 1000 W incandescent lamp. Let the firing angle is $90^{\circ}$, so the lamp is operating at half-power.


Current i


Figure 3.1. Single-Phase Circuit with Triac and Lamp

Assuming that the resistance of the lamp is $\frac{120^{2}}{1000}=14.4 \Omega$, and that the voltage source is $v_{S}(t)=120 \sqrt{2} \sin \left(\omega_{1} t\right)$, then the Fourier series of current in the circuit, truncated at the $5^{\text {th }}$ harmonic, is

$$
i(t)=6.99 \sin \left(\omega_{1} t-32.5^{\circ}\right)+3.75 \sin \left(3 \omega_{1} t-90.0^{\circ}\right)+1.25 \sin \left(5 \omega_{1} t-90.0^{\circ}\right)
$$

If a wattmeter is placed immediately to the left of the triac, the metered voltage is

$$
\begin{aligned}
v_{m}(t) & =v_{s}(t)-i R=120 \sqrt{2} \sin \left(\omega_{1} t\right) \\
& -1 \bullet\left(6.99 \sin \left(\omega_{1} t-32.5^{\circ}\right)+3.75 \sin \left(3 \omega_{1} t-90.0^{\circ}\right)+1.25 \sin \left(5 \omega_{1} t-90.0^{\circ}\right)\right) \\
& =163.8 \sin \left(\omega_{1} t+1.3^{\circ}\right)+3.75 \sin \left(3 \omega_{1} t+90.0^{\circ}\right)+1.25 \sin \left(5 \omega_{1} t+90.0^{\circ}\right)
\end{aligned}
$$

and the average power flowing into the triac-lamp customer is

$$
\begin{aligned}
P_{\text {avg }} & =\frac{163.8 \bullet 6.99}{2} \cos \left(1.3^{\circ}-\left(-32.5^{\circ}\right)\right)+\frac{3.75 \bullet 3.75}{2} \cos \left(90.0^{\circ}-\left(-90.0^{\circ}\right)\right) \\
& +\frac{1.25 \bullet 1.25}{2} \cos \left(90.0^{\circ}-\left(-90.0^{\circ}\right)\right)
\end{aligned}
$$

$$
=475.7-7.03-0.78=467.9 \mathrm{~W}
$$

The first term, 475.7 W , is due to the fundamental component of voltage and current. The 7.03 W and 0.78 W terms are due to the $3^{\text {rd }}$ and $5^{\text {th }}$ harmonics, respectively, and flow back into the power system.

The question now is: should the wattmeter register only the fundamental power, i.e., 475.7 W , or should the wattmeter credit the harmonic power flowing back into the power system and register only $475.7-7.81=467.9 \mathrm{~W}$ ? Remember that the harmonic power produced by the load is consumed by the power system resistance.

### 3.4. True Power Factor

To examine the impact of harmonics on power factor, it is important to consider the true power factor, which is defined as

$$
\begin{equation*}
p f_{\text {true }}=\frac{P_{\text {avg }}}{V_{r m s} I_{r m s}} . \tag{3.9}
\end{equation*}
$$

In sinusoidal situations, (3.9) reduces to the familiar displacement power factor

$$
d p f_{1}=\frac{P_{1, a v g}}{V_{1, r m s} I_{1, r m s}}=\frac{\frac{V_{1} I_{1}}{2} \cos \left(\delta_{1}-\theta_{1}\right)}{\frac{V_{1} I_{1}}{2}}=\cos \left(\delta_{1}-\theta_{1}\right)
$$

When harmonics are present, (3.9) can be expanded as

$$
p f_{t r u e}=\frac{P_{1, a v g}+P_{2, a v g}+P_{3, a v g}+\cdots}{V_{1, r m s} \sqrt{1+T H D_{V}^{2}} \bullet I_{1, r m s} \sqrt{1+T H D_{I}^{2}}}
$$

In most instances, the harmonic powers are small compared to the fundamental power, and the voltage distortion is less than $10 \%$. Thus, the following important simplification is usually valid:

$$
\begin{equation*}
p f_{\text {true }} \approx \frac{P_{1, a v g}}{V_{1, r m s} I_{1, r m s} \sqrt{1+T H D_{I}^{2}}}=\frac{d p f_{1}}{\sqrt{1+T H D_{I}^{2}}} . \tag{3.10}
\end{equation*}
$$

It is obvious in (3.10) that the true power factor of a nonlinear load is limited by its $T H D_{I}$. For example, the true power factor of a PC with $T H D_{I}=100 \%$ can never exceed 0.707 , no matter how good its displacement power is. Some other examples of "maximum" true power factor (i.e., maximum implies that the displacement power factor is unity) are given below in Table 3.1.

Table 3.1. Maximum True Power Factor of a Nonlinear Load.

| Current <br> THD | Maximum <br> $p f_{\text {true }}$ |
| :---: | :---: |
| $20 \%$ | 0.98 |
| $50 \%$ | 0.89 |
| $100 \%$ | 0.71 |

### 3.5. K Factor

Losses in transformers increase when harmonics are present because

1. harmonic currents increase the rms current beyond what is needed to provide load power,
2. harmonic currents do not flow uniformly throughout the cross sectional area of a conductor and thereby increase its equivalent resistance.

Dry-type transformers are especially sensitive to harmonics. The K factor was developed to provide a convenient measure for rating the capability of transformers, especially dry types, to serve distorting loads without overheating. The K factor formula is

$$
\begin{equation*}
K=\frac{\sum_{k=1}^{\infty} k^{2} I_{k}^{2}}{\sum_{k=1}^{\infty} I_{k}^{2}} \tag{3.11}
\end{equation*}
$$

In most situations, $K \leq 10$.

### 3.6. Phase Shift

There are two types of phase shifts pertinent to harmonics. The first is a shift in time, e.g. the $\pm \frac{2 T}{3}$ among the phases of balanced a-b-c currents. To examine time shift, consider Figure 3.2. If the PC waveform is delayed by $\Delta T$ seconds, the modified current is

$$
\begin{align*}
& i(t-\Delta T)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1}(t-\Delta T)+\theta_{k}\right)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t-k \omega_{1} \Delta T+\theta_{k}\right) \\
&=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\left(\theta_{k}-k \omega_{1} \Delta T\right)\right)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}-k \theta_{1}\right) \tag{3.12}
\end{align*}
$$

where $\theta_{1}$ is the phase lag of the fundamental current corresponding to $\Delta T$. The last term in (3.12) shows that individual harmonics are delayed by $k \theta_{1}$.


Figure 3.2. PC Current Waveform Delayed in Time

The second type of phase shift is in harmonic angle, which occurs in wye-delta transformers.
Wye-delta transformers shift voltages and currents by $\pm 30^{\circ}$. ANSI standards require that, regardless of which side is delta or wye, the a-b-c phases must be marked so that the highvoltage side voltages and currents lead those on the low-voltage side by $30^{\circ}$ for positivesequence, and lag by $30^{\circ}$ for negative sequence. Zero sequences are blocked by the three-wire connection so that their phase shift is not meaningful.

### 3.7. Voltage and Current Phasors in a Three-Phase System

Phasor diagrams for line-to-neutral and line-to-line voltages are shown in Figure 3.3. Phasor currents for a delta-connected load, and their relationship to line currents, are shown in Figure 3.4.

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Figure 3.3. Voltage Phasors in a Balanced Three-Phase System
(The phasors are rotating counter-clockwise. The magnitude of line-to-line voltage phasors is $\sqrt{3}$ times the magnitude of line-to-neutral voltage phasors.)

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Figure 3.4. Currents in a Delta-Connected Load
(Conservation of power requires that the magnitudes of delta currents $\mathrm{I}_{\mathrm{ab}}, \mathrm{I}_{\mathrm{c}}$, and $\mathrm{I}_{\mathrm{bc}}$ are $\frac{1}{\sqrt{3}}$ times the magnitude of line currents $\mathrm{I}_{\mathrm{a}}, \mathrm{I}_{\mathrm{b}}, \mathrm{I}_{\mathrm{c}}$.)

### 3.8. Phase Sequence

In a balanced three-phase power system, the currents in phases a-b-c are shifted in time by $\pm 120^{\circ}$ of fundamental. Therefore, since

$$
i_{a}(t)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}\right)
$$

then the currents in phases b and c lag and lead by $\frac{2 \pi}{3}$ radians, respectively. Thus

$$
\begin{aligned}
& i_{b}(t)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}-k \frac{2 \pi}{3}\right) \\
& i_{c}(t)=\sum_{k=1}^{\infty} I_{k} \sin \left(k \omega_{1} t+\theta_{k}+k \frac{2 \pi}{3}\right)
\end{aligned}
$$

Picking out the first three harmonics shows an important pattern. Expanding the above series,

$$
\begin{aligned}
& i_{a}(t)=I_{1} \sin \left(1 \omega_{1} t+\theta_{1}\right)+I_{2} \sin \left(2 \omega_{1} t+\theta_{2}\right)+I_{3} \sin \left(3 \omega_{1} t+\theta_{3}\right) \\
& \begin{aligned}
i_{b}(t) & =I_{1} \sin \left(1 \omega_{1} t+\theta_{1}-\frac{2 \pi}{3}\right)+I_{2} \sin \left(2 \omega_{1} t+\theta_{2}-\frac{4 \pi}{3}\right)+I_{3} \sin \left(3 \omega_{1} t+\theta_{3}-\frac{6 \pi}{3}\right), \text { or } \\
& =I_{1} \sin \left(1 \omega_{1} t+\theta_{1}-\frac{2 \pi}{3}\right)+I_{2} \sin \left(2 \omega_{1} t+\theta_{2}+\frac{2 \pi}{3}\right)+I_{3} \sin \left(3 \omega_{1} t+\theta_{3}-0\right) \\
i_{c}(t) & =I_{1} \sin \left(1 \omega_{1} t+\theta_{1}+\frac{2 \pi}{3}\right)+I_{2} \sin \left(2 \omega_{1} t+\theta_{2}+\frac{4 \pi}{3}\right)+I_{3} \sin \left(3 \omega_{1} t+\theta_{3}+\frac{6 \pi}{3}\right), \text { or } \\
\quad= & I_{1} \sin \left(1 \omega_{1} t+\theta_{1}+\frac{2 \pi}{3}\right)+I_{2} \sin \left(2 \omega_{1} t+\theta_{2}-\frac{2 \pi}{3}\right)+I_{3} \sin \left(3 \omega_{1} t+\theta_{3}-0\right) .
\end{aligned}
\end{aligned}
$$

By examining the current equations, it can be seen that

- the first harmonic (i.e., the fundamental) is positive sequence (a-b-c) because phase $b$ lags phase a by $120^{\circ}$, and phase c leads phase a by $120^{\circ}$,
- the second harmonic is negative sequence (a-c-b) because phase bleads phase a by $120^{\circ}$, and phase c lags phase a by $120^{\circ}$,
- the third harmonic is zero sequence because all three phases have the same phase angle.

The pattern for a balanced system repeats and is shown in Table 2. All harmonic multiples of three (i.e., the "triplens") are zero sequence. The next harmonic above a triplen is positive sequence, the next harmonic below a triplen is negative sequence.

Table 3.2. Phase Sequence of Harmonics in a Balanced Three-Phase System

| Harmonic | Phase <br> Sequence |
| :---: | :---: |
| 1 | + |
| 2 | - |
| 3 | 0 |
| 4 | + |
| 5 | - |
| 6 | 0 |
| $\ldots$ | $\ldots$ |

If a system is not balanced, then each harmonic can have positive, negative, and zero sequence components. However, in most cases, the pattern in Table 3.2 can be assumed to be valid.

Because of Kirchhoff's current law, zero sequence currents cannot flow into a three-wire connection such as a delta transformer winding or a delta connected load. In most cases, systems are fairly well balanced, so that it is common to make the same assumption for third harmonics and other triplens. Thus, a delta-grounded wye transformer at the entrance of an industrial customer usually blocks the flow of triplen harmonic load currents into the power system. Unfortunately, the transformer does nothing to block the flow of any other harmonics, such as $5^{\text {th }}$ and $7^{\text {th }}$.

Zero sequence currents flow through neutral or grounding paths. Positive and negative sequence currents sum to zero at neutral and grounding points.

Another interesting observation can be made about zero sequence harmonics. Line-to-line voltages never have zero sequence components because, according to Kirchhoff's voltage law, they always sum to zero. For that reason, line-to-line voltages in commercial buildings are missing the $3^{\text {rd }}$ harmonic that dominates line-to-neutral voltage waveforms. Thus, the $T H D_{V}$ of line-to-line voltages is often considerably less than for line-to-neutral voltages.

### 3.9. Transformers

Consider the example shown in Figure 3.5 where twin, idealized six-pulse current source ASDs are served by parallel transformers. Line-to-line transformer voltage ratios are identical. The top transformer is wye-wye or delta-delta, thus having no phase shift. The bottom transformer is wye-delta or delta-wye, thus having $30^{\circ}$ phase shift.


Figure 3.5. Current Waveforms of Identical Parallel Six-Pulse Converters Yield a Net Twelve-Pulse Converter

To begin the analysis, assume that the per-unit load-side current of the top transformer is the standard six-pulse wave given by

$$
\begin{aligned}
& i_{\text {top, loadside }}(t)=I_{1} \sin \left(1 \omega_{1} t\right) \\
& \quad+\frac{I_{1}}{5} \sin \left(5 \omega_{o} t+180^{\circ}\right)+\frac{I_{1}}{7} \sin \left(7 \omega_{1} t+180^{\circ}\right) \\
& \quad+\frac{I_{1}}{11} \sin \left(11 \omega_{1} t\right)+\frac{I_{1}}{13} \sin \left(13 \omega_{1} t\right) \\
& \quad+\frac{I_{1}}{17} \sin \left(17 \omega_{1} t+180^{\circ}\right)+\frac{I_{1}}{19} \sin \left(19 \omega_{1} t+180^{\circ}\right)+\cdots
\end{aligned}
$$

Note that the even-ordered harmonics are missing because of half-wave symmetry, and that the triple harmonics are missing because a six-pulse ASD is a three-wire balanced load, having
characteristic harmonics $k=6 n \pm 1, n=1,2,3, \ldots$. Because the transformer has no phase shift, then the line-side current waveform (in per-unit) is the same as the load-side current, or

$$
i_{\text {top,lineside }}(t)=i_{\text {top,loadside }}(t)
$$

Now, because the fundamental voltage on the load-side of the bottom transformer is delayed in time by $30^{\circ}$, then each harmonic of the load-side current of the bottom transformer is delayed by $k \bullet 30^{\circ}$, so that

$$
\begin{aligned}
& i_{\text {bottom, loadside }}(t)=I_{1} \sin \left(1 \omega_{1} t-30^{\circ}\right) \\
& \quad+\frac{I_{1}}{5} \sin \left(5 \omega_{1} t+180^{\circ}-150^{\circ}\right)+\frac{I_{1}}{7} \sin \left(7 \omega_{1} t+180^{\circ}-210^{\circ}\right) \\
& \quad+\frac{I_{1}}{11} \sin \left(11 \omega_{1} t-330^{\circ}\right)+\frac{I_{1}}{13} \sin \left(13 \omega_{1} t-390^{\circ}\right) \\
& \quad+\frac{I_{1}}{17} \sin \left(17 \omega_{1} t+180^{\circ}-510^{\circ}\right)+\frac{I_{1}}{19} \sin \left(19 \omega_{1} t+180^{\circ}-570^{\circ}\right)+\cdots
\end{aligned}
$$

The current waveform through the top transformer is not shifted when going from load-side to line-side, except for its magnitude. However, the various phase sequence components of the current through the bottom transformer are shifted when going to the line-side, so that

$$
\begin{aligned}
& i_{\text {bottom, lineside }}(t)=I_{1} \sin \left(1 \omega_{1} t-30^{\circ}+30^{\circ}\right) \\
& \quad+\frac{I_{1}}{5} \sin \left(5 \omega_{1} t+180^{\circ}-150^{\circ}-30^{\circ}\right)+\frac{I_{1}}{7} \sin \left(7 \omega_{1} t+180^{\circ}-210^{\circ}+30^{\circ}\right) \\
& \quad+\frac{I_{1}}{11} \sin \left(11 \omega_{1} t-330^{\circ}-30^{\circ}\right)+\frac{I_{1}}{13} \sin \left(13 \omega_{1} t-390^{\circ}+30^{\circ}\right) \\
& \quad+\frac{I_{1}}{17} \sin \left(17 \omega_{1} t+180^{\circ}-510^{\circ}-30^{\circ}\right)+\frac{I_{1}}{19} \sin \left(19 \omega_{1} t+180^{\circ}-570^{\circ}+30^{\circ}\right)+\cdots
\end{aligned}
$$

Combining angles yields

$$
\begin{aligned}
& i_{\text {bottom, lineside }}(t)=I_{1} \sin \left(1 \omega_{1} t\right) \\
& \quad+\frac{I_{1}}{5} \sin \left(5 \omega_{1} t\right)+\frac{I_{1}}{7} \sin \left(7 \omega_{1} t\right) \\
& \quad+\frac{I_{1}}{11} \sin \left(11 \omega_{1} t\right)+\frac{I_{1}}{13} \sin \left(13 \omega_{1} t\right) \\
& \quad+\frac{I_{1}}{17} \sin \left(17 \omega_{1} t+180^{\circ}\right)+\frac{I_{1}}{19} \sin \left(19 \omega_{1} t\right)+\cdots
\end{aligned}
$$

Adding the top and bottom line-side currents yields

$$
i_{\text {net }}(t)=i_{\text {top,lineside }}(t)+i_{\text {bottom, lineside }}(t)=2 I_{1} \sin \left(1 \omega_{1} t\right)+\frac{2 I_{1}}{11} \sin \left(11 \omega_{1} t\right)+\frac{2 I_{1}}{13} \sin \left(13 \omega_{1} t\right)+\cdots .
$$

The important observation here is that harmonics $5,7,17,19$ combine to zero at the summing point on the line-side. Recognizing the pattern shows that the remaining harmonics are

$$
k=12 n \pm 1, \quad n=1,2,3, \ldots
$$

which leads to the classification "twelve-pulse converter."
In an actual twelve-pulse ASD, a three winding transformer is used, having one winding on the line-side, and two parallel wye-delta and delta-delta windings on the load-side. The power electronics are in effect divided into two halves so that each half carries one-half of the load power.

Summarizing, since harmonics in a balanced system fall into the predictable phase sequences shown in Table 3.2, it is clear that a wye-delta transformer will advance some harmonics by $30^{\circ}$ and delay other harmonics by $30^{\circ}$. This property makes it possible to cancel half of the harmonics produced by ASDs (most importantly the $5^{\text {th }}$ and $7^{\text {th }}$ ) through a principle known as phase cancellation. The result is illustrated in Figure 3.3, where two parallel six-pulse converters combine to yield a net twelve-pulse converter with much less current distortion. Corresponding spectra are given in the Appendix.

Transformer phase shifting may be used to create net 18-pulse, 24-pulse, and higher-pulse converters.

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## 4. Sources

Harmonics are produced by nonlinear loads or devices that draw nonsinusoidal currents. An example of a nonlinear load is a diode, which permits only one-half of the otherwise sinusoidal current to flow. Another example is a saturated transformer, whose magnetizing current is no sinusoidal. But, by far the most common problem-causing nonlinear loads are large rectifiers and ASDs.

Nonlinear load current waveshapes always vary somewhat with the applied voltage waveshape. Typically, the current distortion of a nonlinear load decreases as the applied voltage distortion increases - thus somewhat of a compensating effect. As a result, most nonlinear loads have the highest current distortion when the voltage is nearly sinusoidal and the connected power system is "stiff" (i.e., low impedance).

In most harmonics simulation cases, these waveshape variations are ignored and nonlinear loads are treated as fixed harmonic current injectors whose harmonic current magnitudes and phase angles are fixed relative to their fundamental current magnitude and angle. In other words, the harmonic current spectrum of a nonlinear load is usually assumed to be fixed in system simulation studies. The fundamental current angle, which is almost always lagging, is adjusted to yield the desired displacement power factor. Harmonics phase angles are adjusted according to the time shift principle to preserve waveshape appearance.

### 4.1 Classical Nonlinear Loads

Some harmonic sources are not related to power electronics and have been in existence for many years. Examples are

- Transformers. For economic reasons, power transformers are designed to operate on or slightly past the knee of the core material saturation curve. The resulting magnetizing current is slightly peaked and rich in harmonics. The third harmonic component dominates. Fortunately, magnetizing current is only a few percent of full-load current. The magnetizing current for a $25 \mathrm{kVA}, 12.5 \mathrm{kV} / 240 \mathrm{~V}$ transformer is shown in Figure 4.1 (see spectrum in the Appendix). The fundamental current component lags the fundamental voltage component by $66^{\circ}$. Even though the 1.54Arms magnetizing current is highly distorted (76.1\%), it is relatively small compared to the rated full-load current of 140Arms.


Figure 4.1. Magnetizing Current for Single-Phase $25 \mathrm{kVA} .12 .5 \mathrm{kV} / 240 \mathrm{~V}$ Transformer. $T H D_{I}=76.1 \%$.

- Machines. As with transformers, machines operate with peak flux densities beyond the saturation knee. Unless blocked by a delta transformation, a three-phase synchronous generator will produce a $30 \%$ third harmonic current.

There is considerable variation among single-phase motors in the amount of current harmonics they inject. Most of them have $T H D_{I}$ in the $10 \%$ range, dominated by the $3^{\text {rd }}$ harmonic. The current waveforms for a refrigerator and residential air conditioner are shown in Figures 4.2 and 4.3 , respectively. The corresponding spectra are given in the Appendix. The current waveform for a 2HP single-phase motor is shown in Figure 5.5 in Section 5.


Figure 4.2. 120V Refrigerator Current.

$$
T H D_{I}=6.3 \%
$$



Figure 4.3. 240V Residential Air Conditioner Current.
$T H D_{I}=10.5 \%$.

- Fluorescent Lamps (with Magnetic Ballasts). Fluorescent lamps extinguish and ignite each half-cycle, but the flicker is hardly perceptable at 50 or 60 Hz . Ignition occurs sometime after the zero crossing of voltage. Once ignited, fluorescent lamps exhibit negative resistive characteristics. Their current waveforms are slightly skewed, peaked, and have a characteristic second peak. The dominant harmonics is the $3^{\text {rd }}$, on the order of $15 \%-20 \%$ of fundamental. A typical waveform is shown in Figure 4.4, and the spectrum is given in the Appendix.


Figure 4.4. 277V Fluorescent Lamp Current (with Magnetic Ballast).

$$
T H D_{I}=18.5 \% .
$$

- Arc Furnaces. These are not strictly periodic and, therefore, cannot be analyzed accurately by using Fourier series and harmonics. Actually, these are transient loads for which flicker is a greater problem than harmonics. Some attempts have been made to model arc furnaces as harmonic sources using predominant harmonics $3^{\text {rd }}$ and $5^{\text {th }}$.


### 4.2. Power Electronic Loads

Examples of power electronic loads are

- Line Commutated Converters. These are the workhorse circuits of AC/DC converters above 500 HP . The circuit is shown in Figure 4.5. These are sometimes described as sixpulse converters because they produce six ripple peaks on Vdc per AC cycle. In most applications, power flows to the DC load. However, if the DC circuit has a source of power, such as a battery or photovoltaic array, power can flow from DC to AC in the inverter mode.

The DC choke smooths Idc, and since Idc has low ripple, the converter is often described as a "current source."

In order to control power flow, each SCR is fired after its natural forward-bias turn-on point. This principle is known as phase control, and because of it, the displacement power factor is poor at medium and low power levels.

The firing order is identified by SCRs 1 through 6 in Figure 4.5. Once fired, each SCR conducts until it is naturally reverse biased by the circuit. The term "line commutated converter" refers to the fact that the load actually turns the SCRs off, rather than having forced-commutated circuits turn them off. Line commutation has the advantage of simplicity.

The idealized AC current $i_{a}(t)$ waveform for a six-pulse converter equals Idc for $120^{\circ}$, zero for $60^{\circ}$, and then -Idc for $120^{\circ}$, and zero for another $60^{\circ}$ (see Figure 4.5 and the field measurement shown in Figure 5.1). The Fourier series is approximately

$$
\begin{aligned}
i(t) & =I_{1}\left[\sin \left(1 \omega_{1} t-1 \theta_{1}\right)-\frac{1}{5} \sin \left(5 \omega_{1} t-5 \theta_{1}\right)-\frac{1}{7} \sin \left(7 \omega_{1} t-7 \theta_{1}\right)\right. \\
& +\frac{1}{11} \sin \left(11 \omega_{1} t-11 \theta_{1}\right)+\frac{1}{13} \sin \left(13 \omega_{1} t-13 \theta_{1}\right) \\
& \left.-\frac{1}{17} \sin \left(17 \omega_{1} t-17 \theta_{1}\right)-\frac{1}{19} \sin \left(19 \omega_{1} t-19 \theta_{1}\right)+\cdots\right],
\end{aligned}
$$

where $I_{1}$ is the peak fundamental current, and $\theta_{1}$ is the lagging displacement power factor angle. The magnitudes of the AC current harmonics decrease by the $1 / \mathrm{k}$ rule, i.e. the fifth harmonic is $1 / 5$ of fundamental, the seventh harmonic is $1 / 7$ of fundamental, etc. The even-ordered harmonics are missing due to half-wave symmetry, and the triple harmonics are missing because the converter is a three-wire load served by a transformer with a delta or ungrounded-wye winding.

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$\mathrm{i}_{\mathrm{a}}{ }^{\prime}$ waveform. $T H D_{I}=30.0 \%$.

$i_{a}$ with delta-wye or wye-delta transformer. $\quad i_{a}$ with delta-delta or wye-wye transformer. $T H D_{I}=30.0 \%$ in both cases.

Figure 4.5. Three-Phase, Six-Pulse Line Commutated Converter

If the converter transformer has no phase shift (i.e., either wye-wye or delta-delta), then the current waveshape on the power system side, i.e., $i_{a}(t)$, is the same as current $i_{a}^{\prime}(t)$ on the converter side of the transformer. If the transformer is wye-delta or delta-wye, then the sign of every other pair of harmonics in $i_{a}(t)$ changes, yielding

$$
\begin{aligned}
i_{a}(t) & =\left[I_{1} \sin \left(1 \omega_{1} t-1 \theta_{1}\right)+\frac{1}{5} \sin \left(5 \omega_{1} t-5 \theta_{1}\right)+\frac{1}{7} \sin \left(7 \omega_{1} t-7 \theta_{1}\right)\right. \\
& +\frac{1}{11} \sin \left(11 \omega_{1} t-11 \theta_{1}\right)+\frac{1}{13} \sin \left(13 \omega_{1} t-13 \theta_{1}\right) \\
& \left.+\frac{1}{17} \sin \left(17 \omega_{1} t-17 \theta_{1}\right)+\frac{1}{19} \sin \left(19 \omega_{1} t-19 \theta_{1}\right)+\cdots\right]
\end{aligned}
$$

Two or more six-pulse converters can be operated in parallel through phase-shifting transformers to reduce the harmonic content of the net supply-side current. This principle is known as phase cancellation. A twelve-pulse converter has two six-pulse converters connected in parallel on the AC side and in series on the DC side. One load-side transformer winding is delta and the other is wye. As a result, half of the harmonic currents cancel (notably, the 5th and 7th), producing an AC current waveform that is much more sinusoidal than that of each individual converter alone. Higher pulse orders (i.e., eighteen pulse, twenty-four pulse, etc.) can also be achieved. The AC current harmonic multiples produced by a P-pulse converter are

$$
\begin{aligned}
& \mathrm{h}=\mathrm{PN} \pm 1, \mathrm{~N}=1,2,3, \cdots, \\
& \mathrm{P}=\text { an integer multiple of } 6 .
\end{aligned}
$$

- Voltage-Source Converters. For applications below 500HP, voltage source converters employing pulse-width modulaters with turn-on/turn-off switches on the motor side are often the choice for ASDs. Since both power and voltage control is accomplished on the load side, the SCRs in Figure 4.5 can be replaced with simple diodes. The circuit is shown in Figure 4.6, and the spectra are given in the Appendix.

The diode bridge and capacitor provide a relatively stiff Vdc source for the PWM drive, hence the term "voltage source." Since voltage-source converters do not employ phase control, their displacement power factors are approximately 1.0.

$\mathrm{i}_{\mathrm{a}}$ with high power. $T H D_{I}=32.6 \% . \quad \mathrm{i}_{\mathrm{a}}$ with low power. $T H D_{I}=67.4 \%$.
(delta-delta or wye-wye)

(delta-delta or wye-wye)


$\mathrm{i}_{\mathrm{a}}$ with high power. $T H D_{I}=32.6 \%$.
(delta-wye or wye-delta)

$\mathrm{i}_{\mathrm{a}}$ with low power. $T H D_{I}=67.4 \%$. (delta-wye or wye-delta)

Figure 4.6. Three-Phase, Six-Pulse Voltage-Source Converter

Unfortunately, current distortion on the power system side is higher for voltage-source converters than for line commutated converters, and the current waveshape varies considerably with load level. Typical waveforms are shown in Figure 4.6. Even though lower load levels have higher $T H D_{I}$, the harmonic amperes do not vary greatly with load level because fundamental current is proportional to load level.

The higher current distortion created by these drives is one of the main reasons that voltage-source inverters are generally not used above 500 HP .

- Switched-Mode Power Supplies. These power supplies are the "front-end" of singlephase 120 V loads such as PCs and home entertainment equipment. Typically, they have a full-wave diode rectifier connected between the AC supply system and a capacitor, and the capacitor serves as a low-ripple "battery" for the DC load. Unfortunately, low ripple means that the AC system charges the capacitor for only a fraction of each half-cycle, yielding an AC waveform that is highly peaked, as shown in Figure 4.7.

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AC Current for Above Circuit. $T H D_{I}=134 \%$.


AC Current on Delta Side of Delta-Grounded Wye Transformer that Serves Three PCs. $T H D_{I}=94.0 \%$.

Figure 4.7. Single-Phase Switched-Mode Power Supply and Current Waveforms

### 4.3. Other Nonlinear Loads

There are many other harmonic sources. Among these are cycloconverters, which directly convert 60 Hz AC to another frequency, static VAr compensators, which provide a variable supply of reactive power, and almost any type of "energy saving" or wave-shaping device, such as motor power factor controllers. Waveforms for three common loads are shown below in Figures 4.8, 4.9, and 4.10, and the corresponding spectra are given in the Appendix.


Figure 4.8. 120V Microwave Oven Current. $T H D_{I}=31.9 \%$.


Figure 4.9. 120V Vacuum Cleaner Current.

$$
T H D_{I}=25.9 \%
$$



Figure 4.10. 277V Fluorescent Lamp Current (with Electronic Ballast). $T H D_{I}=11.6 \%$.

### 4.4. Cumulative Harmonics

Voltage distortion and load level affect the current waveshapes of nonlinear loads. Harmonic magnitudes and phase angles, especially the phase angles of higher-frequency harmonics, are a function of waveshape and displacement power factor. Thus, the net harmonic currents produced by ten or more nearby harmonic loads are not strictly additive because there is some naturallyoccuring phase cancellation. If this phase angle diversity is ignored, then system simulations will predict exaggerated voltage distortion levels.

This net addition, or diversity factor, is unity for the $3^{\text {rd }}$ harmonic, but decreases for higher harmonics. Research and field measurement verifications have shown that the diversity factors in Table 4.1 are appropriate in both three-phase and single-phase studies. Even-ordered harmonics are ignored.

Table 4.1. Current Diversity Factor Multipliers for Large Numbers of Nonlinear Loads

| Current <br> Harmonic | Diversity <br> Factor |
| :---: | :---: |
| 3 | 1.0 |
| 5 | 0.9 |
| 7 | 0.9 |
| 9 | 0.6 |
| 11 | 0.6 |
| 13 | 0.6 |
| 15 | 0.5 |
| Higher Odds | 0.2 |
| All Evens | 0.0 |

A typical application of Table 4.1 is when ten 100HP voltage-source ASDs are located within a single facility, and the facility is to be modeled as a single load point on a distribution feeder. The net ASD is 1000 HP , and the net spectrum is the high-power spectrum of Figure 4.6 but with
magnitudes multiplied by the diversity factors of Table 4.1. The phase angles are unchanged. The composite waveshape is shown in Figure 4.11.


Figure 4.11. Expected Composite Current Waveshape for Large Numbers of High-Power Voltage-Source ASDs. $T H D_{I}=27.6 \%$.

Similiarly, the composite waveshape for one thousand 100 W PCs with the waveform shown in Figure 4.7 would be a single 100 kW load with the waveshape shown in Figure 4.12.


Figure 4.12. Expected Composite Current Waveshape for Large Numbers of PCs.

$$
T H D_{I}=124 \% .
$$

### 4.5. Detailed Analysis of Steady-State Operation of Three-Phase, Six-Pulse, Line Commutated, Current-Source Converters



Figure 4.13. Three-Phase, Six-Pulse, Line Commutated, Current-Source Converter

## Introduction

Line-commutated converters are most-often used in high-power applications such as motor drives (larger than a few hundred kW ) and HVDC (hundreds of MW). These applications require the high voltage and current ratings that are generally available only in thyristors (i.e., silicon-controlled rectifiers, or SCRs). A large series inductor is placed in the DC circuit to lower the ripple content of Idc, which in turn helps to limit the harmonic distortion in the AC currents to approximately $25 \%$.

By adjusting firing angle $\alpha$, the converter can send power from the AC side to the DC side (i.e., rectifier operation), or from the DC side to the AC side (i.e., inverter operation). DC voltage Vdc is positive for rectifier operation, and negative for inverter operation. Because thyristors are unidirectional, DC current always flows in the direction shown.

To understand the operating principles, the following assumptions are commonly made

- Continuous and ripple free Idc
- Balanced AC voltages and currents
- Inductive AC system impedance
- Balanced, steady-state operation with
- firing angle $\alpha, 0^{\circ} \leq \alpha \leq 180^{\circ}$,
- commutation angle $\mu, 0^{\circ} \leq \mu \leq 60^{\circ}$, and
- $0^{\circ} \leq \alpha+\mu \leq 180^{\circ}$.

As a first approximation, when $\alpha<90^{\circ}$, then the circuit is a rectifier. When $\alpha>90^{\circ}$, then the circuit is an inverter. The zero reference for $\alpha$ is the point at which turn-on would naturally occur if a thyristor was replaced by a diode.

The waveform graphs shown in this document are produced by Excel spreadsheets 6P_Waveforms_Rectifier.XLS and 6PLCC_Waveforms.XLS.

## Simple Uncontrolled Rectifier with Resistive Load

A good starting point for understanding the operation of the converter is to consider the circuit shown in Figure 4.14, where the thyristors have been replaced with diodes, the DC circuit is simply a load resistor, and the AC impedance is negligible. Without a large inductor in the DC circuit, the DC current is not ripple-free.


Figure 4.14. Three-Phase Uncontrolled Rectifier
The switching rules for Figure 4.14 are described below.

- Diode \#1 is on when $\mathrm{V}_{\mathrm{an}}$ is the most positive, i.e., $\mathrm{V}_{\mathrm{an}}>\mathrm{V}_{\mathrm{bn}}$ and $\mathrm{V}_{\mathrm{an}}>\mathrm{V}_{\mathrm{cn}}$. Simultaneously, diodes \#3 and \#5 are reverse biased and thus off.
- Likewise, diode \#4 is on when $\mathrm{V}_{\mathrm{an}}$ is the most negative. Simultaneously, diodes \#2 and \#6 are reverse biased and thus off.
- The other diodes work in the same way that \#1 and \#4 do.
- At any time, one (and only one) of top diodes \#1 or \#3 or \#5 is on, and one (and only one) of bottom diodes \#2 or \#4 or \#6 is on.
- The pair of top and bottom diodes that is on determines which line-to-line voltage appears at $\mathrm{V}_{\mathrm{dc}}$.

Using the above rules, waveforms for $\mathrm{i}_{1}, \ldots, \mathrm{i}_{6}, \mathrm{ia}_{\mathrm{a}}, \mathrm{V}_{1}$, and $\mathrm{V}_{\mathrm{dc}}$ can be determined and are shown in Figure 4.15. The figure confirms the natural turn-on sequence for diodes $\# 1, \# 2, \ldots \# 6$.


Figure 4.15. Waveforms for the Three-Phase Uncontrolled Rectifier with Resistive Load (note - the graph contains the phrase "uncontrolled rectifier," but when $\alpha=0^{\circ}$, controlled and uncontrolled rectifiers are essentially the same)

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## Simple Controlled Rectifier with Resistive Load

Now, replace the diodes in Figure 4.14 with SCRs so that power can be controlled. When fired, SCRs will turn on if they are forward biased. The point at which they first become forward biased corresponds to a firing angle $\alpha$ of $0^{\circ}$ - that is the same situation as the diode case of Figure 4.14. If firing angle $\alpha=30^{\circ}$, then firing occurs $30^{\circ}$ past the point at which the SCRs first become forward biased.

Working with the switching rules given for Figure 4.14, and modifying them for $\alpha>0$, the waveforms can be determined and are shown in Figure 4.16.

At this point, it should be noted that if $\alpha$ is greater than $60^{\circ}$, then the load current becomes discontinuous.

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Figure 4.16. Waveforms for the Three-Phase Controlled Rectifier with Resistive Load

## Controlled Operation of Three-Phase, Six-Pulse, Line Commutated, Current-Source Converter

We now return to the circuit shown in Figure 4.13. The DC circuit has a smoothing inductor to remove ripple, and the AC system has an inductance. The significance of the AC inductance means that when SCR \#1 turns on, SCR \#5 does not immediately turn off. Gradually, $\mathrm{I}_{\mathrm{dc}}$ transitions from SCR \#5 to SCR \#1. This transition is known as commutation. In industrial converters, commutation angle $\mu$ may be only $2-3^{\circ}$ of 60 Hz . In HVDC converters, commutation may be intentionally increased to $10-15^{\circ}$ to reduce AC harmonics.

To understand circuit operation, consider the $60^{\circ}$ sequence where
Case 1. \#5 and \#6 are on (just prior to \#1 being fired),
Case 2. Then \#1 is fired, so that \#1 and \#5 commutate (while \#6 stays on),
Case 3. Then \#5 goes off, and \#1 and \#6 are on.
Once this $60^{\circ}$ sequence is understood, then because of symmetry, the firing of the other SCRs and their waveforms are also understood for the other $300^{\circ}$ that completes one cycle of 60 Hz .


Case 1. \#5 and \#6 On (just prior to firing \#1)


Case 2. \#1 and \#5 Commutating (\#6 stays on)


Case 3. \#1 and \#6 On
The analysis for commutation in Case 2 follows. When \#1 comes on, KVL around the loop created by \#1, \#5, and $V_{\text {ac }}$ yields

$$
-V_{a c}+L \frac{d i_{1}}{d t}-L \frac{d i_{5}}{d t}=0
$$

KCL at the top DC node yields

$$
-i_{1}-i_{5}+I_{d c}=0, \text { so that } i_{5}=I_{d c}-i_{1} .
$$

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Substituting the KCL equation into the KVL equation yields

$$
-V_{a c}+L \frac{d i_{1}}{d t}-L \frac{d\left(I_{d c}-i_{1}\right)}{d t}=0
$$

Since $I_{d c}$ is constant, then

$$
\begin{aligned}
& -V_{a c}+L \frac{d i_{1}}{d t}-L \frac{d\left(-i_{1}\right)}{d t}=-V_{a c}+L \frac{d i_{1}}{d t}+L \frac{d\left(i_{1}\right)}{d t}=0, \text { which becomes } \\
& \frac{d i_{1}}{d t}=\frac{V_{a c}}{2 L}
\end{aligned}
$$

Thus

$$
i_{1}=\int \frac{V_{a c}}{2 L} d t=\int \frac{V_{L L P}}{2 L} \sin \left(\omega t-30^{\circ}\right) d t=\frac{-V_{L L P}}{2 \omega L} \cos \left(\omega t-30^{\circ}\right)+\text { const } .
$$

The boundary conditions are $i_{1}\left(\omega t=30^{\circ}+\alpha\right)=0$. Therefore,

$$
\begin{align*}
& \text { const }=\frac{V_{L L P}}{2 \omega L} \cos (\alpha), \text { so that } \\
& i_{1}=\frac{V_{L L P}}{2 \omega L}\left(\cos (\alpha)-\cos \left(\omega t-30^{\circ}\right)\right), 30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu . \tag{4.1}
\end{align*}
$$

From the KCL equation,

$$
\begin{equation*}
i_{5}=I_{d c}-i_{1}=I_{d c}-\frac{V_{L L P}}{2 \omega L}\left(\cos (\alpha)-\cos \left(\omega t-30^{\circ}\right)\right), 30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu \tag{4.2}
\end{equation*}
$$

Anytime that \#1 is on (including commutation), $\mathrm{V}_{1}=0$. The other case of interest to $\# 1$ is when $\# 4$ is on (including commutation). For that case, an examination of Figure 4.13 shows that $\mathrm{V}_{1}=$ $-V_{d c}$.

The above analysis is expanded using symmetry and Figure 4.13 to complete the full cycle. The results are summarized in Table 4.2. Waveforms for several combinations of $\alpha$ and $\mu$ follow the table.

Table 4.2. Firing Regimes and Corresponding Status of Switches

| Angle | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{V d c}$ | $\mathbf{V 1}$ | Comment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case 1. $\omega t=30^{\circ}+\alpha^{-}$ |  |  |  |  | $\bullet$ | $\bullet$ | Vcb | Vac | \#5 on, \#6 on |  |
| Case 2. $30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu$ | C |  |  |  | C | $\bullet$ | $-\frac{3}{2} \mathrm{Vbn}$ | 0 | \#1 turning on, <br> \#5 turning off |  |
| Case $3.30^{\circ}+\alpha+\mu+\leq \omega t \leq 30^{\circ}+\alpha+60^{\circ}$ | $\bullet$ |  |  |  |  | $\bullet$ | Vab | 0 | \#6 on, \#1 on |  |
| $30^{\circ}+\alpha+60^{\circ} \leq \omega t \leq 30^{\circ}+\alpha+60^{\circ}+\mu$ | $\bullet$ | C |  |  |  | C | $\frac{3}{2} \mathrm{Van}$ | 0 | \#2 turning on, <br> \#6 turning off |  |
| $30^{\circ}+\alpha+60^{\circ}+\mu \leq \omega t \leq 30^{\circ}+\alpha+120^{\circ}$ | $\bullet$ | $\bullet$ |  |  |  |  | Vac | 0 | \#1 on, \#2 on |  |
| $30^{\circ}+\alpha+120^{\circ} \leq \omega t \leq 30^{\circ}+\alpha+120^{\circ}+\mu$ | C | $\bullet$ | C |  |  |  | $-\frac{3}{2} \mathrm{Vcn}$ | 0 | \#3 turning on, <br> \#1 turning off |  |
| $30^{\circ}+\alpha+120^{\circ}+\mu \leq \omega t \leq 30^{\circ}+\alpha+180^{\circ}$ |  | $\bullet$ | $\bullet$ |  |  |  | Vbc | Vab | \#2 on, \#3 on <br> $30^{\circ}+\alpha+180^{\circ} \leq \omega t \leq 30^{\circ}+\alpha+180^{\circ}+\mu$$\quad$ | C |
|  | $\bullet$ | C |  |  | $\frac{3}{2} \mathrm{Vbn}$ | -Vdc | \#4 turning on, <br> \#2 turning off |  |  |  |
| $30^{\circ}+\alpha+180^{\circ}+\mu \leq \omega t \leq 30^{\circ}+\alpha+240^{\circ}$ |  |  | $\bullet$ | $\bullet$ |  |  | Vba | -Vdc | \#3 on, \#4 on |  |
| $30^{\circ}+\alpha+240^{\circ} \leq \omega t \leq 30^{\circ}+\alpha+240^{\circ}+\mu$ |  |  | C | $\bullet$ | C |  | $-\frac{3}{2} \mathrm{Van}$ | -Vdc | \#5 turning on, <br> \#3 turning off |  |
| $30^{\circ}+\alpha+240^{\circ}+\mu \leq \omega t \leq 30^{\circ}+\alpha+300^{\circ}$ |  |  |  | $\bullet$ | $\bullet$ |  | Vca | -Vdc | \#4 on, \#5 on |  |
| $30^{\circ}+\alpha+300^{\circ} \leq \omega t \leq 30^{\circ}+\alpha+300^{\circ}+\mu$ |  |  |  | C | $\bullet$ | C | $\frac{3}{2} \mathrm{Vcn}$ | -Vdc | \#6 turning on, <br> \#4 turning off |  |

Notes. Van is the reference angle. $\alpha=0$ when Vac swings positive (i.e., $30^{\circ}$ ). . Firing angle $\alpha$ can be as large as $180^{\circ}$. Symbol C in the above table implies a commutating switch. Symbol $\bullet$ implies a closed switch that is not commutating.







## Kimbark's Equations and the Thevenin Equivalent Circuit

As can be seen in the graphs, Vdc has a period of $60^{\circ}$. Then, the average value, Vdcavg, can be found by integrating over any period. Using the table and the period
$30^{\circ}+\alpha \leq \omega t<30^{\circ}+\alpha+60^{\circ}$,

$$
\begin{align*}
& V_{d c a v g}=\frac{3}{\pi}\left(-\frac{3}{2} \int_{\theta=\frac{\pi}{6}+\alpha}^{\theta=\frac{\pi}{6}+\alpha+\mu} v_{b n}(\theta) d \theta+\int_{\theta=\frac{\pi}{6}+\alpha+\mu}^{\theta=\frac{\pi}{6}+\alpha+\frac{\pi}{3}} v_{a b}(\theta) d \theta\right)  \tag{4.3}\\
& =\frac{3}{\pi}\left(-\frac{3}{2} \frac{V_{L L P}}{\sqrt{3}} \int_{\theta=\frac{\pi}{6}+\alpha}^{\theta=\frac{\pi}{6}+\alpha+\mu} \sin \left(\theta-\frac{2 \pi}{3}\right) d \theta+V_{L L P} \int_{\theta=\frac{\pi}{6}+\alpha+\mu}^{\theta=\frac{\pi}{6}+\alpha+\frac{\pi}{3}} \sin \left(\theta+\frac{\pi}{6}\right) d \theta\right) \\
& =\frac{3 V_{L L P}}{\pi} \frac{\sqrt{3}}{2}\left(\cos \left(\frac{\pi}{6}+\alpha+\mu-\frac{2 \pi}{3}\right)-\cos \left(\frac{\pi}{6}+\alpha-\frac{2 \pi}{3}\right)\right. \\
& \left.\quad-\cos \left(\frac{\pi}{6}+\alpha+\frac{\pi}{3}+\frac{\pi}{6}\right)+\cos \left(\frac{\pi}{6}+\alpha+\mu+\frac{\pi}{6}\right)\right) \\
& =\frac{3 V_{L L P}}{\pi}\left(\frac{\sqrt{3}}{2}\left(\cos \left(\alpha+\mu-\frac{\pi}{2}\right)-\cos \left(\alpha-\frac{\pi}{2}\right)\right)-\left(\cos \left(\alpha+\frac{2 \pi}{3}\right)-\cos \left(\frac{\pi}{3}+\alpha+\mu\right)\right)\right) \\
& =\frac{3 V_{L L P}}{\pi}\left(\frac{\sqrt{3}}{2} \sin (\alpha+\mu)-\frac{\sqrt{3}}{2} \sin (\alpha)-\cos (\alpha) \cos \left(\frac{2 \pi}{3}\right)+\sin (\alpha) \sin \left(\frac{2 \pi}{3}\right)\right. \\
& \left.\quad+\cos (\alpha+\mu) \cos \left(\frac{\pi}{3}\right)-\sin (\alpha+\mu) \sin \left(\frac{\pi}{3}\right)\right) \\
& =\frac{3 V_{L L P}}{\pi}\left(\frac{\sqrt{3}}{2} \sin (\alpha+\mu)-\frac{\sqrt{3}}{2} \sin (\alpha)+\frac{1}{2} \cos (\alpha)+\frac{\sqrt{3}}{2} \sin (\alpha)+\frac{1}{2} \cos (\alpha+\mu)-\frac{\sqrt{3}}{2} \sin (\alpha+\mu)\right) \\
& =\frac{3 V_{L L P}}{\pi}\left(\frac{1}{2} \cos (\alpha+\mu)+\frac{1}{2} \cos (\alpha)\right), \operatorname{leaving} \\
& V_{d c a v g}=\frac{3 V_{L L P}}{2 \pi}(\cos (\alpha)+\cos (\alpha+\mu)) \tag{4.4}
\end{align*}
$$

Now, for current, evaluating (4.1) at the end of commutation, i.e., $\omega t=30^{\circ}+\alpha+\mu$, yields

$$
\begin{align*}
& i_{1}\left(\omega t=30^{\circ}+\alpha+\mu\right)=I_{d c}=\frac{V_{L L P}}{2 \omega L}\left(\cos (\alpha)-\cos \left(30^{\circ}+\alpha+\mu-30^{\circ}\right)\right) \text {, so that } \\
& I_{d c}=\frac{V_{L L P}}{2 \omega L}(\cos (\alpha)-\cos (\alpha+\mu)) \tag{4.5}
\end{align*}
$$

To develop the Thevenin equivalent circuit for the DC side, recognize that at no load, $I_{d c}=0$, and if there is no DC current, then $\mu=0^{\circ}$. Thus, from (4.4), the open-circuit (Thevenin equivalent) voltage is

$$
\begin{equation*}
V_{T H}=\frac{3 V_{L L P}}{\pi} \cos (\alpha) \tag{4.6}
\end{equation*}
$$

If the Thevenin equivalent circuit exists, then it must obey the Thevenin equation

$$
V_{d c a v g}=V_{T H}-R_{T H} I_{d c}
$$

Substituting in for $V_{d c a v g}, V_{T H}$, and $I_{d c}$ yields

$$
\begin{aligned}
& \frac{3 V_{L L P}}{2 \pi}(\cos (\alpha+\mu)+\cos (\alpha))=\frac{3 V_{L L P}}{\pi} \cos (\alpha)-R_{T H} \frac{V_{L L P}}{2 \omega L}(\cos (\alpha)-\cos (\alpha+\mu)) \\
& \cos (\alpha+\mu)+\cos (\alpha)=2 \cos (\alpha)-\frac{R_{T H} \pi}{3 \omega L} \cos (\alpha)+\frac{R_{T H} \pi}{3 \omega L} \cos (\alpha+\mu)
\end{aligned}
$$

Gathering terms,

$$
\cos (\alpha+\mu)\left(1-\frac{R_{T H} \pi}{3 \omega L}\right)+\cos (\alpha)\left(1-2+\frac{R_{T H} \pi}{3 \omega L}\right)=0
$$

Note that if

$$
\begin{equation*}
R_{T H}=\frac{3 \omega L}{\pi} \tag{4.7}
\end{equation*}
$$

then the above equation is satisfied, leaving the DC-side Thevenin equivalent circuit shown below.

$$
\begin{aligned}
& V_{T H} \frac{Q_{T H}}{R_{T H}}+ V_{T H}=\frac{3 V_{L L P}}{\pi} \cos (\alpha) \\
& V_{\text {dcavg }} \\
&- R_{T H}=\frac{3 \omega L}{\pi} \\
& V_{\text {dcavg }}=\frac{3 V_{L L P}}{2 \pi}(\cos (\alpha)+\cos (\alpha+\mu)) \\
& I_{d c}=\frac{V_{L L P}}{2 \omega L}(\cos (\alpha)-\cos (\alpha+\mu))
\end{aligned}
$$

Power is found by multiplying (4.4) and (4.5), yielding

$$
\begin{align*}
& P_{d c a v g}=V_{d c a v g} I_{d}=\frac{3 V_{L L P}}{2 \pi}(\cos (\alpha)+\cos (\alpha+\mu)) \bullet \frac{V_{L L P}}{2 \omega L}(\cos (\alpha)-\cos (\alpha+\mu)), \text { so } \\
& P_{d c}=\frac{3 V_{L L P}^{2}}{4 \pi \omega L}\left(\cos ^{2}(\alpha)-\cos ^{2}(\alpha+\mu)\right) . \tag{4.8}
\end{align*}
$$

Since the converter is assumed to be lossless, then the AC power is the same as (4.10).
To estimate power factor on the AC side, use

$$
\begin{align*}
& \text { pf true }=\frac{P}{3 V_{\text {line-neutral-rms }} I_{r m s}}=\frac{V_{d c a v g} I_{d c}}{3 \frac{V_{L L P}}{\sqrt{3} \sqrt{2}} I_{r m s}}=\frac{\frac{3 V_{L L P}}{2 \pi}(\cos (\alpha)+\cos (\alpha+\mu)) \bullet I_{d c}}{\sqrt{3} \frac{V_{L L P}}{\sqrt{2}} I_{r m s}} \\
& \quad=\frac{\sqrt{3}(\cos (\alpha)+\cos (\alpha+\mu))}{\sqrt{2} \pi} \bullet \frac{I_{d c}}{I_{r m s}} \tag{4.9}
\end{align*}
$$

To approximate the rms value of current, it is very helpful to take advantage of the symmetry of the waveform. The shape of $i_{a}(t)$ is similar to that shown below. Since it is half-wave symmetric, only the positive half-cycle need be shown.


For small $\mu$ (i.e., $\mu<20^{\circ}$ ), the commutating portions of the current waveform can be approximated as straight-line segments.


Remembering that the rms value a triangular wedge of current is $I_{a v g}^{2}+\frac{1}{2} I_{p p}^{2}$, the rms value of the above waveform becomes

$$
\begin{aligned}
I_{r m s}^{2} & \approx \frac{\left(\frac{1}{4} I_{d c}^{2}+\frac{1}{2} I_{d c}^{2}\right) \mu+I_{d c}^{2}\left(\frac{2 \pi}{3}-\mu\right)+\left(\frac{1}{4} I_{d c}^{2}+\frac{1}{2} I_{d c}^{2}\right) \mu}{\pi} \\
& =I_{d c}^{2}\left(\frac{2}{3}+\frac{\mu}{2 \pi}\right) .
\end{aligned}
$$

Therefore,

$$
I_{r m s} \approx I_{d c} \sqrt{\frac{2}{3}+\frac{\mu}{2 \pi}},
$$

and for $\mu<20^{\circ}$, then

$$
\begin{equation*}
I_{r m s} \approx I_{d c} \sqrt{\frac{2}{3}} \tag{4.10}
\end{equation*}
$$

with an maximum error of approximately $5 \%$.
Substituting (4.12) into (4.11) yields

$$
\begin{equation*}
p f_{\text {true }}=\frac{\sqrt{3}(\cos (\alpha)+\cos (\alpha+\mu))}{\sqrt{2} \pi} \cdot \frac{I_{d c}}{I_{d c} \sqrt{\frac{2}{3}}}=\frac{3}{\pi}\left(\frac{\cos (\alpha)+\cos (\alpha+\mu)}{2}\right) . \tag{4.11}
\end{equation*}
$$

True power factor is the product of distortion power factor $p f_{\text {dist }}$ and displacement power factor $p f_{\text {disp }}$. Thus, examining (4.11), the conclusion is that

$$
\begin{equation*}
p f_{d i s t}=\frac{3}{\pi} \tag{4.12}
\end{equation*}
$$

and

$$
\begin{equation*}
p f_{\text {disp }}=\frac{\cos (\alpha)+\cos (\alpha+\mu)}{2} . \tag{4.13}
\end{equation*}
$$

## Analysis of Notching



Nothing is a phenomenon of interest mainly when sensitive loads are operated near a converter and share a portion of the converter's Thevenin equivalent impedance.

Assume that each L of the converter is divided into two inductances, L1 and L2, and that a sensitive load is located at al, b1, c1. Thus, L1 represents the fraction of the Thevenin equivalent impedance that is shared between the converter and the sensitive load. The objective is to determine the voltage notching present in line-to-neutral voltage $\mathrm{V}_{\mathrm{a}} \mathrm{n}$ and in line-to-line voltage $\mathrm{V}_{\mathrm{a} 1 \mathrm{~b} 1}$.

## Line-to-Neutral Voltage Notching

From KVL,

$$
V_{a 1 n}=V_{a n}-L_{1} \frac{d i_{a}}{d t}=V_{a n}-L_{1} \frac{d\left(i_{1}-i_{4}\right)}{d t}=V_{a n}-L_{1} \frac{d i_{1}}{d t}+L_{1} \frac{d i_{4}}{d t}
$$

Current $i_{a}$ is zero or constant, and thus $\mathrm{V}_{\mathrm{a} 1 \mathrm{n}}=\mathrm{V}_{\mathrm{an}}$, except when $i_{1}$ or $i_{4}$ are commutating. As shown previously, these commutation currents and times are

$$
i_{1}=\frac{V_{L L P}}{2 \omega L}\left(\cos (\alpha)-\cos \left(\omega t-30^{\circ}\right)\right), 30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu
$$

$$
i_{1}=I_{d c}-\frac{V_{L L P}}{2 \omega L}\left(\cos \alpha-\cos \left(\omega t-150^{\circ}\right)\right), 150^{\circ}+\alpha \leq \omega t \leq 150^{\circ}+\alpha+\mu
$$

and by symmetry $180^{\circ}$ later when

$$
\begin{aligned}
& i_{4}=\frac{V_{L L P}}{2 \omega L}\left(\cos (\alpha)-\cos \left(\omega t-210^{\circ}\right)\right), 210^{\circ}+\alpha \leq \omega t \leq 210^{\circ}+\alpha+\mu \\
& i_{4}=I_{d c}-\frac{V_{L L P}}{2 \omega L}\left(\cos \alpha-\cos \left(\omega t-330^{\circ}\right)\right), 330^{\circ}+\alpha \leq \omega t \leq 330^{\circ}+\alpha+\mu .
\end{aligned}
$$

Thus, when \#1 is commutating,

$$
\begin{align*}
& V_{a 1 n}=V_{a n}-\frac{L_{1}}{L_{1}+L_{2}} \frac{V_{L L P}}{2} \sin \left(\omega t-30^{\circ}\right)= \\
& V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-30^{\circ}\right)\right) \text { for } 30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu \tag{4.14}
\end{align*}
$$

and similarly,

$$
\begin{align*}
& V_{a 1 n}=V_{L N P}\left(\sin \omega t+\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-150^{\circ}\right)\right)= \\
& V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t+30^{\circ}\right)\right) \text { for } 150^{\circ}+\alpha \leq \omega t \leq 150^{\circ}+\alpha+\mu \tag{4.15}
\end{align*}
$$

When \#4 is commutating,

$$
\begin{aligned}
& V_{a 1 n}=V_{L N P}\left(\sin \omega t+\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-210^{\circ}\right)\right) \text { for } 210^{\circ}+\alpha \leq \omega t \leq 210^{\circ}+\alpha+\mu, \\
& V_{a 1 n}=V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-330^{\circ}\right)\right) \text { for } 330^{\circ}+\alpha \leq \omega t \leq 330^{\circ}+\alpha+\mu
\end{aligned}
$$

Rewriting,

$$
\begin{equation*}
V_{a 1 n}=V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-30^{\circ}\right)\right) \text { for } 210^{\circ}+\alpha \leq \omega t \leq 210^{\circ}+\alpha+\mu \tag{4.16}
\end{equation*}
$$

$$
\begin{equation*}
V_{a 1 n}=V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t+30^{\circ}\right)\right) \text { for } 330^{\circ}+\alpha \leq \omega t \leq 330^{\circ}+\alpha+\mu \tag{4.17}
\end{equation*}
$$

Summarizing

$$
\begin{aligned}
& V_{a 1 n}=V_{a n} \text { except when } \\
& V_{a 1 n}=V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t-30^{\circ}\right)\right)
\end{aligned}
$$

for $30^{\circ}+\alpha \leq \omega t \leq 30^{\circ}+\alpha+\mu$, and for $210^{\circ}+\alpha \leq \omega t \leq 210^{\circ}+\alpha+\mu$,

$$
V_{a 1 n}=V_{L N P}\left(\sin \omega t-\frac{\sqrt{3} L_{1}}{2\left(L_{1}+L_{2}\right)} \sin \left(\omega t+30^{\circ}\right)\right)
$$

$$
\text { for } 150^{\circ}+\alpha \leq \omega t \leq 150^{\circ}+\alpha+\mu \text {, and for } 330^{\circ}+\alpha \leq \omega t \leq 330^{\circ}+\alpha+\mu \text {. }
$$

Sample graphs for $\mathrm{V}_{\mathrm{a} 1 \mathrm{n}}$ are shown on the following pages.

## Line-to-Line Voltage Notching

The easiest way to determine $\mathrm{V}_{\mathrm{a} 1 \mathrm{~b} 1}$ is to recognize that $\mathrm{V}_{\mathrm{b} 1}$ is identical to $\mathrm{V}_{\mathrm{a} 1}$ except for being shifted by $120^{\circ}$, and to then subtract $\mathrm{V}_{\mathrm{b} 1}$ from $\mathrm{V}_{\mathrm{a} 1}$. The expressions are not derived. Rather, sample graphs for $\mathrm{V}_{\mathrm{a} 1 \mathrm{~b} 1}$ using graphical subtraction are shown in the following figures.

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| LS1/(LS1+LS2) $=$ | 1 |
| :---: | :---: |
| alpha $=$ | 30 |
| $\mathrm{mu}=$ | 15 |



| LS1/(LS1+LS2) $=$ | 0.5 |
| :---: | :---: |
| alpha $=$ | 30 |
| $\mathrm{mu}=$ | 15 |



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| LS1/(LS1+LS2) $=$ | 1 |
| :---: | :---: |
| alpha $=$ | 150 |
| $\mathrm{mu}=$ | 15 |




| LS1/(LS1+LS2) $=$ | 0.5 |
| :---: | :---: |
| alpha $=$ | 150 |
| $\mathrm{mu}=$ | 15 |



## 5. Effects and Symptoms

### 5.1. Utility

Harmonics-related problems on electric utility distribution systems are usually created by primary-metered customers. Typically, these problems are due to 500kVA (and larger) ASDs or induction heaters. In weaker systems, or near the end of long feeders, 100 - 200kVA nonlinear loads may be sufficiently large to create problems. The significant harmonics are almost always $5^{\text {th }}, 7^{\text {th }}, 11^{\text {th }}$, or $13^{\text {th }}$, with the $5^{\text {th }}$ harmonic being the problem in most instances.

Classic utility-side symptoms of harmonics problems are distorted voltage waveforms, blown capacitor fuses, and transformer overheating. Capacitors are sensitive to harmonic voltages. Transformers are sensitive to harmonic currents.

Typical utility-side symptoms are described in the next few pages.

- Resonance

Consider the resonant case shown in Figure 5.1, where the rectangular current injection of a 5000 HP six-pulse current-source ASD produced voltage resonance on a 25 kV distribution system. The $13 \% T H D_{V}$ caused nuisance tripping of computer-controlled loads, and the $30 \% T H D_{I}$ distortion caused overheating of parallel $25 \mathrm{kV} / 480 \mathrm{~V}$ 3750 kVA transformers that supplied the ASD. The dominant voltage harmonics are the $13^{\text {th }}(8.3 \%)$, and the $11^{\text {th }}(7.0 \%)$.


Oscilloscope Image of Six-Pulse LCC Current Waveform and the Resulting Voltage Resonance


Fourier Series Reconstruction Using Harmonics through the $17^{\text {th }}$ (Although the phase angles were not recorded, they were manually adjusted in the plotting program to match the oscilloscope waveshapes.)

Figure 5.1. Resonance Due to 5000HP Six-Pulse Line-Commutated ASD

Resonance occurs when the harmonic currents injected by nonlinear loads interact with system impedance to produce high harmonic voltages. Resonance can cause nuisance tripping of sensitive electronic loads and high harmonic currents in feeder capacitor banks. In severe cases, capacitors produce audible noise and sometimes bulge.

To better understand resonance, consider the simple parallel and series cases shown in the one-line diagrams of Figure 5.2. Parallel resonance occurs when the power system presents a parallel combination of power system inductance and power factor correction capacitors at the nonlinear load. The product of harmonic impedance and injection current produces high harmonic voltages.

Series resonance occurs when the system inductance and capacitors are in series, or nearly in series, from the converter point of view.

For parallel resonance, the highest voltage distortion is at the nonlinear load. However, for series resonance, the highest voltage distortion is at a remote point, perhaps miles away or on an adjacent feeder served by the same substation transformer. Actual feeders can have five or ten shunt capacitors, so many parallel and series paths exist, making computer simulations necessary to predict distortion levels throughout the feeder.


Parallel Resonance (high voltage distortion at converter load, low voltage distortion at points sown the feeder)


Series Resonance (low voltage distortion at converter load, high voltage distortion at points down the feeder)

Figure 5.2. Simple Examples of Parallel and Series Resonance

A simple parallel resonant case would be, for example, where the only capacitors on a feeder are at the converter location. In such cases, it is possible to use the parallel resonance approximation formula developed below.

Let $L_{\text {sys }}$ be the total per phase series inductance "seen" at the converter load connection point. $L_{\text {sys }}$ is determined from the short circuit duty at the bus. If $C_{c a p}$ is the capacitance per phase of the power factor correction capacitor, the peak of the parallel resonant curve occurs at

$$
f_{\text {res }}=\frac{\omega_{\text {res }}}{2 \pi}=\frac{1}{2 \pi \sqrt{L_{\text {sys }} C_{c a p}}}
$$

The inductive reactance of the system at the fundamental frequency is

$$
X_{s y s}=\omega_{1} L_{s y s}, \text { so } L_{s y s}=\frac{X_{\text {sys }}}{\omega_{1}} .
$$

The reactance of the capacitor at the fundamental frequency is

$$
X_{c a p}=\frac{1}{\omega_{1} C_{c a p}} \text {, so } C_{c a p}=\frac{1}{\omega_{1} X_{c a p}} .
$$

Substituting into the $f_{\text {res }}$ equation yields

$$
f_{\text {res }}=\frac{1}{2 \pi} \sqrt{\frac{\omega_{1}}{X_{\text {sys }}}} \sqrt{\omega_{1} X_{c a p}}=\frac{\omega_{1}}{2 \pi} \sqrt{\frac{X_{c a p}}{X_{s y s}}}=f_{1} \sqrt{\frac{X_{c a p}}{X_{s y s}}} .
$$

In the per unit system, system short circuit MVA is

$$
M V A_{S C}=\frac{1}{X_{s y s}}
$$

and capacitor MVA is

$$
M V A_{c a p}=\frac{1}{X_{c a p}}
$$

so that the resonant frequency expression becomes

$$
\begin{equation*}
f_{\text {res }}=f_{1} \sqrt{\frac{M V A_{S C}}{M V A_{C A P}}} . \tag{5.1}
\end{equation*}
$$

Thus, "stiff systems" (i.e., relatively high $M V A_{S C}$ ) have higher resonant frequencies. When capacitors are added, the resonant frequency is lowered.

The risk of using (5.1) is that it represents only a small part of the true harmonics situation. Three important points to remember are

1. While (5.1) predicts a resonant frequency, it gives no information about the broadness of the resonant curve. Thus, if a system is resonant at, for example, the $6^{\text {th }}$ harmonic, one might innocently conclude there is no harmonics problem. However, due to the broadness of the resonance curve, the $5^{\text {th }}$ and $7^{\text {th }}$ harmonics will be greatly affected.
2. Anytime there are shunt capacitors, there are resonant frequencies. In fact, almost all distribution feeders are strongly resonant near the $5^{\text {th }}$ and $7^{\text {th }}$ harmonics. However, resonance is a problem only if there are sufficient harmonic amperes to excite harmonic voltages so that $T H D_{v}$ exceeds $5 \%$.
3. Most utility distribution feeders have five or more capacitor banks, so that there are many parallel and series paths. Thus, computer simulations are required to accurately predict distortion levels through the feeder and adjacent feeders connected to the same substation transformer.

To illustrate the broadness of the resonance curve, consider the case shown in Figure 5.3. This curve represents the Thevenin equivalent impedance, also known as the "driving point impedance," at the customer bus. The situation is simple parallel resonance. Note that as the amount of power factor correction is increased by adding additional kVArs, the peak of the resonance curve moves toward lower frequencies.

Figure 5.3 illustrates the following two important facts concerning resonance:

1. The resonance curve is very broad.
2. Typical power factor correction practices to the $0.95-0.98$ DPF range will cause distribution feeders to resonate near the $5^{\text {th }}$ and $7^{\text {th }}$ harmonics.



Figure 5.3. Thevenin Equivalent Impedance at Customer Bus

Figure 5.3 is also useful in estimating the harmonic voltages that will exist at the customer bus. Consider, for example, the 0.95 power factor correction case. At the $5^{\text {th }}$ harmonic, the driving point impedance is approximately $200 \%$ (i.e., 2 pu). If the converter load is 0.18 pu , then the $5^{\text {th }}$ harmonic current will be (assuming the $1 / \mathrm{k}$ rule) $\frac{0.18}{5}=0.036$ pu. The $5^{\text {th }}$ harmonic voltage estimate is then $0.036 \bullet 2=0.072 \mathrm{pu}$. Thus, a $5^{\text {th }}$ harmonic voltage of $7.2 \%$ can be expected, meaning that the $T H D_{v}$ will be at least $7.2 \%$. Of course, the $T H D_{v}$ will be higher after the contributions of the $7^{\text {th }}, 11^{\text {th }}$, and $13^{\text {th }}$ (and higher) harmonics are included.

- Nuisance Tripping of Sensitive Loads

Some computer-controlled loads are sensitive to voltage distortion. Although it is difficult to find reliable data on this subject, one case showed that a $T H D_{V}$ of $5.5 \%$ regularly shut down computerized lathes at a large pipe company heat treatment operation. While voltage distortions of $5 \%$ are not usually a problem, voltage distortions above $10 \%$ will cause significant nuisance tripping.

## - Blown Capacitor Fuses, Failure of Capacitor Cells, and Degredation of Internal Capacitance

A common harmonics-related complaint comes from capacitor crew foremen or other distribution feeder maintenance personnel who complain that "a capacitor bank has to be rebuilt often," or "fuses on a capacitor bank blow regularly," or "a capacitor bank hums," or "the capacitance of a bank is diminishing."

Harmonic voltages produce exaggerated harmonic currents in capacitors because of the inverse relationship between capacitor impedance and frequency. To illustrate this point, the measured current waveform of a $300 \mathrm{kVAr}, 480 \mathrm{~V}$ bank at a commercial bank building is shown in Figure 5.4. The waveform is dominated by an $11^{\text {th }}$ harmonic ( $23.3 \%$ ). The principal distorting load in the building was a large UPS.

Capacitors with excessive harmonic currents often produce a load humming noise. Although the human ear is relatively insensitive to 60 Hz , it is quite sensitive to the $5^{\text {th }}$ harmonic and above (i.e., 300 Hz and above).


Figure 5.4. 300 kVAr, 480V Capacitor Current Waveform at Commercial Bank Building

Since capacitor impedance varies according to $\frac{1}{j \omega C}$, then the impedance for harmonic k is $\frac{1}{j k \omega_{1} C}$, where $\omega_{1}$ is the fundamental radian frequency (e.g., $120 \pi$ radians/sec for 60 Hz systems). Because of this inverse relationship, moderate harmonic voltages can produce large currents in capacitors. For example, if a capacitor has $10 \%$ voltage distortion due entirely to the $5^{\text {th }}$ harmonic, the induced $5^{\text {th }}$ harmonic current is $0.10 \bullet 5=$ 0.50 pu on the capacitor base. The corresponding rms current in the capacitor increases to $\sqrt{1^{2}+0.50^{2}}=1.12$ pu times the fundamental current. A $10 \% 11^{\text {th }}$ harmonic voltage produces an even greater rms current, 1.49 pu .

Now, consider an example where voltage distortion on a capacitor is assumed to be divided among six-pulse characteristic harmonics through the $25^{\text {th }}$, in inverse proportion to frequency. This assumption implies that the harmonic currents have equal magnitudes. Since the voltages are expressed in per unit of fundamental, the squared voltage THD is

$$
T H D_{V}^{2}=V_{5}^{2}+V_{7}^{2}+V_{11}^{2}+V_{13}^{2}+V_{17}^{2}+V_{19}^{2}+V_{23}^{2}+V_{25}^{2} .
$$

Because the harmonic voltages in this example are assumed to vary inversely with frequency, then

$$
\begin{aligned}
& V_{7}=V_{5} \bullet \frac{5}{7}, V_{11}=V_{5} \bullet \frac{5}{11}, \text { etc., so } \\
& T H D_{v}^{2}=V_{5}^{2} \bullet\left(1+\frac{5^{2}}{7^{2}}+\frac{5^{2}}{11^{2}}+\frac{5^{2}}{13^{2}}+\frac{5^{2}}{17^{2}}+\frac{5^{2}}{19^{2}}+\frac{5^{2}}{23^{2}}+\frac{5^{2}}{25^{2}}\right), \\
& T H D_{V}^{2}=V_{5}^{2} \bullet 2.108, \text { so } V_{5}^{2}=\frac{T H D_{v}^{2}}{2.108}
\end{aligned}
$$

Taking the square root, $V_{5}=\frac{T H D_{v}}{1.452}$, then the current on the capacitor base is

$$
I_{5}(p u)=5 V_{5}(p u)=\frac{5 \bullet T H D_{v}}{1.452} .
$$

Since all eight harmonic currents in this example are equal, the total squared rms capacitor current, including fundamental, is

$$
I_{r m s}^{2}(p u)=1^{2}+8 \bullet\left(\frac{5 \bullet T H D_{v}}{1.452}\right)^{2}=1+94.9 \bullet T H D_{v}^{2}
$$

The square root of the above formula is used to computer rms currents for a range of voltage distortion values, and the results are given in Table 5.1.

Table 5.1. RMS Capacitor Current (in pu for harmonics through the $25^{\text {th }}$ ) versus Voltage Distortion (assuming that voltage harmonics decrease in proportion to frequency)

| $T H D_{v}$ | RMS Capacitor Current <br> pu |
| :---: | :---: |
| 0.00 | 1.000 |
| 0.05 | 1.112 |
| 0.10 | 1.396 |

Thus, it is reasonable to expect a $40 \%$ increase in capacitor rms current when voltage distortions are in the $10 \%$ range.

Capacitors may also fail because of overvoltage stress on dielectrics. A 10\% harmonic voltage for any harmonic above the $3^{\text {rd }}$ increases the peak voltage by approximately $10 \%$ because the peak of the harmonic often coincides, or nearly coincides, with the peak of the fundamental voltage.

## - Transformer Overheating That Cannot Be Explained by kVA Load Level Alone

Another common harmonic-related complaint is that "my transformer is only 70\% loaded, but it is too hot to hold my hand on." These cases are usually limited to situations where the transformer serves a large nonlinear load.

There are two reasons for overheating.

1. Losses in a conductor increase when harmonics are present because losses are proportional to the square (at least) of rms current, and rms current increases with current distortion according to

$$
\begin{equation*}
I_{r m s}=\sqrt{1+T H D_{I}^{2}} . \tag{5.2}
\end{equation*}
$$

2. Because of the resistive skin effect and winding proximity effect, one ampere of harmonic current produces more losses than does one ampere of fundamental current.

The impact of harmonic currents on transformers is more serious than on conventional conductors because the resistive skin effect is enhanced within closely-spaced transformer windings. Good engineering practice calls for the derating of transformers that serve nonlinear loads to an equivalent $80 \%$ of nameplate kVA.

To examine the rms current effect, consider a transformer that serves an ideal six-pulse converter with classical $\frac{1}{k}$ harmonic magnitude currents. In terms of the fundamental current $I_{1, r m s}$, the squared rms current is

$$
\begin{equation*}
I_{r m s}^{2}=I_{1, r m s}^{2} \bullet\left(1^{2}+\frac{1}{5^{2}}+\frac{1}{7^{2}}+\frac{1}{11^{2}}+\frac{1}{13^{2}}+\frac{1}{17^{2}}+\frac{1}{19^{2}}+\cdots\right) \tag{5.3}
\end{equation*}
$$

The above infinite series converges to the form

$$
I_{r m s}^{2}=I_{1, r m s}^{2} \bullet \frac{\pi^{2}}{9}=1.0966 I_{1, r m s}^{2}
$$

Since losses increase by the square of rms current, the winding losses automatically increase to at least 1.0966 times the fundamental-only case. Thus, if losses are to be held constant at their rated value so that transformer heating is not excessive, the rms current (and equivalent kVA rating) should be lowered to at least $\sqrt{\frac{1}{1.0966}}=0.955 \mathrm{pu}$. of nameplate. If harmonics above the $25^{\text {th }}$ harmonic are ignored, the equivalent kVA rating is 0.960 pu (i.e., practically the same as the infinite series case).

However, the major transformer derate comes from the resistive skin effect. The resistive skin effect occurs because higher-frequency currents migrate to the outermost portions of a conductor, increasing its equivalent resistance. For power transformers, this phenomenon is usually modeled by dividing resistance into two parts - a non-frequency dependent part, and a frequency-dependent part. The frequency-dependent part is assumed to increase in proportion to the square of frequency, as given by

$$
\begin{equation*}
R_{k}=R_{D C} \bullet\left(1+k^{2} P_{E C-R}\right) \tag{5.4}
\end{equation*}
$$

where $R_{k}$ is the winding resistance at harmonic $\mathrm{k}, R_{D C}$ is the winding resistance at DC, and $P_{E C-R}$ is the winding eddy current loss factor. $P_{E C-R}$ ranges from 0.01 for low voltage service transformers with relatively small conductors to 0.10 for substation transformers having large conductors.

Since heating is proportional to squared current times resistance, the above variation can be incorporated into an equivalent rms current that takes into account skin effect. Incorporating (5.4) into (5.3) yields equivalent rms current

$$
\begin{equation*}
I_{r m s, \text { equiv }}^{2}=I_{1, r m s}^{2} \bullet\left(1^{2}+\sum_{k=5,7,11,13, \ldots}^{\infty} \frac{1}{k^{2}} \bullet \frac{\left(1+k^{2} \bullet P_{E C-R}\right)}{\left(1+1 \bullet P_{E C-R}\right)}\right) . \tag{5.5}
\end{equation*}
$$

The above series does not converge if $P_{E C-R} \neq 0$. Thus, it is appropriate only to discuss a finite number of terms, such as through the $25^{\text {th }}$ harmonic. Using the square root of (5.5) to give the derate, and ignoring harmonics above the $25^{\mathrm{th}}$, the equivalent kVA rating for a realistic range of $P_{E C-R}$ is given in Table 5.2.

Table 5.2. Equivalent kVA Rating for Transformers Serving Six-Pulse Loads
(ignoring harmonics above the $25^{\text {th }}$, and assuming that current harmonics decrease in proportion to frequency)

| $P_{E C-R}$ | Equivalent kVA Rating <br> pu |
| :---: | :---: |
| 0.00 | 0.960 |
| 0.02 | 0.928 |
| 0.04 | 0.900 |
| 0.06 | 0.875 |
| 0.08 | 0.853 |
| 0.10 | 0.833 |

### 5.2. End-User

Symptoms experienced by end-users include the utility symptoms described above, plus the items described on the next few pages.

## - Digital Clocks Gaining Time

Digital clocks work off the principle of counting zero crossings or slope changes in the 60 Hz fundamental voltage. There may be some filtering present in the clock circuitry, but if voltage harmonics are strong enough, then it is possible to have multiple zero crossings or slope changes that cause the clocks to run fast. Older digital clocks are reported to be the most sensitive to voltage harmonics.

An example of a voltage waveform that was responsible for this phenomenon is shown in Figure 5.5. The waveform has a $2 \%, 36^{\text {th }}$ harmonic.


Figure 5.5. Voltage Waveform That Caused Digital Clocks to Gain Time

## - Telephone Interference

Telephone interference has been a harmonics-related concern for many decades, but the gradual phasing out of open-wire telephone circuits has reduced the number of interference problems. While the frequency response of the combined telephone circuit and human ear is largely immune to 60 Hz interference, higher harmonics fall into the low-audio range.

When harmonic currents on power lines inductively couple into nearby phone lines, they can cause significant interference. Typically, the problem harmonics are either characteristic six-pulse harmonics due to large converters, or $9^{\text {th }}$ and higher multiples of three (i.e., zero sequence) due to transformer saturation. All things being equal, zero sequence harmonics are more problematic than positive and negative sequence harmonics because a-b-c zero sequence fields are additive and, therefore, do not decay as rapidly with distance.

The telephone influence factor (i.e., TIF) curve shown in Figure 5.6 gives the relative interference weighting that applies to inductively-coupled harmonic currents flowing in power lines.


Figure 5.6. Telephone Influence Factor (TIF) Curve
'Ielephone interterence problems are usually solved by the telephone company, in cooperation with the electric utility involved. Solutions are often trial-and-error and usually consist of moving or disconnecting capacitor banks that have high harmonic currents, or placing tunable reactors in the ground path of wye-connected capacitor banks that have high harmonic currents. The tuning reactors are invisible to positive and negative sequences, but they can change the zero-sequence resonant frequency of a distribution feeder and often eliminate the resonance problem.

## - Motor Heating

For frequencies higher than fundamental, three-phase induction motors can be approximated by positive/negative shunt impedances,

$$
Z_{k}=R_{\text {winding }}+j k X^{\prime \prime}
$$

where $R_{\text {winding }}$ is the motor winding resistance, and $X^{"}$ is the fundamental frequency subtransient reactance (typically 0.20 pu on motor base). Since most motors are threewire delta or ungrounded-wye connected, motors appear as open circuits to zero sequence harmonics.

Assuming $X^{"}=0.20$, relatively small $R_{\text {winding }}$ with respect to $k X^{"}$, and a $5^{\text {th }}$ harmonic voltage of $10 \%$, the induced $5^{\text {th }}$ harmonic current will be

$$
I_{5, r m s}=\left|\frac{0.10}{5 \bullet 0.20}\right|=0.10 \mathrm{pu} \text { on the motor base. }
$$

Thus, harmonic voltages can create additional rotor winding currents and increase the $I^{2} R_{\text {winding }}$ losses in three-phase motors by several percent.

High efficiency single-phase induction motors are more sensitive to voltage harmonics than are three-phase motors. Auxiliary parallel windings with series-run capacitors create a quasi-sinusoidal flux wave to improve efficiency. The auxiliary winding inductance and run capacitor create a series resonant path in the $4^{\text {th }}-11^{\text {th }}$ harmonic range.

To illustrate this phenomenon, current waveforms for a $2 \mathrm{HP}, 230 \mathrm{~V}$, fully loaded motor were measured, with and without significant $5^{\text {th }}$ harmonic voltage applied. Waveforms for both cases are shown in Figure 5.7. The strong $5^{\text {th }}$ harmonic current causes additional heating and produces noticeable audible noise.


Applied $5^{\text {th }}$ Harmonic Voltage $=1.3 \%$. Resulting $T H D_{I}=11.6 \%$.


Applied $5^{\text {th }}$ Harmonic Voltage $=5.4 \%$. Resulting $T H D_{I}=34.0 \%$.

Figure 5.7. Sensitivity of Fully Loaded 2HP, 230V Single-Phase HighEfficiency Induction Motor Current to $5^{\text {th }}$ Harmonic Voltage

## - Overloaded Neutral Conductors in Commercial Buildings

In a three-phase, four-wire system, the sum of the three phase currents returns through the neutral conductor. Positive and negative sequence components sum to zero at the neutral point, but zero sequence components are purely additive in the neutral.

Power system engineers are accustomed to the traditional rule that "balanced three-phase systems have no neutral currents." However, this rule is not true when zero sequence harmonics (i.e., primarily the $3^{\text {rd }}$ harmonic) are present. In commercial buildings with large numbers of PC loads, the rms neutral current can actually exceed rms phase currents.

Consider the measurements shown in Figure 5.8 for a 120/208V service panel feeding 147 PC workstations plus some miscellaneous linear load. The rms currents for phases a, b , and c are $40.2 \mathrm{~A}, 52.7 \mathrm{~A}$, and 47.8 A , respectively. The neutral current is mainly $3^{\text {rd }}$ harmonic and has rms value 60.9A, which is approximately 1.3 times the average fundamental a-b-c phase current.


Phase A at Service Panel, 40.2Arms Fundamental. $T H D_{I}=77.4 \%$.


Phase B at Service Panel, 52.7Arms Fundamental. $T H D_{I}=54.7 \%$.


Phase C at Service Panel, 47.8Arms Fundamental. $T H D_{I}=60.7 \%$.


Sum of Phases A,B,C in Neutral Wire (bold curve) at Service Panel, 60.9 Arms. $T H D_{I}=633.0 \%$.

Figure 5.8. Current Measurements for a 120/208V Service Panel Feeding 147 PC Workstations

Large numbers of PC loads cause a flattening of phase-to-neutral voltages inside commercial buildings, as shown in Figure 5.9. The spectrum is given in the Appendix.


Figure 5.9. Phase A Voltage at Service Panel for 147 PC Workstations

This commonly-observed flattened voltage waveform has $T H D_{V} 5.1 \%$ and contains $3.9 \%$ of $5^{\text {th }}, 2.2 \%$ of $3^{\text {rd }}$, and $1.4 \%$ of $7^{\text {th }}$ harmonics.

RMS neutral current due to $3^{\text {rd }}$ harmonics can be higher than that of the example shown in Figure 5.8. Many PCs have $3^{\text {rd }}$ harmonic currents greater than $80 \%$. In these cases, the neutral current will be at least $3 \cdot 80 \%=240 \%$ of the fundamental a-b-c phase current.

Thus, when PC loads dominate a building circuit, it is good engineering practice for each phase to have its own neutral wire, or for the common neutral wire to have at least twice the current rating of each phase wire.

Overloaded neutral currents are usually only a local problem inside a building, for example on a service panel. At the service entrance, the harmonic currents produced by PCs and other nonlinear loads are diluted by the many linear loads including air conditioners, pumps, fans, and incandescant lights. The current waveform @ 480V for a large office and classroom building is shown in Figure 5.10. The $T H D_{I}$ is $7.0 \%$, consisting mainly of $4.5 \%$ of $5^{\text {th }}, 3.7 \%$ of $7^{\text {th }}$, and $3.5 \%$ of $3^{\text {rd }}$ harmonics. The spectrum is given in the Appendix.


Figure 5.10. Phase A Current at 480V Service Entrance to College of Business Administration Building, University of Texas at Austin

## 6. Conducting an Investigation

Electric utility engineers may be confronted with harmonic problems on their own distribution feeders, or within customer facilities. Distribution feeder cases are the most difficult to deal with since a large harmonics source can pollute the voltage waveform for many miles, including adjacent feeders connected to the same substation transformer. Customer facility cases are the simplest to investigate because the distances are smaller and the offending load can usually be identified by turning candidates off-and-on while observing area voltage waveforms with an oscilloscope or spectrum analyzer.

The focus of this section is on investigating distribution feeder problems where the combination of harmonic current injection and resonant networks act together to create objectionable harmonic levels.

### 6.1 Field Measurements

In some cases, field measurements alone can be used to identify the source of a harmonics problem. To do this, consider the following:

1. It is important to remember that utility-side harmonics problems are almost always created by primary-metered customers, and the culprit is usually a 500 kVA (or higher) ASD, rectifier, or induction furnace. Therefore, if the problem appears suddently, it is prudent to ask questions within your company to find out which large customers on the feeder (or adjacent feeders) may have added a large distorting load.
2. It is wise to make field measurements before contacting the customer. The basic tool needed is a portable spectrum analyzer that can monitor and record harmonic voltages and currents. Voltage measurements can be made at capacitor control circuits or at metering points. The frequencies of interest (e.g. 1500 Hz and below) are low enough that standard metering, control, and service transformers accurately portray feeder voltage waveforms. Usually, the feeder voltage distortion is high near the harmonics source, but when there are many shunt capacitors, remote points may also have high voltage distortion. It is desirable, but perhaps impossible, to turn off all shunt capacitors when the measurements are made.
3. Next, it is prudent to monitor and record voltage and current harmonics at the customer's metering point for at least two days, and perhaps more. These data will help to correlate the customer's daily work shift patterns or nonlinear loads with distortion levels. $T H D_{V}$, $T H D_{I}$, the $5^{\text {th }}$ and $7^{\text {th }}$ harmonic magnitudes, and if possible, harmonic power, should be recorded. The main indicator is the customer's $T H D_{I}$.
4. While there is debate on the subject, most power quality engineers believe that harmonic power is a good indicator of the source of harmonics. In fact, if a distribution feeder has one large distorting load, then that load is the source of all harmonic power on the feeder. Some spectrum analyzers compute harmonic power. If the customer is the source of harmonic power, then you can expect the net harmonic power (a few percent of
fundamental power) to flow out from the customer onto the feeder, further comfirming that the harmonics source is inside the customer's facility. The $5^{\text {th }}$ harmonic usually has the largest harmonic power.

If Steps 1-4 are inconclusive, then it is sometimes possible to "track down" a harmonics source by taking harmonic power measurements at convenient points along the feeder. For example, voltage measurements can be made at capacitor control boxes. Current measurements can be made with fiber optic-linked current transformers that connect directly to the feeder conductors. Using voltage and current, net harmonic power can be calculated. The expectation is that the net harmonic power flows away from the source.

A wire loop (i.e. "search coil") can be connected to the voltage input of a spectrum analyzer to monitor the current-induced $N \frac{d \varphi}{d t}$ signal that exists below an overhead feeder. The search coil has been used for decades by telephone companies to detect the presence of high harmonic currents. While the search coil gives no power or voltage information, it is useful because large harmonic currents exist on either or both sides of a resonating capacitor bank. Resonating capacitor banks are sometimes turned off, moved, or filtered in an attempt to relieve the harmonics problem.

### 6.2. Computer Simulations

Field measurements are useful when a harmonics problem already exists. However, computer simulations are needed to study potential problems in advance. For example, if a customer desires to add a 1000 HP ASD, then an advance study is definitely needed so that problems can be resolved before the ASD is installed. A harmonics study proceeds in much the same way as a loadflow, short circuit, or motor starting study.

Unless a distribution system is badly unbalanced, or there is a very large single-phase harmonicsproducing load such as an electric train, harmonics analysis can usually be performed using the balanced assumption. The reasons are that

- Most problem-causing loads are large three-phase balanced loads such as ASDs.
- Distribution capacitors are usually applied in the form of three-phase banks, having a balancing effect on harmonics propagation.
- Phase identification of single-phase loads and load levels may not be available or easily obtained.
- The quality of harmonics data for the distorting loads may be poor, so that injection "rules of thumb" must be used.
- Systems are often studied in advance, so that not all of the actual data are available.

In spite of these difficulties, experience has shown that distribution feeder harmonic simulations match "real world" measurements very well, and that simulation is a reliable tool for studying solutions such as passive filtering. The term "accurate" generally means that simulated voltage distortions match field measurements within a few percent (on a $100 \%$ base).

To obtain this accuracy, these eight rules must be obeyed:

1. When modeling a distribution feeder, include in your study all the feeders attached to the same substation transformer, and in equal detail. On the transmission side of the substation transformer, establish a simple Thevenin equivalent using the short circuit impedance. Transmission line capacitance can be added on the substation high-side, but it usually is not important to the study results.
2. Ten to twenty aggregated busses per feeder is usually adequate detail.
3. Load distributions along actual feeders are not known with great accuracy. However, total feeder kVA load and kVA ratings of individual transformers are known. Load distributions are often estimated by assuming that the total feeder kVA load is distributed in proportion to individual load transformer ratings. Adjustments should be made for large customers whose daily load profile does not track the feeder load profile.
4. Harmonics models for conventional loads must be included. These can be simple shunt resistances, where the resistances are sized according to active power.
5. The worst-case for harmonics is usually when the harmonics-producing loads are at full power, and the conventional loads are at low power. Conventional loads add damping and reduce distortion levels, and their sinusoidal currents dilute the nonlinear load currents.
6. Capacitor banks are very important and must be included in the study. Usually this means a case with all capacitors on, and a case with only the fixed capacitors on. Other likely capacitor scenarios may also be needed.
7. If there are significant lengths of underground cables, cable capacitances may be important and should be lumped onto the trunk feeders in the form of shunt capacitors. "Important" is relative to the size of the other shunt capacitors. 100 kVAr is a good rule for being "important." The capacitance of power cables can be estimated using Table 6.1.

Table 6.1. Capacitance and Charging of 12.47 kV and 25 kV Cables

| Cable | Capacitance <br> $(12.47 \mathrm{kV})$ | kVAr <br> $(12.47 \mathrm{kV})$ | Capacitance <br> $(25 \mathrm{kV})$ | kVAr <br> $(25 \mathrm{kV})$ |
| :---: | :---: | :---: | :---: | :---: |
| $1 / 0$ | 0.163 | 9.56 | 0.124 | 29.2 |
| $4 / 0$ | 0.222 | 13.02 | 0.162 | 38.2 |
| 350 kcmil | 0.267 | 15.65 | 0.1914 | 45.1 |
| 500 kcmil | 0.304 | 17.82 | 0.215 | 50.7 |
| 1000 kcmil | 0.401 | 23.5 | 0.278 | 65.5 |

Capacitance: $\quad \mu \mathrm{F}$ per km per phase
kVAr: (three-phase) per km
8. When the system has multiple sources operating at various power levels (i.e., 10 or more sources), then it is important to consider harmonics cancellation brought about by phase angle diversity. The simplest way to consider diversity is to multiply the harmonic injection currents for each load by the following:

- $3^{\text {rd }}$ harmonic, multiply by 1.0 (i.e., no diversity)
- $5^{\text {th }}$ and $7^{\text {th }}$ harmonics, multiply by 0.9
- $11^{\text {th }}$ and $13^{\text {th }}$ harmonics, multiply by 0.6
- Higher harmonics, multiply by 0.2

There are two basic techniques for performing harmonics studies - frequency-domain, and timedomain.

- Frequency-domain modeling is most often used for harmonics studies where the focus of attention is on the network. Approximate models are used for nonlinear loads. Each harmonic is studied individually, and the results are superimposed to produce timedomain waveforms.
- Time-domain modeling is usually performed with full three-phase detail and precise models of nonlinear loads. Time-domain modeling is often used to study small networks where the focus of attention is inside specific equipment such as ASDs.


## Five-Bus Computer Simulation Example

An industrial customer will be served by constructing a three-mile 12.5 kV overhead feeder from a dedicated $138 / 12.5 \mathrm{kV}$ substation transformer. The customer will have 5MW @ $d p f=0.85$ of conventional load and a 2000 HP , six-pulse adjustable-speed drive (ASD). The ASD is connected through a delta-delta transformer (i.e., no phase shift). The customer also has 1800 kVAr of shunt power factor correction capacitors.

The 138 kV substation bus has the following characteristics:

- $\mathrm{Z}^{+}=0.4+\mathrm{j} 2.5 \%$ (100MVA base)
- 50 miles of 138 kV transmission lines are connected to it (line charging $=0.0808 \mathrm{MVAr}$ per km).

The dedicated substation transformer has the following characteristics:

- $\mathrm{P}_{\text {base }}=15 \mathrm{MVA}$
- 138 kV delta $/ 12.5 \mathrm{kV}$ grounded-wye connection
- 0.95 per unit tap on the 138 kV side
- $\mathrm{Z}^{+}=0.5+\mathrm{j} 10.5 \%$ (on 15MVA base).

The overhead feeder will be constructed with 477 ACSR arm-type construction that has the following characteristics:

- $\mathrm{R}^{+}=0.1316 \Omega$ per km
- $\mathrm{X}^{+}=0.387 \Omega$ per km (@ 60 Hz$)$
- $\mathrm{C}^{+}=0.01106 \mu \mathrm{~F}$ per km.

The conventional load transformer is rated at 7.5MVA and has $\mathrm{Z}^{+}=0.50+\mathrm{j} 5.0 \%$ (on 7.5 MVA base).

Once the data have been gathered, the next step is to draw a one-line diagram with all impedances and loads expressed on a common base. The base values are selected as 10MVA throughout, and 12.5 kV on the feeder section. The voltage base varies throughout the circuit according to nominal transformer turns ratios.

The swing bus is effectively grounded for harmonics with a j0.01\% "harmonics-only subtransient impedance." The purpose of this grounding impedance is to model the ability of the "far-distant" system to absorb harmonic currents without incurring appreciable voltage distortion.

Calculations for the above steps are shown below.

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For the transmission system,


For the distribution feeder,

$$
\begin{aligned}
& \mathrm{R}^{+}=0.1316 \Omega \text { per km } \bullet 1.609 \mathrm{~km} / \mathrm{mile} \cdot 3 \text { miles }=0.635 \Omega \\
& \mathrm{X}^{+}=0.387 \Omega \text { per km } \cdot 1.609 \mathrm{~km} / \mathrm{mile} \cdot 3 \text { miles }=1.868 \Omega \\
& \mathrm{C}^{+}=0.01106 \mu \mathrm{~F} \text { per km } \cdot 1.609 \mathrm{~km} / \mathrm{mile} \cdot 3 \text { miles }=0.0534 \mu \mathrm{~F} \\
& \mathrm{Z}_{\text {BASE }}=(12.5)^{2} /(10)=15.625 \Omega \\
& \text { Line Charging } \quad=3\left(\frac{12500}{\sqrt{3}}\right)\left(\frac{12500}{\sqrt{3}}\right)(2 \pi)(60)\left(0.0534 \bullet 10^{-6}\right) \mathrm{VAr} \\
& \quad=3.15 \mathrm{kVAr}=0.0315 \% @ 10 \mathrm{MVA} \\
& \mathrm{R}_{\mathrm{pu}}=\frac{0.635}{15.625} \bullet 100 \%=4.06 \% \\
& \mathrm{X}_{\mathrm{pu}}=\frac{1.868}{15.625} \bullet 100 \%=11.96 \%
\end{aligned}
$$

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For the substation transformer,


For the conventional load transformer


The 1800 kVAr of shunt power factor correction capacitors becomes $18 \%$ on a 10 MVA base.
The final one-line diagram is shown in Figure 6.1.


Figure 6.1. System One-Line Diagram for Five-Bus Example
(Data in PCFLOH files *_FIVE.csv)

### 6.3. Passive Filters

Filters accomplish two objectives - power factor correction, and shunting one or more harmonic currents to ground. A series tuned filter can be constructed in each phase to ground by placing a choke in series with a shunt capacitor, and then tuning the choke so that the inductive and capacitive reactances are equal but opposite at the desired harmonic. Tuning a filter slightly below the desired harmonic, for example at the $4.7^{\text {th }}$ instead of the $5^{\text {th }}$ harmonic, helps to reduce capacitor voltage without significantly degrading filter performance. Often the addition of a $4.7^{\text {th }}$ (i.e., $5^{\text {th }}$ ) filter is sufficient to solve harmonics problems.

Care must taken to dedicate enough kVAr to the filter. In most cases, the filter kVAr should be approximately the amount needed to power factor correct the nonlinear load. Filters with smaller kVAr will have sharp tuning curves and will be easily overloaded by stray harmonics that are present in the network.

Since a filter capacitor usually experiences 1.2 to 1.3 pu rms voltage, plus significant harmonics, care must be taken that the capacitor voltage rating is adequate. The fact that kVArs decrease by the square of voltage must also be taken into consideration.

To illustrate filter design, the five-bus system is modified by converting the 1800 kVAr capacitor bank into a $4.7^{\text {th }}$ harmonic filter. First, a new bus (\#6) is created, and the 1800 kVAr capacitor bank is moved from Bus 4 to Bus 6 . In reality, the 1800 kVAr bank would be replaced with a higher-voltage rated bank, with sufficient kVArs so that it produces 1800 kVAr at system voltage. Then, Bus 4 is connected to Bus 6 with a series choke that has the appropriate reactance.


Figure 6.2. System One-Line Diagram for Five-Bus Example with Filter (Data in PCFLOH files *_FIVE_FILTER.csv)

The tuning formulas for harmonic k are

$$
\begin{aligned}
& \text { Let } \frac{-X_{C}(p u @ 60 \mathrm{~Hz})}{k}=k X_{L}(p u @ 60 \mathrm{~Hz}) \text {, so that } \\
& X_{L}(p u @ 60 \mathrm{~Hz})=\frac{-X_{C}(p u @ 60 \mathrm{~Hz})}{k^{2}} .
\end{aligned}
$$

In this example,

$$
\begin{aligned}
& X_{C}(p u @ 60 \mathrm{~Hz})=\frac{1}{-0.18 p u}=-5.55 p u, \text { so that } \\
& X_{L}(p u @ 60 \mathrm{~Hz})=\frac{5.55 p u}{4.7^{2}}=0.251 p u, \text { or } 25.1 \% .
\end{aligned}
$$

On a $12.5 \mathrm{kV}, 10 \mathrm{MVA}$ base, the choke inductance (each phase of wye connection) is

$$
L=X_{L} \bullet Z_{B A S E} \bullet \frac{1}{2 \pi f}=\frac{0.251 \bullet 15.625}{2 \pi \bullet 60}=10.4 \mathrm{mH}
$$

Assuming

$$
\frac{X_{L}(p u @ 60 H z)}{R}=50
$$

for the choke, the choke resistance is estimated to be $R=\frac{25.1 \%}{50}=0.502 \%$, or $0.0784 \Omega$. The modified one-line diagram is shown in Figure 6.2.

Filter performance is checked using three steps.

1. Impedance scans are performed, without and with the filter. The filter notch should be at the design harmonic.
2. The converter bus voltage waveform, without and with the filter, is examined. $5^{\text {th }}$ harmonic filtering is usually adequate. However, if the voltage distortion is still more than 4 or $5 \%$, it may be necessary to add a larger $5^{\text {th }}$ filter, or possibly $7^{\text {th }}, 11^{\text {th }}$, and $13^{\text {th }}$ filters, in that order. Usually, the higher the harmonic, the fewer kVArs committed to a filter. A good rule for dedicating kVAr when multiple filters are needed is to stairstep the kVAr as follows: if Q kVArs are used for the $5^{\text {th }}$ harmonic, then $\mathrm{Q} / 2$ should be used for the $7^{\text {th }}, \mathrm{Q} / 4$ for the $11^{\text {th }}$, and $\mathrm{Q} / 4$ for the $13^{\text {th }}$. Of course, actual sizes must match standard sizes. The total kVAr should power factor correct the nonlinear load. For best performance, a filter should have at least 300 kVAr (three-phase).
3. The filter current waveform is checked to make sure that it is absorbing the appropriate harmonic and that the filter current is within rating.

Simulation results for the five-bus system, without and with the filter, are given in Figures 6.3 6.5.

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Figure 6.3. Impedance Scans at Converter Bus (Without Filter on Left, With Filter on Right),
(plots produced by program PCFLO_ZBUS_PLOT.XLS)
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Figure 6.4. Converter Bus Voltage Waveforms Bus (Without Filter at Top, With Filter at Bottom), (plots produced by program
PCFLO_VSOLN_PLOT.XLS)

## 7. Standards and Solutions

### 7.1. IEEE 519

The most often quoted harmonics standard in the U.S. is IEEE 519, "Recommended Practices and Requirements for Harmonic Control in Electric Power Systems." IEEE 519 attempts to establish reasonable harmonic goals for electrical systems that contain nonlinear loads. The objective is to propose steady-state harmonic limits that are considered reasonable by both electric utilities and their customers. The underlying philosophy is that

- customers should limit harmonic currents,
- electric utilities should limit harmonic voltages,
- both parties share the responsibility for holding harmonic levels in check.

IEEE 519 applies to all voltage levels, including 120V single-phase residential service. While it does not specifically state the highest-order harmonic to limit, the generally accepted range of application is through the $50^{\text {th }}$ harmonic. Direct current, which is not a harmonic, is also addressed and is prohibited. Since no differentiation is made between single-phase and threephase systems, the recommended limits apply to both.

It is important to remember that IEEE 519 is a recommended practice and not an actual standard or legal document. Rather, it is intended to provide a reasonable framework within which engineeers can address and control harmonic problems. It has been adopted by many electric utilities and by several state public utility commissions.

## Definitions and Terms

THD. Total Harmonic Distortion (or Distortion Factor) of voltage or current is the ratio of the rms value of harmonics above fundamental, divided by the rms value of the fundamental.

PCC. Point of Common Coupling is a point of metering, or any point as long as both the utility and the customer can either access the point for direct measurements of the harmonic indices meaningful to both, or estimate the harmonic indices at the point of interference through mutually agreeable methods. Within an industrial load, the point of common coupling is the point between the nonlinear load and other loads.

There is some flexibility in determining the PCC, but in most instances, it is at the meter. An electric utility might also interpret the PCC to be on the high-voltage side of the service transformer, which would have the effect of allowing a customer to inject higher harmonic currents.

ISC. Maximum short circuit current at the PCC.
IL. Maximum demand load current (fundamental frequency component) at the PCC, calculated as the average current of the maximum demands for each of the preceeding twelve months. For new customers, this value must be estimated.

TDD. Total demand distortion, which is the THD of current (using a 15 or 30 minute averaging measurement period) normalized to the maximum demand load current IL.

## Utility Limits

Electric utilities are responsible for maintaining voltage harmonics and $T H D_{V}$. The limits are divided into two categories: voltages 69 kV and below, and voltages above 69kV. For electric utility distribution systems (i.e., corresponding to 69 kV and below), the limits are
For Voltages 69 kV and Below

| Individual | Total Harmonic |
| :---: | :---: |
| Voltage | Distortion |
| Harmonic | $T H D_{V}$ |
| $\%$ | $\%$ |
| 3.0 | 5.0 |

## Customer Limits

Customers are responsible for maintaining current harmonics and $T H D_{I}$. Again, the limits are divided into two categories: voltages 69 kV and below, and voltages above 69 kV . For 69 kV and below, the limits are

For PCC Voltages 69kV and Below
Maximum $T H D_{I}$ in \% of IL for Odd Harmonics k

| ISC/IL | $\mathrm{k}<11$ | $11 \leq \mathrm{k}<17$ | $17 \leq \mathrm{k}<23$ | $23 \leq \mathrm{k}<35$ | $35 \leq \mathrm{k}$ | TDD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $<20^{*}$ | 4.0 | 2.0 | 1.5 | 0.6 | 0.3 | 5.0 |
| $20-<50$ | 7.0 | 3.5 | 2.5 | 1.0 | 0.5 | 8.0 |
| $50-<100$ | 10.0 | 4.5 | 4.0 | 1.5 | 0.7 | 12.0 |
| $100-<1000$ | 12.0 | 5.5 | 5.0 | 2.0 | 1.0 | 15.0 |
| $\geq 1000$ | 15.0 | 7.0 | 6.0 | 2.5 | 1.4 | 20.0 |

* All power generation equipment is limited to these values of $T H D_{I}$, regardless of the actual ISC/IL.

Even-ordered harmonics are limited to $25 \%$ of the odd harmonic limits given in the tables. Loads that produce direct current offset, e.g. half-wave converters, are not allowed.

### 7.2. Public Utilitity Commission Standards

Several states, including Texas and Oklahoma, have adopted harmonic standards. These standards are based upon IEEE 519. Texas state ruling 25.51, "Power Quality," permits an electric utility to charge a fee for having to investigate and remedy a customer-created excessive
harmonics condition. The fee is limited to actual cost incurred plus a reasonable administrative cost.

### 7.3. Interacting with Customers

It is wise for an electric utility to develop a written document of harmonics policy that can be distributed to large industrial customers as the need arises. While a good basis for the document is IEEE 519, other procedural items should also be addressed. The following key points should be considered for inclusion in the document.

## Modeling

Data are needed to determine whether a proposed customer's facility will cause harmonic limits to be exceeded. These data include

- One-line drawings of the customer's facilities, showing ratings and connections of all electrical equipment,
- Location, connection, size, and control method of capacitors,
- Conductor sizes and impedances,
- Location and type of nonlinear loads,
- Overall plant load and portion that is nonlinear,
- Location, rating, connection, and impedance of transformers.

Customers should be responsible for modeling their systems to project harmonic levels and determine whether the utility's harmonic limits will be exceeded.

The utility should provide information regarding the local power system to support the customer's modeling efforts. This information should include

- Available fault duty at customer’s location,
- Ultimate available fault current,
- Impedance and ratings of service transformers,
- Possible voltage range variation.

Filter modeling should include the utility's background voltage distortion allowed by IEEE 519, which is $3 \%$ for a single harmonic and $5 \% T H D_{V}$. Failure to include this allowed background distortion may result in inadequate filter designs.

The utility may need copies of the customer's harmonic analysis for review prior to approving the customer's proposed facilities. The utility may need the customer to submit manufacturer's documentation and test data demonstrating the harmonic content of nonlinear loads.

## Measurements

The utility should reserve the right to measure the amount of a customer's harmonic current injection at any time at the point of common coupling (normally the electric meter). These measurements are usually spot checks, but additional monitoring may be required.

## Mitigation Devices and Methods

The customer should be responsible for the design, installation, operation, and maintenance of mitigation devices required to meet the utility's harmonic limits. Mitigation devices may include current limiting reactors, passive filters, active filters, or other devices that minimize the flow of harmonic currents onto the utility's distribution system.

The customer should submit mitigation device maintenance records to the utility upon request. The installation and testing of mitigation equipment should be subject to the approval of the utility. The mitigation devices must be capable of handling the IEEE 519 permitted background voltage distortion that can exist on the utility's distribution system.

The utility will likely reconfigure the distribution system regularly in response to load changes and to resolve outages. The mitigation equipment should operate independently of these changes.

### 7.4. Solutions

Solution techniques fall into two broad categories - preventive and remedial.

## Preventive Measures

Preventive measures focus on minimizing the harmonic currents that are injected into power systems. Preventive measures include

- Strict Adherence to IEEE 519.
- Phase Cancellation. The use of twelve-pulse converters instead of six-pulse converters should be encouraged. Most utility harmonic problems are associated with high $5^{\text {th }}$ and $7^{\text {th }}$ harmonic currents, and if they are eliminated through phase cancellation, harmonic problems rarely develop. In situations where there are multiple six-pulse converters, serving half of them (in terms of power) through delta-delta or wye-wye transformers, and the other half through delta-wye or wye-delta transformers, achieves net twelve-pulse operation.
- Encouragement of Low Distorting Loads. Because of IEEE 519, increasing attention is being given to the $T H D_{I}$ of distorting loads. A customer often has a distortion choice in loads. For example, twelve-pulse (or higher) ASDs and low-distortion fluorescent lamp ballasts can be purchased.
- Computer Simulations. It is always better to simulate the impact of a large distorting load before it is ordered and installed. Solutions can be proposed and evaluated "on paper" and perhaps implemented when the load is installed. Once the distorting load is connected, the customer will likely be under considerable pressure to operate it and perhaps less likely to commit additional funds to deal with a distortion problem.


## Remedial Measures

Remedial measures include

- Circuit Detuning. By using only field measurements such as capacitor current waveforms and search coil readings, it is possible to identify the capacitor banks that are most affected by resonance. As a temporary measure to "buy time" before a real solution can be found, the affected capacitor bank can be switched off to see if the resonance problem subsides. Of course, the problem may simply transfer to another capacitor bank, so post-switching measurements at other capacitor banks must be made to see if the temporary solution is satisfactory. If switching a capacitor bank off temporarily solves the problem, computer simulations may be in order to test filtering options and possible re-location of the capacitor bank.
- Passive Filters. These are widely used to control harmonics, especially the $5^{\text {th }}$ and $7^{\text {th }}$ harmonics. Most filters consist of series L and C components that provide a single-tuned notch with a low-impedance ground path. At $50 / 60 \mathrm{~Hz}$, these filters are, for all practical purposes, capacitors. Thus, passive filters provide both power factor correction and voltage distortion control.
$5^{\text {th }}$ harmonic filtering is usually adequate to solve most distribution system harmonic problems. However, in some cases it may be necessary to add $7^{\text {th }}, 11^{\text {th }}$, and $13^{\text {th }}$ harmonic filters, in that order. In general, harmonics may not be "skipped." For example, if the problem harmonic is the $7^{\text {th }}$, both $5^{\text {th }}$ and $7^{\text {th }}$ harmonic filters must be added because the $7^{\text {th }}$ filter alone would aggravate the $5^{\text {th }}$ harmonic voltage. Filters tuned near the $3^{\text {rd }}$ harmonic must be avoided because transformers and machines located throughout distribution feeders are sources of third harmonics, and their currents will easily overwhelm $3^{\text {rd }}$ harmonic filters.

Usually, the higher the harmonic, the fewer kVArs needed for a filter. For multiple filter installations, a good practice is to stairstep the kVAr as follows: if Q kVArs are used for the $5^{\text {th }}$ harmonic, then $\mathrm{Q} / 2$ should be used for the $7^{\text {th }}, \mathrm{Q} / 4$ for the $11^{\text {th }}$, and $\mathrm{Q} / 8$ for the $13^{\text {th }}$. Of course, actual sizes must match standard kVAr sizes. For best performance, a filter should be at least 300 kVAr (three-phase).

It may be possible to add low-voltage filters within the confines of an industrial customer without performing computer simulations, as long as all shunt capacitors in the facility are filtered. However, in a utility distribution system, it is always prudent to perform computer simulations to make sure that a filter does not aggravate the harmonics situation at a remote point. This is especially true if the feeder also has unfiltered capacitors.

Some problems associated with passive filters are that

- their effectiveness diminishes over time as their capacitors age, losing $\mu \mathrm{F}$ and thus raising their notch frequency,
- they attract harmonic currents from all sources in the network - new, known, and unknown, so that they may become overloaded.
- Active Filters. This is a new and promising technology, but there are as yet few distribution feeder installations. Active filters are power electronic converters that inject equal-but-opposite distortion to yield more sinusoidal voltage waveforms throughout a network. Active filters have the advantages of
- time-domain operation so that they automatically "tune" to the problem harmonic or harmonics,
- current limiting capability to prevent overload by new or unknown sources of harmonics on the network,
- multi-point voltage monitoring so that they can simultaneously minimize distortion at local and remote busses.

The performance of mitigation equipment must be verified by extensive monitoring, both before and after commissioning. At least two days of recordings before commissioning, and one week after, should be made to assure that the mitigation equipment is performing as planned. One week of measurements is needed so that the entire weekly load cycle can be observed. Monitoring should include time traces of voltage and current THD, spectra, sample waveforms, power, and harmonic power.

## Economic Justification of Mitigating Measures

From a customer's perspective, the most common economic justification of harmonics mitigation is in minimizing down-time due to nuisance tripping of senstive loads. This cost is totally customer-dependent.

From a loss perspective, harmonics can be considered as a reduction in power factor. In Chapter 3 , true power factor was shown to be

$$
p f_{t r u e} \approx \frac{d p f_{1}}{\sqrt{1+T H D_{I}^{2}}}
$$

Thus, the true power factor of nonlinear loads is limited by $T H D_{I}$. Consider a nonlinear load with perfect displacement power factor $\left(d p f_{1}\right)$. When current distortion is included, the true power factor degrades, as shown in Table 7.1.

Table 7.1. Maximum True Power Factor for Nonlinear Loads

| $T H D_{I}$ <br> $\%$ | Maximum <br> $p f_{\text {true }}$ |
| :---: | :---: |
| 10 | 0.99 |
| 20 | 0.98 |
| 30 | 0.96 |
| 50 | 0.89 |
| 100 | 0.71 |

Since the true power factors given above are for the special case of unity $d p f_{1}$, they represent maximum true power factors for nonlinear loads. "Actual" true power factor is the product of maximum true power factor and displacement power factor, and the product can be significantly lower than $d p f_{1}$.

The power factor comparison presents a rather optimistic picture, because harmonic currents actually cause more losses per ampere than do fundamental currents.

Voltage harmonics have been shown to cause additional losses in motors, especially highefficiency single-phase motors. Voltage harmonics induce harmonic currents that increase motor losses and insulation temperature. Research by Dr. Ewald Fuchs at the University of Colorado at Boulder has shown that voltage distortions in the $6 \%$ range with predominant $3^{\text {rd }}$ and $5^{\text {th }}$ harmonics can reduce the expected lifetime of single-phase motors by $25 \%-30 \%$.

## 8. System Matrices and Simulation Procedures

If a system has more than a few busses, then simulations are too tedious to perform by hand. Thus, a computer-based simulation procedure is needed, and the procedure is usually matrix oriented. This chapter describes the simulation procedure used in PCFLO (and PCFLOH).

### 8.1. Summary of the Procedure Used in PCFLO

We perform harmonic analysis in per unit on a system base. We begin with a conventional loadflow. The loadflow is an excellent debugging tool - if the loadflow does not solve or if the answers seem unreasonable, then the input data are questionable. Once solved, the loadflow establishes the fundamental voltage magnitudes and phase angles through the system. The "swing bus" is assumed to have no distortion, and its fundamental frequency voltage phase angle is always zero. The swing bus voltage angle serves as the phase angle reference for all fundamental and harmonic voltages and currents in the system.

After a successful loadflow, we consider harmonics above the fundamental, one by one, using the standard frequency-domain approach - that is, the principle of current injection in the frequency domain, one harmonic at a time. For each harmonic $k$

- we build the admittance matrix, using the appropriate positive, negative, or zero-sequence network for harmonic k ,
- we use the spectrum and power level of each nonlinear load to determine its $\mathrm{k}^{\text {th }}$ harmonic injection current,
- we simultaneously inject the harmonic k currents for all nonlinear loads into the network,
- we determine bus voltages and branch currents for harmonic k using standard phasor analysis techniques.

After bus voltages and branch currents are determined for all harmonics, the results are combined using Fourier series' to produce time-domain waveforms.

To illustrate the procedure, begin by considering Figure 8.1. Bus m is a "linear bus" with a conventional $P, Q$ linear load represented by impedance $Z_{m}$, linear. For harmonics purposes, in PCFLO we model the linear P load as a resistor to provide harmonics damping, and we ignore the linear Q load. Bus n is a "nonlinear bus" that has a harmonic current injector. The harmonic current injector creates harmonic voltages throughout the power system. Our objective is to determine those harmonic voltages.

There may be many linear and nonlinear busses in a system, and busses can contain both linear loads and nonlinear loads. Mixed busses are easily handled in the solution by simply placing harmonic current injectors in parallel with $Z_{m}$,linear impedance terms.


- For linear loads, $\mathrm{Z}_{\mathrm{m}}$, linear is a resistive load model for the linear P load, in pu at harmonic k .
- At a generator, $\mathrm{Z}_{\mathrm{m}}$,linear is the subtransient impedance, in pu , of the generator.
- At the loadflow swing bus, $Z_{m}$,linear is a very small number, i.e., j 0.0001 pu , so that the voltage at the swing bus remains a perfect, distortion-free sinewave.
- $Z_{m}$,linear may be a parallel combination of loads and subtransient impedances.

Figure 8.1. Illustrative System for Harmonic $k, k>1$

### 8.2. Determining the Magnitudes and Phase Angles of the Harmonic Injection Currents

Once the loadflow is solved, then the local fundamental voltage magnitude and phase angle at bus $\mathrm{n}, V_{n, 1} \angle \delta_{n, 1}$, is known. Nonlinear loads are specified in terms of P , dpf, and spectrum. Define $d_{n, 1}$ as the lagging dpf angle of the nonlinear load. Then the fundamental current magnitude of the nonlinear load in per unit is

$$
\begin{equation*}
\left|I_{n, 1}\right|=\left|\frac{S_{n}}{V_{n, 1}}\right|=\left|\frac{P_{n}}{V_{n, 1} \bullet d p f_{n}}\right| \tag{8.1}
\end{equation*}
$$

and the phase angle of the nonlinear load fundamental current with respect to the swing bus is

$$
\begin{equation*}
\left(\delta_{n, 1}-d_{n, 1}\right) \tag{8.2}
\end{equation*}
$$

Now, we consider the spectrum for the nonlinear load current at harmonic k ,
$I_{\text {spec }, \mathrm{k}}:$ magnitude of harmonic k current, in per unit of fundamental load current, $\theta_{\text {spec }, \mathrm{k}}:$ angle of harmonic k current, with respect to fundamental load current.

The magnitude of the load current for harmonic k is then

$$
\begin{equation*}
\left|I_{n, k}\right|=\left|I_{n, 1}\right| \bullet I_{\text {spec }, k}=\left|\frac{P_{n}}{V_{n, 1} \bullet d p f_{n}}\right| \bullet I_{\text {spec }, k} . \tag{8.3}
\end{equation*}
$$

Time shifting to account for the shift in fundamental phase angle yields the phase angle of the harmonic load current,

$$
\begin{equation*}
\theta_{n, k}=\theta_{\text {spec }, k}+k \bullet\left(\delta_{n, 1}-d_{n, 1}\right) \tag{8.4}
\end{equation*}
$$

Finally, because the harmonic current is typically modeled as an injector (as shown in Figure 8.1) instead of a load, the current magnitude must be negated by adding (or subtracting) $180^{\circ}$ to the phase angle, yielding the final expression for phase angle,

$$
\begin{equation*}
\theta_{n, k}=\theta_{\text {spec }, k}+k \bullet\left(\delta_{n, 1}-d_{n, 1}\right) \pm 180^{\circ} . \tag{8.5}
\end{equation*}
$$

### 8.3. Determining Network Voltages

Network phasor voltages for each harmonic are determined by using Kirchhoff's Current Law (KCL) as follows. Consider the three-bus, five branch, one current-source network shown in Figure 8.2. The objective is to solve for the three bus voltages with respect to ground.


Figure 8.2. Three-Bus Network
For every harmonic, we apply KCL at the three nodes. The equations are
At bus $1, \frac{V_{1}}{Z_{E}}+\frac{V_{1}-V_{2}}{Z_{A}}=0$,
At bus $2, \frac{V_{2}}{Z_{B}}+\frac{V_{2}-V_{1}}{Z_{A}}+\frac{V_{2}-V_{3}}{Z_{C}}=0$,
At bus $3, \frac{V_{3}}{Z_{D}}+\frac{V_{3}-V_{2}}{Z_{C}}=I_{3}$.

Collecting terms and writing the equations in matrix form yields

$$
\left[\begin{array}{ccc}
\frac{1}{Z_{E}}+\frac{1}{Z_{A}} & -\frac{1}{Z_{A}} & 0  \tag{8.6}\\
-\frac{1}{Z_{A}} & \frac{1}{Z_{A}}+\frac{1}{Z_{B}}+\frac{1}{Z_{C}} & -\frac{1}{Z_{C}} \\
0 & -\frac{1}{Z_{C}} & \frac{1}{Z_{C}}+\frac{1}{Z_{D}}
\end{array}\right]\left[\begin{array}{c}
V_{1} \\
V_{2} \\
V_{3}
\end{array}\right]=\left[\begin{array}{c}
0 \\
0 \\
I_{3}
\end{array}\right],
$$

or in abbreviated form,

$$
\begin{equation*}
Y V=I \tag{8.7}
\end{equation*}
$$

where $Y$ is the admittance matrix, $V$ is a vector of phasor bus voltages (with respect to ground), and $I$ is a vector of phasor current injections. Equation (8.7) must be written for every harmonic in the study, and the impedances and current injections vary accordingly.

Voltage sources, if present, can be converted to current sources using the usual Thevenin/Norton conversion rules. If a bus has a zero-impedance voltage source attached to it, then the bus voltage is already known, and the dimension of the problem is reduced by one.

A simple observation of the structure of the above admittance matrix leads to the following rule for building Y:

1. The diagonal terms of $Y$ contain the sum of all branch admittances connected directly to the corresponding bus.
2. The off-diagonal elements of $Y$ contain the negative sum of all branch admittances connected directly between the corresponding busses.

These rules make $Y$ very simple to build using a computer program. For example, assume that the impedance data for the above network has the following form, one data input line per branch:

| From <br> Bus | To <br> Bus | Branch Impedance (Entered <br> as Complex Numbers) |
| :---: | :---: | :---: |
| 1 | 0 | ZE |
| 1 | 2 | ZA |
| 2 | 0 | ZB |
| 2 | 3 | ZC |
| 3 | 0 | ZD |

The following FORTRAN instructions would automatically build $Y$, without the need of manually writing the KCL equations.

```
    COMPLEX Y(3,3),ZB,YB
    DATA Y/9 * 0.0/
1 READ(1,*,END=2) NF,NT,ZB
    YB = 1.0 / ZB
C MODIFY THE DIAGONAL TERMS
    IF(NF .NE. 0) Y(NF,NF) = Y(NF,NF) + YB
    IF(NT .NE. 0) Y(NT,NT) = Y(NT,NT) + YB
    IF(NF .NE. 0 .AND. NT .NE. 0) THEN
C MODIFY THE OFF-DIAGONAL TERMS
    Y(NF,NT) = Y(NF,NT) - YB
    Y(NT,NF) = Y(NT,NF) - YB
    ENDIF
    GO TO 1
2 STOP
    END
```

Of course, error checking is needed in an actual computer program to detect data errors and bus numbers. Also, if bus numbers are not compressed (i.e. bus 1 through bus $N$ ), then additional logic is needed to internally compress the busses, maintaining separate internal and external (i.e., user) bus numbers.

Using harmonic current injections from (8.3) and (8.5), the $\mathrm{k}^{\text {th }}$ harmonic bus voltage phasors for the entire network are found simultaneously using Gaussian elimination and backward substitution on (8.7), or by using direct inversion with

$$
\begin{equation*}
V=Z I \text {, where } Z=Y^{-1} \tag{8.8}
\end{equation*}
$$

Once the bus voltages are known, then branch currents can be easily calculated.

To assist in understanding how $Y$ is built, $Y$ for the five-bus example in Figure 6.1 is given in Table 8.1. For simplicity of presentation here, phase shift due to the substation transformer connection is not included in the table. In reality, PCFLO handles transformer phase shift with a complex transformer tap. Phase shift can also be taken into account by a simple bookkeeping procedure where the phasor voltages are advanced or delayed accordingly, after a solution is obtained.

Also, for simplicity of presentation here, the branch resistances in the table are shown to be independent of harmonic k. In reality, one of the data inputs to PCFLO is for the user to specify the harmonic for which a resistance doubles - with that information, PCFLO varies branch resistance as a function of $\sqrt{k}$.

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Table 8.1. Admittance Matrix for the Five-Bus Example in Figure 6.1 (in per unit) for Harmonic k

| 1 | $\begin{aligned} & \frac{1}{j k 0.0001}+j k \frac{0.65}{2} \\ & +\frac{1}{0.0004+j k 0.0025} \end{aligned}$ | $\frac{-1}{0.0004+j k 0.0025}$ | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $\frac{-1}{0.0004+j k 0.0025}$ | $\begin{gathered} \frac{1}{0.0004+j k 0.0025} \\ +\frac{1}{0.0033+j k 0.07} \\ +j k \frac{0.65}{2} \end{gathered}$ | $\frac{-1}{0.0033+j k 0.07}$ | 0 | 0 |
| 3 | 0 | $\frac{-1}{0.0033+j k 0.07}$ | $\begin{gathered} \frac{1}{0.0033+j k 0.07} \\ +\frac{1}{0.0406+j k 0.1196} \\ +j k \frac{0.000315}{2} \end{gathered}$ | $\frac{-1}{0.0406+j k 0.1196}$ | 0 |
| 4 | 0 | 0 | $\frac{-1}{0.0406+j k 0.1196}$ | $\begin{gathered} \frac{1}{0.0406+j k 0.1196} \\ +\frac{1}{0.0067+j k 0.0667} \\ +j k \frac{0.000315}{2}+j k 0.18 \end{gathered}$ | $\frac{-1}{0.0067+j k 0.0667}$ |
| 5 | 0 | 0 | 0 | $\frac{-1}{0.0067+j k 0.0667}$ | $\begin{gathered} \hline \frac{1}{0.0067+j k 0.0667} \\ +0.50 \end{gathered}$ |

### 8.4. Physical Significance of the Impedance Matrix

Impedance matrix $Z$, evaluated for the fundamental frequency, is the key to short circuit studies. Individual $Z$ matrices for each harmonic are important to harmonic studies. For harmonic studies, the diagonal elements of $Z$ are the Thevenin-equivalent impedances at the system busses, as we will see below.

Individual elements of $Z$ describe how the voltage at a bus is related to the current injection at that bus or at other busses. To see this, examine the $m^{t h}$ row of $V=Z I$, which is

$$
\begin{equation*}
V_{m}=\sum_{n=1}^{N} z_{m, n} I_{n}=z_{m, 1} I_{1}+z_{m, 2} I_{2}+z_{m, 3} I_{3}+\cdots+z_{m, N} I_{N} \tag{8.9}
\end{equation*}
$$

If all current sources are "off" except at bus $n$, then

$$
\begin{equation*}
V_{m}=z_{m, n} I_{n}, \text { where } I_{j}=0 \text { for } j=1,2, \cdots, N, j \neq n . \tag{8.10}
\end{equation*}
$$

The situation is illustrated in Figure 8.3, where $I_{n, k}$ is a current source attached to bus $n, V_{m, k}$ is the resulting voltage at bus $m$, and all other harmonic current sources are turned off. When $m=n, z_{n, n}$ is by definition the Thevenin equivalent impedance at bus $n$ for harmonic k . When an "impedance scan" is requested for a bus (typically a nonlinear bus), PCFLO computes the Thevenin equivalent impedance for that bus in steps of 0.2 of a harmonic (i.e., integer and noninteger harmonics $1.0,1.2,1.4, \ldots, 24.6,24.8,25.0)$.


Figure 8.3. Illustrative System for Harmonic $\mathrm{k}, \mathrm{k}>1$, with All Sources Off Except at Bus n

For interconnected networks (as is the case for positive and negative-sequences), current injection at one bus affects the voltage at every bus in the network. For that reason, $Z$ is a full matrix for positive and negative-sequences. On the other hand, zero-sequence networks are often not interconnected. For example, zero-sequence voltages and currents cannot pass through a transformer with a delta winding. Thus, zero-sequence $Z$ matrices are often sparse.

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### 8.5. Creating Time-Domain Waveforms from the Phasor Solution

Once the harmonic phasors are known for a voltage or current, the corresponding time-domain waveform is easily created. Consider bus m with phasor voltages

| Harmonic | Phasor Voltage |
| :---: | :---: |
| 1 | $\mathrm{~V}_{\mathrm{m}, 1} \underline{\delta}_{\mathrm{m}, 1}$ |
| 5 | $\mathrm{~V}_{\mathrm{m}, 5} \underline{\delta}_{\mathrm{m}, 5}$ |
| 7 | $\mathrm{~V}_{\mathrm{m}, 7} \underline{\delta}_{\mathrm{m}, 7}$ |
| $\ldots$ | $\ldots$ |

If the spectra and phasor references are sine series', then the time domain waveform is

$$
\begin{equation*}
v_{m}(t)=V_{m, 1} \sin \left(\omega_{1} t+\delta_{m, 1}\right)+V_{m, 5} \sin \left(5 \omega_{1} t+\delta_{m, 5}\right)+V_{m, 7} \sin \left(7 \omega_{1} t+\delta_{m, 7}\right)+\cdots \tag{8.11}
\end{equation*}
$$

### 8.6. Comments on Zero-Sequence Networks

Zero-sequence networks are affected by grounding. Ungrounded shunt elements, loads, and capacitors are "invisible" to zero sequence and are not included in the admittance matrix. Delta connections are open circuits to zero-sequence.

Transformer connections are very important in zero-sequence networks. Figure 8.4 is shown below as a reminder of the zero-sequence models for transformers.

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Grounded Wye - Delta


Figure 8.4. Zero Sequence Impedance Equivalent Circuits for Three-Phase Transformers

## 9. Case Studies

Four actual case studies are presented. They are
Case 1. 5000 HP Chiller Motor ASD on 25 kV System. This case has severe voltage resonance with nuisance tripping of sensitive loads and overheating of the ASD transformers. Simulations compare favorably with field measurements.

Case 2. 2000HP Oil Pipeline Pumping Station ASD on 12.5 kV System. There are no serious harmonics problems. Simulations compare favorably with field measurements.

Case 3. Television Broadcast Station on 25 kV System. There is a serious problem where the broadcast picture wobbles due to interaction of a constant-voltage transformer with a distribution system that was upgraded from 12.5 kV to 25 kV . Simulations support a quickly-implemented field solution.

Case 4. 12.5 kV Ski Area with 5150 HP of Distributed Lift Motor ASDs. This is a planning study where simulations are used to design a harmonics mitigation strategy. This case illustrates the effectiveness of phase cancellation and passive filters.

## Case 1. $\mathbf{5 0 0 0 H P}$ Chiller Motor ASD on 25kV System

Overview. This is a serious resonance case where a 3.9 MW six-pulse line commutated ASD driving a chiller motor excites resonance in the 25 kV underground distribution system shown in Figure 9.1. The ASD is served by feeder 2203, but the resonance problem is intense throughout the entire subsystem served by S.W. Substation Transformer \#1 (i.e., feeders 2202, 2203, and 2204). The subsystem load at the time harmonics measurements are taken is $9.5 \mathrm{MW} @ d p f=$ 0.855 , consisting of 5.6 MW of conventional load plus the ASD that draws 3.9MW @ dpf = 0.830 . The subsystem contains 16.27 miles of three-phase underground trunk and lateral cables.

The voltage and current waveforms at the ASD were shown previously in Figure 5.1 of Chapter 5 , and the voltage distortion throughout the 25 kV subsystem is approximately $10 \%$. Voltage distortion is highest at night, when the chiller is running at full power and the conventional load (i.e., harmonics damping load) is at low power. The high voltage distortion regularly trips off a computer-controlled automated train system in the late night hours. Also, there is concern that the resonant overvoltages will cause cable dielectric failure.

The chiller is centrally located so that it can be easily switched from feeder 2203 to feeder 2606, which is served by the N.W. substation. The switching arrangement is shown in Figure 9.2. Unfortunately, field switching of the ASD from the preferred feeder (2203) to the secondary feeder (2606) did not improve the harmonics situation and simply transferred the problem from one subsystem to the other.


Figure 9.1. 25 kV System Serving 5000HP Chiller Motor ASD


Figure 9.2. Local Connections for 5000HP Chiller Motor ASD

Two parallel 3750 kVA transformers, totaling 7500 kVA , serve the ASD. Even though the 5000 HP motor draws less than 5000 kVA , the parallel transformers overheat. Thus, Case 1 contains the following three classic symptoms of a harmonics problem:

1. Resonance.
2. Nuisance tripping of sensitive loads.
3. Overheating of transformers that cannot be explained by kVA load alone.

By making voltage distortion measurements at the substation while the ASD is turned off and on, electric utility engineers confirm that the ASD is the source of the harmonics problem.

The unusual feature of this case is that there is a resonance problem even though no power factor correction capacitors are installed. Capacitors are not needed because the underground cables provide considerable power factor correction, especially during low-load periods.

To examine the impact of cables, consider the perfect coaxial case, where each meter has capacitance

$$
C=\frac{2 \pi \varepsilon_{o} \varepsilon_{r}}{\ln \left(\frac{r_{o}}{r_{i}}\right)} \approx \frac{2 \pi \bullet 8.854 \bullet 2.25}{1}=125 \mathrm{pF} / \mathrm{meter} / \text { phase }(\text { or } 0.125 \mu \mathrm{~F} / \mathrm{km} / \mathrm{phase})
$$

and where $\varepsilon_{o}$ is the permittivity of free space, $\varepsilon_{r}$ is the relative permittivity of the cable dielectric, and $r_{i}, r_{o}$ are the coaxial inner and outer radii of each phase. For three phases, the corresponding kVA at 25 kV is

$$
Q_{3 \phi}=3 V_{L N}^{2} \omega C=3 \bullet\left(\frac{25000}{\sqrt{3}}\right)^{2} \bullet 120 \pi \bullet 125 \bullet 10^{-12}=29.5 \operatorname{VAr}(\text { three-phase) } / \text { meter, or }
$$

29.5 kVAr (three phase)/km.

The net three-phase cable charging of the 16.27 miles of three-phase cables is then 0.772 MVAr .
However, a review of the cable manufacturer's data for this particular case gives the following values per meter for the three types of 25 kV cables:

Table 9.1. Manufacturer-Supplied Electrical Characteristics of 25 kV Underground Cables

| 25 kV Cable | $\mathrm{R}+$ | $\mathrm{X}+$ | C | kVAr |
| :---: | :---: | :---: | :---: | :---: |
| $1 / 0 \mathrm{Al}$ | 0.696 | 0.1581 | 0.161 | 37.9 |
| $4 / 0 \mathrm{Al}$ | 0.352 | 0.1447 | 0.202 | 47.6 |
| 1000 kcmil Al | 0.0827 | 0.1148 | 0.354 | 83.4 |

$\mathrm{R}+, \mathrm{X}+$ : $\quad$ Positive sequence resistance and inductive reactance, $\Omega$ per km
C: $\quad$ Capacitance, $\mu \mathrm{F}$ per km per phase
kVAr: Cable kVAr (three-phase) per km

Thus, the actual cable capacitance, which takes into account actual geometries, is considerably higher than that of the ideal coaxial formula.

It is important to note that the above range of $\mathrm{C}, 0.161-0.354 \mu \mathrm{~F} / \mathrm{km} /$ phase, is very large compared to the approximately $0.01 \mu \mathrm{~F} / \mathrm{km} /$ phase for overhead distribution feeders. The capacitance of overhead distribution feeders can usually be ignored in harmonics studies, especially if the feeders have power factor correction capacitors.

The subsystem contains 6.37 miles of $1000 \mathrm{kcmil} \mathrm{Al}, 5.40$ miles of $1 / 0 \mathrm{Al}$, and 4.50 miles of $4 / 0$ Al cables. Thus, the total cable charging is 1.53 MVAr . Based on the substation P and $d p f$, the P and $d p f$ of the converter, and the 1.53 MVAr of cable charging, the $d p f$ of the conventional load is estimated to be 0.770 (ignoring reactive power losses).

Simulations. (PCFLOH Files *_DFW.csv). Both the S.W. and N.W. systems are measured and simulated, and the comparisons for both systems match quite well. For brevity, only the results for the S.W. system are described here.

A study of the feeder blueprints shows there are 29 major load and circuit branch busses, plus the ASD bus. This set of 30 busses becomes the "retained" load busses for the study. Loads and cable charging for non-retained busses are lumped onto the nearest retained load bus. Retained busses are connected with line segments having the per meter characteristics shown in Table 9.1. The 5.60MW, 4.67MVAr of conventional load is distributed over the retained load busses (excluding the ASD) in proportion to net load transformer rating.

The ASD bus is connected to the feeder by the parallel 3750 kVA transformers. Each transformer has impedance $0.79+\mathrm{j} 5.69 \%$ on its own base, and has connection type high-side grounded-wye, low-side delta. The impedance of S.W. Transformer\#1 is $1.803+\mathrm{j} 40.8 \%$ on a 100MVA base, and the connection is high-side delta, low-side grounded-wye. The short circuit impedance on the 138 kV side of S.W. Transformer\#1 is $0.277+\mathrm{j} 1.588 \%$ on a 100 MVA base. The 138 kV bus is assumed to have voltage 1.025 pu . One-half of the line charging of the 138 kV lines connected to the substation bus is 4.97 MVAr .

Other nonlinear loads in the subsystem are ignored. For harmonics purposes, conventional loads are modeled as resistive elements, sized according to their active power. The ASD injection current waveform employed is that shown in Section 5, Figure 5.1.

An impedance scan (i.e., Thevenin equivalent impedance) at the 25 kV converter bus, shown in Figure 9.3 as "No KVAr added," predicts strong parallel resonance at the $12^{\text {th }}$ harmonic.


Figure 9.3. Thevenin Equivalent Impedance vs. Frequency at 25 kV Converter Bus, with No kVAr Added, and with 1800kVAr Added

The corresponding simulation results show voltage distortions in the 25 kV subsystem to be in the narrow range of $9.1-9.7 \%$. Simulated voltage distortion at the 4160 V ASD bus, behind the additional impedance of the parallel $25 \mathrm{kV} / 4160 \mathrm{~V}$ transformers, is $14.9 \%$.

A comparison with measurements at the S.W. \#1 25 kV substation bus is given in Tables 9.2 and 9.3. Simulations and field measurements match reasonably well.

Table 9.2. Measured and Simulated Harmonic Voltages at S.W. \#1 25kV Substation Bus

|  | No kVAr <br> Added. <br> Measured V <br> $\%$ | No kVAr <br> Added. <br> Simulated V <br> $\%$ | 1800 kVAr <br> Added. <br> Measured V <br> $\%$ | 1800 kVAr <br> Added. <br> Simulated V <br> $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 2.6 | 3.1 | 3.2 | 4.0 |
| 7 | 1.7 | 2.1 | 3.0 | 4.6 |
| 11 | 5.8 | 6.1 | 3.1 | 2.4 |
| 13 | 7.2 | 5.3 | 1.3 | 1.2 |
| 17 | 2.0 | 2.1 | 0.6 | 0.7 |
| $T H D_{V}$ | 9.9 | 9.1 | 5.6 | 6.7 |

Table 9.3. Measured and Simulated Harmonic Currents Through S.W. \#1 25kV Substation Transformer

|  | No kVAr <br> Added. <br> Measured I <br> $\%$ | No kVAr <br> Added. <br> Simulated I <br> $\%$ | 1800 kVAr <br> Added. <br> Measured I <br> $\%$ | 1800 kVAr <br> Added. <br> Simulated I <br> $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 9.3 | 12.7 | 13.0 | 18.5 |
| 7 | 5.0 | 6.2 | 9.7 | 15.0 |
| 11 | 12.0 | 11.4 | 5.9 | 4.9 |
| 13 | 11.0 | 8.4 | 2.0 | 2.2 |
| 17 | 2.6 | 2.5 | 0.8 | 0.9 |
| $T H D_{I}$ | 19.6 | 20.1 | 17.4 | 24.4 |

In an attempt to reduce the resonance problem by detuning the feeder, 1800 kVAr of capacitors are switched on at the converter 25 kV bus. The simulated resonant curve shifts, as shown in Figure 9.3, and the predicted voltage distortions across the 25 kV system lower somewhat to the 6.7-7.6\% range. Individual harmonics at the S.W. \#1 25 kV substation bus are shown in Tables 9.2 and 9.3. Unfortunately, detuning provides only a partial solution because the $5^{\text {th }}$ and $7^{\text {th }}$ harmonic injection currents are strong, and moving the resonant curve too close to them will only make the situation worse.

The major uncertainties in this modeling effort are

1. cumulative effect of other, smaller nonlinear loads that are not included in the study,
2. conventional load level, distribution, and model (for harmonics purposes),
3. cable capacitance.

## Case 2. 2000HP Oil Pipeline Pumping Station ASD on 12.5kV System

Overview. A 2000 HP six-pulse line-commutated ASD is connected to Bus 5 on the 12.5 kV distribution system shown in Figure 9.4. The three-phase trunk portions of the two feeders connected to the substation transformer are entirely overhead construction and consist of

- 1.87 miles of 477 ACSR armless construction,
- 2.02 miles of $4 / 0 \mathrm{ACSR}$ armless construction,
- 4.90 miles of $4 / 0$ ACSR arm construction,
- 0.57 miles of $1 / 0$ ACSR arm construction.

Harmonic measurements are taken on a hot summer day. The transformer load is 10.08 MW , 5.47 MVAr (i.e., $d p f=0.879$ ), which includes 1.5 MW ASD with assumed $d p f=0.83$ (i.e., 1.05MVAr). Subtracting the ASD load from the transformer load leaves $8.58 \mathrm{MW}, 4.42 \mathrm{MVAr}$ for the conventional load plus capacitors. The uncorrected power factor of the conventional load is estimated to be 0.800 , or 6.44 MVAr . Thus, there are likely about $6.44-4.42=2.02 \mathrm{MVAr}$ of shunt capacitors (plus enough to overcome line and transformer reactive power losses) in operation when the measurements are taken.

The two feeders have nine shunt capacitor banks, totaling 7.95MVAr. 5.10MVAr of the capacitors are time-controlled or time-temperature controlled, and 2.85 MVAr are regulated by either voltage, current, or power factor. At the time of the measurements, time-controlled and time-temperature controlled capacitors are supposed to be on, and the regulated capacitors are most likely off due to good voltage and power factor levels. For purposes of comparing simulations to measurements, it is assumed that all of the 5.10MVAr of time and timetemperature capacitors are on-line. In relation to Figure 9.4, these are

- 600 kVAr at Bus 3
- 600 kVAr at Bus 4
- 600 kVAr at Bus 7
- 600 kVAr at Bus 20
- 1200kVAr at Bus 21
- 600 kVAr at Bus 24
- 900 kVAr at Bus 25

Simulations. (PCFLOH Files *_WGAR.csv). The connected load transformer kVA information is not readily available. As an approximation, it is assumed that the 8.58 MW , 4.42MVAr of conventional load is uniformly distributed over all 12.5 kV busses except the substation transformer buss. The conventional loads are modeled for harmonic purposes as shunt resistors. Single-phase laterals, underground cable segments from distribution poles to service transformers, and overhead line capacitances are ignored. The ASD is modeled as a $1 / \mathrm{k}$-rule harmonics injector.


Figure 9.4. 12.5 kV System Serving 2000HP Pipeline ASD

A 10MVA base is chosen. The positive/negative sequence Thevenin equivalent of the 138 kV system (excluding the substation transformer) is $0.05+\mathrm{j} 0.344 \%$, with voltage 1.04 pu . A shunt capacitance equal to one-half of the combined line charging of the 138 kV transmission lines connected to the transformer (i.e., $24 \%$ on 10MVA base) is placed as a shunt element on the transformer 138 kV bus. The substation transformer has positive/negative sequence impedance $0.312+\mathrm{j} 6.75 \%$.

Voltage distortion measurements are taken at the substation 12.5 kV transformer bus and at the ASD. A comparison of measurements and simulations is given in Table 9.4

Table 9.4. Measured and Simulated Voltage Distortion Levels in Oil Pipeline Distribution Feeders

| Measured at | Simulated at | Measured at | Simulated at |
| :---: | :---: | :---: | :---: |
| ASD | ASD | Substation | Substation |
| Bus | Bus | Transformer | Transformer |
| $T H D_{V}-\%$ | $T H D_{V}-\%$ | $T H D_{V}-\%$ | $T H D_{V}-\%$ |
| 3.3 | 3.6 | 2.1 | 1.9 |

The highest simulated $T H D_{V}$ for this case is $4.0 \%$ at Busses 8 and 26. It is interesting to note that Bus 26 is the most distant bus from the converter and is on an adjacent feeder served by the same substation tranformer.

Two other cases are simulated. When all capacitors are on, the highest $T H D_{V}$ is $4.3 \%$ at Bus 7 . When only the regulated capacitors are on, the highest $T H D_{V}$ is $5.7 \%$ at the ASD bus. Hence, when a feeder has switched shunt capacitors,

- a reasonable set of capacitor scenarios should be examined (e.g., all capacitors on; only fixed capacitors on, etc.)
- the highest voltage distortion may be at a remote point from the harmonics source,
- the highest distortion case may be a situation when only a subset of the capacitors are online.

Since the voltage distortions are not objectionable, no additional work is needed in this case.

## Case 3. Television Broadcast Station on 25kV System

Overview. The policy of this electric utility is to avoid 25 kV delta, grounded-wye transformers because of potential ferroresonance problems. When a metropolitian area overhead distribution system is upgraded from 12.5 kV to 25 kV , all delta, grounded-wye service transformers are replaced with grounded-wye, grounded-wye transformers.

In this case, the voltage upgrade and transformer replacement coincides with the appearance of an annoying flicker on the broadcast signal of a television station. The problem is traced to fluctuations on the output of a $480 \mathrm{~V} / 480 \mathrm{~V}$ saturable reactor transformer that is supposed to maintain constant $480 \mathrm{~V}( \pm 1 \%)$ voltage to the transmitter rectifier. Measurements show that the neutral current on the primary of the saturable transformer rises from 30A before the upgrade to 325 A after the upgrade, and that the neutral current is primarily $3^{\text {rd }}$ harmonic. Furthermore, the primary phase current rises to 500 A , while the load current remains 380 A . The saturable transformer hums loudly after the upgrade.

Utility engineers suspect that the problem is harmonics-related. Several capacitor banks on the feeder are switched off, and the situation improves but does not disappear. In an attempt to solve the problem quickly, a special-ordered $25 \mathrm{kV} / 480 \mathrm{~V}$ delta, grounded-wye service transformer is installed in place of the new grounded-wye, grounded-wye transformer, and the problem disappears. Thus, it is speculated that the problem is due to harmonics, and specifically to the zero-sequence $3^{\text {rd }}$ harmonic.

Simulations. (PCFLOH files *_CHIL.csv). At this point, simulations are performed to confirm that the "quick fix" transformer replacement can be explained. While there is no easy way to determine how the control system of the saturable transformer reacts with $3^{\text {rd }}$ harmonic voltages, it is relatively easy to show with simulations that the Thevenin impedance of the 480 V bus is

- not affected by the 25 kV upgrade, but
- greatly affected by the replacement of the service transformer.

With the delta, grounded-wye transformer, the impedance "seen" by the saturable transformer is simply the grounded impedance of the $25 \mathrm{kV} / 480 \mathrm{~V}$ service transformer. However, with the grounded-wye, grounded-wye service transformer, the impedance is the service transformer impedance plus system impedance.

The study system has three feeders served by a substation transformer. The total load is $17.8 \mathrm{MW}, 11.0 \mathrm{MVAr}, \mathrm{dpf}=0.85$. There are 15 power factor correction capacitor banks, totaling 9.3MVAr. The Thevenin equivalent impedance on the 138 kV side of the substation transformer is $0.0376+\mathrm{j} 0.2547 \%$ for positive/negative sequence, and $0.094+\mathrm{j} 0.6368 \%$ for zero sequence, on a 10MVA base. The transformer impedance is $0.354+\mathrm{j} 6.92 \%$ on a 10MVA base, and it is connected delta on the 138 kV side, and grounded-wye on the 25 kV side. The retained 25 kV system has 109 busses. One-half of the combined line charging on the 138 kV side of the substation transformer is 13.5 MVAr .

Plots of the positive/negative and zero sequence impedances at the television 25 kV bus are shown in Figure 9.5. It is clear that the 25 kV system has parallel resonance for zero sequence near the $3^{\text {rd }}$ harmonic. With the delta, grounded-wye service transformer, the saturable transformer does not "see" this $3{ }^{\text {rd }}$ harmonic resonance, but rather the relatively low impedance of the transformer to ground.


Figure 9.5. Thevenin Equivalent Impedance vs. Frequency at 25 kV Television Station Bus

Summarizing, while the control instability problem is not explained, the simulations do confirm that $3^{\text {rd }}$ harmonic resonance coincides with the appearance of the problem, and that by using a delta, grounded-wye transformer, the situation can be returned to normal.

## Case 4. $\mathbf{1 2} .5 \mathrm{kV}$ Ski Area with 5150HP of Distributed Lift Motor ASDs

Overview. This case deals with the proposed expansion of a ski area. The 12.5 kV underground system will eventually have eight ski lifts powered by DC motor drives, totaling 5150 HP . Total load (linear plus nonlinear) will be about 9MW. The DC motors will be driven by six-pulse linecommutated ASDs so that the lifts will have soft-start, soft-stop operation. Measurements of the proposed system are, or course, not possible. Thus, the harmonics situation must be analyzed in advance using simulations.

Simulations. (PCFLOH Files *_SKIA.csv, *_SKIB.csv, *_SKIC.csv). A diagram of the ski area is shown in Figure 9.6. In addition to the ASD loads, the ski area has 6 MVA of linear load. The ASDs are modeled using the $1 / \mathrm{k}$ rule for harmonics through the $25^{\text {th }}$, with no phase angle diversity. The $d p f s$ of the ASDs and linear load are assumed to be 0.85 . Cable capacitance is taken from Table 9.5.

Table 9.5. Capacitance and Charging of 12.47 kV Cables

| Cable | Capacitance | kVAr |
| :---: | :---: | :---: |
| $1 / 0$ | 0.163 | 9.56 |
| $4 / 0$ | 0.222 | 13.02 |
| 350 kcmil | 0.267 | 15.65 |
| 500 kcmil | 0.304 | 17.82 |
| 1000 kcmil | 0.401 | 23.5 |

$$
\text { Capacitance: } \mu \mathrm{F} \text { per km per phase }
$$ kVAr: (three-phase) per km

The point of common coupling (PCC) is Bus \#20, Substation $138 \mathrm{kV} . \mathrm{I}_{\mathrm{Sc}}$ and $\mathrm{I}_{\text {load }}$ at the PCC are 34.4 pu and 1.080 pu , respectively, on a 10MVA base. Twelve-month average $\mathrm{I}_{\text {load }}$ is estimated to be $0.75 \cdot 1.080=0.810 \mathrm{pu}$, so $\mathrm{I}_{\mathrm{Sc}} / \mathrm{I}_{\text {load }}$ at the PCC is 42.5 , and the corresponding IEEE 519 limit for TDD of current is $8.0 \%$.

The three cases studied are

## Case SKIA. No Corrections.

Case SKIB. $\quad 30^{\circ}$ phase shifting transformers added at Apollo and BigBoss ASDs.
Case SKIC. Case SKIB, plus 1800 kVAr of filters.
Bracketed values in Figure 9.6 give solved $T H D_{V}$ s for [Case SKIA, Case SKIB, Case SKIC], except at the substation transformer, where $T H D_{I}$ is given directly under Z .

The results for Case SKIA are shown in Figures 9.7 - 9.11. The highest voltage distortion is an unacceptable $14.1 \%$ at Bus \#12, Apollo.

For Case SKIB, wye-delta transformers are added at approximately one-half of the ASD HP, so that a net twelve-pulse operation for the entire ski area is approximated. Results are shown in Figures 9.12-9.14. The highest voltage distortion reduces to 9.6\% at Bus \#12, Apollo.

Case SKIC builds upon Case SKIB by adding the following passive filters:

- 300 kVAr of $5^{\text {th }}$ at Bus\#6, Base.
- 300 kVAr of $5^{\text {th }}$ at Bus\#10, Taylor.
- 300 kVAr of $7^{\text {th }}$ at Bus\#10, Taylor.
- 300 kVAr of $11^{\text {th }}$ at Bus\#12, Apollo.
- 300 kVAr of $11^{\text {th }}$ at Bus\#15, BigBoss.
- 300 kVAr of $13^{\text {th }}$ at Bus\#13, Jupiter.

Filter $\mathrm{X} / \mathrm{R}$ equals 50 . The $5^{\text {th }}$ and $7^{\text {th }}$ harmonics have only one-half of the dedicated kVArs because the two wye-delta transformers have already reduced $5^{\text {th }}$ and $7^{\text {th }}$ harmonic voltages. Some $5^{\text {th }}$ and $7^{\text {th }}$ filtering is still needed in case one or both of the wye-delta transformers are out of service (simulations for this contingency were made but are not presented here).

Results for Case SKIC are shown in Figures 9.15-9.18. The highest feeder voltage distortion level falls to $2.9 \%$, occurring at Bus \#11, Longs.

A side benefit of the filters is that they correct the ski area power factor from 0.83 to 0.91 , thus providing both a harmonics and power factor solution.


Figure 9.6: Case 4, Ski Area
Mack Grady, Page 9-15


Figure 9.7. Case SKIA Interface Screen
Note - Solved $T H D_{V}$ s are shown. The highest $T H D_{V}, 14.1 \%$ at Bus \#12, Apollo, is identified by the asterisk.

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| Harm k, Vkmag,Vkang (Sine) |
| :--- |
| 1, 12, 100.0, -33.3 <br> 5, 12, 5.4, 105.4 <br> 7, 12, 5.4, -29.8 <br> 11, 12 5.4, -122.8 <br> 13, 12 5.4, 99.7 <br> 17, 12, 5.2, 3.0 <br> 19, 12, 4.9, -135.6 <br> 23, 12, 4.2, 127.8 <br> 25, 12, 3.8, -9.7 |

Figure 9.8. Case SKIA, Voltage Waveform and Spectrum at Bus \#12, Apollo $\left(T H D_{V}=14.1 \%\right)$


| Harm k, Ikmag, Ikang (Sine] |  |  |
| :---: | :---: | :---: |
| 1. 12. | 0,0 , | 100.0, -65.1 |
| 5. 12. | 0, 0, | 20.0, -145.6 |
| 7. 12, | 0,0 , | 14.3, 84.2 |
| 11. 12. | $0,0$. | 9.1, 3.8 |
| 13. 12. | 0, 0, | 7.7. -126.5 |
| 17. 12. | 0.0 , | $5.9,153.1$ |
| 19. 12. | 0,0 , | 5.3, 22.9 |
| 23. 12. | 0, 0, | 4.3, -57.6 |
| 25. 12. | 0, 0, | 4.0, 172.2 |

Figure 9.9. Case SKIA, Current Injection Waveform and Spectrum at Bus \#12, Apollo ( $T H D_{I}=$ 29.0\%)



Figure 9.10. Case SKIA, Voltage Waveform and Spectrum at Bus \#1, Substation 12.5 kV

$$
\left(T H D_{V}=12.2 \%\right)
$$


Harm k,lkmag,lkang (Sine)

| 1, | 20, | 1, | 1, | 100.0, |
| :---: | :---: | :---: | :---: | :---: |
| 5, | 20, | 1, | 1, | 84.3, |
| 7, | 20, | 1, | 1, | 68.9 |
| 11, | 20, | 1, | 1, | 42.1, |
| 13, | 20, | 1, | 1, | 3.4, |
| 17, | 20, | 1, | 1, | 2.5, |
| 19.6 |  |  |  |  |
| 19, | 20, | 1, | 1, | 2.2, |
| 23, | 20, | 1, | 1, | 1.6, |
| 25, | 20, | 1, | 1, | 1.3, |

Figure 9.11. Case SKIA, Substation Transformer Current Waveform and Spectrum on 138 kV Side $\left(T H D_{I}=12.6 \%\right)$

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| Bus, V1mag, THDV |  |
| :---: | :---: |
| 1.Sub 12.47kV | $7 \mathrm{kV}, 100.1 .8 .3$ |
| 2,Near Sub S. . | S. , 100.0, 8.4 |
| 3 Near Sub N. | N. , 100.0, 8.4 |
| 4,PBS . 9 | 99.7. 8.5 |
| $5 . \mathrm{PBN}$, 9 | - 99.7. 8.5 |
| 6,Base . 9 | 99.5, 8.7 |
| 7.Star - 99 | - 99.1, 9.0 |
| 8,Wilderness. | , 98.9, 9.2 |
| 9,Dorsey | 99.0, 9.1 |
| 10, Taylor . | 99.0, 9.1 |
| 11,Longs . | - 98.6. 9.2 |
| 12Apollo . | 98.4, 9.6 |
| 13, Jupiter . | 98.7, 9.4 |
| 14, WipeOut . | , 98.7. 9.3 |
| 15,BigBoss | 98.5, 9.5 |
| 16,Shop | 99.1, 9.1 |
| 20,Sub 138kV | kV , 102.0, 2.4 |

Figure 9.12. Case SKIB, $T H D_{V}$


Figure 9.13. Case SKIB, Voltage Waveform and Spectrum at Bus \#6, Base $\left(T H D_{V}=8.7 \%\right)$

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Figure 9.14. Case SKIC, $T H D_{V}$


Figure 9.15. Case SKIC, Voltage Waveform and Spectrum at Bus \#6, Base $\left(T H D_{V}=2.7 \%\right)$

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Figure 9.15. Case SKIC, Substation Transformer Current Waveform and Spectrum on 138kV Side $\left(T H D_{I}=1.2 \%\right)$

## Problem 1 - Partial Harmonic Current Cancellation Due to Variations in DPF.

Two identical six-pulse phase adjustable-speed motor drives operate side by side. Each draws 500 kVA from a 480 V bus. One drive has DPF $=0.85$, and the other has DPF $=0.75$. Assume that

- the two drive current waveforms have the same standard six-pulse waveform, except that one lags the other in time according to their DPFs.
- the $5^{\text {th }}$ harmonic current for each drive is $20 \%$ of fundamental

What is the net $5^{\text {th }}$ harmonic current drawn by the pair, in rms amperes?

## Problem 2 - Estimating the Filter kVArs Needed to Correct a Seriously Distorted Distribution System.

Highly-distorted distribution systems, such as ski areas with large six-pulse adjustable-speed motor drives, require significant filtering. Conventional shunt capacitors must be avoided - they must be used only as filters. A good "starting point" for developing a filter plan is to:
a. power factor correct the distribution system DPF to 0.95 with harmonic filters,
b. use one-half as many kVArs for $7^{\text {th }}$ harmonic filters as you do for $5^{\text {th }}$,
c. assume that contributions of harmonics above the $7^{\text {th }}$ will not be a serious problem.

If a distribution system has $10 \mathrm{MW}, 5 \mathrm{MVAr}$ of conventional load, and 3MW of six-pulse drives (assume drive $\mathrm{DPF}=0.85$ ), determine a "starting point" estimate for the net kVArs needed for 5 th , and also for $7^{\text {th }}$, harmonic filters.

## Problem 3 - Harmonic Filter Design.

A 12.5 kV distribution feeder is experiencing a problem with high $5^{\text {th }}$ harmonic voltages caused by large six-pulse adjustable-speed motor drives. Your task is to design a "generic" 5 th harmonic, grounded-wye, series-tuned filter. Each phase of the filter will employ a 200 kVAr capacitor and a series inductor. Ignore inductor resistance. (Note - the number of these generic filters needed to solve the harmonic problem is not addressed here)
a. Specify the inductance $(\mathrm{mH})$ of each inductor.
b. Determine the 60 Hz rms current in each inductor.

Now, assuming that the filter will have a $5^{\text {th }}$ harmonic current with rms magnitude equal to the fundamental,
c. Specify the rms ampere rating of each inductor.
d. Determine the rms $5^{\text {th }}$ harmonic voltage that will appear across each capacitor.

## Problem 4 - Filter Design for Ski Area of Case 4 in Chapter 9, "Case Studies."

Step 1. Data files bdat_skia.csv and ldat_skia.csv were prepared on a 10MVA base and using the information given in Case 4. Examine the .csv files and, by comparing their data to the Case 4 description, do the following:

- Verify that the $\mathrm{R}, \mathrm{X}$, and line charging VAr information in ldat_skia.csv for the segment between Bus 6 (Base) and Bus 7 (Star) is correct.
- Verify that the nonlinear load information in bdat_skia.csv for Bus 14 (WipeOut) is correct.
- Verify that the 138 kV transmission system equivalent information given in bdat_skia.csv agrees with the information given in the Case 4 description (i.e., the subtransient values in the file are derived from the $\mathrm{Isc}=34.4 \mathrm{pu}, 10 \mathrm{MVA}$ base, $\mathrm{X} / \mathrm{R}=5.0$ values shown in the box).

Step 2. Run PCFLOH using the given bdat_skia.csv and ldat_skia.csv files and confirm that you obtain the same results given in Figure 9.6 and in the spectral contents given in revised Figures 9.79.11. To run PCFLOH.exe, bdat_skia.csv, and ldat_skia.csv in a directory, click on PCFLOH.exe, and enter _skia.csv in the input field. Follow the instructions by clicking the buttons. View waveforms by clicking on bus numbers or branches.

Step 3. Without using phase shifting transformers, design and test a filter strategy that meets the following criteria:

- Displacement power factor at the substation transformer 138 kV bus is corrected so that it falls between 0.95 (lagging) and 1.0 .
- Filters are either 300 kVAr or 600 kVAr each.
- Tune 0.3 pu Hz (i.e., $0.3 \bullet 60=18 \mathrm{~Hz}$ ) below the harmonic target.
- Assume $\mathrm{X}($ at 60 Hz$) / \mathrm{R}$ ratio of filter inductors $=50$.
- No more than 600 kVAr of filters are added at any single bus.
- Max THDv in the 12.5 kV system $<4.5 \%$ (ignore the contributions of harmonics above the $13^{\text {th }}$ )
- Max fundamental V1 in the 12.5 kV system, excluding filter capacitors, is less than $105 \%$ (note - see Step 4 if you cannot achieve this)

Use impedance scans to confirm that the filters produce impedance dips at the desired frequencies. Note - a filter example is given in Chapter 6, "Conducting an Investigation."

Step 4. If you are unable to satisfy the V1 limit, then consider adding a phase shifting transformer to a large drive. This can be done by specifying a "nonlinear device phase shift" of 30 degrees in the corresponding row of the BDAT file. It may be necessary to remove some of the filters.

Step 5. Your final BDAT and LDAT files, containing your solution, should be named bdat_yourname.csv, ldat_yourname.csv, and emailed to Dr. Grady for checking.

Step 6. Prepare a final report (no more than 3 pages) for your client.

Many engineers have contributed to this document, either through inspiration, examples, or advice. In particular, I would like to thank Arshad Mansoor, Ray Stratford, Michael Doyle, John Soward, Scott Jackson, Diane Ammons, Ewald Fuchs, David Hartmann, Dennis Hansen, Russell Ehrlich, Blaine Leuschner, John Soward, Martin Narendorf, Rao Thallam, Al Schuman, Mark McGranaghan, Alex McEachern, David Fromme, and Matt Rylander.

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| Spectral Data for Figure 2.1, 120V PC Current |  |  |  | Spectral Data for Figure 3.3 (Top), <br> Idealized Six-Pulse Current-Source ASD Current |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Harmonic | Mag - \% Sine Deg. | Harmonic | Ang - Mag - $\%$ Sine Deg. | Harmonic | Ang- Mag - $\%$ Sine Deg. | Harmonic | Mag - \% Sine Deg. |
| 1 | 100.00 5 | 2 |  | 1 | 100.00 0 | 2 |  |
| 3 | 88.90 -177 | 4 |  | 3 |  | 4 |  |
| 5 | 73.60 5 | 6 |  | 5 | $20.00 \quad 180$ | 6 |  |
| 7 | 54.40 -173 | 8 |  | 7 | $14.29 \quad 180$ | 8 |  |
| 9 | 35.60 9 | 10 |  | 9 |  | 10 |  |
| 11 | $19.00-167$ | 12 |  | 11 | $9.09 \quad 0$ | 12 |  |
| 13 | $7.40 \quad 20$ | 14 |  | 13 | 7.69 0 | 14 |  |
| 15 | $1.00-88$ | 16 |  | 15 |  | 16 |  |
| 17 | $2.80 \quad 180$ | 18 |  | 17 | $5.88 \quad 180$ | 18 |  |
| 19 | $3.20 \quad 13$ | 20 |  | 19 | 5.26180 | 20 |  |
| 21 | $2.40-157$ | 22 |  | 21 |  | 22 |  |
| 23 | $1.90 \quad 43$ | 24 |  | 23 | 4.35 0 | 24 |  |
| 25 | $1.80-119$ | 26 |  | 25 | 4.00 0 | 26 |  |
| 27 | 1.90 67 | 28 |  | 27 |  | 28 |  |
| 29 | $2.00-110$ | 30 |  | 29 | $3.45 \quad 180$ | 30 |  |
| 31 | $1.60 \quad 75$ | 32 |  | 31 | $3.23 \quad 180$ | 32 |  |
| 33 | $1.00-85$ | 34 |  | 33 |  | 34 |  |
| 35 | $0.80 \quad 160$ | 36 |  | 35 | 2.86 0 | 36 |  |
| 37 | $1.20 \quad 23$ | 38 |  | 37 | 2.70 0 | 38 |  |
| 39 | 1.50 -140 | 40 |  | 39 |  | 40 |  |
| 41 | 1.30 54 | 42 |  | 41 | $2.44 \quad 180$ | 42 |  |
| 43 | $0.70-111$ | 44 |  | 43 | $2.33-180$ | 44 |  |
| 45 | $0.50 \quad 142$ | 46 |  | 45 |  | 46 |  |
| 47 | $0.70 \quad 15$ | 48 |  | 47 | 2.13 0 | 48 |  |
| 49 | $0.80-157$ | 50 |  | 49 | $2.04 \quad 0$ | 50 |  |

Spectral Data for Figure 3.3 (Bottom Left),
Idealized Six-Pulse Current-Source ASD Current with $30^{\circ}$ Shift

| Harmonic | Ang- Mag - \% Sine Deg. | Harmonic | Mag - \% Sine Deg |
| :---: | :---: | :---: | :---: |
| 1 | 100.00 0 | 2 |  |
| 3 |  | 4 |  |
| 5 | 20.00 0 | 6 |  |
| 7 | 14.29 0 | 8 |  |
| 9 |  | 10 |  |
| 11 | $9.09 \quad 0$ | 12 |  |
| 13 | 7.69 0 | 14 |  |
| 15 |  | 16 |  |
| 17 | 5.88 0 | 18 |  |
| 19 | 5.26 0 | 20 |  |
| 21 |  | 22 |  |
| 23 | 4.35 0 | 24 |  |
| 25 | 4.00 0 | 26 |  |
| 27 |  | 28 |  |
| 29 | 3.450 | 30 |  |
| 31 | 3.230 | 32 |  |
| 33 |  | 34 |  |
| 35 | 2.86 0 | 36 |  |
| 37 | 2.70 0 | 38 |  |
| 39 |  | 40 |  |
| 41 | 2.44 0 | 42 |  |
| 43 | 2.33 0 | 44 |  |
| 45 |  | 46 |  |
| 47 | 2.13 0 | 48 |  |
| 49 | 2.04 0 | 50 |  |

Spectral Data for Figure 3.3 (Left),
Idealized Twelve-Pulse Current-Source ASD Current

| Harmonic | Mag - \% Sine Deg. | Harmonic | Mag - \% Sine Deg |
| :---: | :---: | :---: | :---: |
| 1 | 100.00 0 | 2 |  |
| 3 |  | 4 |  |
| 5 |  | 6 |  |
| 7 |  | 8 |  |
| 9 |  | 10 |  |
| 11 | $9.09 \quad 0$ | 12 |  |
| 13 | 7.69 0 | 14 |  |
| 15 |  | 16 |  |
| 17 |  | 18 |  |
| 19 |  | 20 |  |
| 21 |  | 22 |  |
| 23 | 4.35 0 | 24 |  |
| 25 | 4.00 0 | 26 |  |
| 27 |  | 28 |  |
| 29 |  | 30 |  |
| 31 |  | 32 |  |
| 33 |  | 34 |  |
| 35 | 2.86 0 | 36 |  |
| 37 | 2.70 0 | 38 |  |
| 39 |  | 40 |  |
| 41 |  | 42 |  |
| 43 |  | 44 |  |
| 45 |  | 46 |  |
| 47 | 2.13 0 | 48 |  |
| 49 | 2.04 0 | 50 |  |

Spectral Data for Figure 4.1,

| Harmonic | Mag - \% | Ang - <br> Sine Deg. | Harmonic | Mag - \% Sine Deg. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100.00 | -66 | 2 |  |
| 3 | 63.50 | -69 | 4 |  |
| 5 | 35.90 | -70 | 6 |  |
| 7 | 18.30 | -69 | 8 |  |
| 9 | 10.10 | -73 | 10 |  |
| 11 | 5.40 | -78 | 12 |  |
| 13 | 2.10 | -80 | 14 |  |
| 15 | 0.90 | -101 | 16 |  |
| 17 | 0.40 | -96 | 18 |  |
| 19 | 0.10 | -90 | 20 |  |
| 21 |  |  | 22 |  |
| 23 | 0.20 | -91 | 24 |  |
| 25 | 0.20 | -174 | 26 |  |
| 27 | 0.10 | 126 | 28 |  |
| 29 |  |  | 30 |  |
| 31 | 0.20 | 21 | 32 |  |
| 33 | 0.20 | 99 | 34 |  |
| 35 |  |  | 36 |  |
| 37 | 0.10 | -44 | 38 |  |
| 39 |  |  | 40 |  |
| 41 |  |  | 42 |  |
| 43 | 0.30 | -90 | 44 |  |
| 45 |  |  | 46 |  |
| 47 | 0.40 | 64 | 48 |  |
| 49 | 0.20 | -165 | 50 |  |

Spectral Data for Figure 4.2,
120V Refrigerator Current

| Harmonic | Mag - \% | Ang Sine Deg | Harmonic | Mag - \% | Ang Sine Deg. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 100.00 | -31 | 2 | 3.50 | -48 |
| 3 | 5.10 | 101 | 4 | 0.40 | 98 |
| 5 | 0.50 | 124 | 6 | 0.30 | 152 |
| 7 | 1.00 | 10 | 8 | 0.20 | 55 |
| 9 | 0.40 | -57 | 10 |  |  |
| 11 | 0.20 | -103 | 12 |  |  |
| 13 | 0.20 | -180 | 14 |  |  |
| 15 | 0.10 | 129 | 16 |  |  |
| 17 |  |  | 18 |  |  |
| 19 |  |  | 20 |  |  |
| 21 |  |  | 22 |  |  |
| 23 |  |  | 24 |  |  |
| 25 |  |  | 26 |  |  |
| 27 |  |  | 28 |  |  |
| 29 |  |  | 30 |  |  |
| 31 |  |  | 32 |  |  |
| 33 |  |  | 34 |  |  |
| 35 |  |  | 36 |  |  |
| 37 |  |  | 38 |  |  |
| 39 |  |  | 40 |  |  |
| 41 |  |  | 42 |  |  |
| 43 |  |  | 44 |  |  |
| 45 |  |  | 46 |  |  |
| 47 |  |  | 48 |  |  |
| 49 |  |  | 50 |  |  |

Spectral Data for Figure 4.3,
240V Residential Air Conditioner Current

| Harmonic | Mag - \% | Ang - <br> Sine Deg. | Harmonic | $\begin{array}{r} \text { Ang } \\ \text { Mag - \% Sine Deg. } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100.00 | -23 | 2 |  |
| 3 | 8.00 | 110 | 4 |  |
| 5 | 6.80 | 58 | 6 |  |
| 7 | 0.50 | 165 | 8 |  |
| 9 | 0.60 | 82 | 10 |  |
| 11 | 0.30 | 157 | 12 |  |
| 13 |  |  | 14 |  |
| 15 |  |  | 16 |  |
| 17 | 0.20 | 87 | 18 |  |
| 19 |  |  | 20 |  |
| 21 |  |  | 22 |  |
| 23 |  |  | 24 |  |
| 25 |  |  | 26 |  |
| 27 |  |  | 28 |  |
| 29 |  |  | 30 |  |
| 31 |  |  | 32 |  |
| 33 |  |  | 34 |  |
| 35 |  |  | 36 |  |
| 37 |  |  | 38 |  |
| 39 |  |  | 40 |  |
| 41 |  |  | 42 |  |
| 43 |  |  | 44 |  |
| 45 |  |  | 46 |  |
| 47 |  |  | 48 |  |
| 49 |  |  | 50 |  |

Spectral Data for Figure 4.4,
277V Fluorescent Lamp Current (with Magnetic Ballast)

| Ang - |  |  |  |  |  |
| :---: | ---: | ---: | :---: | :---: | ---: |
| Harmonic | Mag - \% Sine Deg. | Harmonic | Mag - \% Sine Deg. |  |  |
| 1 | 100.00 | -2 | 2 | 2.00 | -117 |
| 3 | 15.80 | -150 | 4 | 0.40 | 168 |
| 5 | 8.60 | -128 | 6 | 0.30 | 47 |
| 7 | 2.90 | -73 | 8 | 0.10 | 8 |
| 9 | 2.00 | -32 | 10 |  |  |
| 11 | 1.40 | 6 | 12 |  |  |
| 13 | 0.80 | 29 | 14 | 0.20 | -49 |
| 15 | 0.40 | 32 | 16 |  |  |
| 17 | 0.20 | -17 | 18 |  |  |
| 19 | 0.50 | -56 | 20 |  |  |
| 21 | 0.40 | -49 | 22 | 0.20 | -133 |
| 23 | 0.20 | -65 | 24 |  |  |
| 25 |  |  | 26 |  |  |
| 27 | 0.10 | 162 | 28 |  |  |
| 29 |  |  | 30 |  |  |
| 31 |  |  | 32 |  |  |
| 33 |  |  | 34 | 0.10 | -166 |
| 35 | 0.10 | -42 | 36 |  |  |
| 37 |  |  | 38 |  |  |
| 39 |  |  | 40 | 0.10 | 149 |
| 41 |  |  | 42 |  |  |
| 43 |  |  | 44 |  |  |
| 45 |  |  | 46 |  |  |
| 47 | 0.10 | -137 | 48 | 0.20 | 25 |
| 49 |  |  | 50 |  |  |

Spectral Data for Figure 4.6 (Top Left),
Six-Pulse Voltage-Source ASD Current (High Power)

| Harmonic | Mag - \% | $\begin{array}{r} \text { Ang - } \\ \text { Sine Deg. } \end{array}$ | Harmonic | Mag - \% Sine Deg |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100.00 | 0 | 2 |  |
| 3 |  |  | 4 |  |
| 5 | 29.81 | 180 | 6 |  |
| 7 | 3.22 | 180 | 8 |  |
| 9 |  |  | 10 |  |
| 11 | 8.95 | 0 | 12 |  |
| 13 | 3.12 | 0 | 14 |  |
| 15 |  |  | 16 |  |
| 17 | 5.05 | 180 | 18 |  |
| 19 | 2.52 | 180 | 20 |  |
| 21 |  |  | 22 |  |
| 23 | 3.48 | 0 | 24 |  |
| 25 | 2.07 | 0 | 26 |  |
| 27 |  |  | 28 |  |
| 29 | 2.64 | 180 | 30 |  |
| 31 | 1.75 | 180 | 32 |  |
| 33 |  |  | 34 |  |
| 35 | 2.13 | 0 | 36 |  |
| 37 | 1.51 | 0 | 38 |  |
| 39 |  |  | 40 |  |
| 41 | 1.78 | 180 | 42 |  |
| 43 | 1.33 | 180 | 44 |  |
| 45 |  |  | 46 |  |
| 47 | 1.53 | 0 | 48 |  |
| 49 | 1.18 | 0 | 50 |  |

Spectral Data for Figure 4.6 (Top Right),
Six-Pulse Voltage-Source ASD Current (Low Power)

| Harmonic | Mag - \% | Ang - <br> Sine Deg. | Harmonic | Ang <br> Mag - \% Sine Deg |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100 | 0 | 2 |  |
| 3 |  |  | 4 |  |
| 5 | 58.89 | 180 | 6 |  |
| 7 | 30.78 | 0 | 8 |  |
| 9 |  |  | 10 |  |
| 11 | 5.3 | 0 | 12 |  |
| 13 | 8.64 | 180 | 14 |  |
| 15 |  |  | 16 |  |
| 17 | 0.34 | 0 | 18 |  |
| 19 | 3.04 | 0 | 20 |  |
| 21 |  |  | 22 |  |
| 23 | 1.43 | 180 | 24 |  |
| 25 | 0.85 | 180 | 26 |  |
| 27 |  |  | 28 |  |
| 29 | 1.48 | 0 | 30 |  |
| 31 | 0.14 | 180 | 32 |  |
| 33 |  |  | 34 |  |
| 35 | 1.2 | 180 | 36 |  |
| 37 | 0.55 | 0 | 38 |  |
| 39 |  |  | 40 |  |
| 41 | 0.84 | 0 | 42 |  |
| 43 | 0.67 | 180 | 44 |  |
| 45 |  |  | 46 |  |
| 47 | 0.49 | 180 | 48 |  |
| 49 | 0.61 | 0 | 50 |  |

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Spectral Data for Figure 4.6 (Bottom Left),
Six-Pulse Voltage-Source ASD Current (High Power)

| Harmonic | Mag - \% Sine Deg. | Harmonic | Ang <br> Mag - \% Sine Deg |
| :---: | :---: | :---: | :---: |
| 1 | 100.00 0 | 2 |  |
| 3 |  | 4 |  |
| 5 | $29.81 \quad 0$ | 6 |  |
| 7 | 3.22 0 | 8 |  |
| 9 |  | 10 |  |
| 11 | 8.95 0 | 12 |  |
| 13 | 3.120 | 14 |  |
| 15 |  | 16 |  |
| 17 | 5.050 | 18 |  |
| 19 | 2.52 0 | 20 |  |
| 21 |  | 22 |  |
| 23 | 3.48 0 | 24 |  |
| 25 | 2.07 0 | 26 |  |
| 27 |  | 28 |  |
| 29 | 2.65 0 | 30 |  |
| 31 | 1.75 0 | 32 |  |
| 33 |  | 34 |  |
| 35 | 2.13 0 | 36 |  |
| 37 | 1.510 | 38 |  |
| 39 |  | 40 |  |
| 41 | 1.78 0 | 42 |  |
| 43 | 1.33 0 | 44 |  |
| 45 |  | 46 |  |
| 47 | 1.53 0 | 48 |  |
| 49 | 1.18 0 | 50 |  |

Spectral Data for Figure 4.6 (Bottom Right),
Six-Pulse Voltage-Source ASD Current (Low Power)

| Harmonic | Mag - \% Sine Deg. | Harmonic | Mag - \% Sine Deg. |
| :---: | :---: | :---: | :---: |
| 1 | 100.00 0 | 2 |  |
| 3 |  | 4 |  |
| 5 | 58.89 0 | 6 |  |
| 7 | 30.78180 | 8 |  |
| 9 |  | 10 |  |
| 11 | 5.30 0 | 12 |  |
| 13 | 8.64180 | 14 |  |
| 15 |  | 16 |  |
| 17 | $0.34 \quad 180$ | 18 |  |
| 19 | $3.04 \quad 180$ | 20 |  |
| 21 |  | 22 |  |
| 23 | 1.43180 | 24 |  |
| 25 | $0.85 \quad 180$ | 26 |  |
| 27 |  | 28 |  |
| 29 | $1.48 \quad 180$ | 30 |  |
| 31 | 0.14 0 | 32 |  |
| 33 |  | 34 |  |
| 35 | $1.20 \quad 180$ | 36 |  |
| 37 | 0.55 0 | 38 |  |
| 39 |  | 40 |  |
| 41 | $0.84 \quad 180$ | 42 |  |
| 43 | 0.67 0 | 44 |  |
| 45 |  | 46 |  |
| 47 | $0.49 \quad 180$ | 48 |  |
| 49 | $0.61 \quad 0$ | 50 |  |

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| Spectral Data for Figure 4.8, 120V Microwave Oven Current |  |  |  |  |  | Spectral Data for Figure 4.9, 120V Vacuum Cleaner Current |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Harmonic | Mag - \% | $\begin{array}{r} \text { Ang - } \\ \text { Sine Deg. } \end{array}$ | Harmonic | Mag - \% | Ang Sine Deg. | Harmonic | Mag - \% | Sine Deg. | Harmonic | Mag - \% | Ang- <br> Sine Deg. |
| 1 | 100.00 | -3 | 2 | 11.60 | -102 | 1 | 100.00 | -18 | 2 | 0.40 | 97 |
| 3 | 27.80 | -149 | 4 | 1.10 | 97 | 3 | 25.70 | 132 | 4 | 0.70 | -118 |
| 5 | 9.50 | -61 | 6 | 1.50 | -109 | 5 | 2.70 | -70 | 6 | 0.20 | 79 |
| 7 | 3.30 | -46 | 8 |  |  | 7 | 1.80 | -126 | 8 | 0.30 | -74 |
| 9 | 1.90 | 45 | 10 | 0.30 | -78 | 9 | 0.40 | 112 | 10 |  |  |
| 11 | 1.10 | 139 | 12 | 0.20 | -21 | 11 | 0.60 | -33 | 12 |  |  |
| 13 | 0.90 | -144 | 14 | 0.10 | 68 | 13 |  |  | 14 |  |  |
| 15 | 0.60 | -60 | 16 | 0.10 | 156 | 15 | 0.20 | 123 | 16 |  |  |
| 17 | 0.50 | 17 | 18 |  |  | 17 |  |  | 18 |  |  |
| 19 | 0.40 | 88 | 20 |  |  | 19 |  |  | 20 |  |  |
| 21 | 0.30 | 162 | 22 |  |  | 21 |  |  | 22 |  |  |
| 23 | 0.30 | -121 | 24 |  |  | 23 |  |  | 24 |  |  |
| 25 | 0.30 | -45 | 26 |  |  | 25 |  |  | 26 | 0.10 | 63 |
| 27 | 0.20 | 25 | 28 |  |  | 27 |  |  | 28 |  |  |
| 29 | 0.20 | 103 | 30 |  |  | 29 |  |  | 30 |  |  |
| 31 | 0.20 | -174 | 32 |  |  | 31 |  |  | 32 |  |  |
| 33 | 0.20 | -111 | 34 |  |  | 33 | 0.20 | 147 | 34 |  |  |
| 35 | 0.20 | -47 | 36 |  |  | 35 | 0.20 | -76 | 36 |  |  |
| 37 | 0.10 | 39 | 38 |  |  | 37 |  |  | 38 |  |  |
| 39 | 0.20 | 114 | 40 |  |  | 39 |  |  | 40 |  |  |
| 41 | 0.10 | -7 | 42 |  |  | 41 |  |  | 42 |  |  |
| 43 | 0.10 | -126 | 44 |  |  | 43 |  |  | 44 |  |  |
| 45 | 0.10 | -43 | 46 |  |  | 45 |  |  | 46 |  |  |
| 47 |  |  | 48 |  |  | 47 |  |  | 48 |  |  |
| 49 |  |  | 50 |  |  | 49 |  |  | 50 |  |  |

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Spectral Data for Figure 4.10,
277V Fluorescent Lamp Current (with Electronic Ballast)

| Harmonic | Mag - \% | Ang - | Harmonic | Mag - \% | Ang- Sine Deg. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 100.00 | 4 | 2 | 0.20 | 32 |
| 3 | 7.30 | -166 | 4 | 0.40 | 111 |
| 5 | 8.00 | -1 | 6 | 0.30 | 84 |
| 7 | 3.00 | -3 | 8 | 0.20 | 67 |
| 9 | 0.20 | -85 | 10 | 0.10 | 86 |
| 11 | 1.40 | -165 | 12 | 0.10 | 60 |
| 13 | 1.10 | 176 | 14 | 0.30 | 60 |
| 15 | 0.90 | -168 | 16 | 0.20 | -65 |
| 17 | 0.40 | -164 | 18 | 0.10 | 19 |
| 19 | 0.30 | -59 | 20 |  |  |
| 21 | 0.70 | -32 | 22 | 0.20 | 61 |
| 23 | 0.30 | -39 | 24 | 0.30 | 162 |
| 25 | 0.40 | -54 | 26 | 0.30 | -87 |
| 27 | 0.20 | -39 | 28 | 0.40 | 95 |
| 29 | 0.20 | -73 | 30 |  |  |
| 31 | 0.30 | -136 | 32 | 0.10 | -81 |
| 33 | 0.60 | -128 | 34 |  |  |
| 35 | 0.40 | -126 | 36 | 0.10 | 0 |
| 37 | 0.30 | -149 | 38 | 0.10 | 148 |
| 39 | 0.20 | -112 | 40 | 0.20 | 102 |
| 41 | 0.50 | -130 | 42 |  |  |
| 43 | 0.40 | -140 | 44 | 0.20 | 93 |
| 45 | 0.60 | 156 | 46 | 0.20 | 51 |
| 47 | 0.30 | 177 | 48 | 0.20 | -130 |
| 49 | 0.30 | 177 | 50 |  |  |

Spectral Data for Figure 5.9,
Voltage at 120V Service Panel for PC Workstations

| Ang |  |  |  |  |
| :---: | :---: | ---: | :---: | ---: |
| Harmonic | Mag $-\%$ Sine Deg. | Harmonic | Mag - \% Sine Deg. |  |
| 1 | 100.00 | 0 | 2 |  |
| 3 | 2.20 | 53 | 4 |  |
| 5 | 3.90 | -150 | 6 |  |
| 7 | 1.40 | -28 | 8 |  |
| 9 | 0.60 | 54 | 10 |  |
| 11 | 0.90 | -176 | 12 |  |
| 13 | 0.50 | -60 | 14 |  |
| 15 | 0.50 | 25 | 16 |  |
| 17 | 0.50 | 139 | 18 |  |
| 19 | 0.30 | -126 | 20 |  |
| 21 | 0.30 | -12 | 22 |  |
| 23 | 0.30 | 112 | 24 |  |
| 25 | 0.30 | -155 | 26 |  |
| 27 | 0.40 | -47 | 28 |  |
| 29 | 0.40 | 38 | 30 |  |
| 31 | 0.40 | 134 | 32 |  |
| 33 | 0.30 | -118 | 34 |  |
| 35 | 0.40 | -44 | 36 |  |
| 37 | 0.20 | 78 | 38 |  |
| 39 | 0.40 | 161 | 40 |  |
| 41 | 0.40 | -102 | 42 |  |
| 43 | 0.40 | -39 | 44 |  |
| 45 | 0.20 | 35 | 46 |  |
| 47 | 0.20 | 163 | 48 |  |
| 49 | 0.20 | -127 | 50 |  |

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Spectral Data for Figure 5.10,
480V Service Entrance Current for College of Business

| Administration, U. T. Austin |  |  |  |  |  |
| :---: | ---: | ---: | :---: | ---: | ---: |
| Harmonic | Mag - \% Sine Deg. | Ang | Harmonic | Mag - \% Sine Deg. |  |
| 1 | 100.00 | -27 | 2 | 0.10 | 0 |
| 3 | 3.50 | -91 | 4 | 0.10 | -42 |
| 5 | 4.50 | -146 | 6 | 0.00 | 0 |
| 7 | 3.70 | -48 | 8 | 0.10 | 61 |
| 9 | 0.40 | 41 | 10 | 0.10 | 145 |
| 11 | 1.00 | 10 | 12 | 0.10 | -30 |
| 13 | 0.70 | 138 | 14 | 0.10 | -134 |
| 15 | 0.00 | 0 | 16 | 0.20 | 38 |
| 17 | 0.40 | -104 | 18 | 0.00 | 0 |
| 19 | 0.30 | -80 | 20 | 0.00 | 0 |
| 21 | 0.00 | 0 | 22 | 0.00 | 0 |
| 23 | 0.20 | -153 | 24 | 0.00 | 0 |
| 25 | 0.00 | 0 | 26 | 0.10 | 54 |
| 27 | 0.00 | 0 | 28 | 0.20 | 173 |
| 29 | 0.40 | -165 | 30 | 0.10 | -67 |
| 31 | 0.20 | 141 | 32 | 0.00 | 0 |
| 33 | 0.00 | 0 | 34 | 0.00 | 0 |
| 35 | 0.00 | 0 | 36 | 0.10 | -162 |
| 37 | 0.00 | 0 | 38 | 0.00 | 0 |
| 39 | 0.10 | 143 | 40 | 0.10 | 6 |
| 41 | 0.20 | 93 | 42 | 0.20 | 58 |
| 43 | 0.00 | 0 | 44 | 0.20 | -99 |
| 45 | 0.30 | 126 | 46 | 0.10 | 176 |
| 47 | 0.00 | 0 | 48 | 0.00 | 0 |
| 49 | 0.20 | 176 | 50 | 0.10 | 169 |

## 1. Summary of PCFLO Files

## Input (note - any lines in input files that begin with a colon in column 1 are treated as comments and are skipped)

- ADAT.CSV: Loadflow area input data. (PCFLO also creates temporary file ADAT.TMP)
- BDAT.CSV: Bus data.
- LDAT.CSV: Line and transformer data.
- OPTIONS.CSV: Solution options.
- SPECTRA.CSV: User-specified harmonic current injection spectra. (PCFLO also creates temporary file SPECTRA.TMP)


## Output

- ASOLN.CSV: Solved area data for loadflows.
- BORDER.CSV: File built by PCFLO that lists the busses in optimal order.
- EXLOG.CSV: Echo print of screen messages.
- FREP.TXT: Output of program FAULTS.
- HPA_LAST_CASE.CSV, HPA_SUMMARY.CSV. Output of harmonic power analyzer program HPA.
- ISOLN.CSV, VSOLN.CSV: Solved branch currents and bus voltages for loadflow and harmonics studies. (Sine series format for Fourier series)
- OUT1.CSV, OUT2.CSV, OUT3.CSV: Echo print of input data for loadflow, short circuit, and harmonics, along with pertinent messages and errors.
- OUT4.CSV: Full loadflow output data (if requested).
- OUT5.CSV: Loadflow summary output used for analyzing the impact of power transactions across a power grid.
- THDV.CSV, THDI.CSV: Solved total harmonic voltage and current distortions for harmonics studies.
- ZBUS0.CSV, ZBUS1.CSV, ZBUS2.CSV: Solved zero/positive/negative impedance matrix elements for short circuit (in rectangular form) or harmonic studies (in polar form), in per unit.


## Temporary Files Created During Execution

- BDAT.TMP: Unformatted bus data file built by PCFLO and read by FAULTS.
- FFREP1.TXT, FFREP2.TXT, FFREP3.TXT. Temporarly files used during short circuit studies (produced by program FAULTS)
- LDAT.TMP: Unformatted line and transformer data file built by PCFLO and read by FAULTS.
- SPECTRA.TMP: Temporary file built by PCFLO during harmonic studies.


## 2. Prepared Cases, Ready to Run

## Loadflow

- 5 Bus Stevenson* Loadflow Example, pp. 200-205. File Extension _S5.
- 4860 Bus Loadflow Case. File Extension _SCREWBEAN. Used for power grid studies.


## Short Circuit

- 6 Bus Grainger-Stevenson** Short Circuit Example (Prob. 3.12, p. 139, and continued with Prob. 11.17, p. 469). File Extension _S6
- 9 Bus Grainger-Stevenson** Short Circuit Example (Prob. 3.13, pp. 139-140, and continued with Prob. 11.18, p. 469). File Extension _S9.


## Harmonics

- 5 Bus Tutorial. File Extensions _FIVE and _FIVE_FILTER.
- 17 Bus Small Ski Area Example. File Extensions _SKIA, _SKIB, _SKIC.
- 454 Bus Large Ski Area Example. File Extensions _OLYMPICS_A (unfiltered), _OLYMPICS_D (filtered).
* William D. Stevenson, Jr., Elements of Power System Analysis, Fourth Edition, McGraw-Hill, New York, 1982.
** John J. Grainger, William D. Stevenson, Jr., Power System Analysis, McGraw-Hill, New York, 1994.


## 3. Content of Input Data Files

## BUS DATA

(File = BDAT.CSV, one record per bus. CSV format)
\(\left.$$
\begin{array}{ll}\text { Variable } & \text { Comments } \\
\text { Number } & \text { Integer } \\
\text { Name } & \text { Up to } 12 \text { characters } \\
\text { Type } & \begin{array}{l}1=\text { Swing Bus } \\
2=\text { PV Bus } \\
3\end{array}
$$ <br>

\& Percent Bus\end{array}\right]\)| Linear P Generation | Percent |
| :--- | :--- |
| Linear Q Generation | Percent |
| Linear P Load | Percent unit |
| Linear Q Load | Percent @ voltage = 1 per unit |
| Desired Voltage | Percent |
| Shunt Reactive Q Load | Percent |
| Maximum Q Generation | Integer |
| Minimum Q Generation |  |

\(\left.$$
\begin{array}{ll}\text { Remote-Controlled Bus Number } & \begin{array}{l}\text { Used for controlling voltage at a remote bus. For these } \\
\text { cases, the desired voltage specified above applies to the } \\
\text { remote bus. }\end{array} \\
\text { Connection Type for Shunt Reactive Q Load } & \begin{array}{l}0 \text { or 1 = Grounded Wye. } \\
\text { Otherwise, ungrounded wye or delta (i.e. no zero } \\
\text { sequence path) }\end{array}
$$ <br>

Subtransient R, X (Pos. Sequence) \& Series impedance of motor or generator, in per unit\end{array}\right\}\)| Subtransient R, X (Neg. Sequence) | Series impedance of motor or generator, in per unit <br> (ignoring connection type and grounding impedances) <br> (do not multiply by 3) |
| :--- | :--- |
| Subtransient R, X (Zero Sequence) | 0 or 1 = Grounded Wye. <br> Otherwise, ungrounded wye or delta (i.e. no zero <br> sequence path) |
| Connection Type for Subtransient Impedancer or generator, in per unit |  |



## CSV Header and Structure for Input File BDAT.CSV (using sample file BDAT_HEADER.csv)

## :Bus Data

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\vdots$ |  |  |  |  |  |  |  |  | Shunt | Maximum

continuing across,

|  |  |  | Connection | Positive | Positive | Negative | Negative | Zero | Zero |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Minimum |  |  | Type for | Sequence | Sequence | Sequence | Sequence | Sequence | Sequence |
| Q |  | Bus | Remote- | Shunt | Subtransient |  | Subtransient | Subtransient | Subtransient |
| Subtransient | Subtransient |  |  |  |  |  |  |  |  |
| Generation | Control | Controlled | Reactive | $R$ | $R$ | R | $R$ | R | X |
| (\%) | Area | Bus No. | Q Load | (pu) | (pu) | (pu) | (pu) | (pu) | (pu) |
| (F) | (I) | (I) | (I) | (F) | (F) | (F) | (F) | (F) | (F) |

continuing across,

|  | Grounding Impedance | Grounding Impedance | Nonlinear | Nonlinear | Nonlinear |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Connection | R for | X for | Device | Device | Device |  | Nonlinear | Linear |
| Type for | Subtransient | Subtransient | P | P | Displacement | Nonlinear | Device | Load |
| Subtransient | Impedances | Impedances | Gen | Load | Power Factor | Device | Phase Shift | Connection |
| R and X | (pu) | (pu) | (\%) | (\%) | (pu) | Type | (Degrees) | Type |
| (I) | (F) | (F) | (F) | (F) | (F) | (I) | (F) |  |

## LINE AND TRANSFORMER DATA

(File = LDAT.CSV, one record per branch. CSV format)

## Variable

FROM BUS Number
TO BUS Number
Circuit Number
R, X (Positive/Negative Sequence)
Charging (Positive/Negative Sequence)
Rating
Minimum Tap, or Minimum Phase Shift Angle

Maximum Tap, or Maximum Phase Shift Angle

Tap
Phase Shift
Voltage-Controlled Bus Number

## Comments

Integer (or blank if neutral)
Integer (or blank, if neutral)
Integer (or blank)
Series impedance, in per unit
Percent, for entire length of line
Percent
Per unit tap, or degrees, FROM BUS side wrt. TO BUS side

Per unit tap, or degrees, on FROM BUS side wrt. TO BUS side

Per unit, or degrees
Per unit, on FROM BUS side
Degrees, FROM BUS side wrt. TO BUS side
Used for controlling voltage at a remote bus. For these cases, the desired voltage specified applies to the remote bus.

| Voltage-Controlled Bus Side | When controlling the voltage at a remote bus, enter 1 when the remote bus is on the FROM BUS side of the transformer. Enter 2 when the remote bus is on the TO BUS side of the transformer. |
| :---: | :---: |
| Desired Voltage for Voltage-Controlled Bus, or Desired Active Power Flow for Phase Shifting Transformer | Per unit Voltage, or Percent Active Power (FROM BUS toward TO BUS) |
| R, X (Zero Sequence) | Series impedance, in per unit (ignoring connection type and grounding impedances), (do not multiply by 3 ) |
| Charging (Zero Sequence) | Percent, for entire length of line |
| Connection Type for Transformers and Shunt Elements | For transformers: |
|  | Type FROM BUS TO BUS |
|  | 0 or $1 \quad$ GY GY |
|  | 2 GY Y |
|  | 3 Y GY |
|  | $4 \quad Y \quad Y$ |
|  | $5 \quad \Delta \quad \Delta$ |
|  | 6 GY $\Delta$ |
|  | $7 \quad Y \quad \Delta$ |
|  | $8 \quad \Delta \quad$ GY |
|  | $9 \quad \Delta$ |
|  | For shunt elements: |
|  | 0 or 1 = Grounded Wye. |
|  | Otherwise, ungrounded wye or delta (i.e. no zero sequence path) |
| Grounding Impedance $\mathrm{R}, \mathrm{X}$ | Series impedance from wye point to ground, in per unit Applies to wye-connected transformers and shunt |

Resistive Skin Effect Factor for Positive/Negative Sequence

Resistive Skin Effect Factor for Zero Sequence

Harmonic h ( $\mathrm{h} \geq 2$, fractional values OK ) at which the conductor resistance is double the fundamental frequency resistance.

Harmonic h ( $\mathrm{h} \geq 2$, fractional values OK ) at which the conductor resistance is double the fundamental frequency resistance. This value applies to the combined conductor and grounding resistance.

continuing across,
Desired Voltage at Voltage Cont. Bus


|  | Voltage- | Voltage- | or Desired $P$ | Zero | Zero | Zero | Type | Series | Series |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Phase | Cont. | Cont. | for | Sequence | Sequence | Sequence for Trans. | Grounding | Grounding |  |
| Shift | Bus | Bus | Phase Shifter | $R$ | R | Charging | and Shunt $R$ | R |  |
| (Degrees) | Number | Side | (\%) | (pu) | (pu) | (\%) | Elements | (pu) | (pu) |
| (F) | (I) | (I) | (F) | (F) | (F) | (F) | (I) | (F) | (F) |

## AREA INTERCHANGE DATA

(File = ADAT.CSV, one record per loadflow area. CSV format)

| Variable | Comments |
| :--- | :--- |
| Number | Integer |
| Tie-Line Loss Assignment |  |
|  |  |
| If non-zero, then power losses on tie lines are assigned |  |
| equally between the two areas. If zero, the TO BUS |  |
| area for each tie line is assigned the loss (i.e., meter at |  |
| the FROM BUS). |  |

## USER-SPECIFIED HARMONIC CURRENT SPECTRAL DATA

(File = SPECTRA.CSV, one record per harmonic per nonlinear load type. CSV format)

## Variable

Type of Series

Nonlinear Load Type
Harmonic Order
Current Harmonic Magnitude

Current Harmonic Phase Angle

## Comments

Must be sin for a sine series, cos for a cosine series. All entries in this file must be either sin, or cos, and cannot be mixed.

Must be 14, 15, 16, . . , 33.
$1,2,3$, etc.
Per unit. If the fundamental is given, its magnitude must be 1.0 , and the other harmonic magnitudes for the same nonlinear load type are assumed to be relative to 1.0. The actual injection currents will be scaled according to bus load/generation.

Important: If the fundamental is not given for a nonlinear load type, then the harmonic magnitudes are assumed to be given on the system base, rather than as a fraction of the P for that bus.

Degrees, using load current convention. If the fundamental angle is given, it must be 0.0 . The actual phase angles will be adjusted internally according to bus power factor and fundamental voltage angle.

If the fundamental is not given, the phase angles are assumed to be given with respect to the bus fundamental frequency voltage phase angle.

| Header :Harmoni | d Structu Current Sp | re for Inp ectral Data | File SP | CTRA.CS |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Current |
| :Type of |  |  | Current | Harmonic |
| :Series | Nonlinear | Harmonic | Harmonic | Phase |
| :(SIN or | Load | Order | Mag. | Angle |
| :COS) | Type | (Integer) | (pu) | (Degrees) |
| :(A) | (I) | (1) | (F) | (F) |

## SOLUTION OPTIONS

(File $=$ OPTIONS.CSV. CSV format. $)$

## For Loadflow (using sample file OPTIONS_HEADER_LOADFLOW.csv)

## Loadflow Study Case - User Title Goes on This Line

:Loadflow Solution Options

|  |  |  |  | Voltage |  |  |  |  | Disable |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| :Optimal |  | P \& Q | Accel. | Update |  | Disable |  |  | Transf. |
| :Bus |  | Mismatch | Factor | Cap | P \& Q | Remote | Disable | Ignore | Tap |
| :Ordering | Gauss- | for Gauss- | for Gauss- | for Gauss- | Mismatch | Volt. Reg. | Area | Q Limits | Adjust. for |
| :Method | Seidel | Seidel | Seidel | Seidel | Solution | by PV | Intrchnge | on PV | Voltage |
| :(Integer) | Start? | Start | Start | Start | Tolerance | Busses? | P Adjust? | Busses? | Control? |
| :(1-2-3) | (T or F) | (0.5 pu) | (1.2 pu) | (0.005 pu) | (5E-06 pu) | (T or F) | ( T or F) | ( T or F) | (T or F) |
| :(1) | (L) | (F) | (F) | (F) | (F) | (L) | (L) | (L) | (L) |
|  | T | 0.5 | 1.2 | 0.005 | 5.00E-06 | F | F | F | F |

continuing across,

| Limit |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Output | and | and | and | and |
| to This | This | This | This | This |
| Control | Control | Control | Control | Control |
| Area? | Area? | Area? | Area? | Area? |
| (Integer) | (Integer) | (Integer) | (Integer) | (Integer) |
| (I) | (I) | (I) | (I) | (I) |
|  | 0 | 0 | 0 | 0 |

```
For Short Circuit (using sample file OPTIONS_HEADER_SHORT_CIRCUIT.csv)
Short Circuit Study Case - User Title Goes on This Line
:Short Circuit Solution Options
    :Optimal
    :Bus
    :Ordering
:Method Enter T for Diagonal and Neighbor ZBUS Elements Only (recommended)
:(Integer) Enter F for All ZBUS Elements (not recommended and not to be followed by FAULTS)
:(1-2-3) (T or F)
(I) (L)
    2 T
```

For Full Harmonic Solution (using sample file OPTIONS_HEADER_FULL_HARMONIC_SOLUTION.csv)
Full Harmonic Solution Study Case - User Title Goes on This Line
:Full Harmonic Solution Options
:

| :Optimal |  | P \& Q | Accel. | Update |  | Highest | Harmonic Load Model for PQ Linear Loads | Global |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| :Bus |  | Mismatch | Factor | Cap | P \& Q | Harmonic | 0 or 1: Resistive-only (recommended) | Linear | Global |
| :Ordering | Gauss- | for Gauss- | for Gauss- | for Gauss- | Mismatch | of | 2: Parallel R \& L Model | Motor | Resistance |
| :Method | Seidel | Seidel | Seidel | Seidel | Solution | Interest | 3: Series R \& L Model | Load | Doubling |
| :(Integer) | Start? | Start | Start | Start | Tolerance | (Integer) | 4: Ignore PQ Loads (i.e. No Model) | Modeling | Harmonic |
| :(1-2-3) | (T or F) | (0.5 pu) | (1.2 pu) | (0.005 pu) | (5E-06 pu) | (1-49) | (0-4) | Fraction | (1) |
| :(1) | (L) | (F) | (F) | (F) | (F) | (1) | (1) | (F) | (I) |


| For Harmonic Impedance Scan (using sample file OPTIONS_HEADER_HARMONIC_IMPEDANHarmonic Impedance Scan Study Case - User Title Goes on This Line |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| :Optimal |  | P \& Q | Accel. | Update |  | Lowest | Highest | Number | Limit the |
| :Bus |  | Mismatch | Factor | Cap | P \& Q | Harmonic | Harmonic | of Steps | Output to |
| :Ordering | Gauss- | for Gauss- | for Gauss- | for Gauss- | Mismatch | of |  |  | Diagonal |
| :Method | Seidel | Seidel | Seidel | Seidel | Solution | Interest | Interest | Harmonic | Elements |
| :(Integer) | Start? | Start | Start | Start | Tolerance | (Integer) | (Integer) | (Integer) | Only? |
| :(1-2-3) | ( T or F) | (0.5 pu) | (1.2 pu) | (0.005 pu) | (5E-06 pu) | (1-49) | (1-49) | (1-100) | ( T or F) |
| :(1) | (L) | (F) | (F) | (F) | (F) | (I) | (I) | (I) | (L) |

continuing across,

|  |  |  |  | Harmonic Load Model <br> for PQ Linear Loads | Global |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## 4. Harmonic-Related and Short-Circuit Related Output Files

(Note - for loadflow studies, the formats of ISOLN and VSOLN are different from below but are self-explanatory when viewing the files. For short circuit studies, the ZBUS files are similar to below, but written in rectangular form)

Commas separate the fields shown below to facilitate their use with Microsoft Excel.

## ISOLN.CSV (for harmonics)

Data Field
(starting from the left)

Description
1 Harmonic number
2 From bus number
3
4
5
6
7
8
To bus number
Circuit number
Current magnitude - per unit
Current phase angle (sine reference) - degrees
From bus name (at the first opportunity only)
To bus name (at the first opportunity only)
Loading level - percent of line rating (for
fundamental frequency only)

## VSOLN.CSV (for harmonics)

Data Field
(starting from
the left)
Description
Harmonic number
Bus number
Voltage magnitude - per unit
$4 \quad$ Voltage phase angle (sine reference) - degrees
5
Nonlinear device load current magnitude per unit
6 Nonlinear device load current phase angle
(sine reference) - degrees
7
Bus name (at the first opportunity only)

## ZBUS0.CSV, ZBUS1.CSV, ZBUS2.CSV (for harmonics and short circuit)

Data Field
(starting from the left)

Description
1 Harmonic number
2 From bus number
3
4
5
6
7
To bus number
Impedance magnitude - per unit
Impedance phase angle - degrees
From bus name (at the first opportunity only)
To bus name (at the first opportunity only)

## THDV.CSV (for harmonics)

Contains a list of bus numbers with their corresponding names and voltage distortions.

## Loadflow Study Example. The Screwbean Wind Farm



The Screwbean 138kV substation is located in west Texas, halfway between Midland/Odessa and El Paso, near Guadalupe Mountains National Park. It is about 400 miles from Austin. This is prime wind country, and several wind farms are already located in the area.

Your job is to examine the feasibility of transporting 50MW of power from a new wind farm near Screwbean to the U.T. Austin campus. In particular, you are to determine the impact of this transaction on the losses in individual control areas, and also determine if any high or low voltages, or line overloads, are created by your transaction.

To perform the analysis, you will use a 5000 bus version of PCFLO, together with a summer peak loadflow case (in which most bus names have been disguised). You should prepare a $1 / 2$ to 1 page summary report of your study, as if you were going to submit it to your client. Tables should be attached as an appendix.

Explain to your client how many MW must be generated at Screwbean to deliver 50MW to U.T. Austin. Quantify the MW needed by each negatively-impacted control area to pay back for their increased losses.

Here are the steps:

1. Go to www.ece.utexas.edu/~grady, and click the PCFLO_V6 link. Follow the download and unzip instructions on the PCFLO page.
2. The _SCREWBEAN case is your "base case." Solve it using PCFLO_V6_Interface.exe. Using Excel, examine the output files produced, notably

- exlog_SCREWBEAN.csv
- asoln_SCREWBEAN.csv
- vsoln_SCREWBEAN.csv
- isoln_SCREWBEAN.csv and
- out5_SCREWBEAN.csv.

3. Print out asoln_SCREWBEAN.csv, using the landscape option. To verify your loadflow result, check your power loss in asoln_SCREWBEAN.csv. It should be about 1215 MW.
4. Find the Screwbean 138 kV substation (SCRWBEAN 138, bus 1095) and the U.T. Austin Harris 69 kV substation (HARRIS 69, bus 9204) in the out5_SCREWBEAN.csv file. Note their voltage magnitudes and phase angles, and the P and Q flows in lines/transformers attached to these busses.
5. Copy files bdat_SCREWBEAN.csv to bdat_mod, ldat_SCREWBEAN.csv to ldat_mod, and adat_SCREWBEAN.csv to adat_mod.
6. Add new PV bus SB WIND as bus 2 to bdat_mod.csv, using 20 for its control area. Put 50MW (i.e., $50 \%$ on 100MVA base) of generation on this new bus, with a Max Q Gen of 25 MVAr , and a Min Q Gen of negative 12.5MVAr. For the desired voltage, put a value that is 0.005 pu higher than the base case voltage at SCRWBEAN 138.
7. Add new PQ bus UT CAMPUS as bus 3 to bdat_mod.csv, using 21 for its control area. Put 50MW, 25MVAr of load on this new bus.
8. Connect new bus SB WIND to SCRWBEAN 138 through a line with impedance $\mathrm{R}=$ $0.001 \mathrm{pu}, \mathrm{X}=0.01 \mathrm{pu}, \mathrm{B}=0 \%$.
9. Connect new bus UT CAMPUS to HARRIS 69 through a line with impedance $\mathrm{R}=0.001 \mathrm{pu}$, $X=0.01 \mathrm{pu}, \mathrm{B}=0 \%$.
10. Add control area SB as area 20 to adat_mod.csv, with a desired export of 50MW (i.e., 50\%). The area control bus number will be that of SB WIND. Use an export solution tolerance of 0.1\%.
11. Add control area UT as area 21 to adat_mod.csv, with a desired import of 50MW (i.e., negative 50MW export). The area control bus number will be that of UT CAMPUS. Use an export solution tolerance of $0.1 \%$.
12. Re-run PCFLO_V6_Interface.exe, using _mod as the input case. Print out the new asoln.csv file (using the landscape option), and using the new asoln file, tabulate area by area the increase/decrease in each control area's losses compared to the base case. The areas with increased losses may reasonably expect MW payment from the wind power company. This can be accomplished by putting in generator larger than 50MW, and exporting some power to the control areas that are negatively impacted.
13. Use the loss increases from Step 12 to estimate how much actual generation would be needed at SB WIND to deliver 50MW to UT CAMPUS and payback the extra losses to the negatively-impacted control areas.
14. Check for any line overloads and high/low voltages created in the vicinity of SCRWBEAN 138 and HARRIS 69. (In an actual study, there would have to be described and remedies proposed. However, do not investigate remedies in your study.)
15. Repeat the above process, but this time reverse the transaction by putting a wind generator at UT CAMPUS, and a 50MW load at SB WIND.
16. Describe the impacts of both transactions in your report.

## Harmonic Study Example. The SKIA Case

Overview. This case deals with the proposed expansion of a ski area. The 12.5 kV underground system will eventually have eight ski lifts powered by DC motor drives, totaling 5150HP. Total load (linear plus nonlinear) will be about 9MW. The DC motors will be driven by six-pulse linecommutated ASDs so that the lifts will have soft-start, soft-stop operation. Measurements of the proposed system are, or course, not possible. Thus, the harmonics situation must be analyzed in advance using simulations.

Simulations. (PCFLOH Files *_SKIA.csv, *_SKIB.csv, *_SKIC.csv). A diagram of the ski area is shown in Figure 9.6. In addition to the ASD loads, the ski area has 6 MVA of linear load. The ASDs are modeled using the $1 / \mathrm{k}$ rule for harmonics through the $25^{\text {th }}$, with no phase angle diversity. The $d p f s$ of the ASDs and linear load are assumed to be 0.85 . Cable capacitance is taken from Table 9.5.

Table 9.5. Capacitance and Charging of 12.47 kV Cables

| Cable | Capacitance | kVAr |
| :---: | :---: | :---: |
| $1 / 0$ | 0.163 | 9.56 |
| $4 / 0$ | 0.222 | 13.02 |
| 350 kcmil | 0.267 | 15.65 |
| 500 kcmil | 0.304 | 17.82 |
| 1000 kcmil | 0.401 | 23.5 |

Capacitance: $\mu \mathrm{F}$ per km per phase
kVAr: (three-phase) per km
The point of common coupling (PCC) is Bus \#20, Substation 138 kV . $\mathrm{I}_{\mathrm{SC}}$ and $\mathrm{I}_{\text {load }}$ at the PCC are 34.4 pu and 1.080 pu, respectively, on a 10MVA base. Twelve-month average $\mathrm{I}_{\text {load }}$ is estimated to be $0.75 \cdot 1.080=0.810 \mathrm{pu}$, so $\mathrm{I}_{\mathrm{SC}} / \mathrm{I}_{\text {load }}$ at the PCC is 42.5 , and the corresponding IEEE 519 limit for TDD of current is $8.0 \%$.

The three cases studied are
Case SKIA. No Corrections.
Case SKIB. $30^{\circ}$ phase shifting transformers added at Apollo and BigBoss ASDs.
Case SKIC. Case SKIB, plus 1800 kVAr of filters.
Bracketed values in Figure 9.6 give solved $T H D_{V}$ s for [Case SKIA, Case SKIB, Case SKIC], except at the substation transformer, where $T H D_{I}$ is given directly under Z .

The results for Case SKIA are shown in Figures 9.7 - 9.11. The highest voltage distortion is an unacceptable 14.1\% at Bus \#12, Apollo.

For Case SKIB, wye-delta transformers are added at approximately one-half of the ASD HP, so that a net twelve-pulse operation for the entire ski area is approximated. Results are shown in Figures 9.12-9.14. The highest voltage distortion reduces to 9.6\% at Bus \#12, Apollo.

Case SKIC builds upon Case SKIB by adding the following passive filters:

- 300 kVAr of $5^{\mathrm{th}}$ at Bus\#6, Base.
- 300 kVAr of $5^{\text {th }}$ at Bus\#10, Taylor.
- 300 kVAr of $7^{\text {th }}$ at Bus\#10, Taylor.
- 300 kVAr of $11^{\text {th }}$ at Bus\#12, Apollo.
- 300kVAr of $11^{\text {th }}$ at Bus\#15, BigBoss.
- 300 kVAr of $13^{\text {th }}$ at Bus\#13, Jupiter.

Filter X/R equals 50. The $5^{\text {th }}$ and $7^{\text {th }}$ harmonics have only one-half of the dedicated kVArs because the two wye-delta transformers have already reduced $5^{\text {th }}$ and $7^{\text {th }}$ harmonic voltages. Some $5^{\text {th }}$ and $7^{\text {th }}$ filtering is still needed in case one or both of the wye-delta transformers are out of service (simulations for this contingency were made but are not presented here).

Results for Case SKIC are shown in Figures 9.15 - 9.18. The highest feeder voltage distortion level falls to $2.9 \%$, occurring at Bus \#11, Longs.

A side benefit of the filters is that they correct the ski area power factor from 0.83 to 0.91 , thus providing both a harmonics and power factor solution.

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Figure 9.6: Case 4, Ski Area


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Short Circuit Example. The 6-Bus Stevenson Prob. 6.15

6.15 The one-line diagram of an unloaded power system is shown
. Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are rated as follows:
Generator 1: $\quad 20 \mathrm{MVA}, 13.8 \mathrm{kV}, X^{\prime \prime}=0.20$ per unit
Generator 2: $\quad 30 \mathrm{MVA}, 18 \mathrm{kV}, X^{\prime \prime}=0.20$ per unit
Generator 3: $\quad 30 \mathrm{MVA}, 20 \mathrm{kV}, X^{\prime \prime}=0.20$ per unit
Transformer $T_{1}$ : $25 \mathrm{MVA}, 220 \mathrm{Y} / 13.8 \Delta \mathrm{kV}, X=10 \%$
Transformer $T_{2}$ : Single-phase units each rated $10 \mathrm{MVA}, 127 / 18 \mathrm{kV}, X=10 \%$
Transformer $T_{3}$ : $35 \mathrm{MVA}, 220 \mathrm{Y} / 22 \mathrm{Y} \mathrm{kV}, X=10 \%$
Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. Use a 100 MVA, 220 kv base in the transmission line.

Draw the negative- and zero-sequence impedance networks for the power system
$\square$ The neutrals of generators 1 and 3 are connected to ground through current-limiting reactors having a reactance of $5 \%$, each on the base of the machine to which it is connected. Each generator has negative- and zero-sequence reactances of 20 and $5 \%$, respectively, on its own rating as base. The zero-sequence reactance of the transmission line is $210 \Omega$ from $B$ to $C$ and $250 \Omega$ from $C$ to $E$.

## Short Circuit Calculations with PCFLO

Balanced Three-Phase Fault, Stevenson Prob. 6.15. A three-phase balanced fault, with $\mathrm{Z}_{\mathrm{F}}=0$, occurs at Bus 4. Determine
a. $\quad I_{4 a}^{F}$ (in per unit and in amps)
b. Phasor abc line-to-neutral voltages at the terminals of Gen 1
c. Phasor abc currents flowing out of Gen 1 (in per unit and in amps)

Line to Ground Fault, Stevenson Prob. 6.15. Repeat the above problem using phase a-to-ground fault at Bus 4, again with $\mathrm{Z}_{\mathrm{F}}=0$.

## Short Circuit BDAT_S6.csv (It is best to view these .csv files with Excel)

```
:Six Bus Stevenson Short Circuit Study Case,,,,,,,r,r,,,,,,r,',,,,r,',,
:Bus Data,,,,,',',',,',',',,,'Grounding,Grounding,,,',
:,,,,,,',,,,,,Connection,Positive,Positive,Negative,Negative,Zero,Zero,,Impedance,Impe
dance,Nonlinear,Nonlinear,Nonlinear,,,
:,,,Linear,Linear,Linear,Linear,,Shunt,Maximum,Minimum, , Type
for,Sequence,Sequence,Sequence,Sequence,Sequence,Sequence,Connection, R for, }\textrm{X
for,Device, Device, Device, ,Nonlinear, Linear
:,,,P,Q,P,Q,Desired,Reactive,Q ,Q ,Bus,Remote-
,Shunt,Subtransient,Subtransient,Subtransient,Subtransient,Subtransient,Subtransient,T
ype for,Subtransient,Subtransient, P, P, Displacement,Nonlinear, Device, Load
:Bus,Bus,Bus,Generation,Generation, Load, Load,Voltage,Q
Load, Generation,Generation, Control, Controlled,Reactive, R, X, R, X, R, X, Subtransient, Impeda
nces,Impedances,Gen, Load, Power Factor,Device,Phase Shift,Connection
:Number,Name,Type,(%), (%),(%), (%),(pu),(%),(%),(%),Area,Bus No.,Q
Load,(pu),(pu),(pu),(pu),(pu),(pu),R and X,(pu),(pu),(%),(%),(pu),Type,(Degrees),Type
:(I),(A),(I),(F),(F),(F),(F),(F),(F),(F),(F),(I),(I),(I),(F),(F),(F),(F),(F),(F),(I),(
F),(F),(F),(F),(F),(I),(F),(I)
1,gen#1,1,,,,,1,,,',,,,1,,1,,0.25,1,,0.25,,,,,,
2,gen#2,3,,,\prime\prime\prime\prime,,,,, 0.6667,,0.6667,,0.1667,1,,0,,,,',
3,gen#3,3,,,,,',',,,,,0.551,,0.551,,0.1377,1,,0.1377,,,,,,
4,bus#4,3,,,',',,,',',,,,,',',,,,,
5,bus#5,3,,,\prime\prime\prime\prime\prime,,\prime\prime\prime\prime\prime,,,\prime\prime\prime\prime\prime,
6,bus#6,3,,,',',',',',',',',',',',
```


## Short Circuit LDAT_S6.csv

:Six Bus Stevenson Short Circuit Study Case,, ,, , , , ,, ,, , , , , , , , ,

:,, , , , , , , , , , , at Voltage, ,',,

:,',',',,',',',(pu),,, Connect.,
:,,,'Positive, Positive, Pos/Neg,,,',, ,Voltage-,Voltage-, or Desired
P, Zero, Zero,Zero, Type, Series, Series
:,, , Sequence, Sequence, Sequence, , Minimum, Maximum, Tap, Fixed, Phase, Cont., Cont., for, Sequen
ce, Sequence, Sequence, for Trans., Grounding, Grounding
:From, To, Circuit, R, X, Charging, Rating, Tap, Tap, Step Size, Tap, Shift, Bus, Bus, Phase
Shifter, R, X, Charging, and Shunt, R, X
: Bus, Bus, Number, (pu), (pu), (\%), (\%), (pu), (pu), (pu), (pu), (Degrees), Number, Side, (\%), (pu), ( pu),(\%),Elements, (pu),(pu)
: (I), (I), (I), (F), (F), (F), (F), (F), (F), (F), (F), (F), (I), (I), (F), (F), (F), (F), (I), (F), (F)
1, 4, , 0.4, ,, , 1, -30, , , , 0.4, $8,1,0$
2, 6, , 0.3333, , , , , $1,-30$, , , , $0.3333,8,1,0$
3,5, , 0.2857, , , , , 1, 0, , , , 0.2857, , 2, , 0
4,5,, 0.1653,, , , , , , , , , , 0.4439, , ,


## Short Circuit FREP_S6.txt (This is the output file produced by PCFLO. It gives results for three-phase, line-to-line, and line-to-ground faults at the requested bus \#4)

```
u.t. austin power system engineering
faults version = 6.0
    ", capabilities = 5000 busses, 12500 lines and transformers
```

three phase fault at bus $=4$, name $=$ bus\#4
(per unit impedances in rectangular form)
(per unit voltages and currents in polar form)

| 012 system impedance $($ pu $)=$ |  |
| :---: | :---: |
| $0.00000 \mathrm{E}+00$ | $0.30553 \mathrm{E}+00$ |
| $-0.93794 \mathrm{E}-08$ | $0.44826 \mathrm{E}+00$ |
| $0.93794 \mathrm{E}-08$ | $0.44826 \mathrm{E}+00$ |


| pu) = 0.00000E+00 0.00000E+00 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 012 voltage = | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| abc voltage $=$ | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| 012 current = | 0.00000 | 0.0 | 2.23085 | -90.0 | 0.00000 | 0.0 |
| abc current = | 2.23085 | -90.0 | 2.23085 | 150.0 | 2.23085 | 30.0 |
| from subtransient impedance |  |  |  |  |  |  |
| 012 current = | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| abc current $=$ | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| at neighboring bus = 1, name $=$ gen\#1 |  |  |  |  |  |  |
| 012 voltage = | 0.00000 | 0.0 | 0.28571 | -30.0 | 0.00000 | 0.0 |
| abc voltage = | 0.28571 | -30.0 | 0.28571 | -150.0 | 0.28571 | 90.0 |
| fault contribution from circuit $=0$ at bus = 4 |  |  |  |  |  |  |
| 012 current = | 0.00000 | 0.0 | 0.71429 | -90.0 | 0.00000 | 0.0 |
| abc current | 0.71429 | -90.0 | 0.71429 | 150.0 | 0.71429 | 30.0 |

v -i impedance ratio for circuit $=0$ at bus $=1$
012 impedance ratio $=\quad$ abc impedance ratio $=$

| $0.00000 \mathrm{E}+00$ | $0.00000 \mathrm{E}+00$ | $0.34641 \mathrm{E}+00$ | $0.20000 \mathrm{E}+00$ |
| :--- | :--- | :--- | :--- |
| $0.34641 \mathrm{E}+00$ | $0.20000 \mathrm{E}+00$ | $0.34641 \mathrm{E}+00$ | $0.20000 \mathrm{E}+00$ |
| $0.00000 \mathrm{E}+00$ | $0.00000 \mathrm{E}+00$ | $0.34641 \mathrm{E}+00$ | $0.20000 \mathrm{E}+00$ |


end of three phase fault report

```
line-to-line fault at bus = 4, name = bus#4
(per unit impedances in rectangular form)
(per unit voltages and currents in polar form)
0 1 2 ~ s y s t e m ~ i m p e d a n c e ~ ( p u ) ~ = ~
        0.00000E+00 0.30553E+00
        -0.93794E-08 0.44826E+00
        0.93794E-08 0.44826E+00
fault impedance (pu) = 0.00000E+00 0.00000E+00
```


end of line-to-line fault report

```
line-to-ground fault at bus = 4, name = bus#4
(per unit impedances in rectangular form)
(per unit voltages and currents in polar form)
```

| 012 system impedance $($ pu $)=$ |  |
| :---: | :---: |
| $0.00000 \mathrm{E}+00$ | $0.30553 \mathrm{E}+00$ |
| $-0.93794 \mathrm{E}-08$ | $0.44826 \mathrm{E}+00$ |
| $0.93794 \mathrm{E}-08$ | $0.44826 \mathrm{E}+00$ |


| fault impedance (pu) | 0.0 | 000E+00 | $0.00000 \mathrm{E}+$ + |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 012 voltage = | 0.25418 | 180.0 | 0.62709 | 0.0 | 0.37291 | 180.0 |
| abc voltage = | 0.00000 | 0.0 | 0.94624 | -113.8 | 0.94624 | 113.8 |
| 012 current = | 0.83191 | -90.0 | 0.83191 | -90.0 | 0.83191 | -90.0 |
| abc current = | 2.49574 | -90.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| from subtransient impedance |  |  |  |  |  |  |
| 012 current = | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| abc current $=$ | 0.00000 | 0.0 | 0.00000 | 0.0 | 0.00000 | 0.0 |
| at neighboring bus = 1, name $=$ gen\#1 |  |  |  |  |  |  |
| 012 voltage = | 0.00000 | 0.0 | 0.73363 | -30.0 | 0.26637 | -150.0 |
| abc voltage = | 0.64324 | -51.0 | 0.64324 | -129.0 | 1.00000 | 90.0 |
| fault contribution from circuit $=0$ at bus = 4 |  |  |  |  |  |  |
| 012 current = | 0.63544 | -90.0 | 0.26637 | -90.0 | 0.26637 | -90.0 |
| abc current $=$ | 1.16817 | -90.0 | 0.36907 | -90.0 | 0.36907 | -90.0 |

v -i impedance ratio for circuit $=0$ at bus $=1$ 012 impedance ratio $=\quad$ abc impedance ratio $=$

```
\begin{tabular}{lrrr}
\(0.00000 \mathrm{E}+00\) & \(0.00000 \mathrm{E}+00\) & \(0.10838 \mathrm{E}+01\) & \(0.87712 \mathrm{E}+00\) \\
\(0.23852 \mathrm{E}+01\) & \(0.13771 \mathrm{E}+01\) & \(0.00000 \mathrm{E}+00\) & \(0.00000 \mathrm{E}+00\) \\
\(0.86603 \mathrm{E}+00\) & \(-0.50000 \mathrm{E}+00\) & \(0.21675 \mathrm{E}+01\) & \(-0.76641 \mathrm{E}-07\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{5}{|l|}{5, name = bus\#5} \\
\hline 012 voltage \(=00.16696180 .0\) & 0.72057 & 0.0 & 0.27943 & 180.0 \\
\hline abc voltage \(=00.274180 .0\) & 0.94878 & -114.1 & 0.94878 & 114.1 \\
\hline fault contribution from circuit = & 0 at bus \(=\) & 4 & & \\
\hline 012 current \(=00.19647-90.0\) & 0.56555 & -90.0 & 0.56555 & -90.0 \\
\hline abc current \(=\quad 1.32756-90.0\) & 0.36907 & 90.0 & 0.36907 & 90.0 \\
\hline v-i impedance ratio for circuit = & 0 at bus = & 5 & & \\
\hline 012 impedance ratio = & abc impedance & ratio = & & \\
\hline -0.35902E-24 -0.84980E+00 & -0.16713E-23 & \(0.20653 \mathrm{E}+00\) & & \\
\hline 0.30662E-07 0.12741E+01 & -0.23465E+01 & \(0.10500 \mathrm{E}+01\) & & \\
\hline -0.37675E-08 -0.49408E+00 & \(0.23465 \mathrm{E}+01\) & \(0.10500 \mathrm{E}+01\) & & \\
\hline
\end{tabular}
```


## Introduction

HAPS was developed for the Office of Naval Research to perform harmonics analysis on industrial voltage networks with small distances between busses, for example a few 100 meters. HAPS computes harmonic distortion levels, allows the user to simulate the effectiveness of filters, and performs IEEE 519 (harmonics standard) compliance checks for ships and other tightly connected three-phase power systems such as industrial facilities.

The fundamental assumptions in HAPS are that

1. Distances between loads and generators in the system are short (e.g., no more than a few hundred meters) so that one bus adequately represents the distribution system.
2. Harmonic loads are current injectors with known spectra.
3. The fundamental bus voltage magnitude is 1.0 per unit.

Zero-sequence harmonics (i.e., triplens) are excluded in this analysis since single-phase distorting loads are not usually major contributors to harmonic problems except in residential load areas or office buildings.

## The Interface Screen

The interface screen is shown in Figure 1. A brief description of the information shown in the figure follows, where the numbered items below are keyed to Figure 1.

1. System impedance, load, and capacitor data are entered via the slide bars.
2. Cases are saved and recalled.
3. Filters are added, and then listed along with their losses.
4. Impedance scans are given for system plus linear load, system plus linear load and capacitors, and system plus linear load, capacitors, and filters.
5. Capacitor voltage and current waveforms for the selected filter are shown.
6. Nonlinear loads are chosen from six typical types. User spectra can also be entered (see Item 9 below and Figure 2).
7. Time-domain plots show a. net or individual nonlinear load current, b. total current produced by the source, c. bus voltage, and d. the spectrum magnitude plot for the selected time-domain plot.
8. IEEE 519 harmonics standard computations and possible violations are noted.
9. A pop-up menu (see Figure 2) allows the user to enter any load spectrum, and then save and recall it for future use.

## The Example Case

HAPS, including the example case shown in Figure 1, can be run as follows:

1. Copy four zipped files in HAPS_Version1.zip into a directory on your hard drive. The zipped files are HAPS.exe, A3_HAPS_USERMAN.pdf, demo.haps, and big_5th.spec
2. Click on HAPS.exe
3. In the lower-left portion of the interface screen, click on file name "demo.haps," and then click "Recall."

Once loaded, you can view the spectrum of the load current, source current, and voltage waveforms by clicking them. Next, you can modify system parameters, add loads, and add filters and see the impact on waveforms and distortion immediately.

To see the user spectra feature, click on "User Spectra" in the nonlinear load section (top right). When the pop-up screen appears, click on file "big_5th.spec," and then click "Recall Spectrum."

## A3_HAPS_USERMAN.doc

## April 2012



Figure 1. HAPS User Interface Screen


Figure 2. Pop-Up Menu to Enter/Save/Recall User Spectra

