CHAPTER 7

THE SOLAR RESOURCE

To design and analyze solar systems, we need to know how much sunlight is available. A fairly straightforward, though complicated-looking, set of equations can be used to predict where the sun is in the sky at any time of day for any location on earth, as well as the solar intensity (or *insolation: in*cident *sol*ar Radi*ation*) on a clear day. To determine average daily insolation under the combination of clear and cloudy conditions that exist at any site we need to start with long-term measurements of sunlight hitting a horizontal surface. Another set of equations can then be used to estimate the insolation on collector surfaces that are not flat on the ground.

7.1 THE SOLAR SPECTRUM

The source of insolation is, of course, the sun—that gigantic, 1.4 million kilometer diameter, thermonuclear furnace fusing hydrogen atoms into helium. The resulting loss of mass is converted into about 3.8×10^{20} MW of electromagnetic energy that radiates outward from the surface into space.

Every object emits radiant energy in an amount that is a function of its temperature. The usual way to describe how much radiation an object emits is to compare it to a theoretical abstraction called a *blackbody*. A blackbody is defined to be a perfect emitter as well as a perfect absorber. As a perfect emitter, it radiates more energy per unit of surface area than any real object at the same temperature. As a perfect absorber, it absorbs all radiation that impinges upon it; that is, none

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is reflected and none is transmitted through it. The wavelengths emitted by a blackbody depend on its temperature as described by *Planck's law*:

$$E_\lambda = \frac{3.74 \times 10^8}{\lambda^5 \left[\exp\left(\frac{14{,}400}{\lambda T}\right) - 1\right]} \tag{7.1}$$

where E_λ is the emissive power per unit area of a blackbody (W/m^2 μm), T is the absolute temperature of the body (K), and λ is the wavelength (μm).

Modeling the earth itself as a 288 K (15°C) blackbody results in the emission spectrum plotted in Fig. 7.1.

The area under Planck's curve between any two wavelengths is the power emitted between those wavelengths, so the total area under the curve is the total radiant power emitted. That total is conveniently expressed by the *Stefan–Boltzmann law of radiation*:

$$E = A\sigma T^4 \tag{7.2}$$

where E is the total blackbody emission rate (W), σ is the Stefan–Boltzmann constant = 5.67×10^{-8} W/m^2-K^4, T is the absolute temperature of the blackbody (K), and A is the surface area of the blackbody (m^2).

Another convenient feature of the blackbody radiation curve is given by *Wien's displacement rule*, which tells us the wavelength at which the spectrum reaches its maximum point:

$$\lambda_{max}(\mu m) = \frac{2898}{T(K)} \tag{7.3}$$

where the wavelength is in microns (μm) and the temperature is in kelvins.

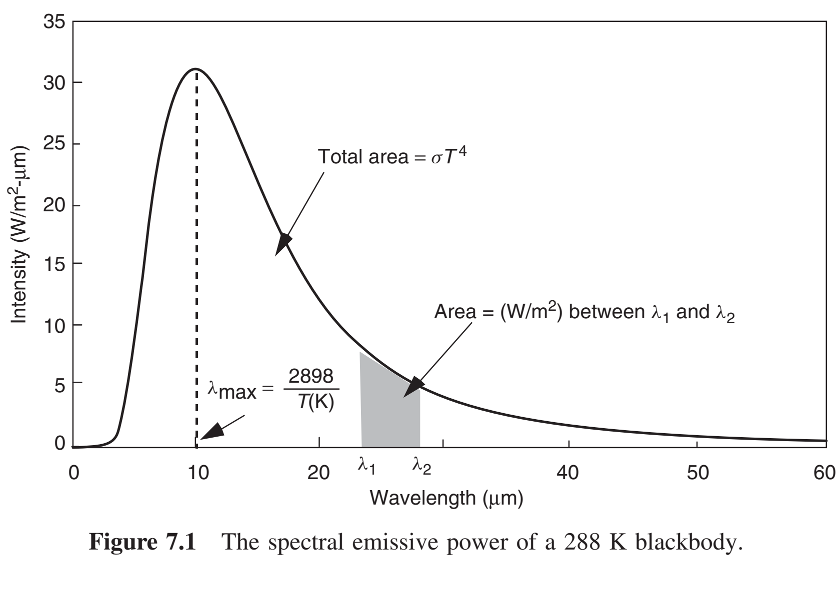


Figure 7.1 The spectral emissive power of a 288 K blackbody.

Example 7.1 The Earth's Spectrum. Consider the earth to be a blackbody with average surface temperature 15°C and area equal to 5.1×10^{14} m^2. Find the rate at which energy is radiated by the earth and the wavelength at which maximum power is radiated. Compare this peak wavelength with that for a 5800 K blackbody (the sun).

Solution. Using (7.2), the earth radiates:

$$E = \sigma A T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4) \times (5.1 \times 10^{14}\ \text{m}^2) \times (15 + 273\ \text{K})^4$$
$$= 2.0 \times 10^{17}\ \text{W}$$

The wavelength at which the maximum power is emitted is given by (5.3):

$$\lambda_{\text{max}}(\text{earth}) = \frac{2898}{T(\text{K})} = \frac{2898}{288} = 10.1\ \mu\text{m}$$

For the 5800 K sun,

$$\lambda_{\text{max}}(\text{sun}) = \frac{2898}{5800} = 0.5\ \mu\text{m}$$

It is worth noting that earth's atmosphere reacts very differently to the much longer wavelengths emitted by the earth's surface (Fig. 7.1) than it does to the short wavelengths arriving from the sun (Fig. 7.2). This difference is the fundamental factor responsible for the greenhouse effect.

While the interior of the sun is estimated to have a temperature of around 15 million kelvins, the radiation that emanates from the sun's surface has a spectral distribution that closely matches that predicted by Planck's law for a 5800 K blackbody. Figure 7.2 shows the close match between the actual solar spectrum and that of a 5800 K blackbody. The total area under the blackbody curve has been scaled to equal 1.37 kW/m^2, which is the solar insolation just outside the earth's atmosphere. Also shown are the areas under the actual solar spectrum that corresponds to wavelengths within the ultraviolet UV (7%), visible (47%), and infrared IR (46%) portions of the spectrum. The visible spectrum, which lies between the UV and IR, ranges from 0.38 μm (violet) to 0.78 μm (red).

As solar radiation makes its way toward the earth's surface, some of it is absorbed by various constituents in the atmosphere, giving the terrestrial spectrum an irregular, bumpy shape. The terrestrial spectrum also depends on how much atmosphere the radiation has to pass through to reach the surface. The length of the path h_2 taken by the sun's rays as they pass through the atmosphere, divided by the minimum possible path length h_1, which occurs when

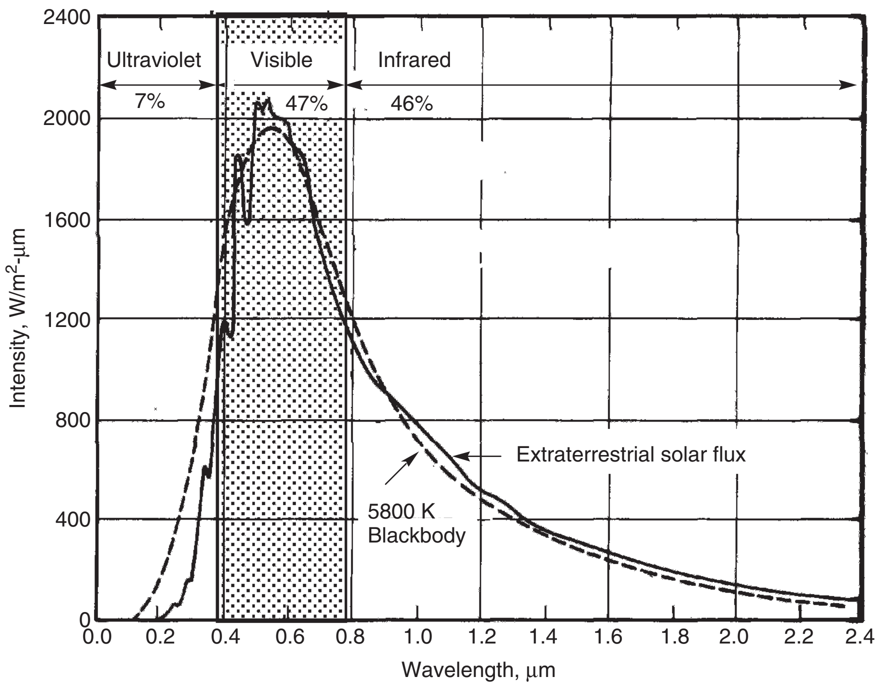


Figure 7.2 The extraterrestrial solar spectrum compared with a 5800 K blackbody.

the sun is directly overhead, is called the *air mass ratio*, m. As shown in Figure 7.3, under the simple assumption of a flat earth the air mass ratio can be expressed as

$$\text{Air mass ratio} \quad m = \frac{h_2}{h_1} = \frac{1}{\sin\beta} \tag{7.4}$$

where h_1 = path length through the atmosphere with the sun directly overhead, h_2 = path length through the atmosphere to reach a spot on the surface, and β = the altitude angle of the sun (see Fig. 7.3).
Thus, an air mass ratio of 1 (designated "AM1") means that the sun is directly overhead. By convention, AM0 means no atmosphere; that is, it is the extraterrestrial solar spectrum. Often, an air mass ratio of 1.5 is assumed for an average solar spectrum at the earth's surface. With AM1.5, 2% of the incoming solar energy is in the UV portion of the spectrum, 54% is in the visible, and 44% is in the infrared.

The impact of the atmosphere on incoming solar radiation for various air mass ratios is shown in Fig. 7.4. As sunlight passes through more atmosphere, less energy arrives at the earth's surface and the spectrum shifts some toward longer wavelengths.

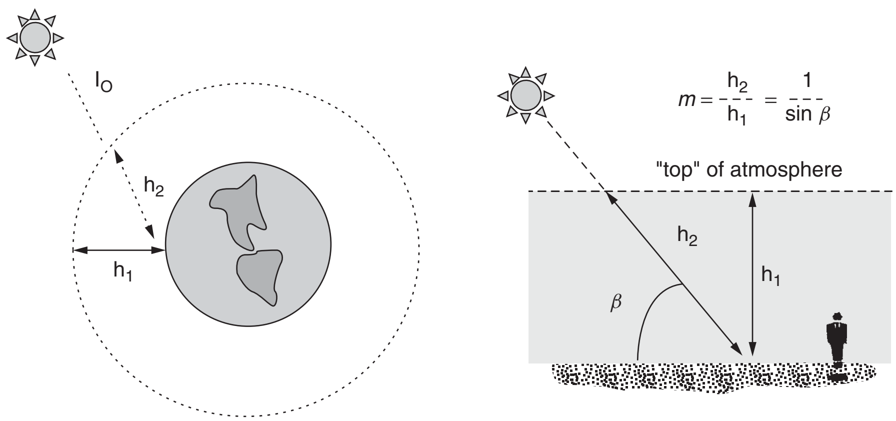


Figure 7.3 The air mass ratio m is a measure of the amount of atmosphere the sun's rays must pass through to reach the earth's surface. For the sun directly overhead, $m = 1$.

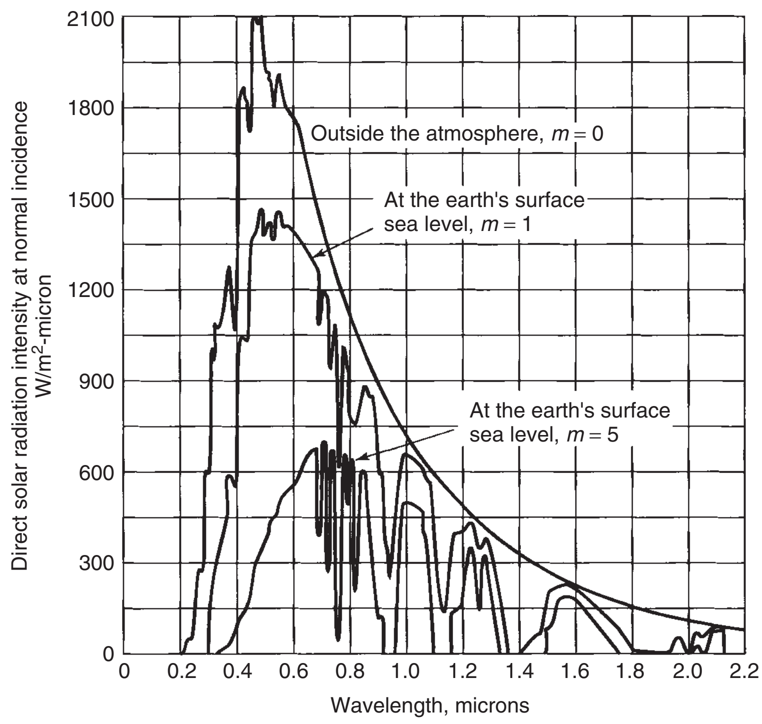


Figure 7.4 Solar spectrum for extraterrestrial ($m = 0$), for sun directly overhead ($m = 1$), and at the surface with the sun low in the sky, $m = 5$. From Kuen et al. (1998), based on *Trans. ASHRAE*, vol. 64 (1958), p. 50.

7.2 THE EARTH'S ORBIT

The earth revolves around the sun in an elliptical orbit, making one revolution every 365.25 days. The eccentricity of the ellipse is small and the orbit is, in fact, quite nearly circular. The point at which the earth is nearest the sun, the perihelion, occurs on January 2, at which point it is a little over 147 million kilometers away. At the other extreme, the aphelion, which occurs on July 3, the earth is about 152 million kilometers from the sun. This variation in distance is described by the following relationship:

$$d = 1.5 \times 10^8 \left\{1 + 0.017 \sin\left[\frac{360(n - 93)}{365}\right]\right\} \text{ km} \tag{7.5}$$

where n is the *day number*, with January 1 as day 1 and December 31 being day number 365. Table 7.1 provides a convenient list of day numbers for the first day of each month. *It should be noted that (7.5) and all other equations developed in this chapter involving trigonometric functions use angles measured in degrees, not radians*.

Each day, as the earth rotates about its own axis, it also moves along the ellipse. If the earth were to spin only 360° in a day, then after 6 months time our clocks would be off by 12 hours; that is, at noon on day 1 it would be the middle of the day, but 6 months later noon would occur in the middle of the night. To keep synchronized, the earth needs to rotate one extra turn each year, which means that in a 24-hour day the earth actually rotates 360.99°, which is a little surprising to most of us.

As shown in Fig. 7.5, the plane swept out by the earth in its orbit is called the ecliptic plane. The earth's spin axis is currently tilted 23.45° with respect to the ecliptic plane and that tilt is, of course, what causes our seasons. On March 21 and September 21, a line from the center of the sun to the center of the earth passes through the equator and everywhere on earth we have 12 hours of daytime and 12 hours of night, hence the term *equinox* (equal day and night). On December 21, the winter *solstice* in the Northern Hemisphere, the inclination of the North Pole reaches its highest angle away from the sun (23.45°), while on June 21 the opposite occurs. By the way, for convenience we are using the

TABLE 7.1 Day Numbers for the First Day of Each Month

January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
May	$n = 121$	November	$n = 305$
June	$n = 152$	December	$n = 335$

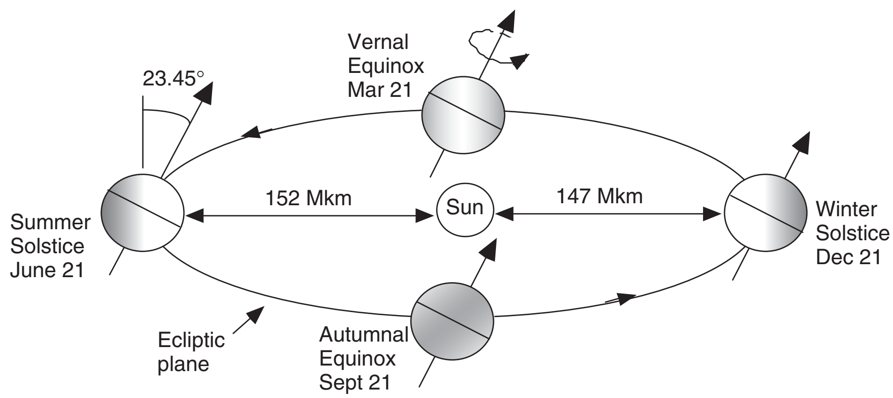


Figure 7.5 The tilt of the earth's spin axis with respect to the ecliptic plane is what causes our seasons. "Winter" and "summer" are designations for the solstices in the Northern Hemisphere.

twenty-first day of the month for the solstices and equinoxes even though the actual days vary slightly from year to year.

For solar energy applications, the characteristics of the earth's orbit are considered to be unchanging, but over longer periods of time, measured in thousands of years, orbital variations are extremely important as they significantly affect climate. The shape of the orbit oscillates from elliptical to more nearly circular with a period of 100,000 years (*eccentricity*). The earth's tilt angle with respect to the ecliptic plane fluctuates from 21.5° to 24.5° with a period of 41,000 years (*obliquity*). Finally, there is a 23,000-year period associated with the *precession* of the earth's spin axis. This precession determines, for example, where in the earth's orbit a given hemisphere's summer occurs. Changes in the orbit affect the amount of sunlight striking the earth as well as the distribution of sunlight both geographically and seasonally. Those variations are thought to be influential in the timing of the coming and going of ice ages and interglacial periods. In fact, careful analysis of the historical record of global temperatures does show a primary cycle between glacial episodes of about 100,000 years, mixed with secondary oscillations with periods of 23,000 years and 41,000 years that match these orbital changes. This connection between orbital variations and climate were first proposed in the 1930s by an astronomer named Milutin Milankovitch, and the orbital cycles are now referred to as *Milankovitch oscillations*. Sorting out the impact of human activities on climate from those caused by natural variations such as the Milankovitch oscillations is a critical part of the current climate change discussion.

7.3 ALTITUDE ANGLE OF THE SUN AT SOLAR NOON

We all know that the sun rises in the east and sets in the west and reaches its highest point sometime in the middle of the day. In many situations, it is quite useful to be able to predict exactly where in the sky the sun will be at any time, at any location on any day of the year. Knowing that information we can, for example, design an overhang to allow the sun to come through a window to help heat a house in the winter while blocking the sun in the summer. In the context of photovoltaics, we can, for example, use knowledge of solar angles to help pick the best tilt angle for our modules to expose them to the greatest insolation.

While Fig. 7.5 correctly shows the earth revolving around the sun, it is a difficult diagram to use when trying to determine various solar angles as seen from the surface of the earth. An alternative (and ancient!) perspective is shown in Fig. 7.6, in which the earth is fixed, spinning around its north–south axis; the sun sits somewhere out in space, slowly moving up and down as the seasons progress. On June 21 (the summer solstice) the sun reaches its highest point, and a ray drawn at that time from the center of the sun to the center of the earth makes an angle of 23.45° with the earth's equator. On that day, the sun is directly over the Tropic of Cancer at latitude 23.45°. At the two equinoxes, the sun is directly over the equator. On December 21 the sun is 23.45° below the equator, which defines the latitude known as the Tropic of Capricorn.

As shown in Fig. 7.6, the angle formed between the plane of the equator and a line drawn from the center of the sun to the center of the earth is called the solar declination, δ. It varies between the extremes of $\pm$ 23.45°, and a simple sinusoidal relationship that assumes a 365-day year and which puts the spring equinox on day $n = 81$ provides a very good approximation. Exact values of declination, which vary slightly from year to year, can be found in the annual publication *The American Ephemeris and Nautical Almanac*.

$$\delta = 23.45 \sin\left[\frac{360}{365}(n - 81)\right] \tag{7.6}$$

Computed values of solar declination on the twenty-first day of each month are given in Table 7.2.

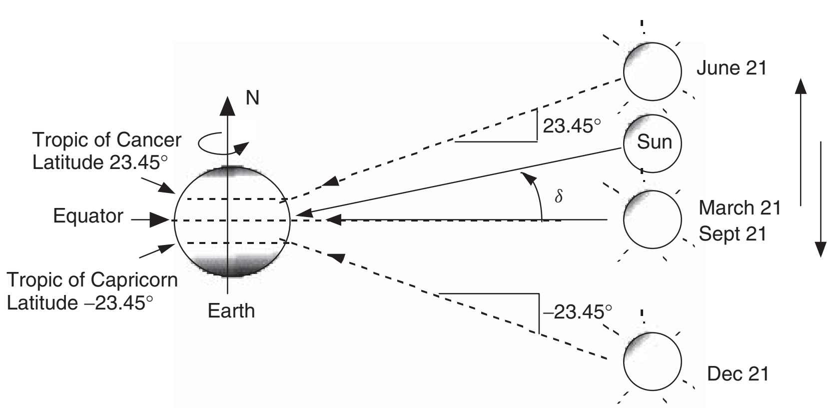


Figure 7.6 An alternative view with a fixed earth and a sun that moves up and down. The angle between the sun and the equator is called the solar declination δ.

TABLE 7.2 Solar Declination δ for the 21st Day of Each Month (degrees)

Month:	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
δ:	−20.1	−11.2	0.0	11.6	20.1	23.4	20.4	11.8	0.0	−11.8	−20.4	−23.4

While Fig. 7.6 doesn't capture the subtleties associated with the earth's orbit, it is entirely adequate for visualizing various latitudes and solar angles. For example, it is easy to understand the seasonal variation of daylight hours. As suggested in Fig. 7.7, during the summer solstice all of the earth's surface above latitude 66.55° (90° − 23.45°) basks in 24 hours of daylight, while in the Southern Hemisphere below latitude 66.55° it is continuously dark. Those latitudes, of course, correspond to the Arctic and Antarctic Circles.

It is also easy to use Fig. 7.6 to gain some intuition into what might be a good tilt angle for a solar collector. Figure 7.8 shows a south-facing collector on the earth's surface that is tipped up at an angle equal to the local latitude, L. As can be seen, with this tilt angle the collector is parallel to the axis of the earth. During an equinox, at *solar noon*, when the sun is directly over the local meridian (line of longitude), the sun's rays will strike the collector at the best possible angle; that is, they are perpendicular to the collector face. At other times of the year the sun is a little high or a little low for normal incidence, but on the average it would seem to be a good tilt angle.

Solar noon is an important reference point for almost all solar calculations. In the Northern Hemisphere, at latitudes above the Tropic of Cancer, solar noon occurs when the sun is due south of the observer. South of the Tropic of Capricorn, in New Zealand for example, it is when the sun is due north. And in the tropics, the sun may be either due north, due south, or directly overhead at solar noon.

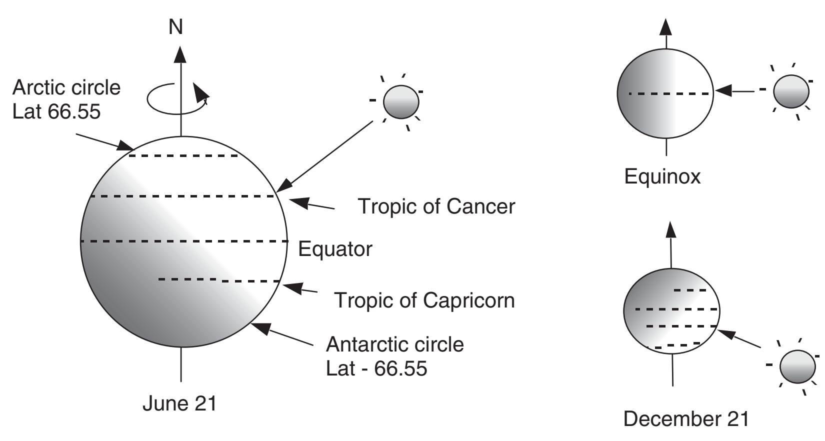


Figure 7.7 Defining the earth's key latitudes is easy with the simple version of the earth−sun system.

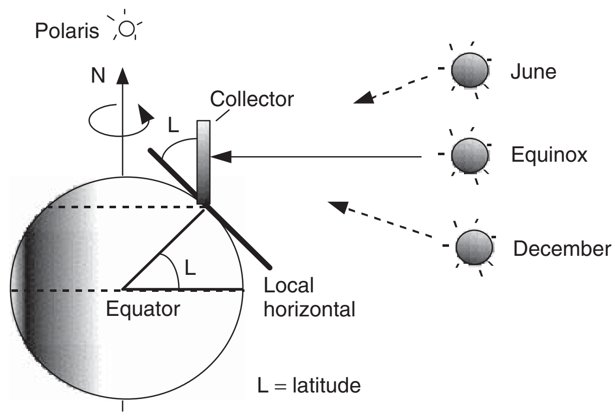


Figure 7.8 A south-facing collector tipped up to an angle equal to its latitude is perpendicular to the sun's rays at solar noon during the equinoxes.

On the average, facing a collector toward the equator (for most of us in the Northern Hemisphere, this means facing it south) and tilting it up at an angle equal to the local latitude is a good rule-of-thumb for annual performance. Of course, if you want to emphasize winter collection, you might want a slightly higher angle, and vice versa for increased summer efficiency.

Having drawn the earth-sun system as shown in Fig. 7.6 also makes it easy to determine a key solar angle, namely the *altitude angle* β_N of the sun at solar noon. The altitude angle is the angle between the sun and the local horizon directly beneath the sun. From Fig. 7.9 we can write down the following relationship by inspection:

$$\beta_N = 90^\circ - L + \delta \tag{7.7}$$

where L is the latitude of the site. Notice in the figure the term *zenith* is introduced, which refers to an axis drawn directly overhead at a site.

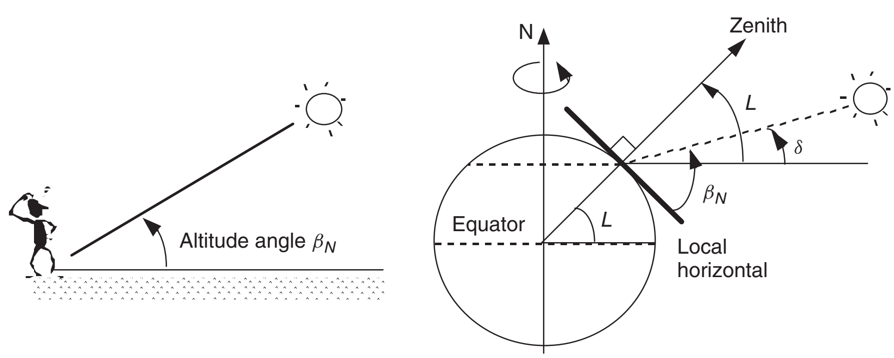


Figure 7.9 The altitude angle of the sun at solar noon.

Example 7.2 Tilt Angle of a PV Module. Find the optimum tilt angle for a south-facing photovoltaic module in Tucson (latitude 32.1°) at solar noon on March 1.

Solution. From Table 7.1, March 1 is the sixtieth day of the year so the solar declination (7.6) is

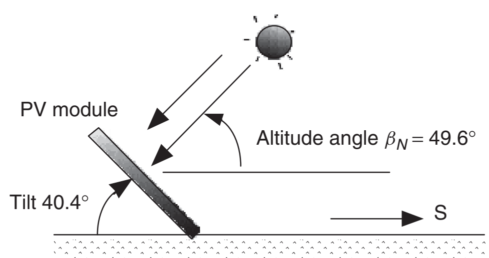
$$\delta = 23.45 \sin\left[\frac{360}{365}(n - 81)\right] = 23.45^\circ \sin\left[\frac{360}{365}(60 - 81)^\circ\right] = -8.3^\circ$$

which, from (7.7), makes the altitude angle of the sun equal to

$$\beta_N = 90^\circ - L + \delta = 90 - 32.1 - 8.3 = 49.6^\circ$$

The tilt angle that would make the sun's rays perpendicular to the module at noon would therefore be

$$\text{Tilt} = 90 - \beta_N = 90 - 49.6 = 40.4^\circ$$



7.4 SOLAR POSITION AT ANY TIME OF DAY

The location of the sun at any time of day can be described in terms of its altitude angle β and its azimuth angle ϕ_s as shown in Fig. 7.10. The subscript s in the azimuth angle helps us remember that this is the azimuth angle of the sun. Later, we will introduce another azimuth angle for the solar collector and a different subscript c will be used. *By convention, the azimuth angle is positive in the morning with the sun in the east and negative in the afternoon with the sun in the west.* Notice that the azimuth angle shown in Fig. 7.10 uses true south as its reference, and this will be the assumption in this text unless otherwise stated. For solar work in the Southern Hemisphere, azimuth angles are measured relative to north.

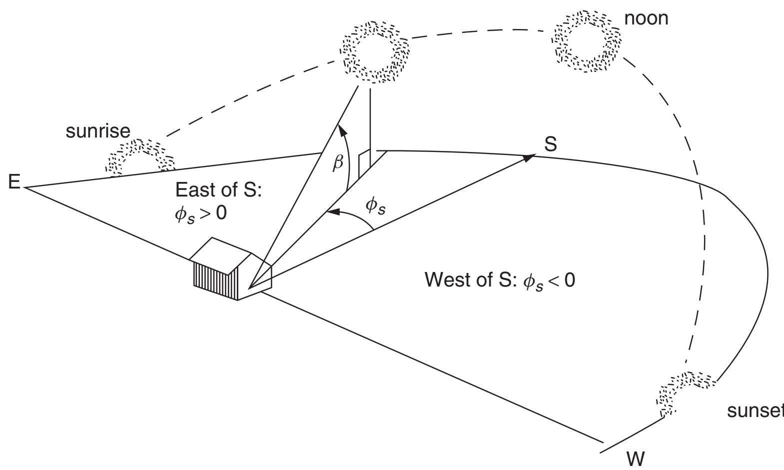


Figure 7.10 The sun's position can be described by its altitude angle β and its azimuth angle ϕ_S. By convention, the azimuth angle is considered to be positive before solar noon.

The azimuth and altitude angles of the sun depend on the latitude, day number, and, most importantly, the time of day. For now, we will express time as the number of hours before or after solar noon. Thus, for example, 11 A.M. *solar time* is one hour before the sun crosses your local meridian (due south for most of us). Later we will learn how to make the adjustment between solar time and local clock time. The following two equations allow us to compute the altitude and azimuth angles of the sun. For a derivation see, for example, T. H. Kuen et al. (1998):

$$\sin\beta = \cos L \cos\delta \cos H + \sin L \sin\delta \tag{7.8}$$

$$\sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta} \tag{7.9}$$

Notice that time in these equations is expressed by a quantity called the *hour angle*, H. The hour angle is the number of degrees that the earth must rotate before the sun will be directly over your local meridian (line of longitude). As shown in Fig. 7.11, at any instant, the sun is directly over a particular line of longitude, called the sun's meridian. The difference between the local meridian and the sun's meridian is the hour angle, with positive values occurring in the morning before the sun crosses the local meridian.

Considering the earth to rotate 360° in 24 h, or 15°/h, the hour angle can be described as follows:

$$\text{Hour angle } H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot \text{(hours before solar noon)} \tag{7.10}$$

Thus, the hour angle H at 11:00 A.M. solar time would be $+15^\circ$ (the earth needs to rotate another 15°, or 1 hour, before it is solar noon). In the afternoon, the hour angle is negative, so, for example, at 2:00 P.M. solar time H would be -30°.

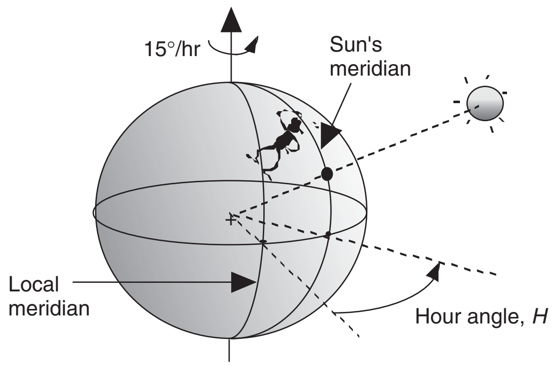


Figure 7.11 The hour angle is the number of degrees the earth must turn before the sun is directly over the local meridian. It is the difference between the sun's meridian and the local meridian.

There is a slight complication associated with finding the azimuth angle of the sun from (7.9). During spring and summer in the early morning and late afternoon, the magnitude of the sun's azimuth is liable to be more than 90° away from south (that never happens in the fall and winter). Since the inverse of a sine is ambiguous, $\sin x = \sin(180 - x)$, we need a test to determine whether to conclude the azimuth is greater than or less than 90° away from south. Such a test is

$$\text{if} \quad \cos H \geq \frac{\tan\delta}{\tan L}, \qquad \text{then } |\phi_S| \leq 90^\circ; \qquad \text{otherwise } |\phi_S| > 90^\circ \tag{7.11}$$

Example 7.3 Where Is the Sun? Find the altitude angle and azimuth angle for the sun at 3:00 P.M. solar time in Boulder, Colorado (latitude 40°) on the summer solstice.

Solution. Since it is the solstice we know, without computing, that the solar declination δ is 23.45°. Since 3:00 P.M. is three hours after solar noon, from (7.10) we obtain

$$H = \left(\frac{15^\circ}{\text{h}}\right) \cdot (\text{hours before solar noon}) = \frac{15^\circ}{\text{h}} \cdot (-3\ \text{h}) = -45^\circ$$

Using (7.8), the altitude angle is

$$\begin{aligned} \sin\beta &= \cos L \cos\delta \cos H + \sin L \sin\delta \\ &= \cos 40^\circ \cos 23.45^\circ \cos(-45^\circ) + \sin 40^\circ \sin 23.45^\circ = 0.7527 \\ \beta &= \sin^{-1}(0.7527) = 48.8^\circ \end{aligned}$$

From (7.9) the sine of the azimuth angle is

$$\sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta}$$
$$= \frac{\cos 23.45^\circ \cdot \sin(-45^\circ)}{\cos 48.8^\circ} = -0.9848$$

But the arcsine is ambiguous and two possibilities exist:

$$\phi_S = \sin^{-1}(-0.9848) = -80^\circ \qquad (80^\circ \text{west of south})$$
$$\text{or} \quad \phi_S = 180 - (-80) = 260^\circ \qquad (100^\circ \text{west of south})$$

To decide which of these two options is correct, we apply (7.11):

$$\cos H = \cos(-45^\circ) = 0.707 \qquad \text{and} \qquad \frac{\tan\delta}{\tan L} = \frac{\tan 23.45^\circ}{\tan 40^\circ} = 0.517$$

Since $\cos H \geq \dfrac{\tan\delta}{\tan L}$ we conclude that the azimuth angle is

$$\phi_S = -80^\circ \qquad (80^\circ \text{west of south})$$

Solar altitude and azimuth angles for a given latitude can be conveniently portrayed in graphical form, an example of which is shown in Fig. 7.12. Similar sun path diagrams for other latitudes are given in Appendix B. As can be seen, in the spring and summer the sun rises and sets slightly to the north and our need for the azimuth test given in (7.11) is apparent; at the equinoxes, it rises and sets precisely due east and due west (everywhere on the planet); during the fall and winter the azimuth angle of the sun is never greater than 90°.

7.5 SUN PATH DIAGRAMS FOR SHADING ANALYSIS

Not only do sun path diagrams, such as that shown in Fig. 7.12, help to build one's intuition into where the sun is at any time, they also have a very practical application in the field when trying to predict shading patterns at a site—a very important consideration for photovoltaics, which are very shadow sensitive. The concept is simple. What is needed is a sketch of the azimuth and altitude angles for trees, buildings, and other obstructions along the southerly horizon that can be drawn on top of a sun path diagram. Sections of the sun path diagram that are covered by the obstructions indicate periods of time when the sun will be behind the obstruction and the site will be shaded.

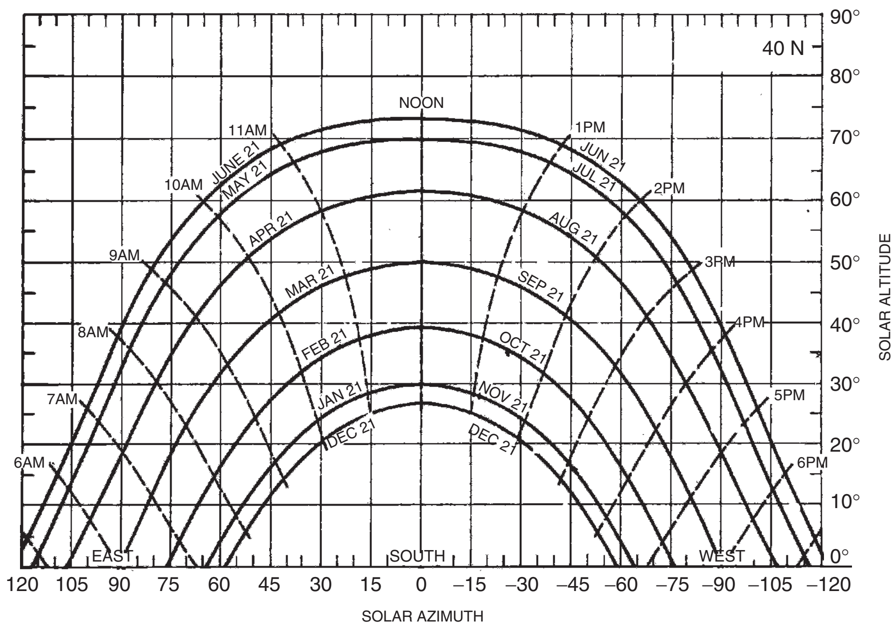


Figure 7.12 A sun path diagram showing solar altitude and azimuth angles for 40° latitude. Diagrams for other latitudes are in Appendix B.

There are several site assessment products available on the market that make the superposition of obstructions onto a sun path diagram pretty quick and easy to obtain. You can do just as good a job, however, with a simple compass, plastic protractor, and plumb bob, but the process requires a little more effort. The compass is used to measure azimuth angles of obstructions, while the protractor and plumb bob measure altitude angles.

Begin by tying the plumb bob onto the protractor so that when you sight along the top edge of the protractor the plumb bob hangs down and provides the altitude angle of the top of the obstruction. Figure 7.13 shows the idea. By standing at the site and scanning the southerly horizon, the altitude angles of major obstructions can be obtained reasonably quickly and quite accurately.

The azimuth angles of obstructions, which go along with their altitude angles, are measured using a compass. Remember, however, that a compass points to magnetic north rather than true north; this difference, called the magnetic declination or deviation, must be corrected for. In the continental United States, this deviation ranges anywhere from about 22°E in Seattle (the compass points 22° east of true north), to essentially zero along the east coast of Florida, to 22 °W at the northern tip of Maine. Figure 7.14 shows this variation in magnetic declination and shows an example, for San Francisco, of how to use it.

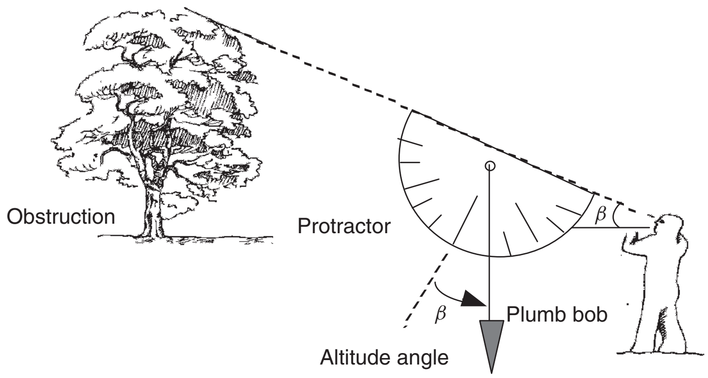


Figure 7.13 Measuring the altitude angle of a southerly obstruction using a plumb bob and protractor.

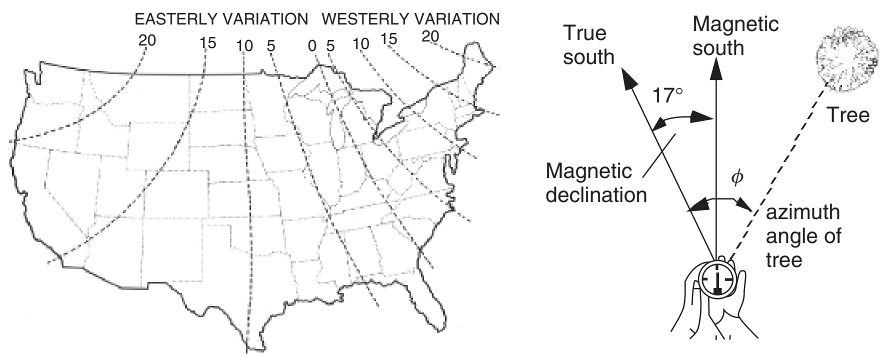


Figure 7.14 Lines of equal magnetic declination across the United States. The example shows the correction for San Francisco, which has a declination of 17°E.

Figure 7.15 shows an example of how the sun path diagram, with a superimposed sketch of potential obstructions, can be interpreted. The site is a proposed solar house with a couple of trees to the southeast and a small building to the southwest. In this example, the site receives full sun all day long from February through October. From November through January, the trees cause about one hour's worth of sun to be lost from around 8:30 A.M. to 9:30 A.M., and the small building shades the site after about 3 o'clock in the afternoon.

When obstructions plotted on a sun path diagram are combined with hour-by-hour insolation information, an estimate can be obtained of the energy lost due to shading. Table 7.3 shows an example of the hour-by-hour insolations available on a clear day in January at 40° latitude for south-facing collectors with fixed tilt angle, or for collectors mounted on 1-axis or 2-axis tracking systems. Later in this chapter, the equations that were used to compute this table will be presented, and in Appendix C there is a full set of such tables for a number of latitudes.

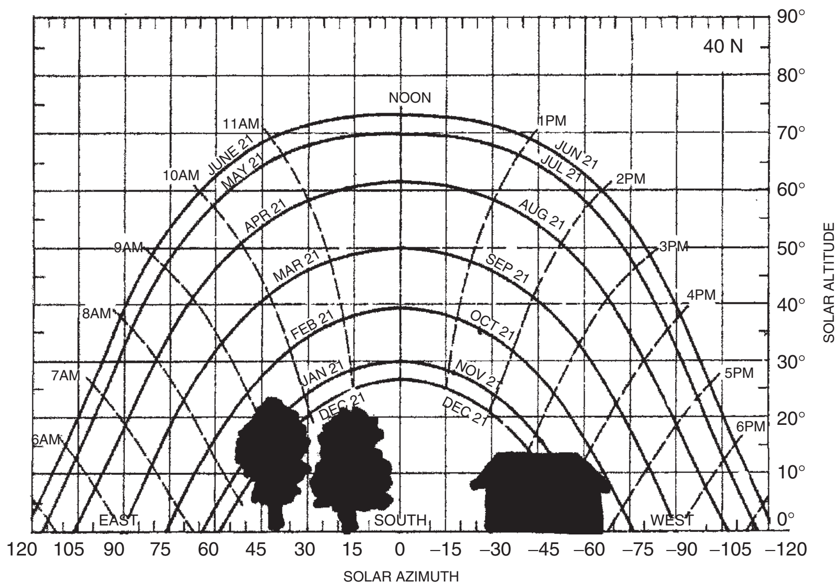


Figure 7.15 A sun path diagram with superimposed obstructions makes it easy to estimate periods of shading at a site.

TABLE 7.3 Clear Sky Beam Plus Diffuse Insolation at 40° Latitude in January for South-Facing Collectors with Fixed Tilt Angle and for Tracking Mounts (hourly W/m^2 and daily kWh/m^2-day)[a]

Solar Time	Tracking		Tilt Angles					Latitude 40°	
	One-Axis	Two-Axis	0	20	30	40	50	60	90
			January 21					(W/m^2)	
7, 5	0	0	0	0	0	0	0	0	0
8, 4	439	462	87	169	204	232	254	269	266
9, 3	744	784	260	424	489	540	575	593	544
10, 2	857	903	397	609	689	749	788	803	708
11, 1	905	954	485	722	811	876	915	927	801
12	919	968	515	761	852	919	958	968	832
kWh/d:	6.81	7.17	2.97	4.61	5.24	5.71	6.02	6.15	5.47

[a] A complete set of tables is in Appendix C.

Example 7.4 Estimate the insolation available on a clear day in January on a south-facing collector with a fixed, 30° tilt angle at the site having the sun path and obstructions diagram shown in Fig. 7.15.

Solution. With no obstructions, Table 7.3 indicates that the panel would be exposed to 5.24 kWh/m^2-day. The sun path diagram shows loss of about 1 h of sun at around 9 A.M., which eliminates about 0.49 kWh. After about 3:30 P.M. there is no sun, which drops roughly another 0.20 kWh. The remaining insolation is

$$\text{Insolation} \approx 5.24 - 0.49 - 0.20 = 4.55\ \text{kWh/m}^2 \approx 4.6\ \text{kWh/m}^2\ \text{per day}$$

Notice it has been assumed that the insolations shown in Table 7.3 are appropriate averages covering the half-hour before and after the hour. Given the crudeness of the obstruction sketch (to say nothing of the fact that the trees are likely to grow anyway), a more precise calculation isn't warranted.

7.6 SOLAR TIME AND CIVIL (CLOCK) TIME

For most solar work it is common to deal exclusively in solar time (ST), where everything is measured relative to solar noon (when the sun is on our line of longitude). There are occasions, however, when local time, called civil time or clock time (CT), is needed. There are two adjustments that must be made in order to connect local clock time and solar time. The first is a longitude adjustment that has to do with the way in which regions of the world are divided into time zones. The second is a little fudge factor that needs to be thrown in to account for the uneven way in which the earth moves around the sun.

Obviously, it just wouldn't work for each of us to set our watches to show noon when the sun is on our own line of longitude. Since the earth rotates 15° per hour (4 minutes per degree), for every degree of longitude between one location and another, clocks showing solar time would have to differ by 4 minutes. The only time two clocks would show the same time would be if they both were due north/south of each other.

To deal with these longitude complications, the earth is nominally divided into 24 1-hour time zones, with each time zone ideally spanning 15° of longitude. Of course, geopolitical boundaries invariably complicate the boundaries from one zone to another. The intent is for all clocks within the time zone to be set to the same time. Each time zone is defined by a *Local Time Meridian* located, ideally, in the middle of the zone, with the origin of this time system passing through Greenwich, England, at 0° longitude. The local time meridians for the United States are given in Table 7.4.

The longitude correction between local clock time and solar time is based on the time it takes for the sun to travel between the local time meridian and the observer's line of longitude. If it is solar noon on the local time meridian, it will be solar noon 4 minutes later for every degree that the observer is west of that meridian. For example, San Francisco, at longitude 122°, will have solar noon 8 minutes after the sun crosses the 120° Local Time Meridian for the Pacific Time Zone.

The second adjustment between solar time and local clock time is the result of the earth's elliptical orbit, which causes the length of a *solar day* (solar noon

TABLE 7.4 Local Time Meridians for U.S. Standard Time Zones (Degrees West of Greenwich)

Time Zone	LT Meridian
Eastern	75°
Central	90°
Mountain	105°
Pacific	120°
Eastern Alaska	135°
Alaska and Hawaii	150°

to solar noon) to vary throughout the year. As the earth moves through its orbit, the difference between a 24-hour day and a solar day changes following an expression known as the *Equation of Time*, E:

$$E = 9.87\sin 2B - 7.53\cos B - 1.5\sin B \qquad \text{(minutes)} \tag{7.12}$$

where

$$B = \frac{360}{364}(n - 81) \qquad \text{(degrees)} \tag{7.13}$$

As before, n is the day number. A graph of (7.12) is given in Fig. 7.16.

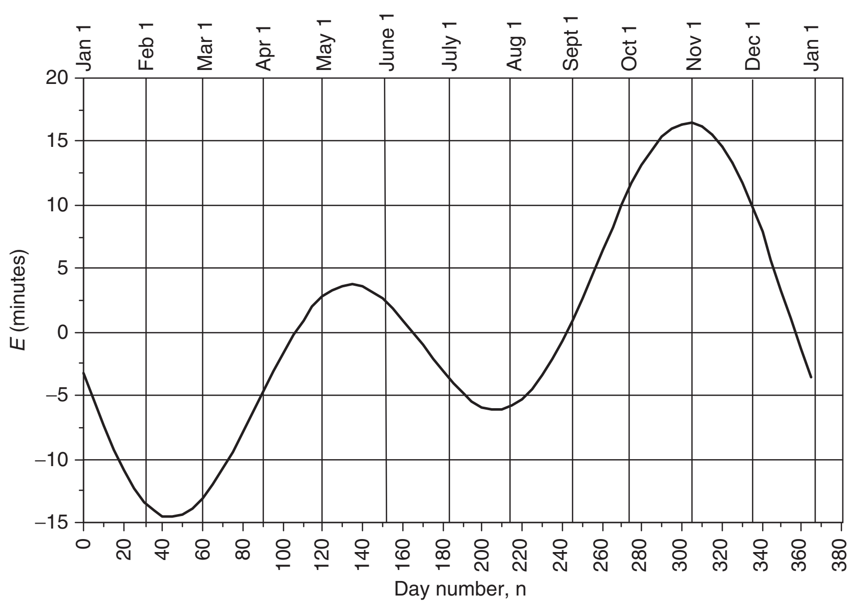


Figure 7.16 The Equation of Time adjusts for the earth's tilt angle and noncircular orbit.

Putting together the longitude correction and the Equation of Time gives us the final relationship between local standard clock time (CT) and solar time (ST).

$$\text{Solar Time (ST)} = \text{Clock Time (CT)} + \frac{4\ \text{min}}{\text{degree}}\ (\text{Local Time Meridian} - \text{Local longitude})^\circ + E(\text{min}) \tag{7.14}$$

When Daylight Savings Time is in effect, one hour must be added to the local clock time ("Spring ahead, Fall back").

Example 7.5 Solar Time to Local Time. Find Eastern Daylight Time for solar noon in Boston (longitude 71.1 °W) on July 1st.

Solution. From Table 7.1, July 1 is day number $n = 182$. Using (7.12) to (7.14) to adjust for local time, we obtain

$$B = \frac{360}{364}(n - 81) = \frac{360}{364}(182 - 81) = 99.89^\circ$$

$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$$

$$= 9.87 \sin[2 \cdot (99.89)] - 7.53 \cos(99.89) - 1.5 \sin(99.89) = -3.5\ \text{min}$$

For Boston at longitude 71.7° in the Eastern Time Zone with local time meridian 75°

$$\text{CT} = \text{ST} - 4(\text{min}/^\circ)(\text{Local time meridian} - \text{Local longitude}) - E(\text{min})$$

$$\text{CT} = 12{:}00 - 4(75 - 71.1) - (-3.5) = 12{:}00 - 12.1\ \text{min}$$

$$= 11{:}47.9\ \text{A.M. EST}$$

To adjust for Daylight Savings Time add 1 h, so solar noon will be at about 12:48 P.M. EDT.

7.7 SUNRISE AND SUNSET

A sun path diagram, such as was shown in Fig. 7.12, can be used to locate the azimuth angles and approximate times of sunrise and sunset. A more careful estimate of sunrise/sunset can be found from a simple manipulation of (7.8). At sunrise and sunset, the altitude angle β is zero, so we can write

$$\sin\beta = \cos L\cos\delta\cos H + \sin L\sin\delta = 0 \tag{7.15}$$

$$\cos H = -\frac{\sin L\sin\delta}{\cos L\cos\delta} = -\tan L\tan\delta \tag{7.16}$$

Solving for the hour angle at sunrise, H_{SR}, gives

$$H_{SR} = \cos^{-1}(-\tan L\tan\delta) \qquad (+\text{ for sunrise}) \tag{7.17}$$

Notice in (7.17) that since the inverse cosine allows for both positive and negative values, we need to use our sign convention, which requires the positive value to be used for sunrise (and the negative for sunset).

Since the earth rotates 15°/h, the hour angle can be converted to time of sunrise or sunset using

$$\text{Sunrise(geometric)} = 12{:}00 - \frac{H_{SR}}{15^\circ/h} \tag{7.18}$$

Equations (7.15) to (7.18) are geometric relationships based on angles measured to the center of the sun, hence the designation *geometric sunrise* in (7.18). They are perfectly adequate for any kind of normal solar work, but they won't give you exactly what you will find in the newspaper for sunrise or sunset. The difference between weather service sunrise and our geometric sunrise (7.18) is the result of two factors. The first deviation is caused by atmospheric refraction; this bends the sun's rays, making the sun *appear* to rise about 2.4 min sooner than geometry would tell us and then set 2.4 min later. The second is that the weather service definition of sunrise and sunset is the time at which the upper limb (top) of the sun crosses the horizon, while ours is based on the center crossing the horizon. This effect is complicated by the fact that at sunrise or sunset the sun pops up, or sinks, much quicker around the equinoxes when it moves more vertically than at the solstices when its motion includes much more of a sideward component. An adjustment factor Q that accounts for these complications is given by the following (U.S. Department of Energy, 1978):

$$Q = \frac{3.467}{\cos L\cos\delta\sin H_{SR}} \qquad (min) \tag{7.19}$$

Since sunrise is earlier when it is based on the top of the sun rather than the middle, Q should be subtracted from geometric sunrise. Similarly, since the upper limb sinks below the horizon later than the middle of the sun, Q should be added to our geometric sunset. A plot of (7.19) is shown in Fig. 7.17. As

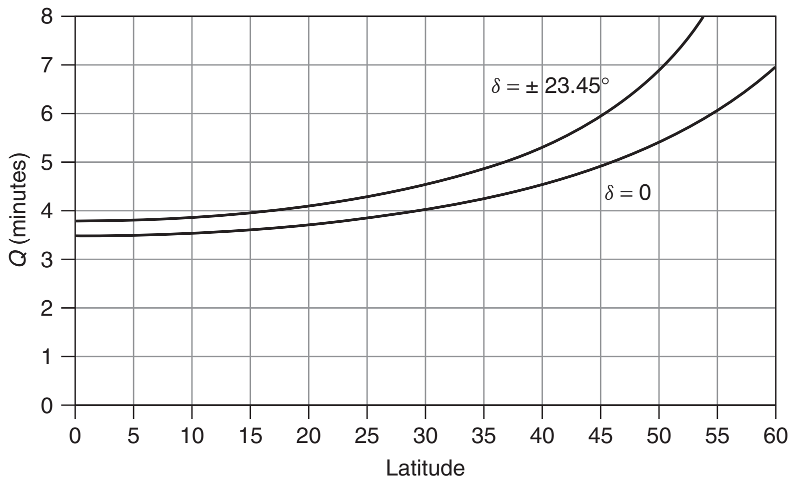


Figure 7.17 Sunrise/sunset adjustment factor to account for refraction and the upper-limb definition of sunrise. The range of solar declinations is shown.

can be seen, for mid-latitudes, the correction is typically in the range of about 4 to 6 min.

Example 7.6 Sunrise in Boston. Find the time at which sunrise (geometric and conventional) will occur in Boston (latitude 42.3°) on July 1 ($n = 182$). Also find conventional sunset.

Solution. From (7.6), the solar declination is

$$\delta = 23.45 \sin\left[\frac{360}{365}(n - 81)\right] = 23.45 \sin\left[\frac{360}{365}(182 - 81)\right] = 23.1^\circ$$

From (7.17), the hour angle at sunrise is

$$H_{SR} = \cos^{-1}(-\tan L \tan\delta) = \cos^{-1}(-\tan 42.3^\circ \tan 23.1^\circ) = 112.86^\circ$$

From (7.18) solar time of geometric sunrise is

$$\begin{aligned}\text{Sunrise (geometric)} &= 12{:}00 - \frac{H_{SR}}{15^\circ/h}\\ &= 12{:}00 - \frac{112.86^\circ}{15^\circ/h} = 12{:}00 - 7.524\ \text{h}\\ &= 4{:}28.6\ \text{A.M. (solar time)}\end{aligned}$$

Using (7.19) to adjust for refraction and the upper-limb definition of sunrise gives

$$Q = \frac{3.467}{\cos L \cos \delta \sin H_{SR}} \text{(min)}$$
$$= \frac{3.467}{\cos 42.3 \cos 23.1^\circ \sin 112.86^\circ} = 5.5 \text{ min}$$

The upper limb will appear 5.5 minutes sooner than our original geometric calculation indicated, so

$$\text{Sunrise} = 4{:}28.6 \text{ A.M.} - 5.5 \text{ min} = 4{:}23.1 \text{ A.M. (solar time)}$$

From Example 7.5, on this date in Boston, local clock time is 12.1 min earlier than solar time, so sunrise will be at

$$\text{Sunrise (upper limb)} = 4{:}23.1 - 12.1 = 4{:}11 \text{ A.M. Eastern Standard Time}$$

Similarly, geometric sunset is 7.524 h after solar noon, or 7:31.4 P.M. solar time. The upper limb will drop below the horizon 5.5 minutes later. Then adjusting for the 12.1 minutes difference between Boston time and solar time gives

$$\text{Sunset (upper limb)} = 7{:}31.4 + (5.5 - 12.1) \text{ min} = 7{:}24.8 \text{ P.M. EST}$$

There is a convenient website for finding sunrise and sunset times on the web at http://aa.usno.navy.mil/data/docs/RS_OneDay.html.

A fun, but fairly useless, application of these equations for sunrise and sunset is to work them in reverse order to navigate—that is, to find latitude and longitude, as the following example illustrates.

Example 7.7 Where in the World Are You? With your watch set for Pacific Standard Time (PST), you travel somewhere and when you arrive you note that the upper limb sunrise is at 1:11 A.M. (by your watch) and sunset is at 4:25 P.M. It is July 1st ($\delta = 23.1^\circ$, E $= -3.5$ min). Where are you?

Solution. Between 1:11 am and 4:25 P.M. there are 15 h and 14 min of daylight (15.233 h). With solar noon at the midpoint of that time—that is, 7 h and 37 min after sunrise—we have

$$\text{Solar noon} = 1{:}11 \text{ A.M.} + 7{:}37 = 8{:}48 \text{ A.M. PST}$$

Longitude can be determined from (7.14):

$$\text{Solar Time} = \text{Clock Time} + \frac{4\ \text{min}}{\text{degree}}(\text{Local Time Meridian} - \text{Local longitude})^\circ + E(\text{min})$$

Using the 120° Local Time Meridian for Pacific Time gives

$$12{:}00 - 8{:}48 = 192\ \text{min} = 4(120 - \text{Longitude}) + (-3.5)\ \text{min}$$

$$\text{Longitude} = \frac{480 - 3.5 - 192}{4} = 71.1^\circ$$

To find latitude, it helps to first ignore the correction factor Q. Doing so, the daylength is 15.233 h, which makes the hour angle at sunrise equal to

$$H_{SR} = \frac{15.233\ \text{hour}}{2} \cdot 15^\circ/\text{hour} = 114.25^\circ$$

A first estimate of latitude can now be found from (7.16)

$$L = \tan^{-1}\left(-\frac{\cos H_{SR}}{\tan\delta}\right) = \tan^{-1}\left(-\frac{\cos 114.25^\circ}{\tan 23.1^\circ}\right) = 43.9^\circ$$

Now we can find Q, from which we can correct our estimate of latitude

$$Q = \frac{3.467}{\cos L\cos\delta\sin H_{SR}} = \frac{3.467}{\cos 43.9^\circ\cos 23.1^\circ\sin 114.25^\circ} = 5.74\ \text{min}$$

Geometric daylength is therefore 2×5.74 min $= 11.48$ min $= 0.191$ h shorter than daylength based on the upper limb crossing the horizon. The geometric hour angle at sunrise is therefore

$$H_{SR} = \frac{(15.233 - 0.191)\text{h}}{2} \cdot 15^\circ/\text{h} = 112.81^\circ$$

Our final estimate of latitude is therefore

$$L = \tan^{-1}\left(-\frac{\cos H_{SR}}{\tan\delta}\right) = \tan^{-1}\left(-\frac{\cos 112.81^\circ}{\tan 23.1^\circ}\right) = 42.3^\circ$$

Notice we are back in Boston, latitude 42.3°, longitude 71.1°. Notice too, our watch worked fine even though it was set for a different time zone.

With so many angles to keep track of, it may help to summarize the terminology and equations for them all in one spot, which has been done in Box 7.1.

BOX 7.1 SUMMARY OF SOLAR ANGLES

δ = solar declination
n = day number
L = latitude
β = solar altitude angle, β_N = angle at solar noon
H = hour angle
H_{SR} = sunrise hour angle
ϕ_S = solar azimuth angle (+ before solar noon, − after)
ϕ_C = collector azimuth angle (+ east of south, − west of south)*
ST = solar time
CT = civil or clock time
E = equation of time
Q = correction for refraction and semidiameter at sunrise/sunset
Σ = collector tilt angle
θ = incidence angle between sun and collector face

$$\delta = 23.45 \sin\left[\frac{360}{365}(n - 81)\right]$$

$$\beta_N = 90^\circ - L + \delta$$

$$\sin\beta = \cos L \cos\delta \cos H + \sin L \sin\delta$$

$$\sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta}$$

$$\text{If } \cos \mathrm{H} \geq \frac{\tan\delta}{\tan L}, \quad \text{then} \quad |\phi_S| \leq 90^\circ; \qquad \text{otherwise } |\phi_S| > 90^\circ$$

$$\text{Hour angle } H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot (\text{Hours before solar noon})$$

$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B \qquad (\text{min})$$

$$B = \frac{360}{364}(n - 81)$$

$$\text{Solar Time (ST)} = \text{Clock Time (CT)} + \frac{4\,\text{min}}{\text{degree}}(\text{Local Time Meridian} - \text{Local Longitude})^\circ + E(\text{min})$$

$$H_{SR} = \cos^{-1}(-\tan L \tan\delta) \qquad (+ \text{ for sunrise})$$

$$Q = \frac{3.467}{\cos L \cos\delta \sin H_{SR}} \qquad (\text{min})$$

$$\cos\theta = \cos\beta \cos(\phi_S - \phi_C) \sin\Sigma + \sin\beta \cos\Sigma$$

*Opposite signs in Southern Hemisphere.

7.8 CLEAR SKY DIRECT-BEAM RADIATION

Solar flux striking a collector will be a combination of *direct-beam* radiation that passes in a straight line through the atmosphere to the receiver, *diffuse* radiation that has been scattered by molecules and aerosols in the atmosphere, and *reflected* radiation that has bounced off the ground or other surface in front of the collector (Fig. 7.18). The preferred units, especially in solar–electric applications, are watts (or kilowatts) per square meter. Other units involving British Thermal Units, kilocalories, and langleys may also be encountered. Conversion factors between these units are given in Table 7.5.

Solar collectors that focus sunlight usually operate on just the beam portion of the incoming radiation since those rays are the only ones that arrive from a consistent direction. Most photovoltaic systems, however, don't use focusing devices, so all three components—beam, diffuse, and reflected—can contribute to energy collected. The goal of this section is to be able to estimate the rate at which just the beam portion of solar radiation passes through the atmosphere

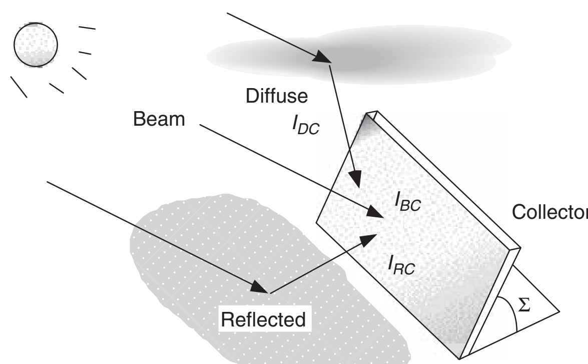


Figure 7.18 Solar radiation striking a collector, I_C, is a combination of direct beam, I_{BC}, diffuse, I_{DC}, and reflected, I_{RC}.

TABLE 7.5 Conversion Factors for Various Insolation Units

1 kW/m^2	=	316.95 Btu/h-ft^2
	=	1.433 langley/min
1 kWh/m^2	=	316.95 Btu/ft^2
	=	85.98 langleys
	=	3.60 × 10^6 joules/m^2
1 langley	=	1 cal/cm^2
	=	41.856 kjoules/m^2
	=	0.01163 kWh/m^2
	=	3.6878 Btu/ft^2

and arrives at the earth's surface on a clear day. Later, the diffuse and reflected radiation will be added to the clear day model. And finally, procedures will be presented that will enable more realistic average insolation calculations for specific locations based on empirically derived data for certain given sites.

The starting point for a clear sky radiation calculation is with an estimate of the extraterrestrial (ET) solar insolation, I_0, that passes perpendicularly through an imaginary surface just outside of the earth's atmosphere as shown in Fig. 7.19. This insolation depends on the distance between the earth and the sun, which varies with the time of year. It also depends on the intensity of the sun, which rises and falls with a fairly predictable cycle. During peak periods of magnetic activity on the sun, the surface has large numbers of cooler, darker regions called *sunspots*, which in essence block solar radiation, accompanied by other regions, called *faculae*, that are brighter than the surrounding surface. The net effect of sunspots that dim the sun, and faculae that brighten it, is an increase in solar intensity during periods of increased numbers of sunspots. Sunspot activity seems to follow an 11-year cycle. During sunspot peaks, the most recent of which was in 2001, the extraterrestrial insolation is estimated to be about 1.5% higher than in the valleys (U.S. Department of Energy, 1978).

Ignoring sunspots, one expression that is used to describe the day-to-day variation in extraterrestrial solar insolation is the following:

$$I_0 = \text{SC} \cdot \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] \qquad (\text{W/m}^2) \tag{7.20}$$

where SC is called the *solar constant* and n is the day number. The solar constant is an estimate of the average annual extraterrestrial insolation. Based on early NASA measurements, the solar constant was often taken to be 1.353 kW/m^2, but 1.377 kW/m^2 is now the more commonly accepted value.

As the beam passes through the atmosphere, a good portion of it is absorbed by various gases in the atmosphere, or scattered by air molecules or particulate matter. In fact, over a year's time, less than half of the radiation that hits the top of the atmosphere reaches the earth's surface as direct beam. On a clear day, however, with the sun high in the sky, beam radiation at the surface can exceed 70% of the extraterrestrial flux.

Attenuation of incoming radiation is a function of the distance that the beam has to travel through the atmosphere, which is easily calculable, as well as factors

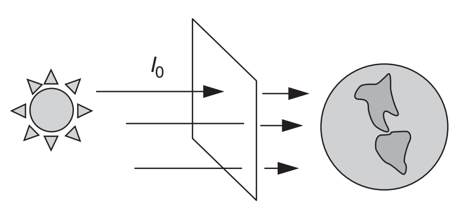


Figure 7.19 The extraterrestrial solar flux.

TABLE 7.6 Optical Depth k, Apparent Extraterrestrial Flux A, and the Sky Diffuse Factor C for the 21st Day of Each Month

Month:	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
A (W/m^2):	1230	1215	1186	1136	1104	1088	1085	1107	1151	1192	1221	1233
k:	0.142	0.144	0.156	0.180	0.196	0.205	0.207	0.201	0.177	0.160	0.149	0.142
C:	0.058	0.060	0.071	0.097	0.121	0.134	0.136	0.122	0.092	0.073	0.063	0.057

Source: ASHRAE (1993).

such as dust, air pollution, atmospheric water vapor, clouds, and turbidity, which are not so easy to account for. A commonly used model treats attenuation as an exponential decay function:

$$I_B = Ae^{-km} \tag{7.21}$$

where I_B is the beam portion of the radiation reaching the earth's surface (normal to the rays), A is an "apparent" extraterrestrial flux, and k is a dimensionless factor called the *optical depth*. The air mass ratio m was introduced earlier as (7.4)

$$\text{Air mass ratio} \quad m = \frac{1}{\sin\beta} \tag{7.4}$$

where β is the altitude angle of the sun.

Table 7.6 gives values of A and k that are used in the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) Clear Day Solar Flux Model. This model is based on empirical data collected by Threlkeld and Jordan (1958) for a moderately dusty atmosphere with atmospheric water vapor content equal to the average monthly values in the United States. Also included is a diffuse factor, C, that will be introduced later.

For computational purposes, it is handy to have an equation to work with rather than a table of values. Close fits to the values of optical depth k and apparent extraterrestrial (ET) flux A given in Table 7.6 are as follows:

$$A = 1160 + 75\sin\left[\frac{360}{365}(n-275)\right] \quad (\text{W/m}^2) \tag{7.22}$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n-100)\right] \tag{7.23}$$

where again n is the day number.

Example 7.8 Direct Beam Radiation at the Surface of the Earth. Find the direct beam solar radiation normal to the sun's rays at solar noon on a clear day in Atlanta (latitude 33.7°) on May 21. Use (7.22) and (7.23) to see how closely they approximate Table 7.6.

Solution. Using Table 7.1 to help, May 21 is day number 141. From (7.22), the apparent extraterrestrial flux, A, is

$$A = 1160 + 75\sin\left[\frac{360}{365}(n - 275)\right] = 1160 + 75\sin\left[\frac{360}{365}(141 - 275)\right]$$
$$= 1104\ \text{W/m}^2$$

(which agrees with Table 7.6).

From (7.23), the optical depth is

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n - 100)\right]$$
$$= 0.174 + 0.035\sin\left[\frac{360}{365}(141 - 100)\right] = 0.197$$

(which is very close to the value given in Table 7.6).

From Table 7.2, on May 21 solar declination is 20.1°, so from (7.7) the altitude angle of the sun at solar noon is

$$\beta_N = 90^\circ - L + \delta = 90 - 33.7 + 20.1 = 76.4^\circ$$

The air mass ratio (7.4) is

$$m = \frac{1}{\sin\beta} = \frac{1}{\sin(76.4^\circ)} = 1.029$$

Finally, using (7.21) the predicted value of clear sky beam radiation at the earth's surface is

$$I_B = Ae^{-km} = 1104\ e^{-0.197\times 1.029} = 902\ \text{W/m}^2$$

7.9 TOTAL CLEAR SKY INSOLATION ON A COLLECTING SURFACE

Reasonably accurate estimates of the clear sky, direct beam insolation are easy enough to work out and the geometry needed to determine how much of that will strike a collector surface is straightforward. It is not so easy to account for the diffuse and reflected insolation but since that energy bonus is a relatively small fraction of the total, even crude models are usually acceptable.

7.9.1 Direct-Beam Radiation

The translation of direct-beam radiation I_B (normal to the rays) into beam insolation striking a collector face I_{BC} is a simple function of the angle of incidence

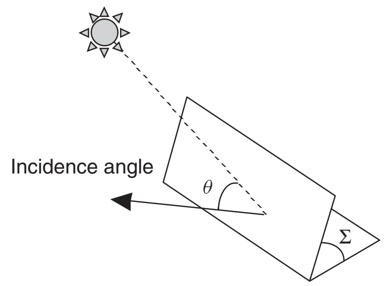


Figure 7.20 The incidence angle θ between a normal to the collector face and the incoming solar beam radiation.

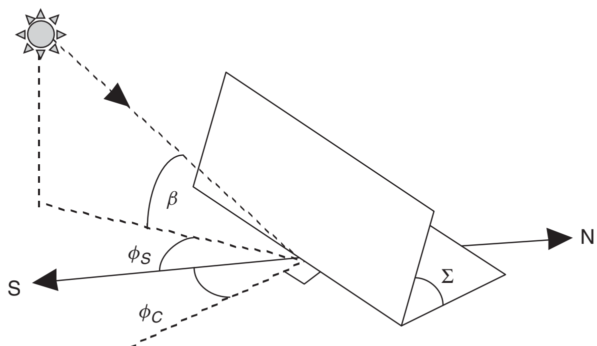


Figure 7.21 Illustrating the collector azimuth angle ϕ_C and tilt angle Σ along with the solar azimuth angle ϕ_S and altitude angle β. Azimuth angles are positive in the southeast direction and are negative in the southwest.

θ between a line drawn normal to the collector face and the incoming beam radiation (Fig. 7.20). It is given by

$$I_{BC} = I_B \cos\theta \tag{7.24}$$

For the special case of beam insolation on a horizontal surface I_{BH},

$$I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin\beta \tag{7.25}$$

The angle of incidence θ will be a function of the collector orientation and the altitude and azimuth angles of the sun at any particular time. Figure 7.21 introduces these important angles. The solar collector is tipped up at an angle Σ and faces in a direction described by its azimuth angle ϕ_C (measured relative to due south, *with positive values in the southeast direction and negative values in the southwest*). The incidence angle is given by

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma \tag{7.26}$$

Example 7.9 Beam Insolation on a Collector. In Example 7.8, at solar noon in Atlanta (latitude 33.7°) on May 21 the altitude angle of the sun was found to be 76.4° and the clear-sky beam insolation was found to be 902 W/m^2. Find the beam insolation at that time on a collector that faces 20° toward the southeast if it is tipped up at a 52° angle.

Solution. Using (7.26), the cosine of the incidence angle is

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$
$$= \cos 76.4^\circ \cos(0 - 20^\circ)\sin 52^\circ + \sin 76.4^\circ \cos 52^\circ = 0.7725$$

From (7.24), the beam radiation on the collector is

$$I_{BC} = I_B\cos\theta = 902\ \text{W/m}^2 \cdot 0.7725 = 697\ \text{W/m}^2$$

7.9.2 Diffuse Radiation

The diffuse radiation on a collector is much more difficult to estimate accurately than it is for the beam. Consider the variety of components that make up diffuse radiation as shown in Fig. 7.22. Incoming radiation can be scattered from atmospheric particles and moisture, and it can be reflected by clouds. Some is reflected from the surface back into the sky and scattered again back to the ground. The simplest models of diffuse radiation assume it arrives at a site with equal intensity from all directions; that is, the sky is considered to be *isotropic*. Obviously, on hazy or overcast days the sky is considerably brighter in the vicinity of the sun, and measurements show a similar phenomenon on clear days as well, but these complications are often ignored.

The model developed by Threlkeld and Jordan (1958), which is used in the ASHRAE Clear-Day Solar Flux Model, suggests that diffuse insolation on a

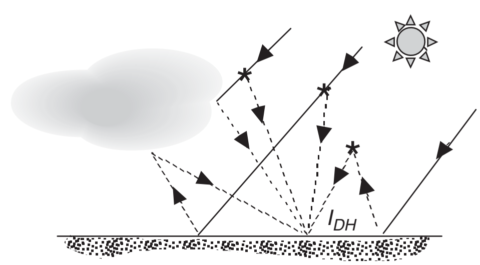


Figure 7.22 Diffuse radiation can be scattered by atmospheric particles and moisture or reflected from clouds. Multiple scatterings are possible.

horizontal surface I_{DH} is proportional to the direct beam radiation I_B no matter where in the sky the sun happens to be:

$$I_{DH} = C\, I_B \tag{7.27}$$

where C is a sky diffuse factor. Monthly values of C are given in Table 7.6, and a convenient approximation is as follows:

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right] \tag{7.28}$$

Applying (7.27) to a full day of clear skies typically predicts that about 15% of the total horizontal insolation on a clear day will be diffuse.

What we would like to know is how much of that horizontal diffuse radiation strikes a collector so that we can add it to the beam radiation. As a first approximation, it is assumed that diffuse radiation arrives at a site with equal intensity from all directions. This means that the collector will be exposed to whatever fraction of the sky the face of the collector points to, as shown in Fig. 7.23. When the tilt angle of the collector Σ is zero—that is, the panel is flat on the ground—the panel sees the full sky and so it receives the full horizontal diffuse radiation, I_{DH}. When it is a vertical surface, it sees half the sky and is exposed to half of the horizontal diffuse radiation, and so forth. The following expression for diffuse radiation on the collector, I_{DC}, is used when the diffuse radiation is idealized in this way:

$$I_{DC} = I_{DH}\left(\frac{1 + \cos\Sigma}{2}\right) = C\, I_B\left(\frac{1 + \cos\Sigma}{2}\right) \tag{7.29}$$

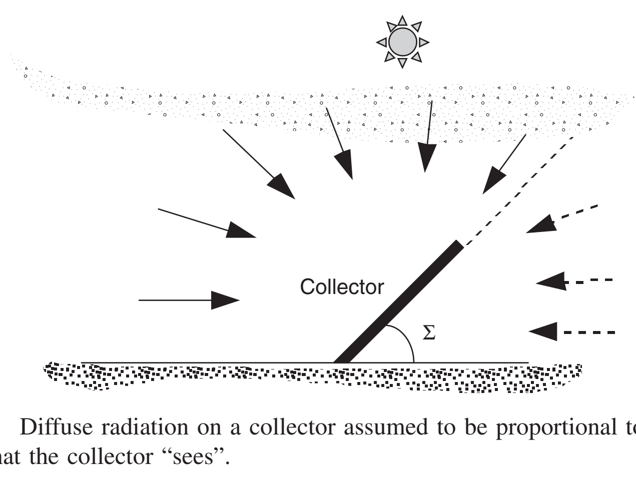


Figure 7.23 Diffuse radiation on a collector assumed to be proportional to the fraction of the sky that the collector "sees".

Example 7.10 Diffuse Radiation on a Collector. Continue Example 7.9 and find the diffuse radiation on the panel. Recall that it is solar noon in Atlanta on May 21 ($n = 141$), and the collector faces 20° toward the southeast and is tipped up at a 52° angle. The clear-sky beam insolation was found to be 902 W/m^2.

Solution. Start with (7.28) to find the diffuse sky factor, C:

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(141 - 100)\right] = 0.121$$

And from (7.29), the diffuse energy striking the collector is

$$I_{DC} = CI_B\left(\frac{1 + \cos\Sigma}{2}\right)$$

$$= 0.121 \cdot 902 \text{ W/m}^2 \left(\frac{1 + \cos 52^\circ}{2}\right) = 88 \text{ W/m}^2$$

Added to the total beam insolation of 697 W/m^2 found in Example 7.9, this gives a total beam plus diffuse on the collector of 785 W/m^2.

7.9.3 Reflected Radiation

The final component of insolation striking a collector results from radiation that is reflected by surfaces in front of the panel. This reflection can provide a considerable boost in performance, as for example on a bright day with snow or water in front of the collector, or it can be so modest that it might as well be ignored. The assumptions needed to model reflected radiation are considerable, and the resulting estimates are very rough indeed. The simplest model assumes a large horizontal area in front of the collector, with a reflectance ρ that is diffuse, and it bounces the reflected radiation in equal intensity in all directions, as shown in Fig. 7.24. Clearly this is a very gross assumption, especially if the surface is smooth and bright.

Estimates of ground reflectance range from about 0.8 for fresh snow to about 0.1 for a bituminous-and-gravel roof, with a typical default value for ordinary ground or grass taken to be about 0.2. The amount reflected can be modeled as the product of the total horizontal radiation (beam I_{BH}, plus diffuse I_{DH}) times the ground reflectance ρ. The fraction of that ground-reflected energy that will

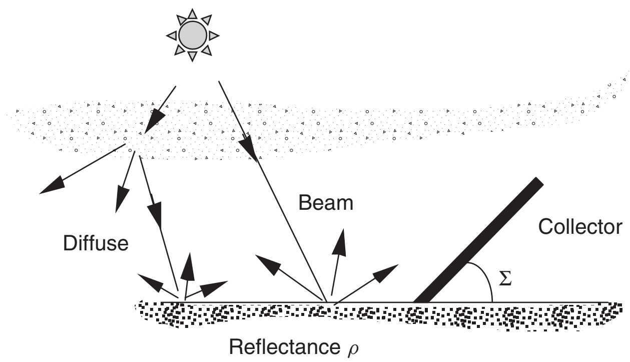


Figure 7.24 The ground is assumed to reflect radiation with equal intensity in all directions.

be intercepted by the collector depends on the slope of the panel Σ, resulting in the following expression for reflected radiation striking the collector I_{RC}:

$$I_{RC} = \rho(I_{BH} + I_{DH})\left(\frac{1 - \cos\Sigma}{2}\right) \tag{7.30}$$

For a horizontal collector ($\Sigma = 0$), Eq. (7.30) correctly predicts no reflected radiation on the collector; for a vertical panel, it predicts that the panel "sees" half of the reflected radiation, which also is appropriate for the model.

Substituting expressions (7.25) and (7.27) into (7.30) gives the following for reflected radiation on the collector:

$$I_{RC} = \rho I_B(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right) \tag{7.31}$$

Example 7.11 Reflected Radiation Onto a Collector. Continue Examples 7.9 and 7.10 and find the reflected radiation on the panel if the reflectance of the surfaces in front of the panel is 0.2. Recall that it is solar noon in Atlanta on May 21, the altitude angle of the sun β is 76.4°, the collector faces 20° toward the southeast and is tipped up at a 52° angle, the diffuse sky factor C is 0.121, and the clear-sky beam insolation is 902 W/m^2.

Solution. From (7.31), the clear-sky reflected insolation on the collector is

$$\begin{aligned}I_{RC} &= \rho I_B(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right)\\ &= 0.2 \cdot 902\ \text{W/m}^2(\sin 76.4^\circ + 0.121)\left(\frac{1 - \cos 52^\circ}{2}\right) = 38\ \text{W/m}^2\end{aligned}$$

The total insolation on the collector is therefore

$$I_C = I_{BC} + I_{DC} + I_{RC} = 697 + 88 + 38 = 823\ \text{W/m}^2$$

Of that total, 84.7% is direct beam, 10.7% is diffuse, and 4.6% is reflected. The reflected portion is clearly modest and is often ignored.

Combining the equations for the three components of radiation, direct beam, diffuse and reflected gives the following for total rate at which radiation strikes a collector on a clear day:

$$I_C = I_{BC} + I_{DC} + I_{RC} \tag{7.32}$$

$$I_C = Ae^{-km}\left[\cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1+\cos\Sigma}{2}\right) + \rho(\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right)\right] \tag{7.33}$$

Equation (7.33) looks terribly messy, but it is a convenient summary, which can be handy when setting up a spreadsheet or other computerized calculation of clear sky insolation.

7.9.4 Tracking Systems

Thus far, the assumption has been that the collector is permanently attached to a surface that doesn't move. In many circumstances, however, racks that allow the collector to track the movement of the sun across the sky are quite cost effective. Trackers are described as being either *two-axis trackers*, which track the sun both in azimuth and altitude angles so the collectors are always pointing directly at the sun, or *single-axis trackers*, which track only one angle or the other.

Calculating the beam plus diffuse insolation on a two-axis tracker is quite straightforward (Fig. 7.25). The beam radiation on the collector is the full insolation I_B normal to the rays calculated using (7.21). The diffuse and reflected radiation are found using (7.29) and (7.31) with a collector tilt angle equal to the complement of the solar altitude angle, that is, $90^\circ - \beta$.

Two-Axis Tracking:

$$I_{BC} = I_B \tag{7.34}$$

$$I_{DC} = CI_B\left[\frac{1+\cos(90^\circ - \beta)}{2}\right] \tag{7.35}$$

$$I_{RC} = \rho(I_{BH} + I_{DH})\left[\frac{1-\cos(90^\circ - \beta)}{2}\right] \tag{7.36}$$

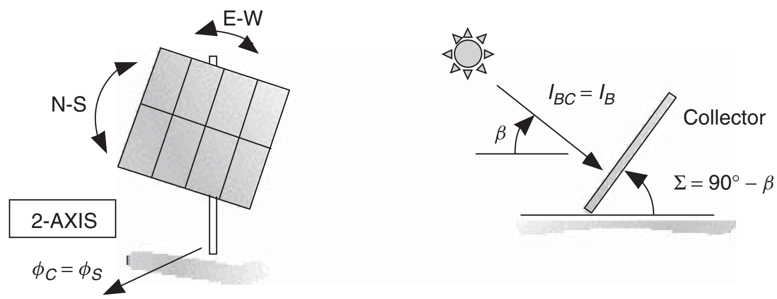


Figure 7.25 Two-axis tracking angular relationships.

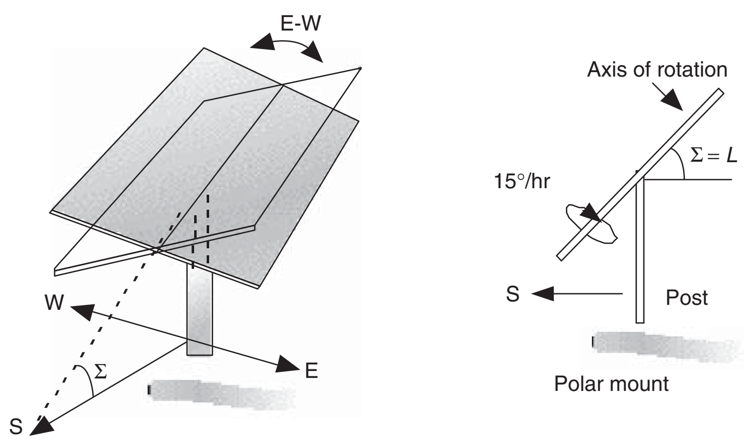


Figure 7.26 A single-axis tracking mount with east–west tracking. A polar mount has the axis of rotation facing south and tilted at an angle equal to the latitude.

Single-axis tracking for photovoltaics is almost always done with a mount having a manually adjustable tilt angle along a north-south axis, and a tracking mechanism that rotates the collector array from east-to-west, as shown in Fig. 7.26. When the tilt angle of the mount is set equal to the local latitude (called a *polar mount*), not only is that an optimum angle for annual collection, but the collector geometry and resulting insolation are fairly easy to evaluate as well.

As shown in Fig. 7.27, if a polar mount rotates about its axis at the same rate as the earth turns, 15°/h, then the centerline of the collector will always face directly into the sun. Under these conditions, the incidence angle θ between a normal to the collector and the sun's rays will be equal to the solar declination δ. That makes the direct-beam insolation on the collector just $I_B \cos \delta$. To evaluate diffuse and reflected radiation, we need to know the tilt angle of the collector. As can be seen in Fig. 7.26, while the axis of rotation has a fixed tilt of $\Sigma = L$, unless it is solar noon, the collector itself is cocked at an odd angle with respect

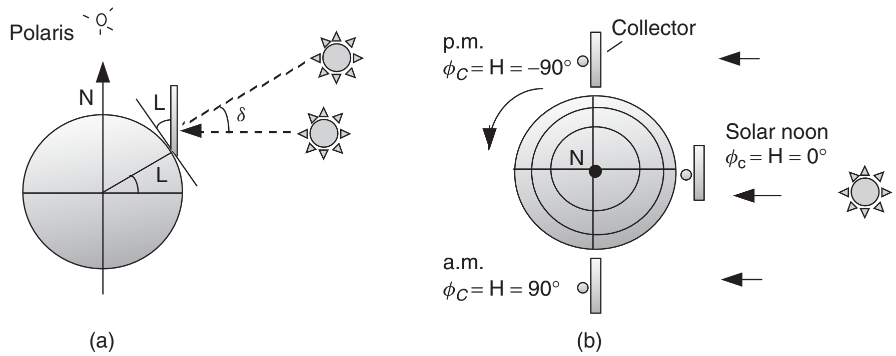


Figure 7.27 (a) Polar mount for a one-axis tracker showing the impact of a 15°/h angular rotation of the collector array. (b) Looking down on North Pole.

to the horizontal plane. The effective tilt, which is the angle between a normal to the collector and the horizontal plane, is given by

$$\Sigma_{\text{effective}} = 90 - \beta + \delta \tag{7.37}$$

The beam, diffuse, and reflected radiation on a polar mount, one-axis tracker are given by

One-Axis, Polar Mount:

$$I_{BC} = I_B \cos\delta \tag{7.38}$$

$$I_{DC} = CI_B \left[\frac{1 + \cos(90^\circ - \beta + \delta)}{2}\right] \tag{7.39}$$

$$I_{RC} = \rho(I_{BH} + I_{DH}) \left[\frac{1 - \cos(90^\circ - \beta + \delta)}{2}\right] \tag{7.40}$$

Example 7.12 One-Axis and Two-Axis Tracker Insolation. Compare the 40° latitude, clear-sky insolation on a collector at solar noon on the summer solstice for a two-axis tracking mount versus a single-axis polar mount. Ignore ground reflectance.

Solution

1. *Two-Axis Tracker:* To find the beam insolation from (7.21) $I_B = Ae^{-km}$, we need the air mass ratio m, the apparent extraterrestrial flux A, and the optical depth k. To find m, we need the altitude angle of the sun. Using (7.7)

with a solstice declination of 23.45°,

$$\beta_N = 90^\circ - L + \delta = 90 - 40 + 23.45 = 73.45^\circ$$

$$\text{Air mass ratio } m = \frac{1}{\sin\beta} = \frac{1}{\sin 73.45^\circ} = 1.043$$

From Table 7.6, or Eqs. (7.22), (7.23) and (7.28), we find $A = 1088\ \text{W/m}^2$, $k = 0.205$, and $C = 0.134$. The direct beam insolation on the collector is therefore

$$I_{BC} = I_B = Ae^{-km} = 1088\ (\text{W/m}^2) \cdot e^{-0.205x1.043} = 879\ \text{W/m}^2$$

Using (7.35) the diffuse radiation on the collector is

$$I_{DC} = CI_B\left[\frac{1+\cos(90^\circ-\beta)}{2}\right]$$
$$= 0.134 \cdot 879\left[\frac{1+\cos(90^\circ-73.45^\circ)}{2}\right] = 115\ \text{W/m}^2$$

The total is $I_C = I_{BC} + I_{DC} = 879 + 115 = 994\ \text{W/m}^2$

2. *One-Axis Polar Tracker:* The beam portion of insolation is given by (7.38)

$$I_{BC} = I_B\cos\delta = 879\ \text{W/m}^2\cos(23.45^\circ) = 806\ \text{W/m}^2$$

The diffuse portion, using (7.39), is

$$I_{DC} = CI_B\left[\frac{1+\cos(90^\circ-\beta+\delta)}{2}\right]$$
$$= 0.134 \cdot 879\ \text{W/m}^2\left[\frac{1+\cos(90-73.45+23.45)}{2}\right] = 104\ \text{W/m}^2$$

The total is $I_C = I_{BC} + I_{DC} = 806 + 104 = 910\ \text{W/m}^2$

The two-axis tracker provides 994 W/m^2, which is only 9% higher than the single-axis mount.

To assist in keeping this whole set of clear-sky insolation relationships straight, Box 7.2 offers a helpful summary of nomenclature and equations. And, obviously, working with these equations is tedious until they have been put onto a spreadsheet. Or, for most purposes it is sufficient to look up values in a table and, if necessary, do some interpolation. In Appendix C there are tables of hour-by-hour clear-sky insolation for various tilt angles and latitudes, an example of which is given here in Table 7.7.

BOX 7.2 SUMMARY OF CLEAR-SKY SOLAR INSOLATION EQUATIONS

I_0	=	extraterrestrial solar insolation
m	=	air mass ratio
I_B	=	beam insolation at earth's surface
A	=	apparent extraterrestrial solar insolation
k	=	atmospheric optical depth
C	=	sky diffuse factor
I_{BC}	=	beam insolation on collector
θ	=	incidence angle
Σ	=	collector tilt angle
I_H	=	insolation on a horizontal surface
I_{DH}	=	diffuse insolation on a horizontal surface
I_{DC}	=	diffuse insolation on collector
I_{RC}	=	reflected insolation on collector
ρ	=	ground reflectance
I_C	=	insolation on collector
n	=	day number
β	=	solar altitude angle
δ	=	solar declination
ϕ_S	=	solar azimuth angle (+ = AM)
ϕ_C	=	collector azimuth angle (+ = SE)

$$I_0 = 1370\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right] \text{ (W/m}^2\text{)}$$

$$m = \frac{1}{\sin\beta}$$

$$I_B = Ae^{-km}$$

$$A = 1160 + 75\sin\left[\frac{360}{365}(n - 275)\right] \text{ (W/m}^2\text{)}$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n - 100)\right]$$

$$I_{BC} = I_B\cos\theta$$

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

$$I_{BH} = I_B\cos(90^\circ - \beta) = I_B\sin\beta$$

$$I_{DH} = CI_B$$

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

$$I_{DC} = I_{DH}\left(\frac{1 + \cos\Sigma}{2}\right) = I_B C\left(\frac{1 + \cos\Sigma}{2}\right)$$

$$I_{RC} = \rho I_B(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right)$$

$$I_C = I_{BC} + I_{DC} + I_{RC}$$

$$I_C = Ae^{-km}\left[\cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1 + \cos\Sigma}{2}\right)\right.$$
$$\left. + \rho(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right)\right]$$

Two-Axis Tracking:

$$I_{BC} = I_B$$

$$I_{DC} = CI_B\left[\frac{1 + \cos(90^\circ - \beta)}{2}\right]$$

$$I_{RC} = \rho(I_{BH} + I_{DH})\left[\frac{1 - \cos(90^\circ - \beta)}{2}\right]$$

One-Axis, Polar Mount:

$$I_{BC} = I_B\cos\delta$$

$$I_{DC} = CI_B\left[\frac{1 + \cos(90^\circ - \beta + \delta)}{2}\right]$$

$$I_{RC} = \rho(I_{BH} + I_{DH})\left[\frac{1 - \cos(90^\circ - \beta + \delta)}{2}\right]$$

7.10 MONTHLY CLEAR-SKY INSOLATION

The instantaneous insolation equations just presented can be tabulated into daily, monthly and annual values that provide considerable insight into the impact of collector orientation. For example, Table 7.8 presents monthly and annual clear sky insolation on collectors with various azimuth and tilt angles, as well as for one- and two-axis tracking mounts, for latitude 40°N. They have been computed as the sum of just the beam plus diffuse radiation, which ignores the usually modest reflective contribution. Similar tables for other latitudes are given in

TABLE 7.7 Hour-by-Hour Clear-Sky Insolation in June for Latitude 40°

Solar Time	Tracking		Tilt Angles, Latitude 40°						
	One-Axis	Two-Axis	0	20	30	40	50	60	90
			June 21					(W/m²)	
6, 6	471	524	188	128	93	57	53	48	32
7, 5	668	742	386	330	289	240	185	126	45
8, 4	772	855	572	538	498	445	380	305	51
9, 3	835	921	731	722	686	632	560	473	147
10, 2	875	961	853	865	834	780	703	607	233
11, 1	898	982	929	956	928	874	795	693	288
12	906	989	955	987	960	906	826	723	308
kWh/d:	9.94	10.96	8.27	8.06	7.62	6.96	6.18	5.23	1.90

Note: Similar tables for other months and latitudes are given in Appendix C

Appendix D. When plotted, as has been done in Fig. 7.28, it becomes apparent that *annual* performance is relatively insensitive to wide variations in collector orientation for nontracking systems. For this latitude, the annual insolation for south-facing collectors varies by less than 10% for collectors mounted with tilt angles ranging anywhere from 10° to 60°. And, only a modest degradation is noted for panels that don't face due south. For a 45° collector azimuth angle (southeast, southwest), the annual clear sky insolation available drops by less than 10% in comparison with south-facing panels at similar tilt angles.

While Fig. 7.28 seems to suggest that orientation isn't critical, remember that it has been plotted for *annual insolation* without regard to monthly distribution. For a grid-connected photovoltaic system, for example, this may be a valid way to consider orientation. Deficits in the winter are automatically offset by purchased utility power, and any extra electricity generated during the summer can simply go back onto the grid. For a stand-alone PV system, however, where batteries or a generator provide back-up power, it is quite important to try to smooth out the month-to-month energy delivered to minimize the size of the back-up system needed in those low-yield months.

A graph of monthly insolation, instead of the annual plots given in Fig. 7.28, shows dramatic variations in the pattern of monthly solar energy for different tilt angles. Such a plot for three different tilt angles at latitude 40°, each having nearly the same annual insolation, is shown in Fig. 7.29. As shown, a collector at the modest tilt angle of 20° would do well in the summer, but deliver very little in the winter, so it wouldn't be a very good angle for a stand-alone PV system. At 40° or 60°, the distribution of radiation is more uniform and would be more appropriate for such systems.

In Fig. 7.30, monthly insolation for a south-facing panel at a fixed tilt angle equal to its latitude is compared with a one-axis polar mount tracker and also

TABLE 7.8 Daily and Annual Clear-Sky Insolation (Beam plus Diffuse) for Various Fixed-Orientation Collectors, Along with one- and Two-Axis Trackers

Daily Clear-Sky Insolation (kWh/m^2) Latitude 40°N																				
Azim:		S						SE/SW						E, W						Tracking	
Tilt:	0	20	30	40	50	60	90	20	30	40	50	60	90	20	30	40	50	60	90	One-Axis	Two-Axis
Jan	3.0	4.6	5.2	5.7	6.0	6.2	5.5	4.1	4.5	4.7	4.9	4.9	4.0	2.9	2.8	2.7	2.6	2.4	1.7	6.8	7.2
Feb	4.2	5.8	6.3	6.6	6.8	6.7	5.4	5.3	5.6	5.7	5.7	5.5	4.2	4.1	3.9	3.7	3.5	3.3	2.2	8.2	8.3
Mar	5.8	6.9	7.2	7.3	7.1	6.8	4.7	6.5	6.6	6.6	6.4	6.0	4.1	5.5	5.3	5.0	4.6	4.3	2.8	9.5	9.5
Apr	7.2	7.7	7.7	7.4	6.9	6.2	3.3	7.5	7.4	7.1	6.6	6.1	3.7	6.9	6.6	6.2	5.7	5.2	3.3	10.3	10.6
May	8.1	8.0	7.7	7.1	6.4	5.5	2.3	8.0	7.6	7.2	6.5	5.8	3.2	7.7	7.3	6.8	6.2	5.5	3.5	10.2	11.0
Jun	8.3	8.1	7.6	7.0	6.2	5.2	1.9	8.0	7.6	7.1	6.4	5.6	3.0	7.8	7.4	6.9	6.3	5.6	3.4	9.9	11.0
July	8.0	7.9	7.6	7.0	6.3	5.5	2.2	7.9	7.5	7.1	6.4	5.7	3.2	7.6	7.2	6.7	6.1	5.5	3.4	10.0	10.7
Aug	7.1	7.5	7.5	7.2	6.7	6.0	3.2	7.3	7.2	6.9	6.5	5.9	3.6	6.7	6.4	6.0	5.5	5.0	3.2	9.8	10.1
Sept	5.6	6.7	6.9	7.0	6.9	6.5	4.5	6.3	6.4	6.3	6.1	5.8	4.0	5.4	5.2	4.9	4.5	4.1	2.7	9.0	9.0
Oct	4.1	5.5	6.0	6.3	6.4	6.4	5.1	5.0	5.3	5.4	5.4	5.2	4.0	3.9	3.7	3.6	3.3	3.1	2.1	7.7	7.8
Nov	2.9	4.5	5.1	5.5	5.8	5.9	5.3	3.9	4.3	4.6	4.7	4.7	3.9	2.8	2.7	2.6	2.5	2.3	1.6	6.5	6.9
Dec	2.5	4.1	4.7	5.2	5.5	5.7	5.2	3.6	3.9	4.2	4.4	4.4	3.8	2.4	2.3	2.2	2.1	2.0	1.4	6.0	6.5
Total	2029	2352	2415	2410	2342	2208	1471	2231	2249	2216	2130	1997	1357	1938	1848	1738	1612	1467	960	3167	3305

Tables for other latitudes are in Appendix D

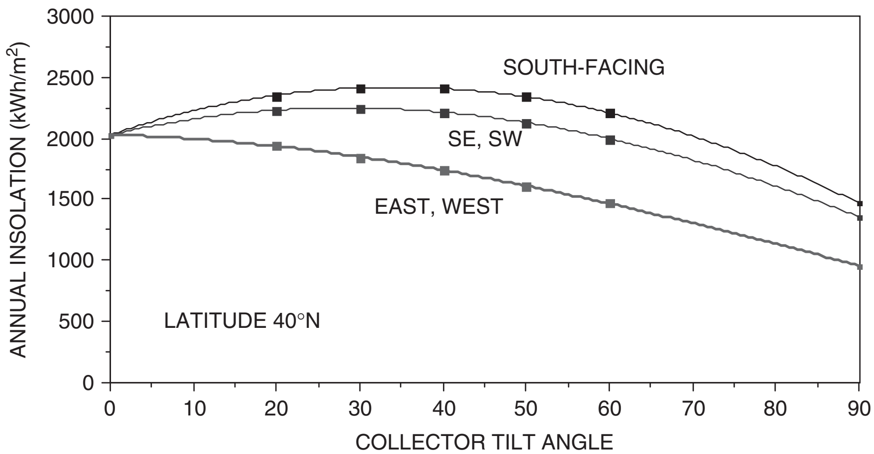


Figure 7.28 Annual insolation, assuming all clear days, for collectors with varying azimuth and tilt angles. Annual amounts vary only slightly over quite a range of collector tilt and azimuth angles.

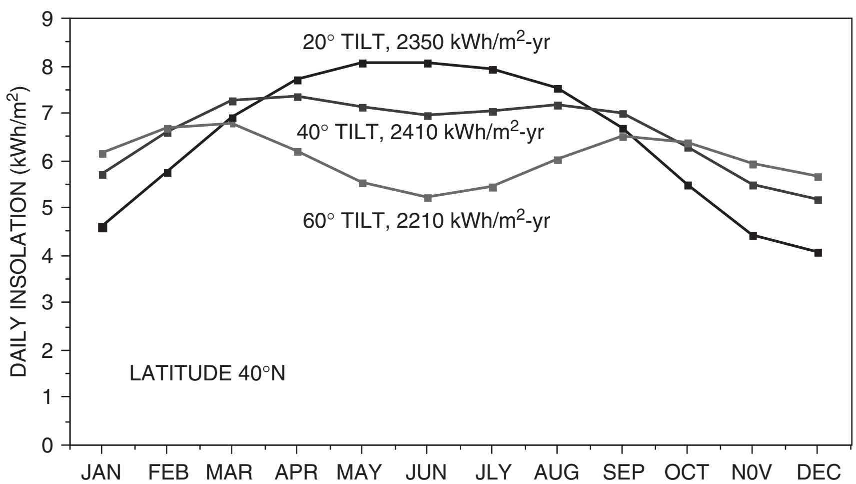


Figure 7.29 Daily clear-sky insolation on south-facing collectors with varying tilt angles. Even though they all yield roughly the same annual energy, the monthly distribution is very different.

a two-axis tracker. The performance boost caused by tracking is apparent: Both trackers are exposed to about one-third more radiation than the fixed collector. Notice, however, that the two-axis tracker is only a few percent better than the single-axis version, with almost all of this improvement occurring in the spring and summer months.

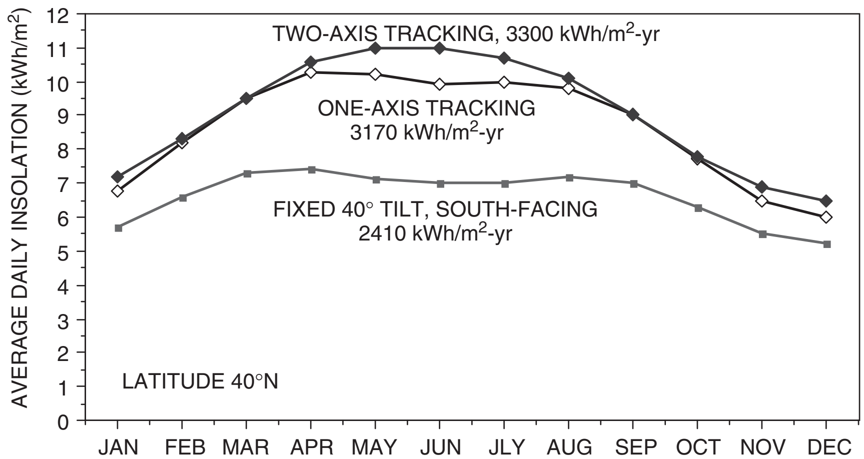


Figure 7.30 Clear sky insolation on a fixed panel compared with a one-axis, polar mount tracker and a two-axis tracker.

7.11 SOLAR RADIATION MEASUREMENTS

Creation of solar energy data bases began in earnest in the United States in the 1970s by the National Oceanic and Atmospheric Administration (NOAA) and later by the National Renewable Energy Laboratory (NREL). NREL has established the National Solar Radiation Data Base (NSRDB) for 239 sites in the United States. Of these, only 56 are primary stations for which long-term solar measurements have been made, while data for the remaining 183 sites are based on estimates derived from models incorporating meteorological data such as cloud cover. Figure 7.31 shows these 239 sites. The World Meteorological Organization (WMO), through its World Radiation Data Center in Russia, compiles data for hundreds of other sites around the world. Cloud mapping data taken by satellite are now a very important complement to the rather sparse global network of ground monitoring stations.

There are two principal types of devices used to measure solar radiation. The most widely used instrument, called a *pyranometer*, measures the total radiation arriving from all directions, including both direct and diffuse components. That is, it measures all of the radiation that is of potential use to a solar collecting system. The other device, called a *pyrheliometer*, looks at the sun through a narrow collimating tube, so it measures only the direct beam radiation. Data collected by pyrheliometers are especially important for focusing collectors since their solar resource is pretty much restricted to just the beam portion of incident radiation.

Pyranometers and pyrheliometers can be adapted to obtain other useful data. For example, as shall be seen in the next section, the ability to sort out the direct from the diffuse is a critical step in the conversion of measured insolation on a horizontal surface into estimates of radiation on tilted collectors. By temporarily affixing a shade ring to block the direct beam, a pyranometer can be used to

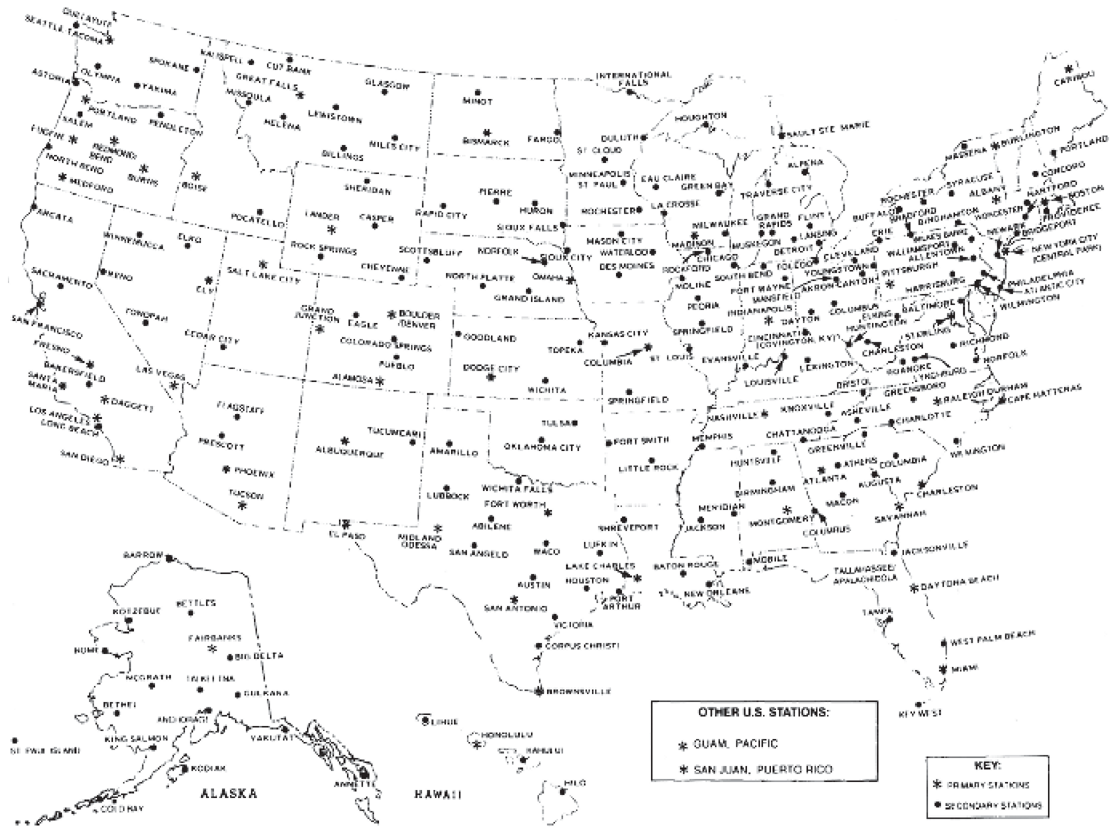


Figure 7.31 Map showing the 239 National Solar Radiation Data Base stations. From NREL (1994).

measure just diffuse radiation (Fig. 7.32). By subtracting the diffuse from the total, the beam portion can then be determined. In other circumstances, it is important to know not only how much radiation the sun provides, but also how much it provides within certain ranges of wavelengths. For example, newspapers now routinely report on the ultraviolet (UV) portion of the spectrum to warn us about skin cancer risks. This sort of data can be obtained by fitting pyranometers or pyrheliometers with filters to allow only certain wavelengths to be measured.

The most important part of a pyranometer or pyrheliometer is the detector that responds to incoming radiation. The most accurate detectors use a stack of thermocouples, called a thermopile, to measure how much hotter a black surface becomes when exposed to sunlight. The most accurate of these incorporate a sensor surface that consists of alternating black and white segments (Fig. 7.33). The thermopile measures the temperature difference between the black segments, which absorb sunlight, and the white ones, which reflect it, to produce a voltage that is proportional to insolation. Other thermopile pyranometers have sensors that are entirely black, and the temperature difference is measured between the case of the pyranometer, which is close to ambient, and the hotter, black sensor.

The alternative approach uses a photodiode sensor that sends a current through a calibrated resistance to produce a voltage proportional to insolation. These pyranometers are less expensive but are also less accurate than those based on thermopiles. Unlike thermopile sensors, which measure all wavelengths of incoming radiation, photoelectric sensors respond to only a limited portion of the solar spectrum. The most popular devices use silicon photosensors, which means that any photons with longer wavelengths than their band gap of 1100 μm don't contribute to the output. Photoelectric pyranometers are calibrated to produce very accurate results under clear skies, but if the solar spectrum is altered, as for

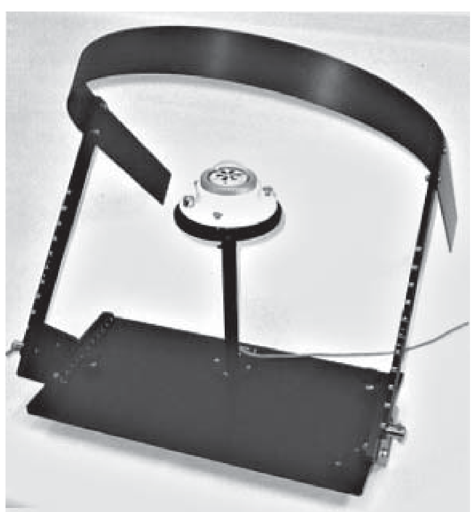
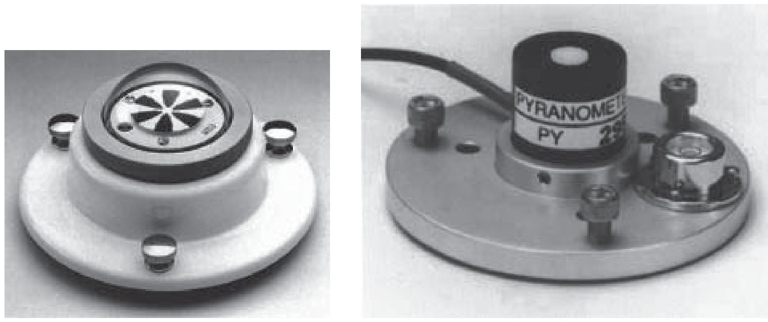


Figure 7.32 Pyranometer with a shade ring to measure diffuse radiation.



(a) (b)

Figure 7.33 (a) A thermopile-type, black-and-white pyranometer and (b) a Li-Cor silicon-cell pyranometer.

example when sunlight passes through glass or clouds, they won't be as accurate as a pyranometer that uses a thermopile sensor. Also, they don't respond accurately to artificial light.

7.12 AVERAGE MONTHLY INSOLATION

It is one thing to be able to compute the insolation on a tilted surface when the skies are clear, but what really is needed is a procedure for estimating the average insolation that can be expected to strike a collector under real conditions at a particular site. The starting point is site-specific, long-term radiation data, which is primarily insolation measured on a horizontal surface. Procedures used to convert these data into expected radiation on a tilted surface depend on being able to sort out what portion of the total measured horizontal insolation $\overline{I}_H$ is diffuse $\overline{I}_{DH}$ and what portion is direct beam, $\overline{I}_{BH}$.

$$\overline{I}_H = \overline{I}_{DH} + \overline{I}_{BH} \tag{7.41}$$

Once this decomposition has been estimated, adjusting the resulting horizontal diffuse radiation into diffuse and reflected radiation on a collecting surface is straightforward and uses equations already presented. Converting horizontal beam radiation is a little trickier.

Procedures for decomposing total horizontal insolation into its diffuse and beam components begin by defining a clearness index K_T, which is the ratio of the average horizontal insolation at the site $\overline{I}_H$ to the extraterrestrial insolation on a horizontal surface above the site and just outside the atmosphere, $\overline{I}_0$.

$$\text{Clearness index } K_T = \frac{\overline{I}_H}{\overline{I}_0} \tag{7.42}$$

A high clearness index corresponds to clear skies in which most of the radiation will be direct beam while a low one indicates overcast conditions having mostly diffuse insolation.

The average daily extraterrestrial insolation on a horizontal surface $\overline{I}_0$ (kWh/m^2-day) can be calculated by averaging the product of the normal radiation (7.20) and the sine of the solar altitude angle (7.8) from sunrise to sunset, resulting in

$$\overline{I}_0 = \left(\frac{24}{\pi}\right) \mathrm{SC}\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right](\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta) \tag{7.43}$$

where SC is the solar constant and the sunrise hour angle H_{SR} is in radians.

Usually the clearness index is based on a monthly average, and (7.43) can be computed daily and those values averaged over the month or a day in the middle of the month can be used to represent the average monthly condition. The solar constant SC used here will be 1.37 kW/m^2.

A number of attempts to correlate clearness index and the fraction of horizontal insolation that is diffuse have been made, including Liu and Jordan (1961), and Collares-Pereira and Rabl (1979). The Liu and Jordan correlation is as follows:

$$\frac{\overline{I}_{DH}}{\overline{I}_H} = 1.390 - 4.027K_T + 5.531{K_T}^2 - 3.108{K_T}^3 \tag{7.44}$$

From (7.44), the diffuse portion of horizontal insolation can be estimated. Then, adjusting (7.29) and (7.30) to indicate average daylong values, the average diffuse and reflected radiation on a tilted collector surface can be found from

$$\overline{I}_{DC} = \overline{I}_{DH}\left(\frac{1 + \cos\Sigma}{2}\right) \tag{7.45}$$

and

$$\overline{I}_{RC} = \rho\overline{I}_H\left(\frac{1 - \cos\Sigma}{2}\right) \tag{7.46}$$

where Σ is the collector slope with respect to the horizontal. Equations (7.45) and (7.46) are sufficient for our purposes, but it should be noted that more complex models that don't require the assumption of an isotropic sky are available (Perez et al., 1990).

Average beam radiation on a horizontal surface can be found by subtracting the diffuse portion $\overline{I}_{DH}$ from the total $\overline{I}_H$. To convert the horizontal beam radiation into beam on the collector $\overline{I}_{BC}$, begin by combining (7.25)

$$I_{BH} = I_B\sin\beta \tag{7.25}$$

with (7.24)

$$I_{BC} = I_B\cos\theta \tag{7.24}$$

to get

$$I_{BC} = I_{BH}\left(\frac{\cos\theta}{\sin\beta}\right) = I_{BH}R_B \tag{7.47}$$

where θ is the incidence angle between the collector and beam, and β is the sun's altitude angle. The quantity in the parentheses is called the *beam tilt factor* R_B.

Equation (7.47) is correct on an instantaneous basis, but since we are working with monthly averages, what is needed is an average value for the beam tilt factor. In the Liu and Jordan procedure, the beam tilt factor is estimated by simply averaging the value of cos θ over those hours of the day in which the sun is in front of the collector and dividing that by the average value of sin β over those hours of the day when the sun is above the horizon. For south-facing collectors at tilt angle Σ, a closed-form solution for those averages can be found and the resulting average beam tilt factor becomes

$$\overline{R}_B = \frac{\cos(L-\Sigma)\cos\delta\sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta} \tag{7.48}$$

where H_{SR} is the sunrise hour angle (in radians) given in (7.17):

$$H_{SR} = \cos^{-1}(-\tan L\tan\delta) \tag{7.17}$$

H_{SRC} is the sunrise hour angle for the collector (when the sun first strikes the collector face, $\theta = 90^\circ$):

$$H_{SRC} = \min\{\cos^{-1}(-\tan L\tan\delta), \cos^{-1}[-\tan(L-\Sigma)\tan\delta]\} \tag{7.49}$$

Recall that L is the latitude, Σ is the collector tilt angle, and δ is the solar declination (7.6).

To summarize the approach, once the horizontal insolation has been decomposed into beam and diffuse components, it can be recombined into the insolation striking a collector using the following:

$$\overline{I}_C = \overline{I}_H\left(1 - \frac{\overline{I}_{DH}}{\overline{I}_H}\right)\cdot\overline{R}_B + \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) + \rho\overline{I}_H\left(\frac{1-\cos\Sigma}{2}\right) \tag{7.50}$$

where $\overline{R}_B$ can be found for south-facing collectors using (7.48).

Example 7.13 Average Monthly Insolation on a Tilted Collector. Average horizontal insolation in Oakland, California (latitude 37.73°N) in July is 7.32 kWh/m^2-day. Estimate the insolation on a south-facing collector at a tilt angle of 30° with respect to the horizontal. Assume ground reflectivity of 0.2.

Solution. Begin by finding mid-month declination and sunrise hour angle for July 16 ($n = 197$):

$$\delta = 23.45 \sin\left[\frac{360}{365}(n-81)\right] = 23.45 \sin\left[\frac{360}{365}(197-81)\right]$$
$$= 21.35^\circ \qquad (7.6)$$
$$H_{SR} = \cos^{-1}(-\tan L \tan\delta) \qquad (7.17)$$
$$= \cos^{-1}(-\tan 37.73^\circ \tan 21.35^\circ) = 107.6^\circ = 1.878 \text{ radians}$$

Using a solar constant of 1.37 kW/m^2, the E.T. horizontal insolation from (7.43) is

$$\overline{I}_0 = \left(\frac{24}{\pi}\right)\text{SC}\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right](\cos L \cos\delta \sin H_{SR} + H_{SR}\sin L \sin\delta)$$
$$= \left(\frac{24}{\pi}\right)1.37\left[1 + 0.034\cos\left(\frac{360\cdot 197}{365}\right)^\circ\right](\cos 37.73 \cos 21.35^\circ \sin 107.6^\circ$$
$$+ 1.878 \sin 37.73^\circ \sin 21.35^\circ)$$
$$= 11.34 \text{ kWh/m}^2\text{-day}$$

From (7.42), the clearness index is

$$K_T = \frac{\overline{I}_H}{\overline{I}_0} = \frac{7.32 \text{ kWh/m}^2\cdot\text{day}}{11.34 \text{ kWh/m}^2\cdot\text{day}} = 0.645$$

From (7.44) the fraction diffuse is

$$\frac{\overline{I}_{DH}}{\overline{I}_H} = 1.390 - 4.027K_T + 5.531{K_T}^2 - 3.108{K_T}^3$$
$$= 1.390 - 4.027\ (0.645) + 5.531\ (0.645)^2 - 3.108\ (0.645)^3 = 0.258$$

So, the diffuse horizontal radiation is

$$\overline{I}_{DH} = 0.258 \cdot 7.32 = 1.89 \text{ kWh/m}^2\text{-day}$$

The diffuse radiation on the collector is given by (7.45)

$$\overline{I}_{DC} = \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = 1.89\left(\frac{1+\cos 30^\circ}{2}\right) = 1.76 \text{ kWh/m}^2\text{-day}$$

The reflected radiation on the collector is given by (7.46)

$$\overline{I}_{RC} = \rho\,\overline{I}_H\left(\frac{1-\cos\Sigma}{2}\right) = 0.2\cdot 7.32\left(\frac{1-\cos 30^\circ}{2}\right) = 0.10 \text{ kWh/m}^2\text{-day}$$

From (7.41), the beam radiation on the horizontal surface is

$$\overline{I}_{BH} = \overline{I}_H - \overline{I}_{DH} = 7.32 - 1.89 = 5.43 \text{ kWh/m}^2\text{-day}$$

To adjust this for the collector tilt, first find the sunrise hour angle on the collector from (7.49)

$$\begin{aligned} H_{SRC} &= \min\{\cos^{-1}(-\tan L \tan\delta),\ \cos^{-1}[-\tan(L-\Sigma)\tan\delta]\} \\ &= \min\{\cos^{-1}(-\tan 37.73^\circ \tan 21.35^\circ),\ \cos^{-1}[-\tan(37.73-30)^\circ \tan 21.35^\circ]\} \\ &= \min\{107.6^\circ, 93.0^\circ\} = 93.0^\circ = 1.624 \text{ radians} \end{aligned}$$

The beam tilt factor (7.48) is thus

$$\begin{aligned} \overline{R}_B &= \frac{\cos(L-\Sigma)\cos\delta \sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L \cos\delta \sin H_{SR} + H_{SR}\sin L \sin\delta} \\ &= \frac{\cos(37.73-30)^\circ \cos 21.35^\circ \sin 93^\circ + 1.624 \sin(37.73-30)^\circ \sin 21.35^\circ}{\cos 37.73^\circ \cos 21.35^\circ \sin 107.6^\circ + 1.878 \sin 37.73^\circ \sin 21.35^\circ} \\ &= 0.893 \end{aligned}$$

So the beam insolation on the collector is

$$\overline{I}_{BC} = \overline{I}_{BH}\overline{R}_B = 5.43 \cdot 0.893 = 4.85 \text{ kWh/m}^2\text{-day}$$

Total insolation on the collector is thus

$$\overline{I}_C = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC} = 4.85 + 1.76 + 0.10 = 6.7 \text{ kWh/m}^2\text{-day}$$

Clearly, with calculations that are this tedious it is worth spending the time to set up a spreadsheet or other computer analysis or, better still, use precomputed data available on the web or from publications such as the *Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors* (NREL, 1994). An example of the sort of data available from NREL is shown in Table 7.9. Average total radiation data are given for south-facing collectors with various fixed-tilt angles as well as for one-axis and two-axis tracking mounts. In addition, the range of insolations each month is presented, which, along with the figure, gives a good sense of how variable insolation has been during the period in which the actual measurements were made. Also included are values for just the direct-beam portion of radiation for concentrating collectors that can't focus diffuse radiation. The direct-beam data are presented for horizontal collectors in which the tracking rotates about a north–south axis or an east–west axis as well as for tilted, tracking mounts. Horizontal mounts are common in solar–thermal systems that focus sunlight using parabolic troughs (Fig. 7.34).

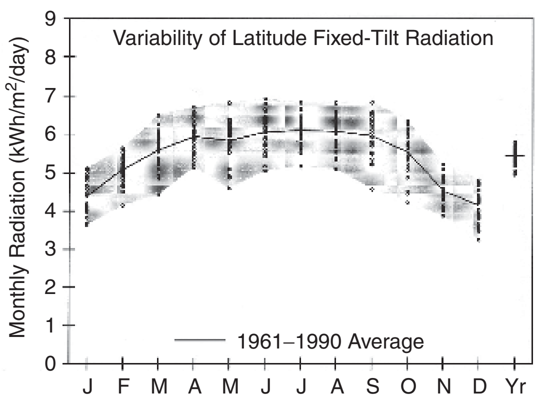
TABLE 7.9 Average Solar Radiation for Boulder, CO (kWh/m^2-day) for South-Facing, Fixed-Tilt Collectors, Tracking Collectors, and Tracking/Focusing Collectors that Operate on Just the Beam Portion of Insolation

Boulder, CO

WBAN NO. 94018

LATITUDE: 40.02° N
LONGITUDE: 105.25° W
ELEVATION: 1634 meters
MEAN PRESSURE: 836 millibars

STATION TYPE: Primary



Solar Radiation for Flat-Plate Collectors Facing South at a Fixed Tilt (kWh/m^2/day), Uncertainty ±9%

Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	2.4	3.3	4.4	5.6	6.2	6.9	6.7	6.0	5.0	3.8	2.6	2.1	4.6
	Min/Max	2.1/2.7	2.8/3.5	3.7/5.0	4.8/6.1	5.1/7.2	5.7/7.8	5.6/7.4	5.2/6.6	4.0/5.5	3.1/4.2	2.3/2.8	1.9/2.3	4.3/4.8
Latitude −15	Average	3.8	4.6	5.4	6.1	6.2	6.6	6.6	6.3	5.9	5.1	4.0	3.5	5.4
	Min/Max	3.2/4.4	3.8/5.1	4.3/6.2	5.3/6.8	4.9/7.3	5.5/7.6	5.6/7.4	5.3/7.1	4.6/6.7	4.0/5.8	3.4/4.6	2.8/4.1	4.9/5.7
Latitude	Average	4.4	5.1	5.6	6.0	5.9	6.1	6.1	6.1	6.0	5.6	4.6	4.2	5.5
	Min/Max	3.6/5.1	4.2/5.7	4.4/6.5	5.2/6.7	4.6/6.8	5.1/6.9	5.2/6.8	5.1/6.8	4.6/6.8	4.2/6.4	3.9/5.2	3.2/4.8	5.0/5.8
Latitude +15	Average	4.8	5.3	5.6	5.6	5.2	5.2	5.3	5.5	5.8	5.7	4.8	4.5	5.3
	Min/Max	3.9/5.6	4.3/5.9	4.4/6.5	4.8/6.2	4.1/6.0	4.4/5.9	4.5/5.9	4.6/6.2	4.4/6.6	4.2/6.5	4.1/5.6	3.5/5.3	4.8/5.6
90	Average	4.5	4.6	4.3	3.6	2.8	2.6	2.7	3.2	4.0	4.6	4.4	4.3	3.8
	Min/Max	3.6/5.4	3.7/5.2	3.5/5.0	3.0/4.0	2.3/3.1	2.2/2.8	2.3/2.9	2.7/3.6	3.1/4.6	3.4/5.3	3.7/5.1	3.4/5.2	3.4/4.1

Solar Radiation for 1-Axis Tracking Flat-Plate Collectors with a North–South Axis (kWh/m^2/day), Uncertainty ±9%

Axis Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	3.7	4.9	6.2	7.6	8.2	9.1	9.0	8.2	7.1	5.7	4.0	3.3	6.4
	Min/Max	3.0/4.4	4.1/5.5	4.6/7.4	6.2/8.7	6.2/10.0	7.4/10.9	7.1/10.2	6.7/9.3	5.3/8.3	4.2/6.6	3.4/4.4	2.6/3.9	5.7/6.9
Latitude −15	Average	4.8	5.9	7.0	8.1	8.4	9.1	9.1	8.6	7.9	6.7	5.0	4.4	7.1
	Min/Max	3.8/5.6	4.8/6.7	5.1/8.4	6.6/9.2	6.3/10.2	7.4/10.9	7.1/10.3	7.0/9.8	5.8/9.2	4.8/7.8	4.2/5.7	3.3/5.2	6.2/7.6
Latitude	Average	5.2	6.2	7.2	8.0	8.1	8.8	8.7	8.4	7.9	7.1	5.5	4.9	7.2
	Min/Max	4.2/6.2	5.1/7.1	5.2/8.6	6.6/9.2	6.1/9.9	7.1/10.4	6.8/10.0	6.8/9.6	5.8/9.3	5.0/8.2	4.6/6.3	3.6/5.8	6.3/7.8
Latitude +15	Average	5.5	6.4	7.1	7.7	7.7	8.2	8.2	8.0	7.8	7.1	5.7	5.2	7.1
	Min/Max	4.4/6.6	5.2/7.3	5.2/8.6	6.3/8.9	5.8/9.4	6.6/9.8	6.4/9.3	6.5/9.2	5.6/9.1	5.0/8.3	4.8/6.6	3.8/6.2	6.1/7.6

Solar Radiation for 2-Axis Tracking Flat-Plate Collectors (kWh/m^2/day), Uncertainty ±9%

Tracker		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
2-Axis	Average	5.6	6.4	7.2	8.1	8.5	9.4	9.2	8.6	8.0	7.1	5.7	5.3	7.4
	Min/Max	4.5/6.7	5.2/7.3	5.2/8.6	6.7/9.3	6.4/10.4	7.6/11.1	7.2/10.5	7.0/9.8	5.8/9.3	5.1/8.3	4.8/6.6	3.9/6.3	6.5/8.0

Direct Beam Solar Radiation for Concentrating Collectors (kWh/m^2/day), Uncertainty ±8%

Tracker		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
1-Axis, E–W	Average	3.5	3.7	3.7	4.0	4.2	5.0	4.9	4.5	4.4	4.3	3.6	3.4	4.1
Horiz Axis	Min/Max	2.3/4.6	2.8/4.5	2.1/4.8	2.9/5.0	2.9/5.7	3.5/6.4	3.8/6.1	3.4/5.4	2.8/5.5	2.5/5.2	2.7/4.7	2.0/4.3	3.4/4.5
1-Axis, N–S	Average	2.6	3.4	4.2	5.3	5.6	6.6	6.5	6.0	5.4	4.3	2.8	2.3	4.6
Horiz Axis	Min/Max	1.6/3.4	2.5/4.2	2.2/5.7	3.6/6.4	3.8/7.6	4.8/8.5	4.8/8.1	4.5/7.1	3.4/6.7	2.4/5.3	2.2/3.6	1.3/3.0	3.7/5.1
1-Axis, N–S	Average	3.9	4.5	5.0	5.6	5.5	6.2	6.2	6.1	6.0	5.5	4.1	3.6	5.2
Tilt = Latitude	Min/Max	2.5/5.1	3.4/5.5	2.7/6.6	3.8/6.8	3.7/7.5	4.5/8.0	4.6/7.7	4.6/7.3	3.8/7.6	3.1/6.7	3.1/5.3	2.0/4.6	4.2/5.7
2-Axis	Average	4.1	4.6	5.0	5.7	5.8	6.8	6.7	6.3	6.1	5.6	4.3	4.0	5.4
	Min/Max	2.7/5.4	3.5/5.7	2.7/6.6	3.9/6.9	4.0/7.9	4.9/8.7	4.9/8.3	4.8/7.5	3.8/7.6	3.2/6.8	3.3/5.6	2.2/5.0	4.3/6.0

Note: Additional tables are in Appendix E.
Source: NREL (1994).

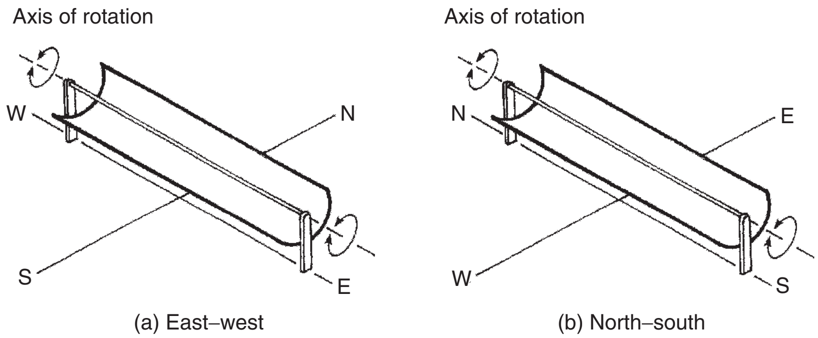


Figure 7.34 One-axis tracking parabolic troughs with horizontal axis oriented east–west or north–south. Most are oriented north–south.

TABLE 7.10 Sample of the Solar Data from Appendix E

	Los Angeles, CA: Latitude 33.93°N												
Tilt	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
Lat − 15	3.8	4.5	5.5	6.4	6.4	6.4	7.1	6.8	5.9	5.0	4.2	3.6	5.5
Lat	4.4	5.0	5.7	6.3	6.1	6.0	6.6	6.6	6.0	5.4	4.7	4.2	5.6
Lat + 15	4.7	5.1	5.6	5.9	5.4	5.2	5.8	6.0	5.7	5.5	5.0	4.5	5.4
90	4.1	4.1	3.8	3.3	2.5	2.2	2.4	3.0	3.6	4.2	4.3	4.1	3.5
1-Axis (Lat)	5.1	6.0	7.1	8.2	7.8	7.7	8.7	8.4	7.4	6.6	5.6	4.9	7.0
Temp (°C)	18.7	18.8	18.6	19.7	20.6	22.2	24.1	24.8	24.8	23.6	21.3	18.8	21.3

Solar data from the NREL Solar Radiation Manual have been reproduced in Appendix E, a sample of which is shown in Table 7.10.

Radiation data for Boulder are plotted in Fig. 7.35. As was the case for clear-sky graphs presented earlier, there is little difference in annual insolation for fixed, south-facing collectors over a wide range of tilt angles, but the seasonal variation is significant. The boost associated with single-axis tracking is large, about 30%.

Maps of the seasonal variation in insolation, such as that shown in Fig. 7.36, provides a rough indication of the solar resource and are useful when more specific local data are not conveniently available. Analogous figures for the entire globe are included in Appendix F. The units in these figures are average kWh/m^2-day of insolation, but there is another way to interpret them. On a bright, sunny day with the sun high in the sky, the insolation at the earth's surface is roughly 1 kW/m^2. In fact, that convenient value, 1 kW/m^2, is defined to be *1-sun of insolation*. That means, for example, that an average daily insolation of say 5.5 kWh/m^2 is equivalent to 1 kW/m^2 (1-sun) for 5.5 h; that is, it is the same as

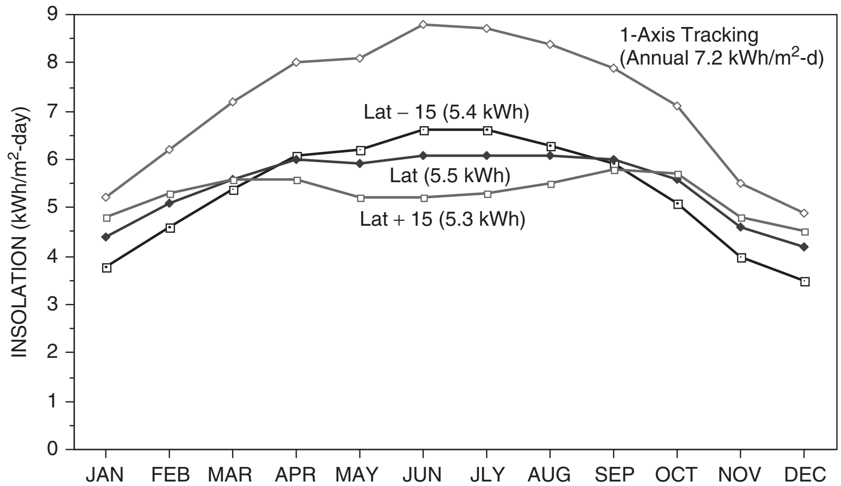


Figure 7.35 Insolation on south-facing collectors in Boulder, CO, at tilt angles equal to the latitude and latitude ±15°. Values in parentheses are annual averages (kWh/m^2-day). The one-axis tracker with tilt equal to the latitude delivers about 30% more annual energy.

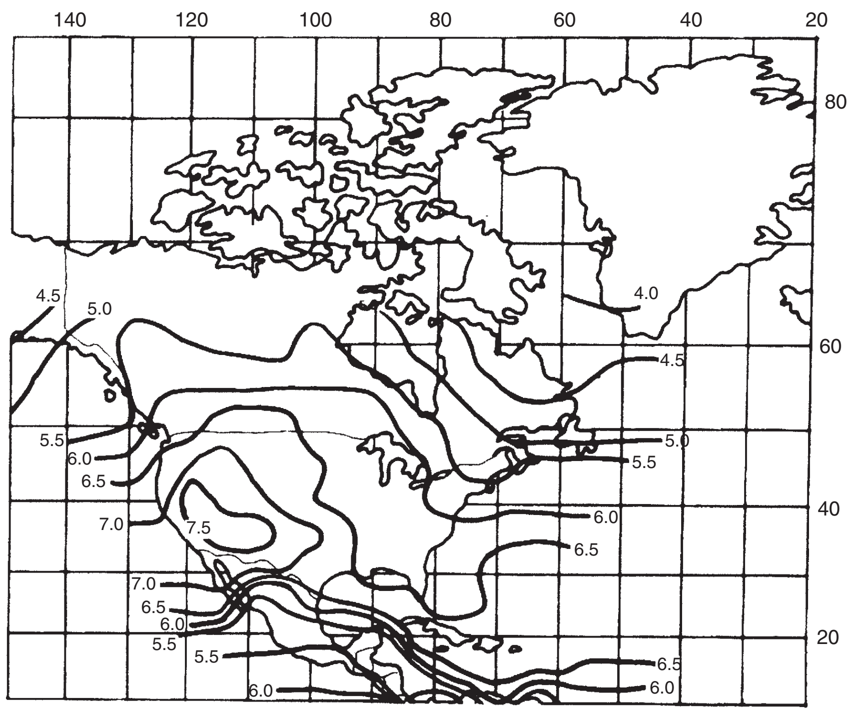


Figure 7.36 Solar radiation for south-facing collectors with tilt angle equal to L − 15° in summer (kWh/m^2-day). From Sandia National Laboratories (1987).

5.5 h of full sun. The units on these radiation maps can therefore be thought of as "hours of full sun." As will be seen in the next chapters on photovoltaics, the hours-of-full-sun approach is central to the analysis and design of PV systems.

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PROBLEMS

7.1 Using (7.5), determine the following:

- **a.** The date on which the earth will be a maximum distance from the sun.
- **b.** The date on which it will be a minimum distance from the sun.

7.2 What does (7.6) predict for the date of the following:

- **a.** The two equinox dates
- **b.** The two solstice dates

7.3 At what angle should a South-facing collector at 36° latitude be tipped up to in order to have it be normal to the sun's rays at solar noon on the following dates:

- **a.** March 21
- **b.** January 1
- **c.** April 1

7.4 Consider June 21st (the solstice) in Seattle (latitude 47°).

a. Use (7.11) to help find the time of day (solar time) at which the sun will be due West.

b. At that time, what will the altitude angle of the sun be?

c. As a check on the validity of (7.11), use your answers from (a) and (b) in (7.8) and (7.9) to be sure they yield an azimuth angle of 90°.

7.5 Find the altitude angle and azimuth angle of the sun at the following (solar) times and places:

a. March 1st at 10:00 A.M. in New Orleans, latitude 30°N.

b. July 1st at 5:00 P.M. in San Francisco, latitude 38°

c. December 21st at 11 A.M. at latitude 68°

7.6 Suppose you are concerned about how much shading a tree will cause for a proposed photovoltaic system. Standing at the site with your compass and plumb bob, you estimate the altitude angle of the top of the tree to be about 30° and the width of the tree to have azimuth angles that range from about 30° to 45° West of South. Your site is at latitude 32°.
Using a sun path diagram (Appendix B), describe the shading problem the tree will pose (approximate shaded times each month).

7.7 Suppose you are concerned about a tall thin tree located 100 ft from a proposed PV site. You don't have a compass or protractor and plumb bob, but you do notice that an hour before solar noon on June 21, it casts a 30-ft shadow directly toward your site. Your latitude is 32°N.

a. How tall is the tree?

b. What is its azimuth angle with respect to your site?

c. What are the first and last days in the year when the shadow will land on the site?

7.8 Using Figure 7.16, what is the greatest difference between local standard time for the following locations and solar time? At approximately what date would that occur?

a. San Francisco, CA (longitude 122°, Pacific Time Zone)

b. Boston, MA (longitude 71.1°, Eastern Time Zone)

c. Boulder, CO (longitude 105.3°, Mountain Time Zone)

d. Greenwich, England (longitude 0°, Local time meridian 0°)

7.9 Using Figure 7.16, roughly what date(s) would local time be the same as solar time in the cities described in Problem 7.8?

7.10 Find the local Daylight Savings Time for geometric sunrise in Seattle (latitude 47°, longitude 123 °W) on the summer solstice ($n = 172$).

7.11 Find the local Daylight Savings time at which the upper limb of the sun will emerge at sunrise in Seattle (latitude 47°, longitude 123°) on the summer solstice.

7.12 Equations for solar angles, along with a few simple measurements and an accurate clock, can be used for rough navigation purposes. Suppose you know it is June 22 ($n = 172$, the solstice), your watch, which is set for Pacific Standard Time tells you that (geometric) sunrise occurred at 4:23 A.M. and sunset was 15 hs 42 min later. Ignoring refraction and assuming you measured geometric sunrise and sunset (mid-point in the sun), do the following calculations to find your latitude and longitude.

a. Knowing that solar noon occurs midway between sunrise and sunset, at what time would your watch tell you it is solar noon?

b. Use (7.14) to determine your longitude.

c. Use (7.17) to determine your latitude.

7.13 Following the procedure outlined in Example 7.7, you are to determine your location if on January 1, the newspaper says sunrise is 7:50 A.M. and sunset is at 3:50 P.M. (both Central Standard Time). Solar declination $\delta = -23.0^\circ$ and Figure 7.16 (or by calculation) $E = -3.6$ minutes.

a. What clock time is solar noon?

b. Use (7.14) to determine your longitude.

c. Using (7.16), estimate the latitude without using the Q correction

d. Estimate Q and from that find the clock time at which geometric sunrise occurs.

e. Use (7.17) to determine your latitude.

7.14 A south-facing collector at latitude 40° is tipped up at an angle equal to its latitude. Compute the following insolations for January 1^{st} at solar noon:

a. The direct beam insolation normal to the sun's rays.

b. Beam insolation on the collector.

c. Diffuse radiation on the collector.

d. Reflected radiation on the collector with ground reflectivity 0.2.

7.15 Create a "Clear Sky Insolation Calculator" for direct and diffuse radiation using the following spreadsheet as a guide. In this example, the insolation has been computed to be 964 W/m^2 for a South-facing collector tipped up at 45° at noon on November 7 at latitude 37.5°. Note the third column simply adjusts angles measured in degrees to radians.

Use the calculator to compute clear sky insolation under the following conditions:

a. January 1, latitude 40°, horizontal insolation, solar noon

b. March 21, latitude 20°, South-facing collector with tilt 20°, 11:00 A.M. (solar time)

c. July 1, latitude 48°, South-East facing collector (azimuth 45°), tilt 20°, 2 P.M. (solar time)

Clear Sky Insolation Calculator:

		radians	Radians = 0.017453292| × degrees
Day number n	311		ENTER (excel uses radians)
Latitude (L)	37.5	0.6545	ENTER
Collector azimuth (ϕ_C)	0	0	ENTER (+ is east of south)
Collector tilt (Σ)	45	0.7854	ENTER
Solar time (ST) (24 h)	12		ENTER
Hour angle H	0	0	$H = 15°/\text{h} \times (12 - \text{ST})$ (7.10)
Declination (δ)	−17.11	−0.2986	$\delta = 23.45 \sin(360/365\ (n - 81))$ (7.6)
Altitude angle (β)	35.39	0.6177	$\beta = \text{ASIN}((\cos L \cos\delta \cos H + \sin L \sin\delta)$ (7.8)
Solar azimuth (ϕ_S)	0.000	0	$\phi_S = \text{ASIN}(\cos\delta \sin H / \cos\beta)$ (7.9)
Air mass ratio (m)	1.727		$\text{m} = 1/\sin\beta$ (7.4)
A (W/m^2)	1204		$\text{A} = 1160 + 75 \sin(360/365\ (n - 275))$ (7.22)
k	0.158		$k = 0.174 + 0.035 \sin(360/365(n - 100))$ (7.23)
I_B (W/m^2)	917		$I_B = A \exp(-km)$ (7.21)
$\cos\theta$	0.986		$\cos\theta = \cos\beta \cos(\phi_s - \phi_c) \sin(\Sigma) + \sin\beta \cos(\Sigma)$ (7.26)
I_{BC} (W/m^2)	904		$I_{BC} = I_B \cos\theta$ (7.24)
C	0.076		$C = 0.095 + 0.04 \sin(360/365(n - 100))$ (7.28)
I_{DC} (W/m^2)	60		$I_{DC} = C I_B (1 + \cos\Sigma)/2$ (7.29)
$I_C = I_{BC} + I_{DC}$ (W/m^2)	964		$I_C = I_{BC} + I_{DC}$ (ignoring reflection) (7.32)

7.16 Air Mass AM1.5 is supposedly the basis for a standard 1-sun insolation of 1 kW/m^2. To see whether this is reasonable, compute the following for a clear day on March 21st:

a. What solar altitude angle gives AM1.5?

b. What would be the direct beam radiation normal to the sun's rays?

c. What would be the diffuse radiation on a collector normal to the rays?

d. What would be the reflected radiation on a collector normal to the rays with $\rho = 0.2$?

e. What would be the total insolation normal to the rays?

7.17 Consider a comparison between a south-facing photovoltaic (PV) array with a tilt equal to its latitude located in Los Angeles versus one with a polar-mount, single-axis tracker. Assuming the PVs are 10% efficient at converting sunlight into electricity:

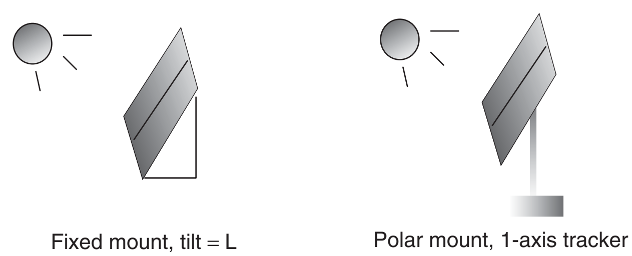

Figure P7.17

a. For a house that needs 4000-kWh per year, how large would each array need to be?

b. If the PVs cost \$400/m^2 and everything else in the two systems has the same cost except for the extra cost of the tracker, how much can the tracker cost (\$) to make the systems cost the same amount? How much per unit area of tracker (\$/m^2)?

c. Derive a general expression for the justifiable extra cost of a tracker per unit area (\$/m^2) as a function of the PV cost (\$/m^2) and the ratio of tracker insolation I_T to fixed insolation I_F.