ELC 4350: Principles of Communication

Digital Modulation Techniques

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Communication System Architecture

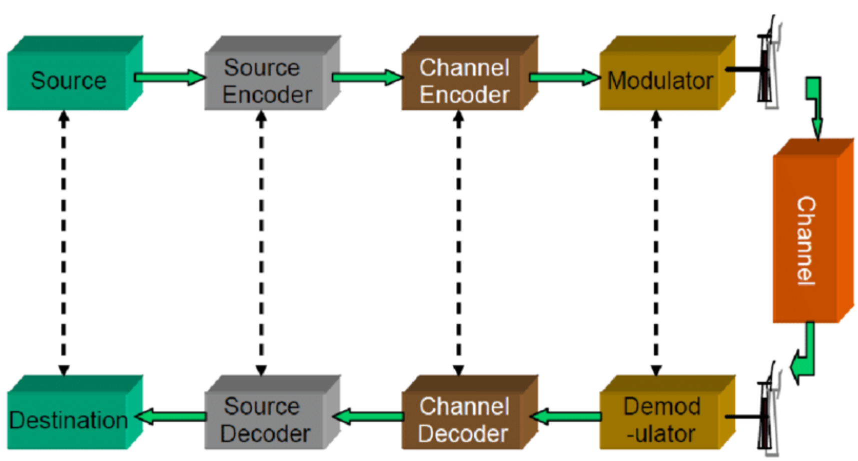


Figure: Typical Communication System.

Digital Modulation and Demodulation

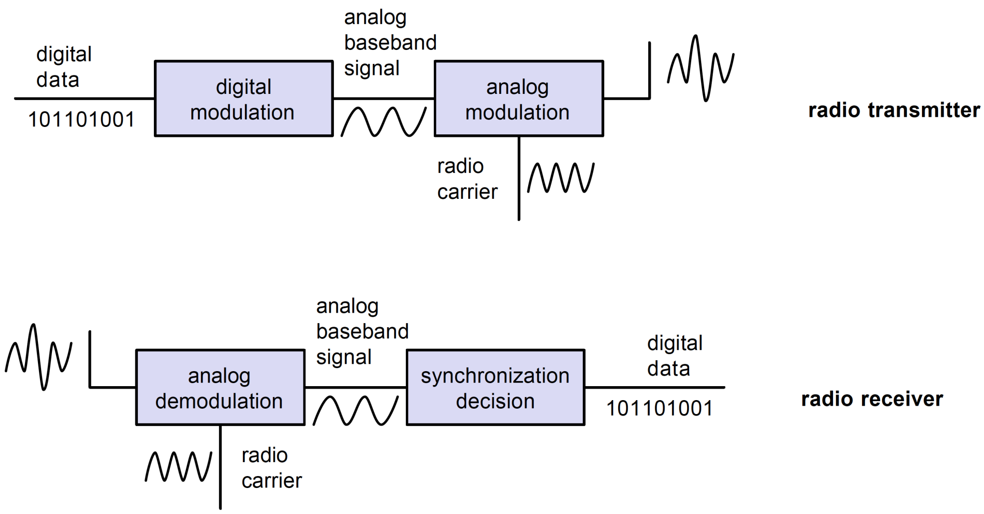


Figure: Digital Modulation and Demodulation.

Modulation

- Modulation: Converting digital data to analog waveform suitable for transmission over the communication medium.
- The message information is represented by the varying components of a (sinusoidal) carrier waveform:

$$c(t) = \underbrace{A_c}_{\text{amplitude}} \cos(2\pi \underbrace{f_c}_{\text{frequency}} t + \underbrace{\phi_c}_{\text{phase}})$$

- Basic techniques:
  1. Amplitude Modulation (AM) $\longrightarrow$ Amplitude Shift Keying (ASK)
  2. Frequency Modulation (FM) $\longrightarrow$ Frequency Shift Keying (FSK)
  3. Phase Modulation (PM) $\longrightarrow$ Phase Shift Keying (PSK)

Digital Modulation

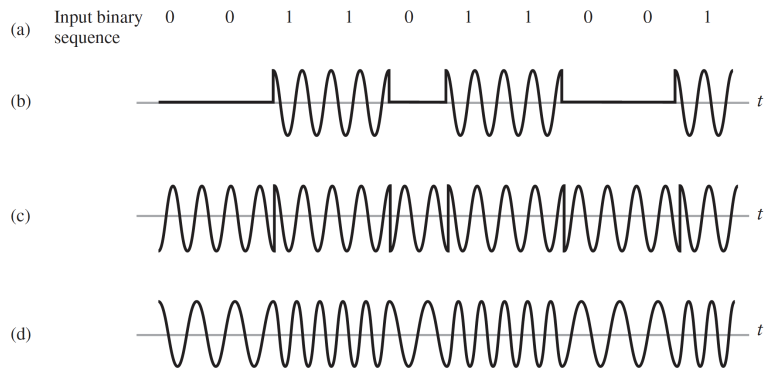


Figure: (a) Binary data. (b) ASK. (c) PSK. (d) FSK.

- ASK: Change amplitude with each symbol – Susceptible to interference
- PSK: Change phase with each symbol – Robust against interference
- FSK: Change frequency with each symbol – Larger bandwidth needed

Binary Amplitude Shift Keying (BASK)

- A binary data stream $b(t)$ of the On-Off signaling variety

$$b(t) = \begin{cases} \sqrt{E_b} & \text{, for binary symbol 1} \\ 0 & \text{, for binary symbol 0} \end{cases}$$

- The BASK signal is (with $\phi_c = 0$)

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{, for symbol 1} \\ 0 & \text{, for symbol 0} \end{cases}$$

where T_b is the bit duration.

- If the two binary symbols are equal-probable, $E_{\text{avg}} = E_b/2$.

Binary Phase Shift Keying (BPSK)

- The BPSK signal is

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{, for symbol 1} \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{, for symbol 0} \end{cases}$$

- The average transmitted power is constant.

Binary Phase Shift Keying (BPSK)

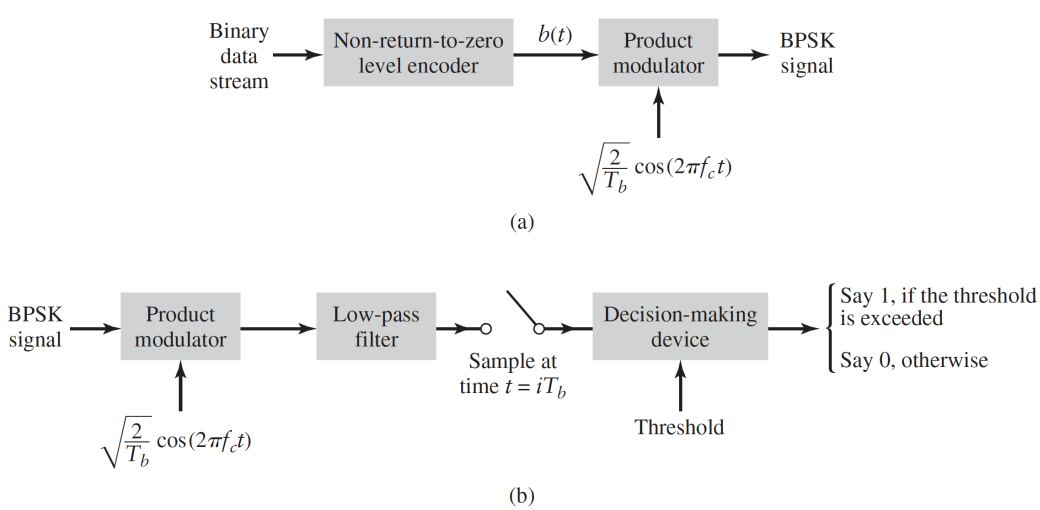


Figure: (a) BPSK modulator. (b) Coherent detector for BPSK.

A special case of DSB-SC modulation.

Quadrature Phase Shift Keying (QPSK)

- Efficient utilization of channel bandwidth
- The QPSK signal is

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \qquad i = 1, 2, 3, 4$$

  where E_s is the energy per symbol and T_s is the symbol duration.

- Each symbol s_i corresponds to one dibit from the Gray encoded set of dibits: $10, 00, 01, 11$.
- $T_s = 2T_b$.

Quadrature Phase Shift Keying (QPSK)

- Using trigonometric identity, we have

$$\begin{aligned} s_i(t) &= \sqrt{\frac{2E_s}{T_s}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) \\ &\quad - \sqrt{\frac{2E_s}{T_s}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t) \end{aligned}$$

- In the in-phase part

$$\sqrt{E_s} \cos\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} \sqrt{E_s/2} & \text{for } i = 1, 4 \\ -\sqrt{E_s/2} & \text{for } i = 2, 3 \end{cases}$$

Quadrature Phase Shift Keying (QPSK)

- Using trigonometric identity, we have

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T_s}}\sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t)$$

- In the quadrature part

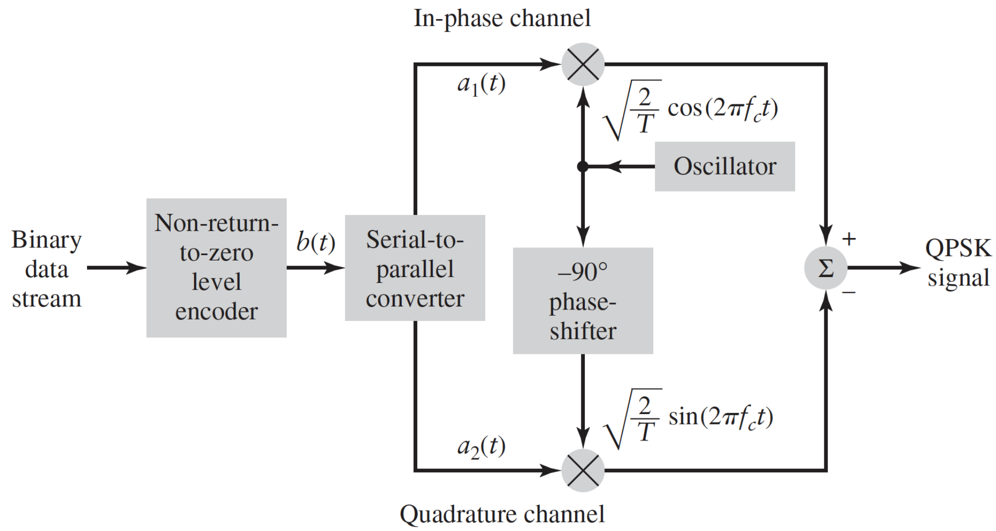
$$-\sqrt{E_s}\sin\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} -\sqrt{E_s/2} & \text{for } i=1,2 \\ \sqrt{E_s/2} & \text{for } i=3,4 \end{cases}$$

Quadrature Phase Shift Keying (QPSK)

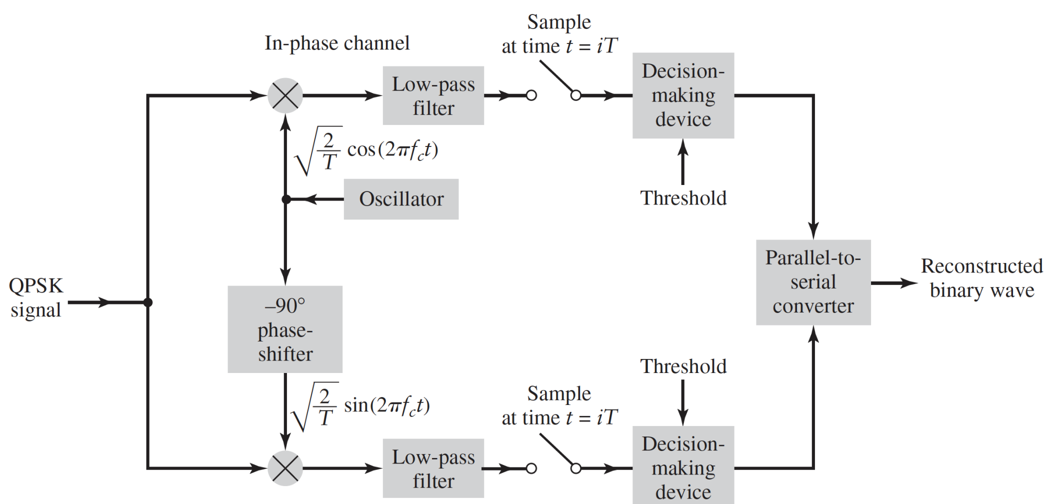
Index i	Phase of QPSK signal (radians)	Amplitudes of constituent binary waves: Binary wave 1 $a_1(t)$	Binary wave 2 $a_2(t)$	Input dibit $0 \le t \le T$
1	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	10
2	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	00
3	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	01
4	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	11

$$E_s/2 = E_b$$

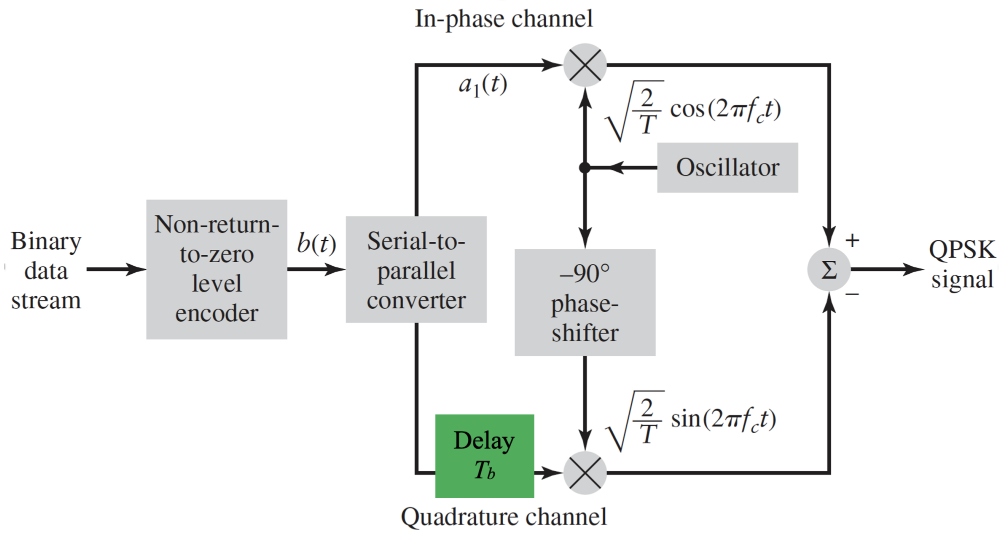
QPSK Transmitter



Coherent QPSK Receiver

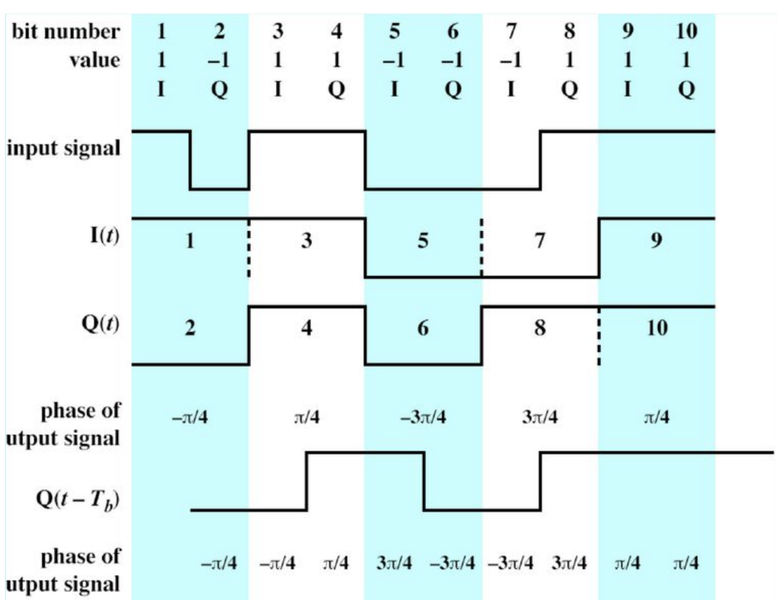


Offset QPSK (OQPSK)

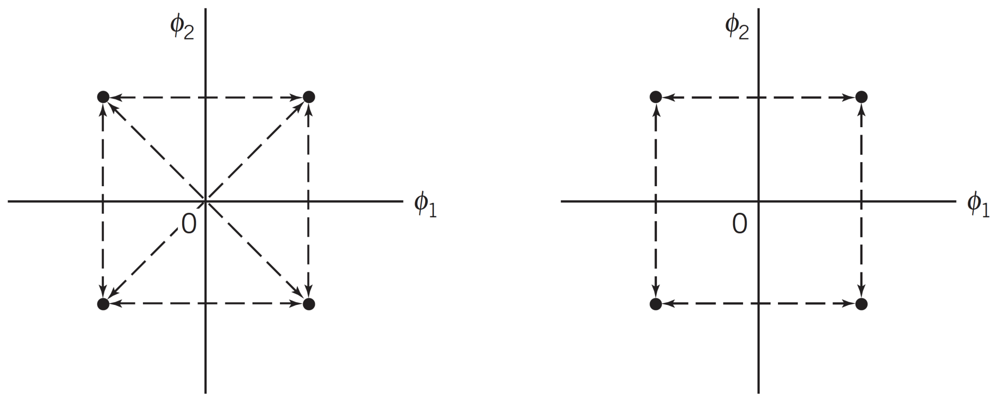


$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} I(t)\cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T_s}} Q(t - T_b)\sin(2\pi f_c t)$$

Offset QPSK (OQPSK)



Offset QPSK (OQPSK)



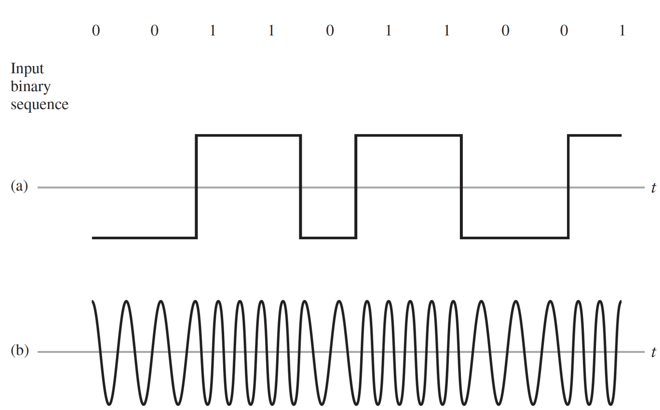
- Left: In QPSK, the carrier phase undergoes jumps of $0°, \pm 90°, \pm 180°$ every $2T_b$ seconds.
- Right: In OQPSK, the carrier phase undergoes jumps of $0°, \pm 90°$ every T_b seconds.
- In OQPSK, amplitude fluctuation due to nonlinear filtering is smaller.

Binary Frequency Shift Keying (BFSK)

- The BFSK signal is

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), \qquad i = 1, 2$$

- Sunde's BFSK when $|f_1 - f_2| = 1/T_b$. This modulated signal is a continuous-phase signal.



Minimum-Shift Keying (MSK)

- The frequency excursion is one half the bit rate

$$\Delta f = f_1 - f_2 = \frac{1}{2T_b}$$

- We have

$$\begin{aligned} f_1 &= f_c + \Delta f/2, \quad \text{for symbol 1} \\ f_2 &= f_c - \Delta f/2, \quad \text{for symbol 0} \end{aligned}$$

- The MSK signal as the angle-modulated signal

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)]$$

Minimum-Shift Keying (MSK)

- The MSK signal as the angle-modulated signal

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)]$$

- The phase $\theta(t)$ of the MSK signal

$$\theta(t) = \begin{cases} 2\pi \frac{\Delta f}{2} t = \frac{\pi t}{2T_b} & \text{, for symbol 1} \\ -2\pi \frac{\Delta f}{2} t = -\frac{\pi t}{2T_b} & \text{, for symbol 0} \end{cases}$$

- At $t = T_b$, $\theta = \pi/2$ for symbol 1 and $\theta = -\pi/2$ for symbol 0.
- In MSK, Δf is the minimum frequency spacing between symbols 0 and 1 that makes their FSK signals to be coherently orthogonal.

M-ary Digital Modulation

- A block of m bits to produce one symbol, with $M = 2^m$.
- Symbol duration $T_s = mT_b$, and the bandwidth required is proportional to $1/T_s = 1/(mT_b)$.
- Bandwidth conservation – A reduction in transmission bandwidth by a factor $m = \log_2 M$ over binary keying.

M-ary Phase Shift Keying

- M-ary PSK signal is a phase-modulated signal

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right), \qquad i = 0, 1, \ldots, M-1$$

- Using trigonometric identity, we have

$$\begin{aligned} s_i(t) &= \underbrace{\left[\sqrt{E_s}\cos\left(\frac{2\pi}{M}i\right)\right]}_{\text{Ii: in-phase}} \left[\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)\right] \\ &\quad \underbrace{-\left[\sqrt{E_s}\sin\left(\frac{2\pi}{M}i\right)\right]}_{\text{Qi: quadrature}} \left[\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)\right] \end{aligned}$$

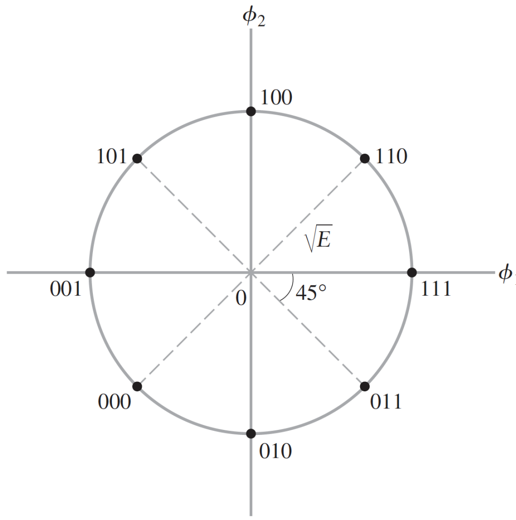
- The envelope of the signal is constant: $I_i^2 + Q_i^2 = E_s$

M-ary Phase Shift Keying

- 2D signal-space diagram with the horizontal and vertical axes representing the orthonormal functions

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t), \quad \phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t), \quad 0 \le t \le T_s$$

- Gray-encoded 8-PSK:



M-ary Quadrature Amplitude Modulation (M-ary QAM)

- The M-ary QAM is a hybrid form of ASK and PSK.

$$s_i(t) = \sqrt{\frac{2E_0}{T_s}} a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T_s}} b_i \sin(2\pi f_c t),$$

$$i = 0, 1, \ldots, M-1$$

where a_i in the in-phase component and b_i in the quadrature component are independent. E_0 is the lowest symbol energy.

M-ary Quadrature Amplitude Modulation (M-ary QAM)

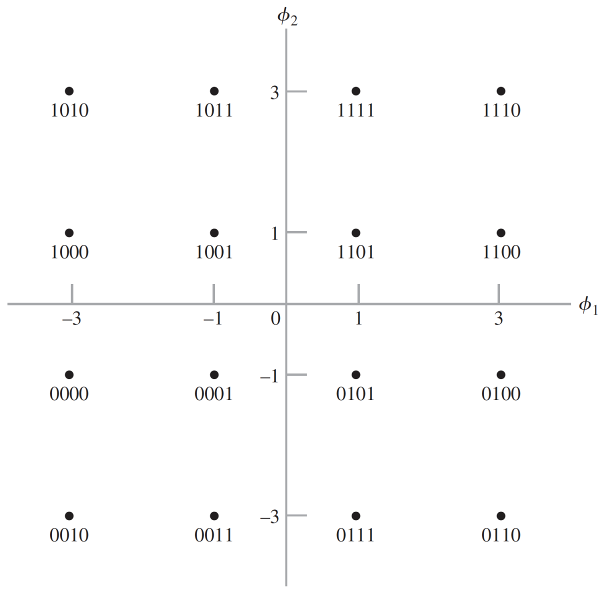


Figure: Signal constellation of Gray-encoded 16-QAM.

M-ary Frequency Shift Keying

- The M-ary FSK signal is

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[\frac{\pi}{T_s}(n+i)t\right], \qquad i = 0, 1, \ldots, M-1$$

- If $\Delta f = 1/(2T_s)$, the signals are orthogonal. That is

$$\int_0^{T_s} s_i(t)s_j(t)dt = \begin{cases} E_s & \text{, for } i = j \\ 0 & \text{, for } i \neq j \end{cases}$$

- Therefore, a complete set of orthonormal functions:

$$\phi_i(t) = \frac{1}{\sqrt{E_s}} s_i(t), \qquad i = 0, 1, \ldots M-1$$

M-ary Frequency Shift Keying

- M-dimensional signal space

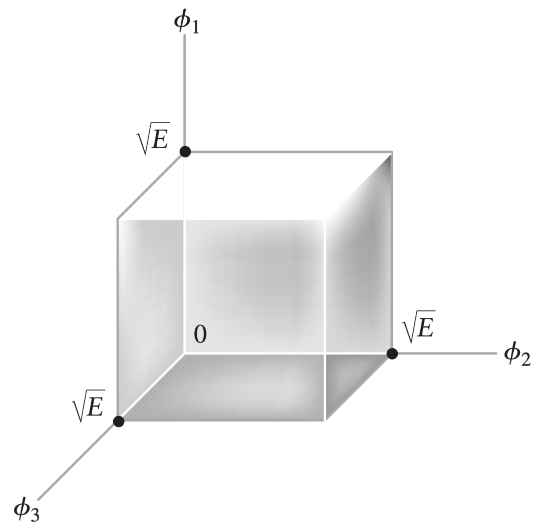


Figure: Signal constellation of 3-FSK.

Coherent Detection for Binary Phase-Shift Keying (BPSK)

- BPSK signals

$$
\begin{aligned}
s_1(t) &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t), \quad 0 \le t \le T_b \\
s_2(t) &= -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t), \quad 0 \le t \le T_b
\end{aligned}
$$

- With basis function

$$
\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \quad 0 \le t \le T_b
$$

- BPSK signals become

$$
\begin{aligned}
s_1(t) &= \sqrt{E_b}\phi_1(t), \quad 0 \le t \le T_b \\
s_2(t) &= -\sqrt{E_b}\phi_1(t), \quad 0 \le t \le T_b
\end{aligned}
$$

Coherent Detection for BPSK

- A (one-dimensional) signal constellation consists of two message points

$$s_1 = \int_0^{T_b} s_1(t)\phi_1(t)dt = +\sqrt{E_b}$$
$$s_2 = \int_0^{T_b} s_2(t)\phi_1(t)dt = -\sqrt{E_b}$$

Coherent Detection for BPSK

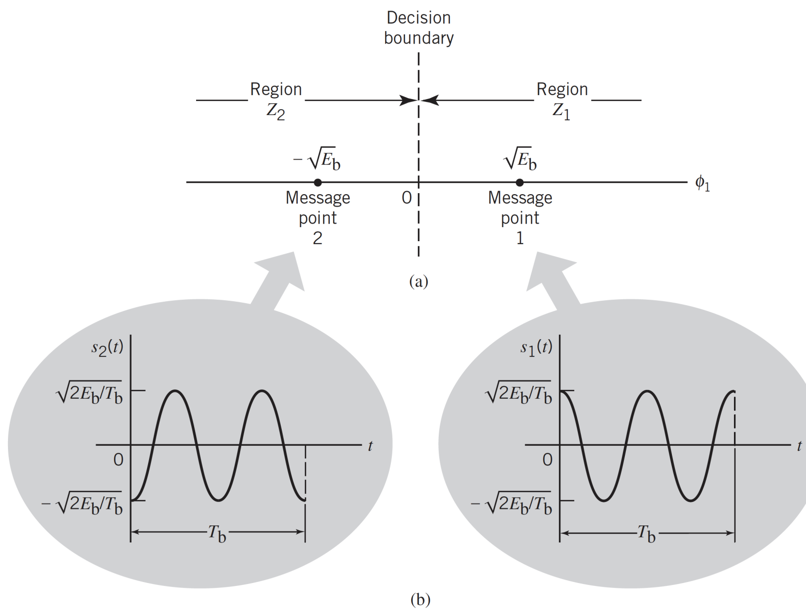


Figure: (a) Signal constellation of BPSK. (b) The transmitted waveforms.

Coherent Detection for BPSK

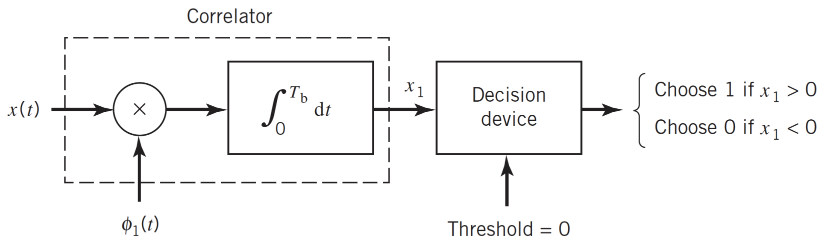


Figure: Coherent BPSK Receiver.

BPSK system operating on an AWGN channel,

$$x(t) = s_i(t) + w(t), \quad 0 \le t \le T_b,\ i = 1, 2$$

where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$.

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)dt$$

Coherent Detection for BPSK

- The conditional pdf of random variable X_1, given that symbol 0 (signal s_2) was transmitted, is

$$\begin{aligned} f_{X_1}(x_1 \mid 0) &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_2)^2\right] \\ &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \end{aligned}$$

- Therefore, the error probability of receiver deciding in favor of symbol 1 but symbol 0 was actually transmitted is

$$\begin{aligned} p_{10} &= \int_0^{\infty} f_{X_1}(x_1 \mid 0)dx_1 \\ &= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \end{aligned}$$

Coherent Detection for BPSK

- The error probability of receiver deciding in favor of symbol 1 but symbol 0 was actually transmitted is

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \\ &= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N0}}^\infty \exp\left[-\frac{z^2}{2}\right] dz \end{aligned}$$

where $z = \sqrt{\frac{2}{N_0}}(x_1 + \sqrt{E_b})$.

- Using the Q-function of Gaussian distribution, we have

$$p_{10} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Coherent Detection for BPSK

- The error probability of receiver deciding in favor of symbol 1 but symbol 0 was actually transmitted is

$$p_{10} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- Similarly, the error probability of receiver deciding in favor of symbol 0 but symbol 1 was actually transmitted is

$$p_{01} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- Therefore, the average probability of symbol error for BPSK (equivalently BER) is

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Coherent Detection for Quadri Phase-Shift Keying (QPSK)

- QPSK signals

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right), \quad 0 \le t \le T_s,\ i = 1, 2, 3, 4$$

$$\begin{aligned} s_i(t) &= \sqrt{\frac{2E_s}{T_s}} \cos\left((2i-1)\frac{\pi}{4}\right) \cos(2\pi f_c t) \\ &\quad -\sqrt{\frac{2E_s}{T_s}} \sin\left((2i-1)\frac{\pi}{4}\right) \sin(2\pi f_c t) \end{aligned}$$

Coherent Detection for QPSK

- With orthonormal basis function

$$\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \quad 0 \le t \le T_s \\ \phi_2(t) &= \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t), \quad 0 \le t \le T_s \end{aligned}$$

- There are four message points, defined by the two-dimensional signal vector

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E_s} \cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E_s} \sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix}, \quad i = 1, 2, 3, 4$$

- QPSK has two-dimensional signal constellation and four message points.

Coherent Detection for QPSK

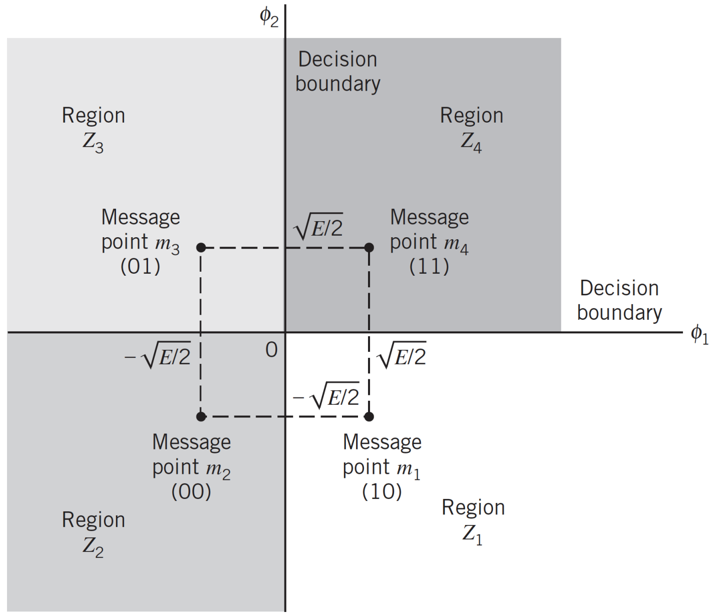


Figure: Signal constellation of QPSK.

Coherent Detection for QPSK

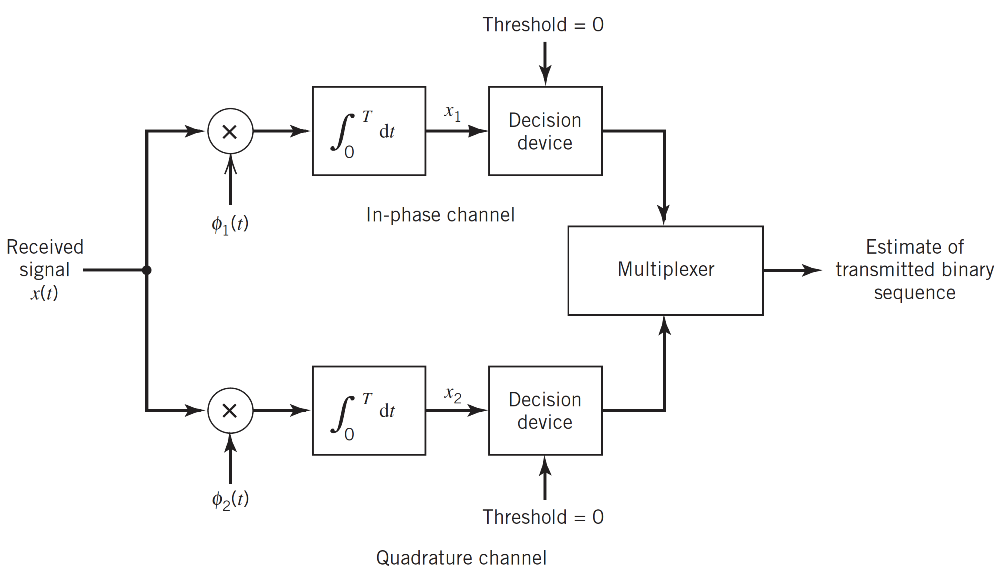


Figure: Coherent QPSK receiver.

Coherent Detection for QPSK

- QPSK system operating on an AWGN channel, the received signal is

$$x(t) = s_i(t) + w(t), \quad 0 \le t \le T_s,\ i = 1, 2, 3, 4$$

where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$.

- In-phase channel

$$x_1 = \int_0^{T_s} x(t)\phi_1(t)dt = \sqrt{E_s}\cos\left((2i-1)\frac{\pi}{4}\right) + w_1 = \pm\sqrt{\frac{E_s}{2}} + w_1$$

- Quadrature channel

$$x_2 = \int_0^{T_s} x(t)\phi_2(t)dt = \sqrt{E_s}\sin\left((2i-1)\frac{\pi}{4}\right) + w_2 = \mp\sqrt{\frac{E_s}{2}} + w_2$$

Coherent Detection for QPSK

- Similar to BPSK, we can find the probability of bit error in each of the in-phase and quadrature paths of QPSK receiver is

$$P' = Q\left(\sqrt{\frac{E_s}{N_0}}\right), \quad E_s = 2E_b$$

- In-phase and quadrature components are independent. The average probability of a correct detection is

$$\begin{aligned} P_c &= (1-P')^2 = \left[1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2 \\ &= 1 - 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q^2\left(\sqrt{\frac{E_s}{N_0}}\right) \end{aligned}$$

Coherent Detection for QPSK

- The average probability of symbol error for QPSK is

$$P_e = 1 - P_c = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)$$

- When $E_s/N_0 \gg 1$, $P_e \approx 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$.

- With Gray encoding, the bit-error-rate (BER) of QPSK is

$$\mathrm{BER} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- For the same E_b/N_0, QPSK can transmit information at twice the bit rate of BPSK for the same channel bandwidth with the same BER.

Coherent Detection for M-ary PSK

- M-ary PSK signal

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + (i-1)\frac{2\pi}{M}\right], \quad i = 1, 2, \ldots, M$$

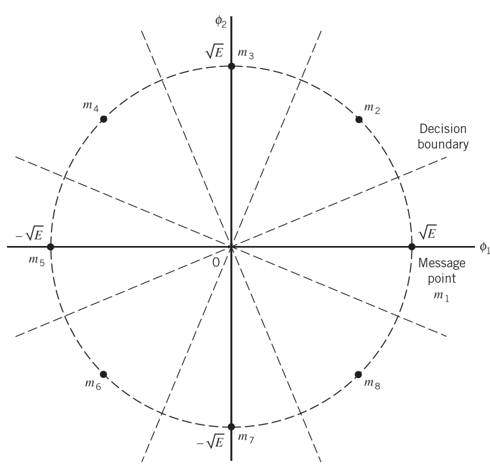


Figure: Signal constellation of octaphase-shift keying.

Coherent Detection for 8PSK

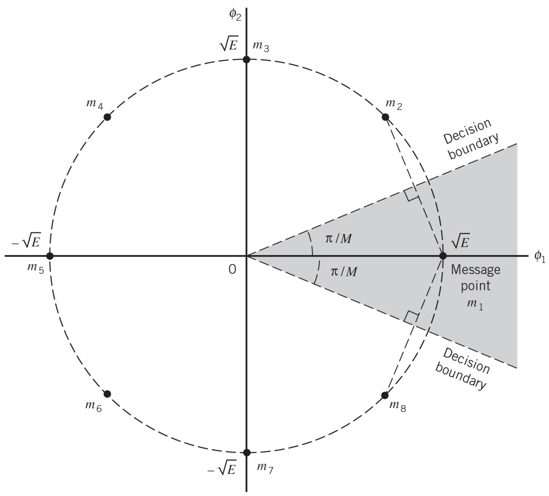


Figure: Signal constellation of octaphase-shift keying.

The Euclidean distances:

$$d_{12} = d_{18} = 2\sqrt{E_s}\sin\left(\frac{\pi}{M}\right)$$

The average probability of symbol error for coherent M-ary PSK:

$$P_e \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

Coherent Detection for M-ary Quadrature Amplitude Modulation

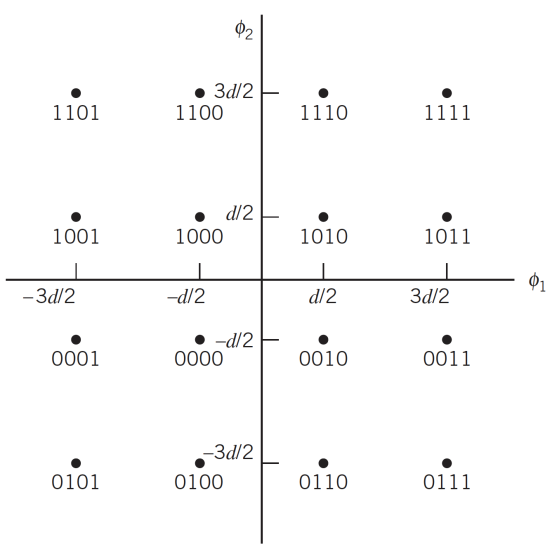


Figure: Signal constellation of 16-QAM.

The probability of symbol error of L-PAM ($L = \sqrt{M}$)

$$P'_e = 2\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$$

The probability of symbol error for M-ary QAM

$$P_e = 1 - (1 - P'_e)^2 \approx 2P'_e$$

$$P_e \approx 4\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$$

where $\sqrt{E_0} = d_{\min}/2$.

Comparison of Digital Demodulation over AWGN Channels

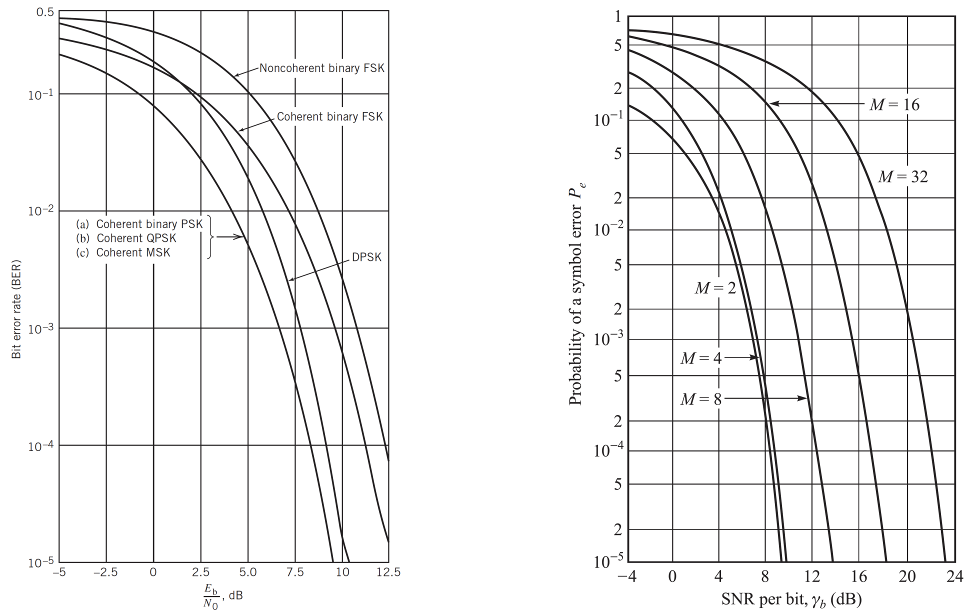


Figure: Performance comparison of different PSK and FSK signaling over AWGN channel.