Abstract—Quantum-dot cellular automata (QCA) was conceptualized to provide low-power, high-speed, general-purpose computing in the post-CMOS era. Here, an elementary device, called a “cell” is a system of quantum dots and a few mobile charges. The configuration of charge on a cell encodes a binary state, and cells are networked locally using the electrostatic field. Layouts of QCA cells on a substrate provide non-von-Neumann circuits in which digital logic, interconnections, and memory are intermingled. QCA supports reversible, adiabatic computing for arbitrarily low levels of dissipation. Here, we focus on a molecular implementation of QCA and describe the promise this holds. This discussion includes an outline of an architecture for clocked molecular QCA circuits and some technical challenges remaining before molecular QCA computation may be realized. This work focuses on the challenge of using macroscopic devices to write-in bits to nanoscale QCA molecules. We use an electric field established between electrodes fabricated using standard, mature lithographic processes, and the field need not feature single-molecule specificity. An intercellular Hartree approximation is used to model the state of an \(N\)-molecule circuit. Simulations of a method for providing bit inputs to clocked molecular circuits are shown.

I. INTRODUCTION

The design and operation of transistors at the physical limits of scaling has led to significant power dissipation [1], the real and very present limit to progress. Present fabrication technologies support device densities of over \(10^9\) cm\(^{-2}\), but full device utilization will dissipate levels of heat that can destroy the chip. While design techniques such as multicore processors and dark silicon mitigate actual levels of dissipation, the ability to sink this heat dissipation has limited commercial computer clock speeds to 3-4 GHz since 2003. Additionally, power dissipation contributes directly to the significant and growing percentage of global electrical power consumed by information and computing technologies, calculated at 10% in 2010 and estimated to reach 30-50% in 2030 [2]. Clearly, to achieve energy-efficient, next-generation computing technologies, solutions beyond CMOS must be considered.

To answer the concerns raised by power dissipation in CMOS, a low-power, general-purpose, digital computational paradigm known as quantum-dot cellular automata (QCA) was developed [3]. In particular, a molecular implementation of QCA promises nanometer-scale devices [4]–[6], THz-or-faster switching speeds [7], and room-temperature operation. While QCA devices have been fabricated [8] and the concept of information processing using QCA circuits has been established [9], [10], technical challenges persist in the realization of molecular QCA.

In this paper, we provide simulations as \textit{in silico} demonstrations of a technology for writing bits to nanoscale molecular QCA circuits. The discussion begins with Section II, which provides a brief overview of the QCA concept and emphasizes the molecular implementation. The concept of clocking in molecular QCA circuits is presented. The conclusion to Section II sets the context for our work by briefly overviewing the technical challenges which must be overcome if computation using molecular QCA is to be achieved, including: molecule design; molecular QCA circuit layout; bit inputs to the QCA molecules; and bit readout from molecular QCA circuits. In particular, own work proposes a solution for bit inputs to clocked molecular QCA circuits using electrodes which may be fabricated using mature nanolithographic processes. Section III describes the model used for describing a QCA circuit immersed in an electric field \(\vec{E}(\vec{r})\) which provides both bit inputs and clocking. Simulation results of bits selected on clocked molecular QCA circuits are presented in Section IV. This solution for bit write-in is important because it may enable the experimental demonstration of controlled switching of QCA molecules, which in turn, will support the development and testing bit read-out technologies. This discussion is concluded in Section V by underscoring the fact that novel solutions to one technical challenge in realizing molecular QCA can help enable solutions to other challenges.

II. OVERVIEW OF QCA

In QCA, the elementary device is a cell, a structure with a set of quantum dots which provide charge localization sites for a few mobile charges. The configuration of charge on these dots encodes a bit, and device switching occurs via the quantum tunneling of charge between the dots.

Fig. 1 depicts three states of a cell with six dots and two mobile electrons. The two states, labeled “0” and “1” are designated active states, and the third state is designated as “Null,” conveying no information. The cell can be clocked to the Null state by applying a positive voltage sufficient to attract the electrons to the central null dots (dots 2 and 5). We refer to dots 1, 3, 4, and 6 as active dots. For this cell, it is
a design feature that direct tunneling between “0” and “1” is suppressed: the transition between either of the active states requires an intermediate transition to the “Null” state.

![Charge-localized states of a six-dot QCA cell.](image)

When arranged on a substrate, neighboring cells are networked locally via the electrostatic field, enabling general-purpose computation. Basic QCA circuits are shown in Fig. 2. Cells arranged in a row tend to align via simple Coulomb repulsion, and diagonal coupling can be used to achieve a bit inversion. The natural logic gate in QCA is the majority gate, for which three inputs have each an equal influence over a device cell. The bit in the majority on the inputs appears on the device cell and is copied to the output. Any one of the three inputs can function as a control bit, making the majority gate function as a programmable two-input AND/OR gate between the other two inputs. These devices provide a logically-complete set, from which more complex devices may be formed. QCA circuits such as adders and a Simple-12 processor have been designed [9], [10].

![Basic QCA devices.](image)

A. Molecular QCA

There are various implementations for QCA. The earliest implementation of QCA used metallic dots patterned on an insulating substrate [8], [11]. Here, tunnel junctions allow inter-dot charge transfer. Later, semiconductor quantum dots were used [12], [13]. Also, cells have been written on a hydrogen-passivated silicon surface using a scanning tunneling microscope (STM) tip: individual H atoms were removed, exposing single dangling bonds, each of which functions as a dot [14]. By virtue of nanometer-scale dimensions, these atomic-scale QCA cells have bit energies of the electron-volt scale and are robust at room temperature. Molecular QCA also support room-temperature operation and are the focus of this paper. Here, mixed-valence molecules function as QCA cells, and redox centers on those molecules provide dots [5], [6]. Molecular QCA also may support THz-scale or better device operation [7].

The zwitterionic nido carborane molecule (\(\text{Fc}^+\text{FcC}_2\text{B}_9^-\)) was designed specifically for use as a QCA molecule [15]. \(\text{Fc}^+\text{FcC}_2\text{B}_9^-\) is a self-doping molecule with a net-zero charge which provides three dots, as shown in Fig. 3. In \(\text{Fc}^+\text{FcC}_2\text{B}_9^-\), the mobile charge is a single hole. Two such molecules can be paired to function as a six-dot QCA cell like the one depicted in Fig. 1.

![A zwitterionic nido carborane (\(\text{Fc}^+\text{FcC}_2\text{B}_9^-\)) molecule is designed](image)
Fig. 4. The bit energy for a molecular two-dot cells is the kink energy, the difference in electrostatic energies of the two-cell favored configuration [subfigure (a)] and the kinked configuration [subfigure (b)], given that the driver is fixed as shown.

dots, and the cell takes the null state regardless of neighbor interactions. Since direct tunneling between active states is suppressed, clocked molecular QCA support latching. Once the cell is clocked to an active state $X$, it is latched in $X$ until the clock is lowered: a bit flip $X \rightarrow \bar{X}$ first requires a reset to the null state, which is prevented by the clock.

B. Clocked Molecular QCA

Molecular six-dot cells may be clocked using an externally-applied electric field. Light-blue spheres represent the quantum dots of a six-dot cell, and a red sphere represents a single mobile electron. The cell is adsorbed onto the substrate at the central (“null”) dots such that the active dots used to represent “0” and “1” are elevated above the substrate. An electric field with a vertical component can be applied to the molecule using a buried conducting slab. A negative voltage establishes a field that repels the mobile electrons from the null dots. With this bias, the cell is forced to the active state favored by neighbor interactions. A positive voltage, on the other hand, attracts the electrons to the null dots so that the cell takes the null state regardless of the states of neighboring molecules.

Arrays of independently-charged conductors can be used to create an inhomogeneous electric field at the device layer, as shown in Fig. 7. Some domains of the field activate cells; other regions clock cells to the null state. Calculations will take place in the transition region between active and null domains. It is worth noting that the clocking conductors may be much larger than the molecules themselves. QCA molecules are of the 1-nm scale, and it would be onerous if not intractable to wire each individual molecule, a task that this clocking scheme avoids.

Calculations become dynamic when the excitation applied to the clocking wires is time-dependent. Consider the plan view of the clocking wires shown in Fig. 8. Here, an array of parallel clocking wires is laid out like ties supporting a railroad track. A four-phase voltage is applied to this set of wires, resulting in a time-varying electric field at the device layer with active domains which propagate rightward along the track. A calculation of $E_z(\vec{r})$ (pointing out of the page) at the device plane is visualized in the lower part of Fig. 8.

Fig. 9 demonstrates how active domains from the field of Fig. 8 can be used to drive bit packets through circuitry. Here, we zoom in from the length scale of the clocking wires (100 nm in length) to the length scale of several cells (each cell is a 1-nm-by-1-nm square) to see an active domain propagating along a binary wire. The strong clocking field at the center of the active domain has latched cells in a given state. Cells at the leading edge of the active domain are activated to their immediate left (more internal to the active domain). Computation—in this case, a bit copy—occurs at the leading edge of the moving active domain. At the trailing edge of the active domain, cells are released to the null state in preparation
Multiple independently-charged conductors can establish an electric field with an inhomogeneous vertical component at the QCA device plane. In some domains of this field, cells will be driven to active states, whereas in other regions, cells will be driven to the null state. Calculations and erasures will occur in the transitions between active and null domains.

for another computation. Since the action of the binary wire is clocked rather than ballistic, we refer to this as a shift register. Computations may be more complex than a mere copy, however: logic gates such as AND, OR, NOT, and XOR have been simulated.

In this clocking paradigm for molecular QCA, clock parameters determine circuit operation. For example, the frequency of the clock signal determines the number of calculations driven through a particular circuit per unit time. The pitch of the clocking conductors will determine the wavelength of the clock, or the length of the active domains. The amplitude of the clocking voltage along with any DC bias applied will determine the strength of active and null domains.

C. Realizing Molecular QCA Computation

Technical challenges remain in the path to realizing molecular QCA. We briefly outline these challenges here, and discuss the state of solutions to these challenges. Technical challenges include:

1) **Molecule design.** The design of QCA molecules is an ongoing endeavor involving physicists, synthetic chemists, and physical chemists. New molecules are being designed and studied as QCA candidates [15], [17]. Models are being developed to predict the performance of molecules as QCA devices. Such predictive models must treat relevant quantum phenomena for particular QCA candidates such as electron localization and the self-trapping of charge [18]–[20], vibration-coupled electron transfer [7], as well as environmentally-driven quantum decoherence [21], [22], power dissipation, and disentanglement [23]. Candidate molecules may be functionalized with a ligand to facilitate adsorption on a substrate.

Some helpful advances have come from this work. The development of net-neutral, self-doping QCA candidate molecules [15] eliminate the need for additional counterions associated with each cell. Theoretical work suggests that environmental interactions strengthen molecular QCA bits [21]. Models of field-driven, vibration-coupled electron transfer suggest QCA can switch at THz or faster speeds [7]. Controlled switching has not yet been observed in any molecule. Thus, efforts on designing, synthesizing, and testing individual molecular QCA species will be efforts well-spent. Models of circuit-level behaviors for particular molecular species may also be developed.

2) **Circuit layout.** The layout of molecular circuitry is another technical challenge. Homogenous monolayers of devices, while the most simple to achieve, perform no useful calculation. Therefore, the preferential layout of molecules in well-defined, specific arrangements is necessary. For circuit layout, self-assembled techniques are promising. Researchers have demonstrated exquisite control over processes to form 2D and 3D structures from DNA [24], [25]. In particular, DNA tiles may be used as molecular circuit boards [26]. Here, DNA tiles will be programmed with specific sites to conjugate with a docking ligand built into the QCA molecule. It may be possible to control the location of DNA tiles on a substrate using lithographic techniques.

One challenge with DNA tiles is localized negative charges due to phosphate groups on the DNA itself. Stray charge has been shown to adversely affect the performance of QCA devices [27], [28]. Therefore, tiles formed from pep-
Fig. 8. A four-phase voltage is applied to an array of wires to create a time-dependent, inhomogeneous clocking electric field. Middle: a plan view of a layout of clocking wires is shown. Wires (colored lines) are laid out in parallel, like railroad ties. A four-phase voltage is applied to these wires, and the color of each wire indicates one of the four particular voltage phases. Upper right: one phase of the four-phase clocking voltage is shown here, with clock period $T_{\text{clk}}$. Bottom: $E_z$, the $\hat{z}$-component (coming out of the page) of the electric field at the device layer is coded in the grayscale background. White regions in $E_z$ represent active domains, and black regions in $E_z$ drive cells to the null state. White lines mark the position of clocking wires, and red arrows indicate that the active domains propagate rightward in time.

Tide nucleic acid (PNA) [29] may better serve as molecular QCA circuit boards, since PNA has no such stray charge.

3) **Bit write-in for molecular circuits.** The nanometer scale of molecular QCA makes bit input more challenging than in conventional devices. It is neither possible nor desirable to form contacts that address individual molecules, because lithographically-feasible devices are an order of magnitude larger than molecular QCA. Optical control of molecular QCA is challenging because optical wavelengths are much longer than the length scale of desirable QCA molecules, and optical interactions are weak at this scale. Additionally, beam sizes are macroscopic, so they can only illuminate large groups of molecular QCA.

Some solutions to the bit input problem have been proposed. One system proposed complementary pairs of fixed-state molecules as bit sources to a shift register [30]. The clock is used to selectively couple one of the sources to a binary wire. In addition to the molecular species required to implement the clocked molecular QCA, this solution likely requires the use of a second molecular species to provide the fixed input sources. Another concept modeled by Pulimen, et al, was an input electric field applied to a single molecule at the end of a row of clocked, three-dot bisferrocene molecules [31]. While this eliminates the need for fixed-state molecules, nanoelectrodes are required to generate a field with single-molecule specificity.

4) **Bit read-out from molecular QCA circuits.** One promising approach to the detection of the state of QCA molecules is the use of single-electron devices, such as single-electron transistors (SETs) [32] and single-electron boxes (SEB) [33]. In particular, SETs have demonstrated sensitivity to single-electron tunneling events over distances less than one nanometer [34]. SETs and SEBs require cryogenic temperatures for operation. The readout of molecular QCA states using SETs or SEBs has not yet been achieved. This will be enabled by the development of a method for controllably switching QCA molecules which is amenable to SET- or SEB-based read-out.

The focus of this paper is to present models of a solution for bit write-in to clocked molecular QCA circuits which requires neither fixed-state QCA molecules nor fields with single-molecule specificity. This extends an earlier work that proposed an input system for asynchronous molecular QCA.
circuits, in which lithographically-formed electrodes establish an input field $E_{\text{in}}$ to select the state of multiple unclocked two-dot molecules [35]. The selected input bit packet may then be transmitted via a binary wire to QCA circuitry for processing. It was shown that single-molecule specificity is not required of $E_{\text{in}}$ for the input circuits to work; however, if the interaction of $E_{\text{in}}$ with the binary wire portion can be suppressed, the circuits will be more robust.

III. MODEL FOR WRITE-IN TO MOLECULAR QCA CIRCUITS

A single clocked molecular QCA circuit is depicted in Fig. 10. Here, we apply a constant field $E(\vec{r}) = E_{\text{in}} + E_{\text{clk}}$. Input field $E_{\text{in}}(\vec{r})$ selects bits on cells 1-3, and the field $E_{\text{clk}}(\vec{r})$ for clocking the entire molecular circuit. The input field $E_{\text{in}}$ is idealized to

$$E_{\text{in}}(\vec{r}) = \begin{cases} E_y \hat{y}, & \text{in the volume between electrodes,} \\ 0, & \text{elsewhere,} \end{cases}$$

with

$$E_y = \frac{v_{\text{in}}}{d}.$$  

(2)

Thus, present results are calculated within a limit where interaction between $E_{\text{in}}$ and the shift register (cells 4-8) may be suppressed. The clock field $E_{\text{clk}}(\vec{r})$ is assumed to be uniform: $E_{\text{clk}}(\vec{r}) = E_z \hat{z}$ over all space. Cells 4-8 shift the selected input bit to computational QCA circuits. With this approach, a single molecular species can provide both input devices and computational QCA circuits.

Fig. 10. An input voltage $v_{\text{in}}$ is applied to electrodes to select a QCA input bit. Each three-dot QCA molecule is schematically drawn as system of three quantum dots (black circles). The middle dot is drawn smaller than the other two to indicate that it sits on the substrate $(z = 0)$, while the other dots are elevated above the substrate in the $+\hat{z}$ direction. Each cell is assigned the number next to its middle dot. The applied $v_{\text{in}}$ establishes an electric field $E = E_y \hat{y}$, which immerses the column of molecules aligned in the $\pm \hat{y}$ direction (cells 1-3). Thus, $E_y$ will select the state of molecules 1-3, which are activated when a negative clock (the $z$-component, $E_z \hat{z}$) is applied. The row of molecules aligned in the $\pm \hat{z}$ direction (cells 4-8) will function as a shift register and can transmit the input bit to other QCA circuitry.

A three-dot molecule like $\text{Fc}^+\text{FcC}_2\text{B}_9^-$ is modeled as a quantum three-state system. Fig. 11 shows the localized states of a single mobile electron for this molecule. These states form a basis $B = \{ |0\rangle, |N\rangle, |1\rangle \}$ for the molecule’s electronic quantum state. Here, $a$ is a displacement vector from dot 1 to dot 0 of the cell, and $\vec{h}$ is a displacement from dot zero to a line passing through both dots 0 and 1 perpendicular to $\vec{a}$. All charges are treated as point charges.

The Hamiltonian for a single cell immersed in electrostatic field $E$ may be written as

$$H = H_o + H_E + H_{\text{un}}$$  

with

$$H_o = -\gamma \left( \hat{P}_{1,N} + \hat{P}_{N,1} + \hat{P}_{0,N} + \hat{P}_{N,0} \right),$$

$$H_E = -\frac{q_e E \cdot \vec{a}}{2} \left( \hat{P}_{1,1} - \hat{P}_{0,0} \right) - q_e \vec{E} \cdot \vec{h} \hat{P}_{N,N},$$

$$H_{\text{un}} = \sum_{j \in \{0, N, 1\}} U_j^{(l)} \hat{P}_{j,j}.$$  

(5)

(6)

Inter-dot electron transfers are described by $\hat{H}_{\text{un}}$, in which $\gamma$ is the hopping energy and $\hat{P}_{j,j} \equiv |j\rangle \langle k|$ are transition operators. The projection operators $\hat{P}_{j,j} \equiv |j\rangle \langle j|$ are used to form $H_E$, which includes the effects of the applied field. The first term of $H_E$ describes the field-driven bias between the active states $|0\rangle$ and $|1\rangle$, and the second term describes the field-driven bias between $|N\rangle$ and the active states. Finally, $H_{\text{un}}$ describes the interaction between the cell and any neighbors, each labeled with an integer $l \in \{1, 2, \ldots \}$. The electrostatic potential energy $U_j^{(l)}$ is calculated by setting the target cell to state $|j\rangle$ and then evaluating the electrostatic energy of interaction between the charges from neighbor $l$ with the charges in the target cell. $U_j^{(l)}$ depends on the state of the $l$-th neighbor cell.

The ground state of a circuit of $M$ cells maps to the desired calculational result. To calculate the ground state, an intercellular Hartree approximation (IHCA) is made [36]. This involves making an initial guess for the ground state of each cell in the $M$-cell circuit, and then iterating through the circuit,
relaxing each cell to its ground state. This process is repeated until a self-consistent configuration is achieved for the circuit.

IV. RESULTS

A simulation of a clocked QCA input system of Fig. 10 is shown in Fig. 12. Specifying a bit on the input cells is a matter of specifying the sign of \( E_y \) of an appropriate magnitude and then applying a clock voltage \( \vec{E}_{clk} = E_z \hat{z} \). The field is specified in units of \( E_o \), the input field strength required to induce a kink between two molecules:

\[
E_o = \frac{|E_k|}{q_e a}.
\]  \hspace{1cm} (7)

Here, even an input field \( E_y < E_o \) is sufficient to select an input bit, and an applied clock \( E_z = -1.5E_o \) activates the system of cells. In this case, cells 1 and 3 could be eliminated; however, they are necessary in the case where \( \vec{E}_{in} = E_y \hat{y} \) is uniform and is applied to all cells (molecules 1-8) (see [35]). Cells 1 and 3 also may be helpful when fringing field effects are considered. These results were calculated using an IHCA approach with only nearest-neighbor and next-nearest neighbor interactions.

The result of Fig. 12 is extended simply to a simulated ground state for a majority gate, shown in Fig. 13. Here, three separate bit inputs are supplied to a majority gate, and the two majority bits dominate the minority bit.

While this demonstrates in silico the viability of clocked majority logic using three-dot molecules, a more realistic simulation of the electrode/molecular-circuit input system requires further development of the model’s software implementation. A larger circuit footprint would be required to accommodate large input electrodes, and the majority logic will likely require isolation from fringing fields due to input electrodes. This will require longer shift registers between inputs and the majority gate. A larger circuit with more molecules will likely require a non-uniform, time-dependent clock \( \vec{E}_{clk}(\vec{r},t) \).

More detailed explorations of molecular QCA input systems are needed. Some modifications to the molecular input circuitry could provide a more robust response. Fringing-field effects should be considered, and an inhomogeneous, time-dependent clock field \( \vec{E}_{clk}(\vec{r},t) \) may be designed to optimize circuit performance, along with a time-dependent, inhomogeneous input field \( \vec{E}_{in}(\vec{r},t) \). This will be necessary to better isolate logic from input fields, and it will require further development of the software implementing our model.

V. CONCLUSION

Molecular QCA is a promising candidate for post-CMOS computation because this paradigm can provide high operating speeds, high device densities, and low power dissipation. In order to realize molecular QCA computation, some strategic advances are required in molecular design, synthesis and testing; circuit layout; and bit input and readout. Solutions to these technical challenges have been conceived, and, with some strategically focused effort, these challenges may be overcome.

In this paper, an input system for clocked, molecular three-dot QCA cells has been introduced, and simple calculations for this system are provided. This system does not require fixed-state input molecules, which would necessitate a separate species of molecule in addition to the computational molecules. This input system may be achieved using electrodes easily fabricated using mature photolithographic processes.
Only a uniform clocking field was considered here, and fringing input fields were neglected. Present efforts and future work will focus on developing the software implementation of the model to support less-idealized fields. The work presented here is focused on addressing the challenge of bit write-in for molecular QCA. This solution could provide insights into as well as helpful constraints for solutions to other challenges. This input concept may prove useful in enabling the demonstration of controlled switching in QCA molecules, as well as the read-out of QCA molecules using SETs or SEBs. Additionally, the features of the proposed input system may also identify capabilities desirable in a solution to the problem of molecular circuit layout. It is our hope that discussions such as the one presented here can inspire dialog and collaborations between diverse groups of researchers and help advance the field of QCA computation closer to realization.

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