Problem 1. Following the steps and templates given below, use Gaussian Elimination to solve for unknowns a, b, c for the following equations. Show your arithmetic.

Eq. 1, \(4a - 2b + 7c = 48\)
Eq. 2, \(1a + 2b + 3c = 7\)
Eq. 3, \(7a - 3b - 2c = 25\)

**Step 1.** Put the equations into standard matrix form

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>-2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2.** Normalize Row 1, so that its diagonal element is unity

\[
R_1' = \frac{R_1}{4} = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{4} \\ 1 & 2 & 3 \\ 7 & -3 & -2 \end{bmatrix}
\]

\[
\begin{align*}
a &= \frac{48}{9} = \frac{16}{3} \\
\frac{b}{4} &= \frac{-7}{2} = -\frac{7}{2} \\
c &= 25
\end{align*}
\]

**Step 3.** Perform row operations on Rows 2 and 3 to create zeros in their first columns

\[
R_2 - R_1 = \begin{bmatrix} 0 & \frac{7}{2} & -\frac{5}{2} \\ 0 & \frac{5}{2} & -1 \end{bmatrix}
\]

\[
R_3 - 3R_1 = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{5}{2} \\ 0 & \frac{1}{2} & -\frac{5}{2} \end{bmatrix}
\]

**Step 4.** Normalize Row 2, so that its diagonal element is unity

\[
R_2 = \frac{2}{5} R_2 = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{4} \\ 0 & \frac{1}{2} & -\frac{5}{2} \end{bmatrix}
\]

\[
\begin{align*}
a &= 12 \\
b &= 5 \cdot \frac{2}{5} = -2 \\
c &= -59
\end{align*}
\]

*Make Step 3 & 4 cells taller*
Problem 1, cont.

Step 5. Perform row operations on Row 3 to create a zero in its second column

\[
\begin{array}{ccc|c}
1 & -\frac{1}{2} & \frac{7}{4} & a \\
0 & 1 & \frac{1}{2} & b \\
0 & \frac{1}{2} & 0 & c \\
\end{array}
\]

\[
\begin{array}{c|c}
a & 12 \\
b & -2 \\
c & -58 \\
\end{array}
\]

\[
R_3 - \frac{1}{2} R_2
\]

Step 6. Normalize Row 3, so that its diagonal element is unity

\[
\begin{array}{ccc|c}
1 & -\frac{1}{2} & \frac{7}{4} & a \\
0 & 1 & \frac{1}{2} & b \\
0 & 0 & 1 & c \\
\end{array}
\]

\[
\begin{array}{c|c}
a & 12 \\
b & -2 \\
c & -58 \cdot \frac{4}{58} = 4 \\
\end{array}
\]

\[
R_3 \cdot \frac{4}{58}
\]

Step 7. Use backward substitution to solve for c, then b, then a.

\[
R_3, \quad 4c = 4, \quad c = 1
\]

\[
R_2, \quad 2b + c = -2, \quad b = -2 - \frac{1}{2}c = -2 - \frac{1}{2}(4) = -4
\]

\[
R_3, \quad a - \frac{1}{2}b + \frac{7}{4}c = 12, \quad a = 12 + \frac{1}{2}b - \frac{7}{4}c \\
= 12 + \frac{1}{2}(-4) - \frac{7}{4}(4) = 12 - 2 - 7 = 3
\]

Step 8. Substitute your computed values for a, b, c into the original equation from Step 1, and demonstrate that your answers are correct.

\[
\begin{array}{ccc|c}
4 & -2 & 7 & 3 \\
1 & 2 & 3 & -4 \\
7 & -3 & -2 & 4 \\
\end{array}
\]

\[
\begin{array}{c|c}
12 + 8 + 28 = 48 \\
3 - 8 + 12 = 7 \\
21 + 12 - 8 = 25 \\
\end{array}
\]
Problem 2. Conventional U.S. units. Ignore the weight of unistrut and rope.
A 10 foot piece of steel unistrut holds up a 500 pound weight at its center. The unistrut is suspended in air by two long vertical ropes. Each rope is rated at 400 pounds. Angle $\Theta$ is constant. If the weight very slowly slides down the unistrut, at what value of $x$ will one of the ropes break?

\[ \sum F_y = 0 \]
\[ T_1 + T_2 = 500 \]

\[ \sum M_x = 0 \text{ at the left corner} \]
\[ -500x + T_2 (10) \cos 20^\circ = 0 \]

\[
\text{IF the left rope breaks, } T_1 = 400 \text{ lb}
\]

\[ T_2 = 500 - T_1 = 500 - 400 = 100 \text{ lb} \]

\[ \sqrt{500x^2} = T_2 (10) \cos 20^\circ = 100 (10) \cos 20^\circ \]

\[ x = \frac{1000 \cos 20^\circ}{500} = 1.879 \text{ feet} \]

Should the weight be moved up the unistrut so that right rope breaks?

Then $T_2 = 400$, $T_1 = 100$,

\[ T_2 (10 \cos 20^\circ) - 500x = 0 \]

\[ x = \frac{4000 \cos 20^\circ}{500} = 8 \cos 20^\circ = 7.52 \text{ feet} \]
Problem 3.

**Part 1.** 12V connected. For each of the following, compute the voltage displayed by a voltmeter connected: Red probe on B, black probe on D, Red probe on K, black probe on F; Red probe on D, black probe on E.

\[ V_{BD} = 12 \left[ \frac{220 + 330}{2180} \right] = 3.03V \]

\[ V_{KG} = 12 \left[ \frac{330 + 150 + 470 + 100}{1370} \right] = 9.20V \]

\[ V_{FG} = 12 \left[ \frac{680}{2180} \right] = 3.74V \]

\[ V_{KF} = (V_{KG} - V_{FG}) - (V_{E} - V_{E}) = V_{KG} - V_{FG} = 9.20 - 3.74 = 5.46V \]

**Part 2.** Determine the Thevenin equivalent circuit for the PC Board, with 12V connected, as seen by an external circuit connected between D and E.

**Part 2.** For terminals DE,

\[ V_{TH} = V_{DE} \text{ for open circuit} \]

(Where nothing external is connected between D & E)

\[ V_{DE} = 12 \left[ \frac{470}{2180} \right] = 2.59V \]

\[ V_{TH} = V_{DE} = 2.59V \]

Attach DMMeter to D & E, Reads 470\| (330 + 220 + 330 + 150 + 680) = 1710 = 369.52 \]

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Page 5 is blank
Problem 4. Two pieces of basswood support rigid horizontal top and bottom aluminum plates. The basswood supports are connected via swivels and sleeves so there is no glue to break loose. Thus, any failures will be in the basswood itself, not at a joint.

![Diagram of basswood and aluminum plates]

The base supports are 25 inches apart. The basswood has square cross section with side dimension = 0.5 inch. The yield strength is 4000 pounds per square inch. With $\Theta = 35$ degrees, a downward testing force is slowly applied at the top. How much downward testing force causes the basswood to yield? (Hint – analyze the yield condition).

Symmetry, $R_y = \frac{F}{2}$

$\Sigma F_y = 0$, $F_c \sin \Theta = \frac{F}{2}$, Breaks when $F_c = (4000)(\frac{1}{2})^2 = 1000 \text{ lbs}$

$1000 \sin \Theta = \frac{F}{2}$

$F = 2000 \sin \Theta$

$= 2000 \sin 35^\circ = 1147 \text{ lbs}$
Problem 5.
Step 1. Use the nodal method to write equations for V1 and V2, then put them into matrix form.
Step 2. Use Gaussian Elimination to solve for V1 and V2.
Step 3. Use your solved voltages to compute the current in each branch leaving Node #2. Show direction arrows and magnitudes on the circuit.

\[
\begin{align*}
\text{Node 1}, & \quad \frac{V_1 + 125}{45} - \frac{V_2}{45} + \frac{V_1}{12} = 0 \\
\text{Node 2}, & \quad \frac{V_2}{55} + \frac{V_2}{33} + \frac{V_2 - (V_1 + 125)}{25} + \frac{V_2 - (V_1 + 125)}{45} = 0
\end{align*}
\]

\[
\begin{bmatrix}
\frac{125}{45} + \frac{125}{12} & \frac{1}{45} + \frac{1}{25} \\
\frac{1}{25} - \frac{1}{45} & \frac{55}{33} + \frac{1}{25} - \frac{1}{45}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= 
\begin{bmatrix}
\frac{-125}{45} - \frac{125}{25} \\
\frac{125}{25} + \frac{125}{45}
\end{bmatrix}
\]

Multiply each term by 225, 225/45 = 5,
\[
\begin{bmatrix}
5 + 9 + \frac{225}{12} & -5 - 9 \\
-9 - 5 & 4.091 + 0.818 + 9 + 5
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= 
\begin{bmatrix}
-625 - 1125 \\
1125 + 625
\end{bmatrix}
\]

\[
\begin{bmatrix}
32.175 & -14 \\
-14 & 24.91
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= 
\begin{bmatrix}
-1.750 \\
1.750
\end{bmatrix}
\]

Reduced Step 4.
Problem 5, additional work area

Gaussian Elim

\[
\begin{bmatrix}
1 & -0.4275 \\
-1 & 2.4911
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
-53.44 \\
1750
\end{bmatrix}
\]

\[
R_2 + 14R_2
\begin{bmatrix}
1 & -0.4275 \\
0 & 18.93
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
-53.44 \\
1002
\end{bmatrix}
\]

\[
R_2 / 18.93
\begin{bmatrix}
1 & -0.4275 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
-53.44 \\
52.93
\end{bmatrix}
\]

Thus, \(V_2 = 52.93 \text{ V}\)

\[
V_1 = -0.4275V_2 = -53.44, \quad V_1 = -53.44 + 0.4275(52.93)
\]

\[
V_1 = -30.81 \text{ V}
\]

Check currents leaving Node 2

\[
0.96I + 1.1604 - 1.651 - 0.917 = -0.002 \quad \text{(close enough)}
\]
Problem 1. Following the steps and templates given below, use Gaussian Elimination to solve for unknowns a, b, c for the following equations. Show your arithmetic.

Eq. 1, 2a - 2b + 7c = 49
Eq. 2, 1a + 2b + 3c = 16
Eq. 3, 7a - 3b - 2c = 31

Step 1. Put the equations into standard matrix form

\[
\begin{align*}
\text{Eq. 1} & : \begin{bmatrix} 2 & -2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 49 \end{bmatrix} \\
\text{Eq. 2} & : \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16 \end{bmatrix} \\
\text{Eq. 3} & : \begin{bmatrix} 7 & -3 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 31 \end{bmatrix}
\end{align*}
\]

Step 2. Normalize Row 1, so that its diagonal element is unity

\[
\begin{align*}
R_1 & : \begin{bmatrix} 1 & -1 & 7/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 49/2 \end{bmatrix} \\
R_2 & : \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16 \end{bmatrix} \\
R_3 & : \begin{bmatrix} 7 & -3 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 31 \end{bmatrix}
\end{align*}
\]

Step 3. Perform row operations on Rows 2 and 3 to create zeros in their first columns

\[
\begin{align*}
R_2 & = R_2 - R_1 \\
R_3 & = R_3 - 7R_1
\end{align*}
\]

\[
\begin{align*}
\text{R}_2 & : \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16 - 49/2 = -17/2 \end{bmatrix} \\
\text{R}_3 & : \begin{bmatrix} -8 & 21 & -53 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 31 - 49(2/3) = -23 \end{bmatrix}
\end{align*}
\]

Step 4. Normalize Row 2, so that its diagonal element is unity

\[
\begin{align*}
R_2 & : \begin{bmatrix} 1 & -1/3 & 7/2 \\ 0 & 3/3 = 1 & -1/2 \cdot 1/3 = -1/6 \\ 0 & 4 & -53/2 \\
\end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 49/2 \\ -17 \cdot 1/3 = -17/6 \\ -281/2 \\
\end{bmatrix}
\end{align*}
\]
Problem 1, cont.

Step 5. Perform row operations on Row 3 to create a zero in its second column

\[
\begin{array}{ccc|c}
1 & -1 & \frac{7}{2} & a \\
0 & 1 & -\frac{1}{6} & b \\
0 & -4 & 0 & c \\
\end{array}
\]

Step 6. Normalize Row 3, so that its diagonal element is unity

\[
\begin{array}{ccc|c}
1 & -1 & \frac{7}{2} & a \\
0 & 1 & -\frac{1}{6} & b \\
0 & 0 & \frac{155}{6} & c \\
\end{array}
\]

Step 7. Use backward substitution to solve for \( c \), then \( b \), then \( a \).

\[
R_3: \quad c = \frac{\frac{171}{6}}{155} = \frac{171}{155} = 5
\]

\[
R_2: \quad b - \frac{1}{6}c = -\frac{17}{6}, \quad b = -\frac{17}{6} + \frac{1}{6}(5) = -\frac{12}{6} = -2
\]

\[
R_1: \quad a - b + \frac{7}{2}c = \frac{49}{2}, \quad a = \frac{49}{2} + (-2) - \frac{7}{2}(5) = \frac{49 - 35}{2} = \frac{10}{2} = 5
\]

Step 8. Substitute your computed values for \( a, b, c \) into the original equation from Step 1, and demonstrate that your answers are correct.

\[
\begin{array}{ccc|c}
2 & -2 & 7 & 5 \\
1 & 2 & 3 & -2 \\
7 & -3 & -2 & 5 \\
\end{array}
\]

\[
\begin{array}{c|c}
2(5) - 2(-2) + 7(5) = 10 + 4 + 35 = 49 \\
1(5) + 2(-2) + 3(5) = 5 - 4 + 15 = 16 \\
7(5) - 3(-2) - 2(5) = 35 + 6 - 10 = 31 \\
\end{array}
\]

ORIG MATRIX SOLUTION

\[\text{OK}\]
Problem 2. Conventional U.S. units. Ignore the weight of unistrut and rope.
A 10 foot piece of steel unistrut holds up a 500 pound weight at its center. The unistrut is suspended in air by two long vertical ropes. Each rope is rated at 350 pounds. Angle Θ is constant. If the weight very slowly slides down the unit strut, at what value of x will one of the ropes break?

\[
\begin{align*}
\text{Weight} & \quad \Theta = 20^\circ \\
\text{350 lb} & \quad \text{Horizontal} \\
\text{x} & \quad (\text{starts at } 5 \cos 20 = 4.170 \text{ feet}) \\
\text{(at center of the weight)}
\end{align*}
\]

See Key A
\[
\sum F_y = 0
\]
\[
T_1 + T_2 = 500 \text{ lb}
\]
\[
T_1 = 350 \text{ when it breaks, so } T_2 = 500 - 350 = 150
\]

Moment at left corner = 0
\[
-350x + 150(10 \cos 20^\circ) = 0
\]
\[
x = \frac{1500 \cos 20^\circ}{350} = 4.103 \text{ feet}
\]
Part 1. 12V connected. For each of the following, compute the voltage displayed by a voltmeter connected
Red probe on B, black probe on D,
Red probe on K, black probe on F,
Red probe on D, black probe on E.

Part 2. Determine the Thevenin equivalent circuit for the PC Board, with 12V connected, as seen by an external circuit connected between D and E.

\[ \Sigma R's \text{ on right} = 2320 \, \Omega \]
\[ \Sigma R's \text{ on left} = 1370 \, \Omega \]

\[ V_{BD} = 12 \left( \frac{470 + 220}{2320} \right) = 3.57 \, \text{V} \]

\[ V_{KG} = \frac{220 + 150 + 330 + 100}{1370} = 1.01 \, \text{V} \]

\[ V_{FG} = 12 \left( \frac{680}{2320} \right) = 3.52 \, \text{V} \]

\[ V_{KE} = 1.01 - 3.52 = 3.49 \, \text{V} \]

\[ V_{DE} = 12 \left( \frac{330}{2320} \right) = 1.707 \, \text{V} = V_{TH} \text{ for terminals D-E} \]

For TheV, short D to E and compute
\[ I_{sc} = \frac{12}{2320 - 330} = 0.00603 \, \text{A} \]

\[ R_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{1.707 \, \text{V}}{0.00603 \, \text{A}} = 283.5 \, \Omega \]
Problem 4. Two pieces of basswood support rigid horizontal top and bottom aluminum plates. The basswood supports are connected via swivels and sleeves so there is no glue to break loose. Thus, any failures will be in the basswood itself, not at a joint.

![Uniform Tester Force Across Top Aluminum Plate](image)

The base supports are 25 inches apart. The basswood has square cross section with side dimension = 0.5 inch. The yield strength is 4000 pounds per square inch. With $\Theta = 40$ degrees, a downward testing force is slowly applied at the top. How much downward testing force causes the basswood to yield? (Hint – analyze the yield condition).

![As in Test 4](image)

$F_c$ at breaking condition is $(4000)(0.5)^2 = 1000 \text{ lbs}$

$\sum F_y = 0$

$F = F_c \sin \Theta$

$F = 2F_c \sin \Theta = 2(1000) \sin 40^o$

$= 1286 \text{ lbs}$. 

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Problem 5.
Step 1. Use the nodal method to write equations for $V_1$ and $V_2$, then put them into matrix form.
Step 2. Use Gaussian Elimination to solve for $V_1$ and $V_2$.
Step 3. Use your solved voltages to compute the current in each branch leaving Node #2. Show direction arrows and magnitudes on the circuit.

Node 1, \[
\frac{V_1 - V_2}{45} + \frac{V_1 - V_2}{25} + \frac{(V_1 - 125)}{12} = 0
\]

Node 2, \[
\frac{V_2}{55} + \frac{V_2}{47} + \frac{V_2 - V_1}{25} + \frac{V_2 - V_1}{45} = 0
\]

\[
\begin{bmatrix}
\frac{1}{45} + \frac{1}{25} + \frac{1}{12} & -\frac{1}{45} - \frac{1}{25} \\
-\frac{1}{25} & -\frac{1}{45} + \frac{1}{25} + \frac{1}{45}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
\frac{125}{12} \\
0
\end{bmatrix}
\]

Multiply by $45$

\[
\begin{bmatrix}
4 + \frac{9}{5} + 3.95 & -1 - \frac{9}{5} \\
-1 - \frac{9}{5} & 0.8182 + 0.9574 + \frac{9}{5} + 1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
468.75 \\
0
\end{bmatrix}
\]
Problem 5, additional work area

\[
\begin{bmatrix}
6.55 & -2.8 \\
-2.8 & 4.576 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
468.175 \\
0 \\
\end{bmatrix}
\]

\[
R_1/6.55
\begin{bmatrix}
1 & -0.4244 \\
-2.8 & 4.576 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
71.56 \\
0 \\
\end{bmatrix}
\]

\[
R_2+2.8R_1
\begin{bmatrix}
1 & -0.4244 \\
0 & 3.379 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
71.56 \\
200.4 \\
\end{bmatrix}
\]

\[
V_2 = \frac{200.4}{3.379} = 59.3 \text{ V}
\]

\[
V_1 + 0.4244V_2 = 71.56
\]

\[
V_1 = 71.56 + (0.4244)(59.3) = 96.9 \text{ V}
\]

\[
\text{KCL for # 2, } \quad 1.078 + 1.262 - 1.504 - 0.836 = 0
\]
Problem 1. Following the steps and templates given below, use Gaussian Elimination to solve for unknowns $a$, $b$, $c$ for the following equations. Show your arithmetic.

Eq. 1, $3a - 2b + 7c = 67$
Eq. 2, $1a + 2b + 3c = 19$
Eq. 3, $7a - 3b - 2c = 23$

Step 1. Put the equations into standard matrix form

\[
\begin{align*}
\text{Eq. 1} & \quad \begin{array}{ccc|c}
3 & -2 & 7 & a \\
1 & 2 & 3 & b \\
7 & -3 & -2 & c \\
\end{array} = \begin{array}{c}
67 \\
19 \\
23 \\
\end{array} \\
\end{align*}
\]

Step 2. Normalize Row 1, so that its diagonal element is unity

\[
\begin{align*}
\frac{1}{3}a & = \frac{67}{3} \\
1 & = \frac{2}{3} \\
7 & = \frac{7}{3} \\
\end{align*}
\]

Step 3. Perform row operations on Rows 2 and 3 to create zeros in their first columns

\[
\begin{align*}
\text{R2} - \frac{1}{3} \text{R1} & \quad \begin{align*}
1 & = \frac{2}{3} \\
-1 & = 0 \\
7 & = \frac{7}{3} \\
\end{align*} \\
\text{R3} - 7 \text{R1} & \quad \begin{align*}
1 & = \frac{2}{3} \\
-1 & = 0 \\
7 & = \frac{7}{3} \\
\end{align*} \\
\end{align*}
\]

Step 4. Normalize Row 2, so that its diagonal element is unity

\[
\begin{align*}
\text{R2} \cdot \frac{3}{8} & \quad \begin{align*}
1 & = \frac{2}{3} \\
0 & = \frac{3}{8} \\
0 & = \frac{5}{3} \\
\end{align*} \\
\text{R3} \cdot \frac{3}{8} & \quad \begin{align*}
0 & = \frac{3}{8} \\
0 & = \frac{5}{3} \\
0 & = \frac{-100}{3} \\
\end{align*} \\
\end{align*}
\]
Problem 1, cont.
Step 5. Perform row operations on Row 3 to create a zero in its second column

\[
\begin{array}{ccc|c}
1 & -\frac{2}{3} & \frac{7}{3} & a \\
0 & 1 & \frac{1}{4} & b \\
0 & \frac{5}{3} & \frac{5}{3} & 0 & c \\
\end{array}
\]

\[\begin{aligned}
R_3 & \rightarrow \frac{-5}{3}R_2 \\
R_3 & \rightarrow \frac{-12}{225}R_3 \\
\end{aligned}\]

Step 6. Normalize Row 3, so that its diagonal element is unity

\[
\begin{array}{ccc|c}
1 & -\frac{2}{3} & \frac{7}{3} & a \\
0 & 1 & \frac{1}{4} & b \\
0 & 0 & 1 & c \\
\end{array}
\]

Step 7. Use backward substitution to solve for c, then b, then a.

Row 3, \[c = 7\]
Row 2, \[b + \frac{1}{3}c = \frac{-5}{4}, b = \frac{-5}{4} - \frac{1}{3}(7) = \frac{-5 - 7}{4} = \frac{-12}{4} = -3\]
Row 1, \[a - \frac{2}{3}b + \frac{7}{3}c = \frac{67}{3}, a = \frac{67}{3} + \frac{2}{3}(-3) - \frac{7}{3}(7) = \frac{67 - 6 - 49}{3} = \frac{12}{3} = 4\]

Step 8. Substitute your computed values for a, b, c into the original equation from Step 1, and demonstrate that your answers are correct.

\[
\begin{align*}
3 & a - 2b + 7c = 3(4) - 2(-3) + 7(7) = 12 + 6 + 49 = 67 \\
1 & b = -3 \\
7 & c = 7 \\
\end{align*}
\]

\[
\begin{align*}
3(4) - 2(-3) + 7(7) = 12 + 6 + 49 = 67 \\
1(4) + 2(-3) + 3(7) = 4 - 6 + 21 = 19 \\
7(4) - 3(-3) - 2(7) = 28 + 9 - 14 = 23
\end{align*}
\]
Problem 2. Conventional U.S. units. Ignore the weight of unistrut and rope. A 10 foot piece of steel unistrut holds up a 500 pound weight at its center. The unistrut is suspended in air by two long vertical ropes. Each rope is rated at 300 pounds. Angle $\Theta$ is constant. If the weight very slowly slides down the unitstrut, at what value of $x$ will one of the ropes break?

\[ \sum F_y = T_1 + T_2 = 500 \text{lb}, \]

\[ \sum M = 0, -500x + T_2[10 \cos 20^\circ] = 0 \]

Use break condition $T_1 = 300$, so $T_2 = 500 - 300 = 200$

\[ -500x + 2000 \cos 20^\circ = 0 \]

\[ x = \frac{500}{2000 \cos 20^\circ} = 3.76 \text{ ft} \]
Problem 3.

Part 1. 12V connected. For each of the following, compute the voltage displayed by a voltmeter connected
Red probe on B, black probe on D,
Red probe on K, black probe on F,
Red probe on D, black probe on E.

Part 2. Determine the
Thevenin equivalent circuit
for the PC Board, with 12V
connected, as seen by an
external circuit connected
between D and E.

\[ \sum R's \ on \ right = 2640 \ \Omega \]
\[ \sum R's \ on \ left = 1260 \ \Omega \]

\[ V_{BD} = 12 \left( \frac{470 + 330}{2640} \right) = 3.64 \text{ V} \]

\[ V_{KF} = V_{KG} - V_{FG} = 12 \left( \frac{220 + 220 + 470 + 150}{1260} \right) - 12 \left( \frac{680}{2640} \right) \]

\[ V_{KF} = 6.53 \text{ V} \]

\[ V_{DE} = 12 \left( \frac{330}{2640} \right) = 1.500 \text{ V} \]

Also this is \( V_{TH} \) for
terminals D-E.

\[ V_{off} = \frac{680 + 470 + 330}{680 + 150} = 1.480 \]

Ohmmeter attached DE measures

\[ 330 \parallel (1480 + 830) = 330 \parallel 2310 = \frac{R_{TH}}{289 \Omega} \]
Problem 3.

Part 1. 12V connected. For each of the following, compute the voltage displayed by a voltmeter connected
Red probe on B, black probe on D,
Red probe on K, black probe on F,
Red probe on D, black probe on E.

Part 2. Determine the Thevenin equivalent circuit for the PC Board, with 12V connected, as seen by an
external circuit connected between D and E.

ΣR's on right = 2640Ω
Σ on left = 1260Ω

\[ V_{BD} = 12 \left[ \frac{470 + 330}{2640} \right] = 3.64 \text{ V}. \]

\[ V_{KF} = V_{KG} - V_{FG} = 12 \left[ \frac{220 + 220 + 470 + 100}{1260} \right] - 12 \left[ \frac{680}{2640} \right] \]

\[ V_{KF} = 6.53 \text{ V} \]

\[ V_{DE} = 12 \left[ \frac{330}{2640} \right] = 1.500 \text{ V}. \]

Also, this is V_{TH} for terminals D-E.

\[ (680+470+330) = 1480 \]

\[ 330 || (1480+830) = 330 || 2310 = \frac{289}{2} \text{ Ω} \]
Problem 4. Two pieces of basswood support rigid horizontal top and bottom aluminum plates. The basswood supports are connected via swivels and sleeves so there is no glue to break loose. Thus, any failures will be in the basswood itself, not at a joint.

The base supports are 25 inches apart. The basswood has square cross section with side dimension = 0.5 inch. The yield strength is 4000 pounds per square inch. With $\theta = 45$ degrees, a downward testing force is slowly applied at the top. How much downward testing force causes the basswood to yield? (Hint – analyze the yield condition).

\[ \sum F_y = 0, \quad F_c \sin \theta = F/2 \]

\[ F_c \text{ breaks at } (4000)(0.5)^2 = 1000 \text{ lbs} \]

\[ F = 2F_c \sin \theta = 2(1000) \sin 45^\circ \]

\[ F = 1414 \text{ lbs} \]
Problem 5.
Step 1. Use the nodal method to write equations for V1 and V2, then put them into matrix form.
Step 2. Use Gaussian Elimination to solve for V1 and V2.
Step 3. Use your solved voltages to compute the current in each branch leaving Node #2. Show direction arrows and magnitudes on the circuit.

\[ \text{Node 2: } \frac{V_2}{55} + \frac{V_2}{68} + \frac{V_2 - V_1}{25} + \frac{V_2 - V_1}{45} = 0 \]

\[ \text{Node 1: } \frac{V_1 - V_2}{45} + \frac{V_1 - V_2}{25} + \frac{V_1 - 125}{12} = 0 \]

\[
\begin{bmatrix}
\frac{1}{45} & \frac{1}{25} & \frac{1}{12} \\
\frac{-1}{45} & \frac{-1}{25} & \frac{1}{12} \\
\frac{-1}{25} & \frac{-1}{45} & \frac{1}{25} \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix}
=
\begin{bmatrix}
\frac{125}{12} \\
0 \\
\end{bmatrix}
\]

Multiply by 45

\[
\begin{bmatrix}
1 + \frac{9}{5} + 3.75 \\
-1 - \frac{9}{5} \\
-1 - \frac{9}{5} \\
\end{bmatrix}
\begin{bmatrix}
\frac{8182}{150.6618} + 0.6618 \\
0.8182 + 0.6618 \\
+ \frac{9}{5} + 1 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix}
=
\begin{bmatrix}
468.75 \\
0 \\
\end{bmatrix}
\]
Problem 5, additional work area

\[
\begin{bmatrix}
6.55 & -2.8 \\
-2.8 & 4.28
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
468.75 \\
0
\end{bmatrix}
\]

\[
R_1/6.55
\begin{bmatrix}
1 & -0.4275 \\
-2.8 & 4.28
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
71.56 \\
0
\end{bmatrix}
\]

\[
R_2 + 2.8\Omega
\begin{bmatrix}
1 & -0.4275 \\
0 & 3.083
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
71.56 \\
200.4
\end{bmatrix}
\]

\[v_2 = \frac{200.4}{3.083} = 65.0\text{V}\]

\[v_1 - 0.4275v_2 = 71.56\]

\[v_1 = 71.56 + 0.4275(65.0) = 99.3\text{V}\]

\[\text{KCL #2}\]

\[1.182 + 0.956 - 1.372 - 0.762 = 0.004\]

(close enough)