Quantum Mechanics for Engineers

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That chart explained the quantum Hall effect. Now, if you'll bear with me for a moment, this next graph shows rainfall over the Amazon Basin...

If you keep saying "bear with me for a moment", people take a while to figure out that you're just showing them random slides.
1 What is Classical Physics?
Outline

1. What is Classical Physics?
2. What is Quantum Physics?
3. How Can This Apply to Computers?
Knowing the (scalar) expressions for the kinetic energy $T(\dot{q})$ and the potential energy $V(q)$ for a system, we can define the [Lagrangian](#) as

$$L(q, \dot{q}) \equiv T - V$$
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The **Action** $S[q]$ is then defined as the integral over time,

$$S[q] \equiv \int dt \ L(q, \dot{q})$$
The equations of motion are then given by the zeros of the *Euler Lagrange Derivative*,

\[
\frac{\delta L}{\delta q} \equiv \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0
\]
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This is called the principle of **Least Action**.
Projectile Motion

\[ T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2), \quad V = mgy \]
Projectile Motion

- \[ T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2), \quad V = mgy \]
- \[ L = T - V = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - mgy \]
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\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{d}{dt} m\dot{x} = m\ddot{x}, \quad \frac{\partial L}{\partial x} = 0 \]

\[ \Rightarrow m\ddot{x} = 0 \]
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\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} = \frac{d}{dt} m\dot{y} = m\ddot{y}, \quad \frac{\partial L}{\partial y} = -mg \]

\[ \Rightarrow m\ddot{y} = -mg \]
Concluding Thoughts on Classical Physics

For a given initial condition, only one path is possible - the path which satisfies the Euler-Lagrange Equation.

The nature of classical physics can be thought of as follows:

INPUT: Ask question.
OUTPUT: Get answer.
Concluding Thoughts on Classical Physics

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So What is Quantum Physics?
Principle of Least Action … *Probably*

Classical physics demanded $\delta L \delta q = 0$

Quantum physics allows any value on the right hand side $\delta L \delta q = \lambda$ where $\lambda$ can be anything.
Classical physics demanded

\[ \frac{\delta L}{\delta q} = 0 \]
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  where \( \lambda \) can be anything.
However, not all paths are equally probable.
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Each possible path has a statistical weight/probability equal to

$$e^{-S[q]/\hbar} \quad \text{or} \quad e^{iS[q]/\hbar}$$

(these are the same under the change of variables $dt \to idt$)
But it gets worse
Interference

\[ q \rightarrow e^{-S[q]/\hbar} \]
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\[ q + \delta q \rightarrow e^{-S[q+\delta q]/\hbar} \]
Interference

\begin{align*}
q & \rightarrow e^{-S[q]/\hbar} \\
q + \delta q & \rightarrow e^{-S[q+\delta q]/\hbar} \\
\frac{e^{-S[q+\delta q]/\hbar}}{\hbar} &= e^{-\left(S[q] + \delta q \frac{\delta S[q]}{\delta q}\right)/\hbar} = e^{-S[q]/\hbar} e^{-\frac{\delta q}{\hbar} \frac{\delta S[q]}{\delta q}}
\end{align*}
Interference

- $q \rightarrow e^{-S[q]/\hbar}$
- $q + \delta q \rightarrow e^{-S[q+\delta q]/\hbar}$

$e^{-S[q+\delta q]/\hbar} = e^{-(S[q]+\delta q\frac{\delta S[q]}{\delta q})/\hbar} = e^{-S[q]/\hbar} e^{-\frac{\delta q}{\hbar} \frac{\delta S[q]}{\delta q}}$

Most likely path is where

$$\frac{\delta S[q]}{\delta q} = 0$$

(which is the same as where $\frac{\delta L}{\delta q} = 0$)
Equation of State

If $\psi \propto e^{iS/\hbar}$, then

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar} \frac{\partial S}{\partial t} \psi$$

$$\int_{a}^{b} \psi^* \psi \ dx = \int_{a}^{b} |\psi|^2 \ dx$$
If \( \psi \propto e^{iS/\hbar} \), then

\[
\frac{\partial \psi}{\partial t} = \frac{i}{\hbar} \frac{\partial S}{\partial t} \psi
\]

\[
H\psi = i\hbar \frac{\partial \psi}{\partial t}
\]

(where \( H \equiv T + V \))
Equation of State

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Solving this differential equation (called Schroedinger’s Equation) for $\psi$ will give the (complex) Wave Function for the particle.
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What is Classical Physics?
What is Quantum Physics?
How Can This Apply to Computers?
Principle of Least Action ... Probably
Superposition
So How Do We Do Quantum Mechanics?
Observation
Concluding Thoughts on Quantum Physics

\[ V = \infty \quad \text{for} \quad x = 0 \]

\[ V = \infty \quad \text{for} \quad x = L \]
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\[
\Phi = 0
\]

\[
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QUANTUM MECHANICAL TUNNELING
### Outline

- What is Classical Physics?
- What is Quantum Physics?
- How Can This Apply to Computers?

<table>
<thead>
<tr>
<th>Principle of Least Action</th>
<th>Probably</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superposition</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td></td>
</tr>
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<td></td>
</tr>
</tbody>
</table>

## Hydrogen Atom
Hydrogen Atom

\[ Y_0^0 = 1 \quad Y_1^0 = \cos \theta \quad Y_2^0 = 3\cos^2 \theta - 1 \]

\[ \psi Y_1^2 = \cos \theta \sin \phi \quad Y_3^0 = 5\cos^3 \theta - 3\cos \theta \]

\[ \psi Y_2^1 = (5\cos^2 \theta - 1) \sin \theta \cos \phi \]
Observation

But it gets *even* worse ...
Observation

- A system exists in every possible state it can be in prior to observation (superposition).
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- These possibilities all interfere with each other.
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- A system exists in every possible state it can be in prior to observation (superposition).
- These possibilities all interfere with each other.
- Once an observation is made, the system "collapses" into one of the possible states, according to some probability distribution which depends on the system.
Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
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- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
- How often you will *observe* that path/state is determined by the wave function, which is a probability distribution.
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- For a given initial condition, any path/state is possible - no matter how improbable it may be.
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- The classical path/state will be the most likely, which is why macroscopic systems appear classical.
Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
- How often you will *observe* that path/state is determined by the wave function, which is a probability distribution.
- The classical path/state will be the most likely, which is why macroscopic systems appear classical.
- The nature of quantum physics can be thought of as follows:

  INPUT: Ask question, suggest an answer.
  OUTPUT: How often that answer will be right.
Classical computer works using bits, which can either be 0 or 1.
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A quantum computer uses **quantum bits**, which can be 0 *and* 1.

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]
Classical Bits and Quantum Bits

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- Measurement produces:
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- Classical computer works using bits, which can either be 0 or 1
- A quantum computer uses quantum bits, which can be 0 and 1

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

- Measurement produces:
  0 with probability \( |\alpha|^2 \)
  1 with probability \( |\beta|^2 \)
Manipulating Q-Bits

Any (invertible) transformation done to $|\psi\rangle$ will act on every term in the superposition:

\[
\text{NOT} |\psi\rangle = \text{NOT}(\alpha |0\rangle + \beta |1\rangle) = \alpha |1\rangle + \beta |0\rangle
\]

But, all operations done to a q-bit must be reversible.
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- But, all operations done to a q-bit must be reversible
Illustration of Why Quantum Computing Is So Powerful

Consider the Hadamard transformation $H$, which takes $|0⟩ \rightarrow \frac{1}{\sqrt{2}} (|0⟩ + |1⟩)$ and $|1⟩ \rightarrow \frac{1}{\sqrt{2}} (|0⟩ - |1⟩)$. This transformation is a key example of how quantum computing can be more powerful than classical computing.
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$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
Consider the *Hadamard* transformation $H$, which takes

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

and

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
Consider $n$ copies of $|0\rangle$.
Illustration of Why Quantum Computing Is So Powerful

- Consider $n$ copies of $|0\rangle$.
- Act on each with $H$. 
Illustration of Why Quantum Computing Is So Powerful

- Consider $n$ copies of $|0\rangle$.
- Act on each with $H$.
- Result is

$$\sum_{j=1}^{n} |j\rangle$$
Illustration of Why Quantum Computing Is So Powerful

- Consider $n$ copies of $|0\rangle$.
- Act on each with $H$.
- Result is 

$$\frac{1}{\sqrt{2^n}} \sum_{j=1}^{2^n} |j\rangle$$
Consider $n = 3$. Acting on three $|0\rangle$'s with $H$ gives
Illustration of Why Quantum Computing Is So Powerful

- Consider \( n = 3 \). Acting on three \( |0\rangle \)'s with \( H \) gives

\[
\frac{1}{\sqrt{8}} (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)
\]
Consider $n = 3$. Acting on three $|0\rangle$'s with $H$ gives

$$\frac{1}{\sqrt{8}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{8}}(|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + \cdots + |1\rangle|1\rangle|1\rangle)$$
Consider $n = 3$. Acting on three $|0\rangle$’s with $H$ gives

$$\frac{1}{\sqrt{8}} (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{8}} (|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + \cdots + |1\rangle|1\rangle|1\rangle)$$

So, any operation done to this state with $n$ q-bits will produce a superposition of $2^n$ outcomes.