A Maximum Loading Margin Method for Static Voltage Stability in Power Systems

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***Abstract*—In this paper, the maximum loading margin (MLM) approach is proposed in finding generation directions to maximize the static voltage stability margin, where the MLM is evaluated at various possible generation directions in the generation direction space. An approximate and simple model representing the relationship between the generation direction and the LM is used to obtain the MLM point. The proposed method is validated in the modified IEEE 14-bus test system and applied to the Thailand power system. LMs of the system with the generation directions are compared for different generator combinations using the proposed technique.**

***Index Terms*—Generation pattern, maximum loading margin (MLM), Thailand power system, voltage instability.**

I. Introduction

IN the recent past, one of the problems that received wide attention among utilities is the voltage instability [1]–[4]. The lack of new generation and transmission facilities and over-exploitation of the existing facilities geared by increase in load demand makes this type of problem more likely to happen in the present power systems.

Voltage instability due to "visability" of the contingencies was the main reasons for the recent and worst North American power interruption on August 14th, 2003. In this incident, reports indicate that approximately 50 million people were interrupted from the continuous supply of power for more than 15 h [5]. Moreover, with an open-access market, poorly scheduled generation for the competitive bidding is one of many reasons for the voltage instability problem in the deregulated electricity environment. Thus, in order to relieve or at least minimize the system from the voltage instability problem, many electric utilities have made a great deal of effort in system studies related to static voltage stability [6].

Major contributory factors to voltage instability are power system configuration, generation pattern, and load pattern [1]–[4], [7], [8]. The power system network can be modified to ease voltage instability by adding shunt capacitors and/or flexible ac transmission system (FACTS) controllers, at the weakest bus of the system [8], [9]. The generation pattern is much easier to control by system operators compared to load pattern. Customarily, the generation of each participating generator is raised at the same rate, at a predefined rate, or according to their spinning reserves. However, it would be useful to know what kind of generation pattern could provide the best solution in terms of the loading margin (LM). This will be beneficial for today's competitive market when a feasibility of scheduling generation is required to meet the voltage stability criteria, subject to the commercial realities of scheduling.

In [10], linear and quadratic estimates of the LM with respect to system parameters, including power generation, are computed, by using the sensitivity method to locally predict the new location of the maximum loading margin (MLM) points. However, the method illustrates the effect of LM with respect to only one generation parameter. Apart from computational complexity, i.e., computing eigenvectors for these estimates, the estimates would perform poorly near the nose of the PV curve. Finding appropriate "generation direction" (which generators need to be loaded by what percentage) to operate the system under normal and stressed conditions is not possible using this approach. The proposed method provides the generation direction that maximizes LM with respect to any number of generation parameters. It can globally approximate the LM in the "generation direction space" based on the continuation of the power flow process. The proposed method can provide an approximate equation of LM and then the plot of the approximate LM surface, which provides the information on the shape of LM and also in the vicinity around the maximum point.

A modal analysis based on generator participation factor is proposed in [11] to determine the impacts of generators on system capability. The generator participation factors need to be updated whenever there is a change in loading level and load flow equations, due to power limits. This method, however, depends upon the operating condition near the collapse point. Although, this technique is an effective technique, it does not provide the best generation directions (GDs) or pattern and the MLM.

A method to increase a power system's security margin by re-dispatching generator outputs is proposed using a normal vector found at a voltage collapse boundary [12]. A normal vector is used as an indicator to change the GD so that more power can be transferred before reaching the boundary of a critical point. Although this method provides the maximum power generation, it gives the solution only at a local maximum point [12]. Moreover, it is necessary to write its own software to apply this methodology, and it can not be solved with the currently available software. Although this method provides the maximum power generation, it can provide the global solution only when the LM surface has a single maximum point. Alternatively, an optimization technique can be used to calculate the maximum loading point and corresponding GD. However, it involves solving the necessary conditions and may

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not be practical for utilities to use existing software for their studies. In addition, it gives only the solution at the maximum point, which is not useful in the operation of an intermediate loading point, between base and peak cases.

Based on the above observation, attention drawn in this paper is to propose a new generation pattern from the MLM approach, which provides the MLM or static voltage stability margin. The proposed methodology provides an equation that represents the LM surface in a function of GD. The proposed technique, the MLM approach, is based on a simple approach where the LMs in various GDs are combined to cover the whole generation direction space. It is a general method in that the effect of LM with respect to any number of generation parameters is considered. It guarantees the MLM for single and multiple MLM surfaces.

The rest of the paper is organized as follows. Section II introduces existing methods that identify generation directions in static voltage stability study. The proposed methodology called the MLM approach is presented in Section III. An IEEE test system and a real Thailand Power System are briefly stated in Section IV. In Section V, some interesting results are presented along with detailed discussion. Finally, major contributions and conclusions are summarized in Section VI.

II. Generation Direction

Generation pattern or "direction" is defined as the portions of generation increase in each participating generator to serve the desired load increase and losses in the system. Let K_{Gi} be the factor for active power increase at generator i and $P_{Gi,o}$ be the generation at the base case; then, the generation P_{Gi} at a higher loading point can be written as

$$P_{Gi} = P_{Gi,o}(1 + K_{Gi}) \tag{1}$$

where $i = 1, 2, \ldots, n$, for all participating generators.

The factor K_{Gi} can be viewed as the generation direction (GD_i) and is very crucial to voltage stability. Existing methods to identify generation directions in a voltage stability study are summarized below.

A. Conventional Approach

Conventionally, the generation of the system is increased by a fixed percentage, as pre-specified in the planning stage, e.g., according to the spinning reserve [8]. The power generation of generator i after the load increase can be written as

$$P_{Gi} = P_{Gi,o}(1 + K_{Gi}) = P_{Gi,o} + \Delta P_{Gi} \tag{2}$$

and

$$\sum_{NG} \Delta P_{Gi} = \Delta P_D + \Delta P_{\text{loss}} \tag{3}$$

where

P_{Gi}	power generation of generator i;
$P_{Gi,o}$	generation of generator i at base load;
ΔP_{Gi}	increase of power generation at generator i;
ΔP_D	total load increase;
ΔP_{Loss}	total loss increase;
NG	number of generators.

B. Optimal Power Flow Approach

Traditional optimal power flow (OPF) can be formulated to include voltage stability criteria as follows [13]:

Minimize

$$C(P_{Gi}) = \sum_{NG} \left(a_{Gi}P_{Gi}^2 + b_{Gi}P_{Gi} + c_{Gi}\right) \tag{4}$$

subject to

$$P_{Gi} - (1+\lambda)P_{Di,o} - \sum_{j=1}^{n} |U_i||U_j|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) = 0 \tag{5}$$

$$Q_{Gi} - (1+\lambda)Q_{Di,o} - \sum_{j=1}^{n} |U_i||U_j|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) = 0 \tag{6}$$

$$|P_{Gi}|_{\min} \le |P_{Gi}| \le |P_{Gi}|_{\max} \tag{7}$$

$$|U_i|_{\min} \le |U_i| \le |U_i|_{\max} \tag{8}$$

$$S_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2} \le S_{ij,\max} \tag{9}$$

where

C	total operating cost of the system;
a_{Gi}, b_{Gi}, c_{Gi}	cost coefficients of generator i;
λ	load incremental parameter or loading factor (LF);
P_{Gi}, Q_{Gi}	real and reactive power generation at bus i;
$P_{Di,o}, Q_{Di,o}$	real and reactive power demand at bus i at base load;
n	number of buses in the system;
$\lvert P_{Gi}\rvert_{min}, \lvert P_{Gi}\rvert_{max}$	lower and upper power limits of generator i;
$\lvert U_i\rvert_{min}, \lvert U_i\rvert_{max}$	lower and upper limits of voltage magnitude at bus i;
P_{ij}, Q_{ij}, S_{ij}	real, reactive, and apparent power in line ij;
$S_{ij,max}$	MVA (thermal) limit of line ij.

GD, in this approach, can be worked out by subtracting the new dispatch from the old dispatch for individual generators.

C. Cost Participation Factor Approach

The cost participation factor is viewed as the easiest method to identify the amount of power generation with economic load dispatch consideration. It is calculated based on generators' incremental cost [14]

$$\Delta P_{Gi} = \frac{\left(\frac{1}{C_i''}\right)}{\sum_{j=1}^{NG}\left(\frac{1}{C_j''}\right)} \Delta P_D \tag{10}$$

where

C_i	cost function of generator i;
C_i''	second derivative of the cost function i;
ΔP_{Gi}	increase in power generation for generator i;
ΔP_D	total load increase.

Among the existing methods, very few of them can provide the highest LM of the system. Hence, in the following section, a new GD is proposed to maximize the LM by searching through the "GD space."

III. MLM Approach

The MLM method is based on the surface approximation. In the next subsection, the theory on curve and surface approximation is reviewed. Then, the separability condition of LM is introduced. Finally, in the last subsection, the MLM method is proposed.

A. Curve and Surface Approximation

Any regular embedded smooth curve can be written in a Taylor series expansion [15]

$$f(x) = J_{B,n}(x) + O(x^{n+1}) \tag{11}$$

with

$$J_{B,n}(x) = B_0 + B_1 x + B_2 x^2 + B_3 x^3 + \cdots + B_n x^n \tag{12}$$

where $f(x)$ is a smooth function in a defined range of x, B_i is a coefficient of x_i, n is the order of the approximation, and $O(x^{n+1})$ is the remainder.

Similarly, for a surface [15]

$$f(x,y) = J_{B,n}(x,y) + O\left(\|(x,y)\|^{n+1}\right) \tag{13}$$

with

$$J_{B,n}(x,y) = \sum_{j=0}^{n} B_{n-j,j} x^{n-j} y^j \tag{14}$$

where $f(x,y)$ is a continuous function of variables x and y defined over some ranges, $B_{n-j,j}$ is a coefficient of $x^{n-j}y^j$ term, n is the order of approximation, and $O(\|(x,y)\|^{n+1})$ is the remainder.

In particular, if the function is separable, one can approximate a multivariable surface by an algebraic equation, which combines all polynomial equations in the form of (12) defined for each variable with only one constant (B_o) term [15], [16]. Quadric equation $z = x^2 + y^2 + 1$ is an example of surface approximation, which is the combination of equations $z = x^2 + 1$ and $z = y^2 + 1$.

B. Separability Condition of LM

In a voltage stability study, LM or singular points can be solved by using the direct method [3]. The direct method involves solving the following three equations:

$$F(z,\lambda) = 0 \tag{15}$$

$$D_z F(z,\lambda)^T \omega = 0 \tag{16}$$

$$\|\omega\| = 1 \tag{17}$$

where z, λ, and ω are load flow state variables, LM, and left eigenvector, respectively.

Equations (15)–(17) are load flow equations, the singularity condition at the collapse point, and the nonzero left eigenvector at the collapse point, respectively. The load flow equation for real power balance is represented by (5). Due to the reasons that $\partial P_i/\partial \delta_i$ (diagonal) terms are dominant in the submatrix of the load flow Jacobian, δ_i is a dominant function of P_{Gi}, and it is a function of GD_i. Therefore, (5) can be approximated as

$$P_{Gi} - (1+\lambda)P_{Di,o} - f(\delta_i) = 0 \tag{18}$$

$$\text{LM} = \lambda \approx f_i(P_{Gi}) + B_o = f_i(\text{GD}_i) + B_o \tag{19}$$

where f_i is a function of GD_i. These assumptions are valid at the singular point since load flow equations are the ones of necessary conditions. From (19), LM is a separable and continuous function of GD_i, which can be represented by (12). The multivariable surface of LM can be approximated by combining all polynomial equations in the form of (12) with one B_o term

$$\text{LM} \approx f_2(\text{GD}_2) + \ldots + f_j(\text{GD}_j) + B_o \tag{20}$$

where j is a bus where a generator is connected. From (20), the LM is a separable function of each GD. This condition is called the "separability condition of LM." An analytical study with a three-generator case is given in the Appendix.

C. MLM Approach

The MLM approach is a method to identify a vector of the GDs of generators that gives MLM by approximating the surface of the LM as a function of the GD. If one can approximate the LM surface as a function of all generation direction variables (GD_i), any optimization techniques can be used to provide the highest LM point [14]. The MLM approach can be explained in the following three steps.

Step 1) The first step is to find the relationship of LM with respect to GD of each generator. An LM plot is obtained from a series of P-V curves for different GDs for a particular generator. This could be considered as a single dimensional curve as only one generator is considered, apart from the swing bus. If the one-dimensional continuous curve between the LM and GD is plotted, one can approximate this curve by a polynomial equation (12) [14]

$$\begin{aligned}\text{LM}_j &= B_{j,0} + B_{j,1}\text{GD}_j + \ldots + B_{j,n}\text{GD}_j^n \\ &= B_{j,0} + \sum_{p=1}^{n} B_{j,p}\text{GD}_j^p = B_0 + \text{LM1}_j\end{aligned} \tag{21}$$

where LM_j is a polynomial approximation representing the LM curve for the two-generator case (the swing generator and another generator connected to bus j), $B_{j,p}$ are the coefficients in the polynomial approximation, n is number of coefficients, and LM1 is the LM equation without the constant term B_0, where $B_0 = B_{j,0}$ for all j, as the initial dispatches are the same.

Step 2) The LM surface is approximated for the multidimensional case based on the separability condition.

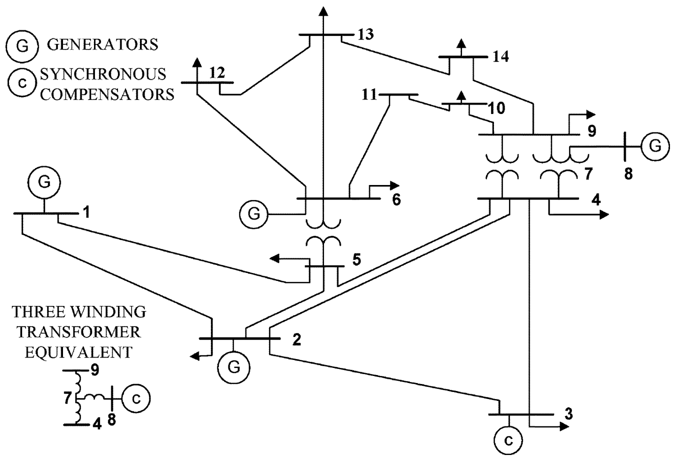

Fig. 1. Single line diagram of the IEEE 14-bus system.

For all participating generators, the LM surface can be defined by

$$\mathrm{LM} = B_0 + \sum \mathrm{LM1}_j. \tag{22}$$

Equation (22) represents the LM surface for all possible GDs of all participating generators. It is found from the combination of all polynomial equations (21) with one constant B_0 term, which are obtained from Step 1).

Step 3) The final step of the proposed technique is to determine the best GD vector that gives the highest LM. The best GD vector can be found by maximizing (22) subject to the following constraints:

$$\sum \mathrm{GD}_j = 1$$
$$0 \leq \mathrm{GD}_j \leq 1.$$

In Step 1), the study can be done mainly by using commercial voltage stability software, while Steps 2) and 3) require developed software. This separates the studies based on the commercial from developed software, which may be easy for utilities to implement. In this paper, since the LM is represented by a polynomial equation (22), a conventional optimization technique, the "Lagrangian" technique, is used to find a global maximum in a simple way. The best GD can be used to find a more accurate LM by using the CPF or direct method. The proposed methodology is tested and validated through various cases in both test and practical power systems.

IV. Test Power Systems

A. IEEE 14-Bus Test System

The modified IEEE 14-bus test system is used first to validate the proposed method, and then, the methodology is applied to a practical system, namely, the Thailand Power System. A single line diagram of the IEEE 14-bus test system is depicted in Fig. 1, which consists of five synchronous machines, including one synchronous compensator used only for reactive power support and four generators located at buses 1, 2, 6, and 8. The modification from the original IEEE 14-bus test system is that generators located at buses 6 and 8 were changed from synchronous compensators to generators. In the system, there are 20 branches and 14 buses with 11 loads, totaling 259 MW and 81.4 Mvar. The value of 259 MW is used for the base MVA of the IEEE 14-bus system.

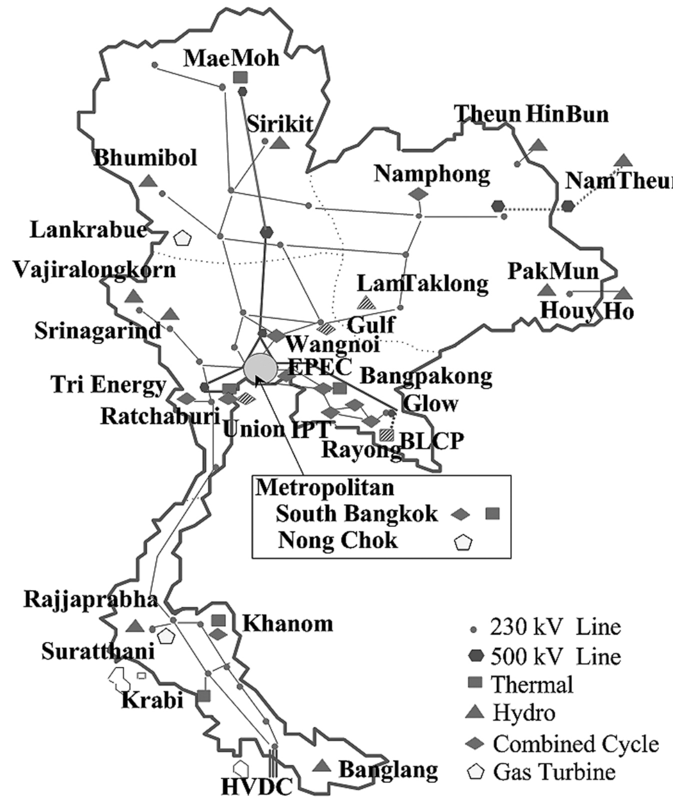

Fig. 2. Power stations and 230/500-kV transmission lines of the Thailand Power System.

B. Thailand Power System [17]

Thailand is a country located in the southeastern region of Asia. The electric supply industry in Thailand consists of three utilities, namely, the Electricity Generating Authority of Thailand (EGAT), the Metropolitan Electricity Authority (MEA), and the Provincial Electricity Authority (PEA). EGAT is responsible for generation and transmission grids of high voltage levels, while MEA and PEA are responsible for distribution levels. Fig. 2 shows power stations and 230/500-kV transmission lines of the Thailand Power System.

EGAT consists of 196 substations and a total of 28 330.8 circuit-kilometer transmission system. There are 820 buses with voltage levels ranging from 500 down to 22 kV. The total installed capacity of the system was 25 324.92 MW, including generations from EGAT power plants, independent power producers (IPPs), small power producers (SPPs), and power imports from neighboring countries. The maximum electrical demand was 19 325.8 MW, which occurred at 14:30 hours on March 30, 2004. The maximum demand is used for the base MVA of Thailand Power System.

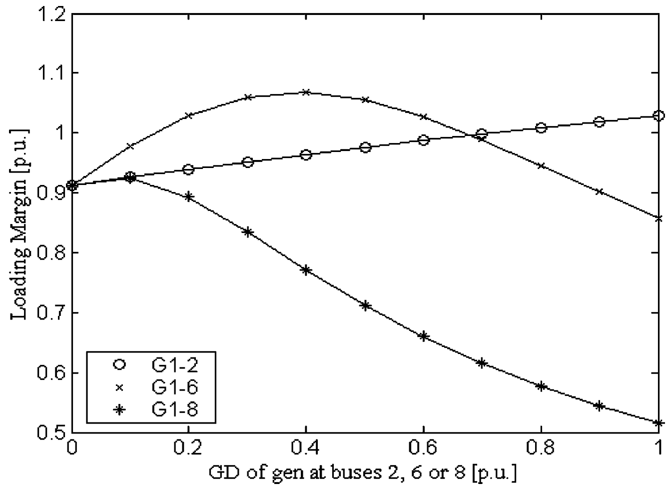

Fig. 3. LMs in the two-generator cases.

V. NUMERICAL RESULTS

The MLM method is based on the LM of the system at various possible GDs in the GD space. The size of GD space is in proportion to the number of dispatchable generators considered in the study. To limit the number of generators in this study, a total of four generators are used for the IEEE 14-bus test system. In the beginning, two-generator cases are examined; then, three- and four-generator cases are investigated to demonstrate the practical usefulness and to validate the proposed approach. After that, the proposed technique is validated in the Thailand Power System. The generation set points of generators at buses 1, 2, 6, and 8 are 150, 77.94, 40, and 40 MW, respectively, at the base load of 259 MW for the IEEE 14-bus test system. The base case of Thailand Power System is the operating condition at the maximum demand on March 30, 2004. Simulations and discussion for all cases are presented in the following subsections.

A. Two-Generator Cases

In the case of two generators, three cases having generators at buses 1 and 2 (G1-2), buses 1 and 6 (G1-6), and buses 1 and 8 (G1-8) are studied. In each case, bus 1 is considered as a swing bus, which delivers the balance of the power. The plots of LMs in the GD spaces for all two-generator cases are illustrated in Fig. 3. These curves are obtained from PV curves, with various GDs. For example, in the G1-2 case, if the GD of the generator at bus 2 is varied from 0 to 1, the LM curve can be plotted as a function of GD_2. Note that the total GDs in all cases were assumed to be one (i.e., $\mathrm{GD}_1+\mathrm{GD}_2=1$, $\mathrm{GD}_1+\mathrm{GD}_6=1$, and $\mathrm{GD}_1+\mathrm{GD}_8=1$). Therefore, the swing bus also participates in sharing the load. For example, in the G1-2 case, 0.1 represents a GD_2 of 0.1 and $\mathrm{GD}_1=0.9$. A $\mathrm{GD}=0$ means that the generator does not participate in dispatching for load increase but remains at its output based on the base generation.

From Fig. 3, it is obvious that the MLMs for the three cases occurred at different generation directions. The basic curve fitting approach can be applied to identify coefficients of polynomial equations representing each LM curve. If the coefficients are known, one can approximate the LM curves by the polynomial equations. Note from Fig. 3 that higher LMs can be achieved by increasing GD at generators 2 or 6. Increasing the generation at bus 6 allows the highest LM, if it is no more than 0.4 p.u.

TABLE I
COEFFICIENTS OF THE POLYNOMIAL APPROXIMATIONS OF THE LM CURVES

Coeffts\Cases	G1-2 (j=2)	G1-6 (j=6)	G1-8 (j=8)
$B_{j,0}$	0.9131	0.9131	0.9130
$B_{j,1}$	0.1356	0.6620	0.4373
$B_{j,2}$	-0.0206	0.0721	-3.6198
$B_{j,3}$	0.0005	-3.0990	4.8798
$B_{j,4}$		3.5001	-1.4056
$B_{j,5}$		-1.1907	-1.6883
$B_{j,6}$			0.9992

Table I shows the coefficients of polynomial equations that approximate the corresponding LM curves shown in Fig. 3. From the table, the one-dimensional polynomial approximations of the loading margin for the cases G1-2, G1-6, and G1-8 are, respectively

$$\mathrm{LM}_2=B_0+\sum_{p=1}^{3}B_{2,p}\mathrm{GD}_2^p=B_0+\mathrm{LM1}_2 \tag{23}$$

$$\mathrm{LM}_6=B_0+\sum_{p=1}^{5}B_{6,p}\mathrm{GD}_6^p=B_0+\mathrm{LM1}_6 \tag{24}$$

$$\mathrm{LM}_8=B_0+\sum_{p=1}^{6}B_{8,p}\mathrm{GD}_8^p=B_0+\mathrm{LM1}_8. \tag{25}$$

These polynomial equations are then used to obtain the surface approximation in the three- and four-generator cases.

B. Three-Generator Cases

Three cases of three generators located at buses 1, 2, and 6 (G1-2-6), at buses 1, 2, and 8 (G1-2-8), and at buses 1, 6, and 8 (G1-6-8) are considered in this section. The LM surface in each case can be approximated by a surface equation obtained by combining the polynomial approximations of the two-generator cases. In the G1-2-6 case, the surface of LMs can be represented, by combining the LM curves for generators 2 and 6, (23) and (24), with only one B_0 term, as

$$\mathrm{LM}=B_0+\sum_{p=1}^{3}B_{2,p}\mathrm{GD}_2^p+\sum_{p=1}^{5}B_{6,p}\mathrm{GD}_6^p \tag{26}$$

where $B_{j,p}$ are the coefficients of the polynomial equations shown in Table I, and GD_j is the GD of the generator connected at bus j.

Similarly, the mathematical formulations for the G1-2-8 and G-1-6-8 cases are, respectively, given by

$$\mathrm{LM}=B_0+\sum_{p=1}^{3}B_{2,p}\mathrm{GD}_2^p+\sum_{p=1}^{6}B_{8,p}\mathrm{GD}_8^p \tag{27}$$

$$\mathrm{LM}=B_0+\sum_{p=1}^{5}B_{6,p}\mathrm{GD}_6^p+\sum_{p=1}^{6}B_{8,p}\mathrm{GD}_8^p. \tag{28}$$

TABLE II
COMPARISON OF GDs AND LMs

Cases	Solutions obtained from			
	MLM Approach		The Actual LM plot	
	GDs	LM (p.u.)	LM (p.u.)	GDs
G1-2-6	(0,0.7,0.3)	1.1655	1.1655	(0,0.7,0.3)
G1-2-8	(0,1,0)	1.0286	1.0286	(0,1,0)
G1-6-8	(0.5,0.4,0.1)	1.0676	1.0686	(0.6,0.4,0)

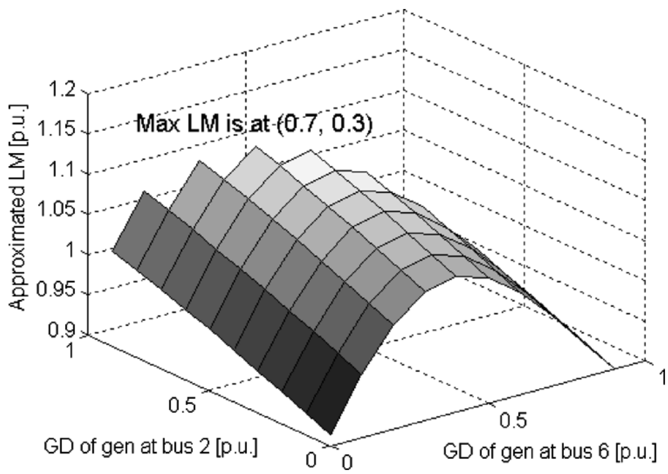

Fig. 4. Approximated LMs in case G1-2-6 using the MLM approach.

From (26)–(28), an optimization technique can be used to find the GD that allows the MLM for each case. For example, in the G1-2-6 case, the formulation for optimization process is

Max.

$$\mathrm{LM} = B_0 + \sum_{p=1}^{3} B_{2,p}\mathrm{GD}_2^p + \sum_{p=1}^{5} B_{6,p}\mathrm{GD}_6^p \qquad (29)$$

Subject to

$$0 \le \mathrm{GD}_j \le 1; \quad j = 2, 6 \qquad (30)$$

$$\mathrm{GD}_1 + \mathrm{GD}_2 + \mathrm{GD}_6 = 1. \qquad (31)$$

The solutions of the MLM for all three-generator cases are shown in Table II. For comparison, the table also provides the actual MLMs and corresponding GDs found from exhaustive simulation using UWPFLOW. The UWPFLOW is a research tool that has been designed to calculate the LM of the power system for a given load and GD [18]. From the table, we observe that the MLM approach can provide the best GDs in all cases, which are the same as those that were obtained from the actual LM plot by UWPFLOW. In G1-6-8 cases, however, the solution occurred near the best GD point found in the actual LM plot.

The approximate plots obtained from (26)–(28) for the G1-2-6, G1-2-8, and G1-6-8 cases are shown in Figs. 4–6, respectively. The corresponding plots for the actual LMs by UWPFLOW are shown in Figs. 7–9, respectively. Clearly, the plots obtained from the MLM approach are almost the same as those obtained from the actual LM plot.

Fig. 4 shows the LM plot, including the best GD, for the case G1-2-6 obtained using the MLM approach. As can be seen from Fig. 4, the MLM occurred at 0.7 and 0.3 GDs for generators

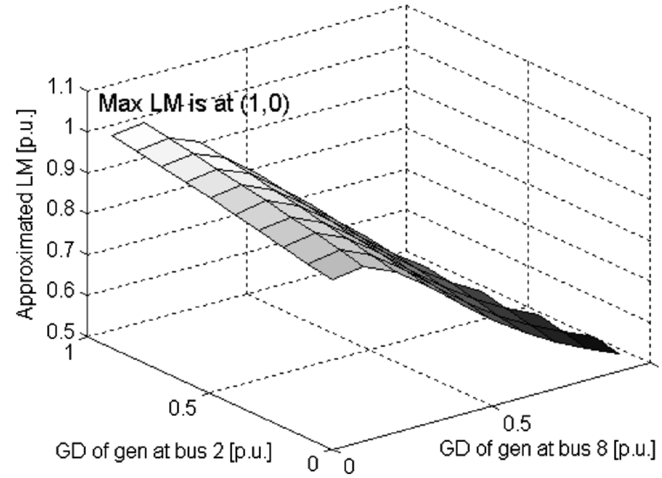

Fig. 5. Approximated LMs in case G1-2-8 using the MLM approach.

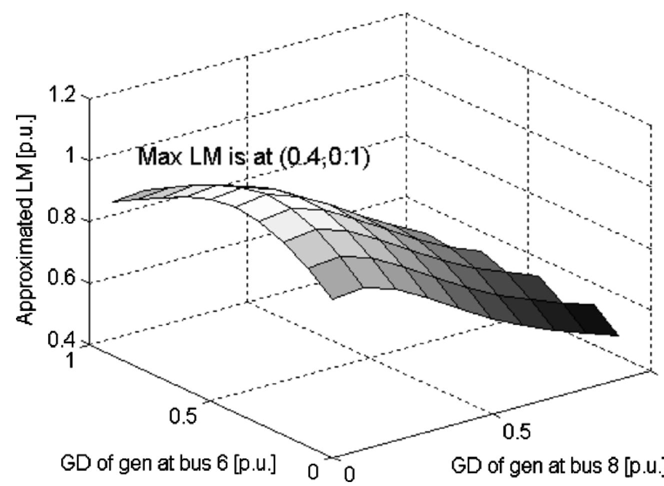

Fig. 6. Approximated LMs in case G1-6-8 using the MLM approach.

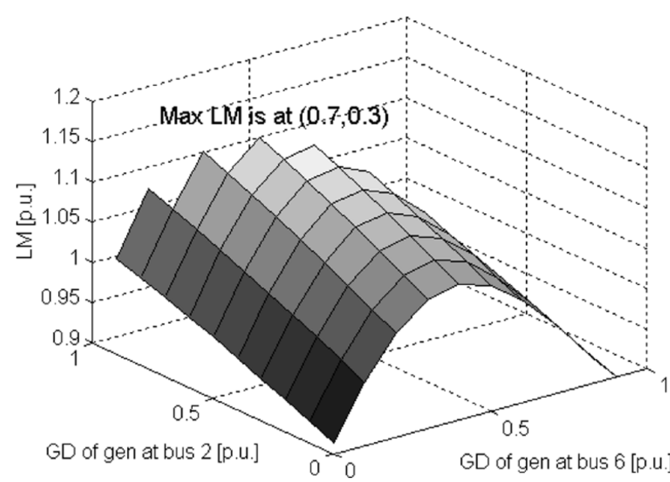

Fig. 7. Actual LMs in case G1-2-6.

2 and 6, respectively. The result is corroborated with the help of an actual (exhaustive) LM plot, as depicted in Fig. 7. The best GDs for the G1-2-8 and G1-6-8 cases obtained from the MLM approach are given in Figs. 5 and 6, respectively. Once again, these results are compared well with actual LMs for the respective cases, as shown in Figs. 8 and 9.

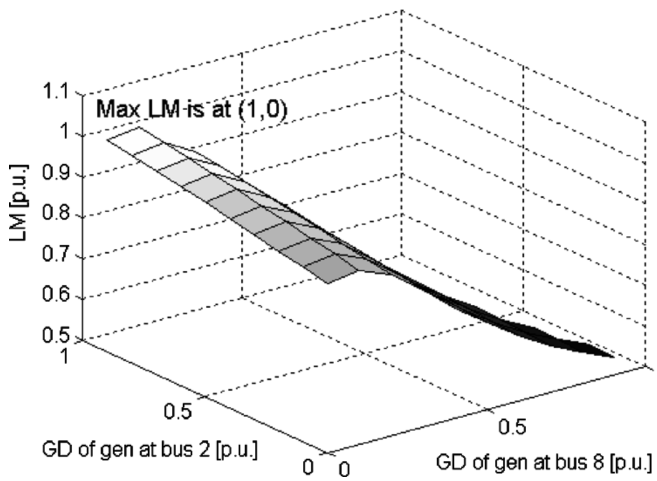

Fig. 8. Actual LMs in case G1-2-8.

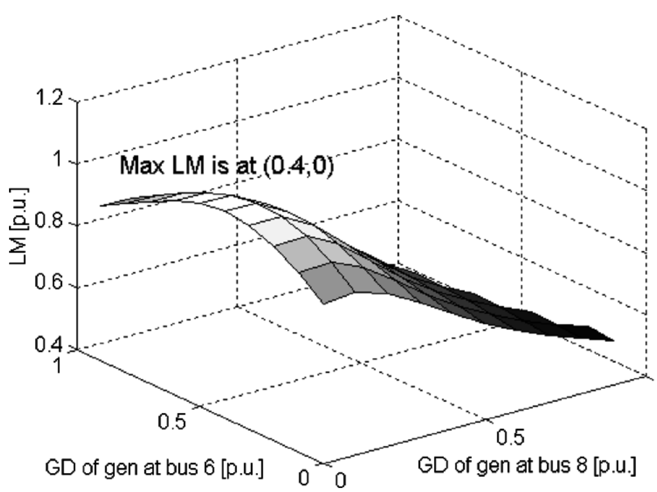

Fig. 9. Actual LMs in case G1-6-8.

If the maximum power output is limited, the rest of the power has to be served by other generators at other buses. This means the generation direction is changed to favor another point in the LM surface. The MLM method can solve the problem of active power limit. A detailed explanation about how it can be done is given below.

In the MLM method, the LM surface has to be plotted without considering the active power limit first. If the active power of generator at bus i is over the limit, we can find a fixed GD_i corresponding to maximum active power. For example, in Fig. 4, for the G1-2-6 case, the MLM of the system is 1.1655. If generators are redispatched from minimum to maximum LM points, the LM increase (LM_{increase}) is $1.1655 - 0.9131 = 0.2524$ p.u. or 65.37 MW with 259 base MVA; at the minimum LM point or the base case, the generator at bus 6 delivers 40 MW. This load increase has to be served by the generators at buses 2 and 6 by 70% and 30%, respectively. This means the generator at bus 6 has to increase the generation by $\mathrm{LM\_increase} * \mathrm{GD}_6 = 0.2524 * 259 * 0.3 = 19.6115$ MW. If the power generation limit of this generator is 50 MW (or a 10-MW increase from the base generation), the fixed GD_6 is then $10/65.37 = 0.1530$ and GD of GD_2 is 1-0.1530. From (22), one more constraint of fixed $\mathrm{GD}_6 = 0.1530$ is added in the optimization process. The MLM with fixed GD_6 is the point that considers the active power limit of this generator. It is noticed that the corresponding LM at the fixed GD_6 can be approximated directly from Fig. 4.

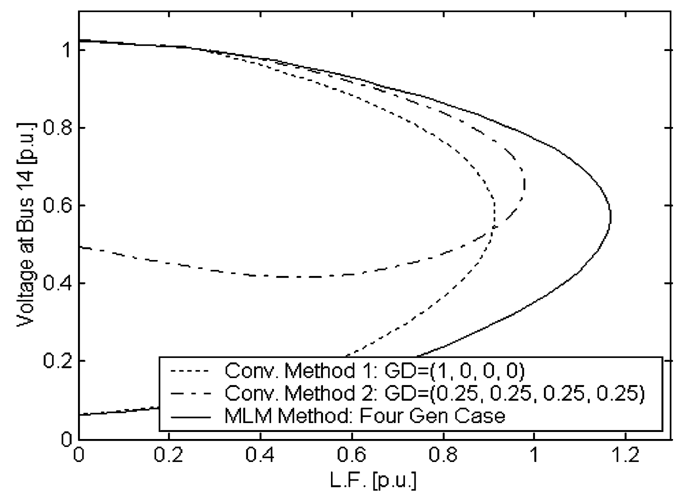

Fig. 10. P-V curves of the conventional and the MLM methods for four-generator case.

C. Four-Generator Case

The same idea of surface approximation can be applied to the case of four generators located at buses 1, 2, 6, and 8. An equation representing the LM surface can be approximated as

$$\mathrm{LM} = B_0 + \sum_{p=1}^{3} B_{2,p}\mathrm{GD}_2^p + \sum_{p=1}^{5} B_{6,p}\mathrm{GD}_6^p + \sum_{p=1}^{6} B_{8,p}\mathrm{GD}_8^p. \tag{32}$$

An optimization technique can be used to obtain the best GDs for the system. In this case, the MLM occurred at GDs 0, 0.6, 0.3, and 0.1 for generators at buses 1, 2, 6, and 8, respectively. At this point, the MLM is 1.1494. To compare the result with the actual LM by UWPFLOW, the simulations were carried out for all possible GDs, and the MLM of 1.1655 occurred at GDs 0, 0.7, 0.3, and 0 for the corresponding generators. The actual LM is almost equal to the LM from the MLM approach. However, there is a small difference in the margins. The difference in LMs is due to the fact that higher order and cross product terms are neglected in the approximate equation. However, the GDs and the corresponding LM resulting from the MLM method are close to the actual values. In practice, a quick estimate of the MLM and corresponding GDs are useful in avoiding a possible voltage collapse.

Fig. 10 illustrates the PV curves of the *conventional* and MLM methods. In the conventional approach, two cases are presented with Conventional Method 1, where only the generator at swing bus is dispatched, and in Conventional Method 2, all four generators are dispatched to meet the load increase. It can be seen that the MLM approach gives about 30% higher LM than the conventional methods. If the MLM approach is used, 61 MW (0.236) more loading margin can be obtained compared to Conventional Method 1.

TABLE III
POWER STATIONS WITH MORE THAN 1000 MW CAPACITY

Name	RB	MM	BPK	SB	WN
Capacity (MW)	2600	2238.4	2060.2	1739.9	1399.9

TABLE IV
GDS AND LMS OF THE THAILAND POWER SYSTEM

Method	GD (RB,MM,BPK,SB,WN)	LM (p.u.)
Conv. Meth.1	(0, 1, 0, 0, 0)	0.0992
Conv. Meth.2	(0.34, 0.33, 0.33, 0, 0)	0.1116
MLM Method	(0, 0.3, 0.7, 0, 0)	0.1134

D. Thailand Power System

Thailand Power System is composed of 20 power stations, including EGAT power stations and IPPs. In the actual case at the peak load, spinning reserve is distributed among five power stations; each of them has more than 1000 MW capacity. The rest of the stations, including SPPs and most of the IPPs, delivers the maximum generation due to their constraint obligations. Table III shows these five power stations, including their capacities. In the next five years, there will be more generators located at or near these stations due to many reasons, such as fuel supply, availability of space, etc.

Thus, in this paper, only five power stations shown in Table III are considered as participating generators. To apply the MLM approach to Thailand Power System, the maximum capacities of these five power stations are relaxed due to the low spinning reserve at the peak load. There is no limitation on the number of generators that could be considered in this approach. The computational cost would increase in proportion to the number of generators considered in the system, as it would in other approaches. However, the computational cost of the proposed method would be much less compared to that of a full optimization technique.

The GDs and LMs using conventional and the MLM approaches for Thailand Power System are given in Table IV. The Conventional method is a business-as-usual case in that few selected high-capacity stations are dispatched to serve the load increase. In Conventional Method 1, only MM station is considered to serve the load increase, whereas in Conventional Method 2, RB, MM, and BPK stations are considered to serve the load increase.

The MLM method gives almost 15% higher LM than Conventional Method 1. This improvement based on maximizing LM is about 300 MW. This capacity relief is almost equal to the capacity of a medium-size thermal power station. Conventional Method 2 provides LM close to the MLM based on the MLM approach. This is because the LM surface of the Thailand Power System is almost flat due to a "well-behaved" system performance related to voltage stability. However, the MLM approach can provide the highest LM point in the Thailand Power System.

PV curves of the Thailand Power System with the MLM method and Conventional Method 1 are illustrated in Fig. 11. From the figure, it is obvious that the MLM approach provides

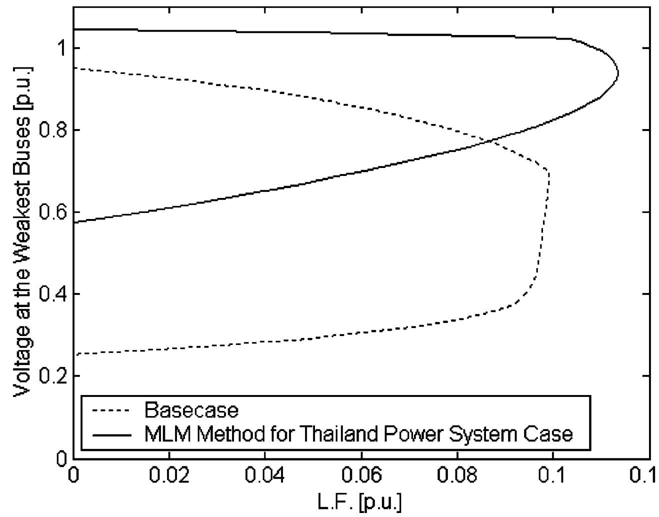

Fig. 11. PV curves at the weakest buses in the Thailand Power System.

higher LM and better voltage profiles. In the figure, the difference in the voltage at $\mathrm{LF} = 0$ is due to the change in the weakest bus. The weakest bus in the base case, conventional GD, is located in the northern area, and it is shifted to the central area with the MLM GD. The new weakest bus with the MLM approach is the next-weakest bus in the base case.

In this paper, the reactive power limits of participating generators have been considered. The reactive power of all generators, except the generator at the swing bus, is at the limits for all cases at the LM, before reaching the nose point. The CPU time for the whole process for three- and four-generator and Thailand power system cases are about 40 s, 80 s, and 12 min, respectively, using AMD 1.4-GHz computer. Most of the CPU time is used by Step 1) based on a conventional voltage stability study.

From the results, it can be concluded that the proposed methodology gives the solutions, both maximum LM and corresponding GDs, closer to the best solutions. As demonstrated through several examples, the method can be applied to any number of generator combinations. The method provides a very good approximation of the GD, which would give the MLM for a given case. Since LM surface may have multiple maximum, as it can be approximated by polynomial equations, the MLM method may be required to find the global maximum. However, if the LM surface has only one maximum, the method of [12] may give a more accurate solution.

Since the approximated model of LM surface with respect to various GDs is available in the MLM approach, feasible GDs can be predicted for any desired LM up to the maximum. Apart from the simple approach, another benefit of the proposed technique is that any existing commercially available software tools can be used to obtain the best GD. In addition, important information such as "voltage stability region" based on generation space can be found by projecting the desired LM on the LM surface onto the GD plane.

VI. CONCLUSION

This paper presents a new methodology for maximizing the LM in a power system by forming an equation representing LM as a function of GDs. The proposed methodology is validated through several examples, both in the test system and in a practical power system. Moreover, the results and 3-D figures presented in the three-generator cases are of particular interest, as they present the closeness of the results of the proposed method and to the actual ones.

The proposed MLM method finds a good approximation of the best GDs and corresponding LM using the surface approximation. Other useful information can be obtained by using the equation. The information helps a system operator to operate the system in a safe and secure manner. Moreover, this approach can be applied in today's competitive market when a feasibility of scheduling generation is required to meet the MLM criteria, subject to the commercial realities of scheduling. Given this information, it is also possible for the independent system operator to adjust the transaction in an appropriate way when the system is in a stressful condition.

APPENDIX

Consider a simple power system with three buses, and each bus is connected with another bus through a transmission line with $j0.2$ p.u. impedance. Each bus contains one generator and load. The voltage magnitude at each bus is assumed to be 1.0 p.u. Equations required to solve singular bifurcation are

$$\begin{aligned} 0 =& P_{G2} - (1+\lambda)P_{D2,o} \\ & - 5\sin\delta_2 - 5\sin(\delta_2-\delta_3) \qquad \text{(A1)} \\ 0 =& P_{G3} - (1+\lambda)P_{D3,o} \\ & - 5\sin\delta_3 - 5\sin(\delta_3-\delta_2) \qquad \text{(A2)} \\ 0 =& \omega_1\left[-5\cos\delta_2 - 5\cos(\delta_2-\delta_3)\right] \\ & + \omega_2\left[5\cos(\delta_3-\delta_2)\right] \qquad \text{(A3)} \\ 0 =& \omega_1\left[5\cos(\delta_2-\delta_3)\right] \\ & + \omega_2\left[-5\cos\delta_3 - 5\cos(\delta_3-\delta_2)\right] \qquad \text{(A4)} \\ 0 =& -1 - P_{D2,o}\omega_1 - P_{D3,o}\omega_2. \qquad \text{(A5)} \end{aligned}$$

Equations (A1) and (A2) can be rearranged and approximated as, respectively

$$\begin{aligned} \mathrm{LM} = \lambda =& \frac{P_{G2}}{P_{D2,o}} - \frac{5\sin\delta_2}{P_{D2,o}} - \frac{5\sin(\delta_2-\delta_3)}{P_{D2,o}} - 1 \\ \approx& \frac{P_{G2}}{P_{D2,o}} - f(\delta_2) - 1 \approx f_2(P_{G2}) - 1 \\ =& f_2(P_{G2}) + B_o = f_2(\mathrm{GD}_2) + B_o \qquad \text{(A6)} \\ \mathrm{LM} = \lambda =& \frac{P_{G3}}{P_{D3,o}} - \frac{5\sin\delta_3}{P_{D3,o}} - \frac{5\sin(\delta_3-\delta_2)}{P_{D3,o}} - 1 \\ \approx& f_3(GD_3) + B_o. \qquad \text{(A7)} \end{aligned}$$

LM surface can be found by combining (A6) and (A7) with one constant B_o term

$$\mathrm{LM} \approx f_2(\mathrm{GD}_2) + f_3(\mathrm{GD}_3) + B_o. \qquad \text{(A8)}$$

Equation (A8) shows that LM is a separable function of GD_2 and GD_3. These equations are valid for the entire GD space.

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