An Auxiliary LQG/LTR Robust Controller Design for Cogeneration Plants

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Abstract - A robust controller is designed to reduce the sensitivity of the frequency and the extraction steam pressure to the disturbance imposed on a cogeneration plant by using the LQG/LTR methodology. Natural frequency of generator swing is analyzed to determine the frequency band of the disturbance due to the variation in load and utilized as a design specification for the LQG/LTR controller. The singular-value Bode diagram of the sensitivity transfer function matrix is shaped to reduce disturbance effects on the power system frequency and steam pressure according to the design specification. The high order LQG/LTR controller is reduced to a low order controller and implemented in a nonlinear cogeneration plant model. Simulation shows the performance of the LQG/LTR controller in disturbance rejection.

Keywords - Cogeneration plant control, LQG/LTR robust control, Disturbance rejection, Low frequency oscillation.

I. INTRODUCTION

A cogeneration plant, which supplies electricity and steam to an industrial process plant, often experiences considerable variations in the power system frequency and extraction steam pressure due to external disturbances such as abrupt changes in electrical and steam loads. Off-nominal frequency excursions cause mechanical and thermal damages in rotating machines. In order to prevent an extended operation at frequencies lower than normal frequencies, load shedding schemes are commonly employed in industrial plants [1]. Although the load shedding can secure the system frequency to normal, it interrupts the continuous operation of a process plant resulting in a significant loss of production and economy. In a similar manner the off-nominal steam pressure causes unnecessary steam blow-out at high pressure and the shortage of thermal energy supply at low pressure. Engineers have made every endeavor to resolve these problems with empirical and physical improvement of power system protection logic, but have not yet fully explored the use of modern optimal control theory. This paper presents the design procedure of a robust multi-input multi-output (MIMO) controller using the Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) to reduce the sensitivity of the frequency and the steam pressure to the external disturbances imposed on the cogeneration plant.

Among many approaches developed for the robust control problem, LQG/LTR approach is preferred due to its systematic design procedure in resolving design issues such as stability robustness and the trade-offs between performance and allowable control effort. Since the methodology has been developed by Doyle, Stein and Athans [2], [3], it has been applied to various problems in energy systems [4], [5] and aerospace systems [6].

One important step in the LQG/LTR methodology is to determine the frequency band of disturbances as a design specification for the robust controller. In this paper, the frequency band is obtained by analyzing the natural frequency of the generator swing as an undamped low frequency oscillation due to external disturbances, such as, variation in electrical load. Another practical issue in the LQG/LTR design is the controller-order reduction. This paper demonstrates the use of the Hankel-norm approximation in designing a reduced-order controller which compares well with the full-order controller.

The paper is organized as follows. Section II presents the modeling of a cogeneration plant possessing extraction

condensing steam turbine. Section III presents the LQG/LTR controller design specification including the analysis of frequency band of disturbance. In section IV a full order compensator is designed. A reduced order compensator is then obtained using the Hankel-Norm optimization method [7] and its performance for disturbance rejection is evaluated. Time domain simulation results and discussions are also given, applying the designed controller to the nonlinear model of a cogeneration plant in Section V. Conclusions are given in Section VI.

II. COGENERATION PLANT MODEL

A robust controller is to be designed for a cogeneration plant in a refinery plant in Korea. Fig. 1 shows the conceptual block diagram of the cogeneration plant controlled by the classical PID controllers. The model includes generator, single extraction steam turbine, actuators and valves, rationers and limiters of high pressure (HP) and low pressure (LP) turbine valves, and speed and pressure PID controllers. The governor employs the Woodward 505E type [8], and has two reference input adjusters to remove steady state errors of the turbine speed and the steam pressure. The mathematical model of the extraction steam turbine is physically based and its parameters are calculated in reference to the manufacturer's design data in steady state condition [9], [10]. The overall cogeneration plant model is represented as a 15-th order nonlinear model.

The objective of controller design is to improve the performance of frequency and extraction steam pressure of a cogeneration plant in the presence of external disturbances by adding a controller as an auxiliary compensator to the existing PID controllers. This State Feedback Assisted Control (SFAC) configuration [4] has an advantage of robustness by modifying the demand signal for an embedded classical output feedback PID controller.

A linearized cogeneration plant model is used for controller design and analysis, while demonstrations are given by applying the designed controller to the original nonlinear system. The linearized cogeneration plant model is in the following form:

\[ A_p x_p(t) + B_p u_p(t) \]
\[ C_p x_p(t) + D_p u_p(t) \]

where \( x_p = [\Delta \omega \ 
\Delta p]^T \), \( u_p = [u_1 \ 
\ u_2]^T \),

\( \Delta \omega \) : frequency variation in per unit,

\( \Delta p \) : extraction steam pressure variation in per unit,

\( u_1 \) : HP valve input,

\( u_2 \) : LP valve input.

Table 1. Operating point of linearization of cogeneration plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical load</td>
<td>0.667 pu / 18 MW*3</td>
</tr>
<tr>
<td>Steam load</td>
<td>0.564 pu / 56 ton/hour</td>
</tr>
<tr>
<td>Turbine speed</td>
<td>1800 rpm</td>
</tr>
<tr>
<td>Extraction steam pressure</td>
<td>43.2 kg/cm² abs</td>
</tr>
</tbody>
</table>

III. DESIGN SPECIFICATIONS

A. Specification of LQG/LTR Controller

An LQG/LTR based stabilizing controller \( K(s) \) is incorporated into the cogeneration plant dynamics \( G(s) \) to establish a MIMO feedback control system, which is shown in Fig. 2. The performance and stability of the MIMO feedback control system can be evaluated by the analysis of the following Transfer Function Matrices (TFMs) in the frequency domain:

\[ T(s) = G(s)K(s) \] : Loop TFM (2)

\[ S(s) = (I + T(s))^{-1} \] : Sensitivity TFM (3)

\[ C(s) = (I + T(s))^{-1}T(s) \] : Closed Loop TFM (4)

The performances of command following, disturbance rejection and sensor noise insensitivity are achieved by singular value shapings of the TFMs, \( T(s) \), \( S(s) \), \( C(s) \), respectively. The design specification of the stabilizing controller can be prepared by frequency domain analysis of disturbances in the cogeneration plant.

Since the classical PID controllers combined with
In order to analyze the natural frequency of generator torque angle, the equations (5) and (6) are linearized:

\[ M\dot{\delta} = \Delta T_M - \Delta T_E - \Delta T_D, \quad (7) \]

\[ \dot{\delta} = \omega_1 \Delta \omega, \quad (8) \]

where

\[ \Delta T_M = -D_M \Delta \omega, \quad \Delta T_E = K_1 \Delta \delta + D_E \Delta \omega, \quad \Delta T_D = D \Delta \omega, \quad (9) \]

or equivalently,

\[ (M\omega_0^2 + (D_M + D_E + D)s + \omega_0^2 K_1)\Delta \delta = 0, \quad (10) \]

where \( M \) is the inertia constant in seconds, \( D \) is the generator damping coefficient, \( D_M \) is the sensitivity of the mechanical torque \( T_M \) with respect to \( \omega \), \( D_E \) is the sensitivity of the electric torque \( T_E \) with respect to \( \omega \) and \( K_1 \) is the sensitivity of the electric torque \( T_E \) with respect to the torque angle [11].

The solution of equation (10) gives the natural frequency of the synchronous generator.

\[ s = (-\xi_n \pm j\sqrt{1 - \xi_n^2})\omega_n \quad (11) \]

\[ \omega_n = \sqrt{\omega_0^2 K_1 / M}, \quad \xi_n = (D_M + D_E + D) / 2\omega_n M, \quad (12) \]

where \( \omega_n \) is the undamped natural frequency of the mechanical mode in radians per second and \( \xi_n \) is the damping coefficient in p.u.

The natural frequency \( \omega_n \) of the generator for the sample cogeneration plant is calculated to be 4.43 radians/second at the steady-state operating condition and has a low frequency oscillation mode in the frequency band of 3 to 5 radians/second. In general, the external utility power system has a low frequency oscillation mode with small mechanical damping. Therefore a cogeneration plant connected to a utility system also has a sustained low frequency oscillation as the generator is affected by disturbances such as utility generator outages, line outages, and abrupt changes in local loads at the cogeneration plant. As an important first step, the natural frequency band calculated within the operation range is considered as the frequency band of disturbances in setting up the design specification for the LQG/LTR controller.

IV. LQG/LTR CONTROLLER DESIGN

A. Design Plant model (DPM) and Scaling

The linear model of cogeneration plant has transmission zeros at the left half s-plane and it is a minimum phase plant. However one of the transmission zeros stays at \( s = 0 \). Because of this the singular values of the open loop TFM of the cogeneration plant are very small at the low frequency band. Thus it is desirable to increase the singular values at the low frequency band by adding integrators to the cogeneration plant [12]. The integrator prevents high frequency oscillation.
of control input to the turbine valve owing to its characteristic as a low-pass filter. The integral control action is provided by appending two integrators to the 15-th order cogeneration plant model, one in each control input channel. The augmented plant becomes the 17-th order design plant model (DPM) with augmented system matrices \( \{ A, B, C, D \} \), augmented state \( x \) and augmented input \( u \) corresponding to equation (1).

The singular value-based stability robustness of the MIMO control system is sensitive to the scaling [5]. The cogeneration plant in the normal operation has a range of variation in frequency about \( |\Delta \omega| = 0.01 \) p.u. and in steam pressure about \( |\Delta P| = 0.1 \) p.u. While a small change in \( |\Delta P| \) will be insignificant, the same change in \( |\Delta \omega| \) will be considered as a large perturbation. Thus prior to designing the LQG/LTR controller, it is important to normalize the plant inputs and outputs with proper scaling factors. Since the scaling prevents a wide spread in the open loop singular value, it is easier to design the Target Filter Loop. The scaling brings together the maximum and minimum singular values close together at the crossover frequency.

The LQG/LTR procedure consists of two steps: first, designing a Kalman filter with a target filter loop (TFL) that satisfies the performance and robustness requirements and second, recovering the TFL asymptotically by tuning the optimal regulator. After the filter gain matrix \( H \) has been obtained, the control gain matrix \( F \) is calculated through the Loop Transfer Recovery.

### B. Target Filter Loop (TFL) Design

The target filter loop transfer function matrix \( G_f(s) \) has the form:

\[
G_f(s) = C\Phi(s)H
\]  

where \( \Phi(s) = (sI - A)^{-1} \) and \( H \) is the filter gain matrix.

In the Kalman filter design process, the input disturbance matrix and the sensor noise intensity are used as design parameters to determine the singular value shape of the target filter loop (TFL) that would meet the desired performance and robustness specification [12]. After specifying a satisfactory shape for singular values of the TFL, the Kalman filter gain matrix \( H \) is calculated from the Kalman Filter Frequency Domain Equivalent (KFDE) and the Filter Algebraic Riccati Equation (FARE).

### C. Loop Transfer Recovery (LTR)

After the filter gain matrix \( H \) has been obtained, the control gain matrix \( F \) is calculated through the Loop Transfer Recovery. The matrix \( F \) is calculated as the full state feedback regulator gain matrix using the Linear Quadratic Regulator (LQR) technique:

\[
u = -Fx, \quad (14)
\]

where the control gain matrix \( F \) is calculated from the Control Algebraic Riccati Equation (CARE). The control gain \( F \) is iteratively determined by adjusting the input weighting matrix of the performance index for the LQR to recover the singular values of the loop TFM to the design specification of the TFL.

The LQG/LTR controller transfer function matrix \( K(s) \) is then given by

\[
K(s) = F(sI - A + HC + BF)^{-1}H
\]  

and the overall loop transfer function matrix \( T(s) \) with the loop broken at the output is given by

\[
T(s) = G(s)K(s),
\]

where \( G(s) \) is the transfer function matrix of the augmented scaled plant, e.g., \( C(sI - A)^{-1}B \).

In order to perform the loop transfer recovery, the LQR is designed with varying input weighting matrix in the performance index. As the input weighting matrix approaches to zero, the control input \( u \) increases very high. The control input given by the LQG/LTR controller is furnished to the steam turbine valve actuators through the integrator. Since there are physical limits to the valve opening and closing, control input \( u \) will be limited. During the simulation, the controller inputs are managed to be relatively low compared with the governor inputs.

### D. LQG/LTR Controller Order Reduction and Evaluation

The LQG/LTR controller designed in the above section is of the 17-th order, which is the order of the integrator-augmented cogeneration plant model. Various methods for model reduction of large scale linear systems have been studied [13], which are for example, Padé approximations, modal approximation, or continued fraction expansions. In this study the method of optimal Hankel-norm approximations is utilized [7] to reduce the order of the LQG/LTR controller. This technique makes it possible to calculate the achievable error between the frequency responses of the full order model and any reduced-order model. Its objective is to minimize the Hankel-norm

\[
\| K(s) - \hat{K}_r(s) \|_H
\]

where \( \hat{K}_r(s) \) is the reduced controller with order \( r \).

During the order reduction process of the LQG/LTR controller, the Bode diagrams of the singular values of the TFMs are examined and compared for the full and reduced order models to determine the order of reduced controller. Then the 17-th order LQG/LTR controller is reduced to the 11-th order controller while keeping the desirable performance and stability robustness of the controlled plant. Further reduction of the model degrades the performance.
Bode diagrams of the full and reduced order models closely match with each other in the frequency region of interest.

Fig. 3 compares the command following performance by means of the shape of the singular values of the loop TFM, $\sigma[T(j\omega)]$. The minimum singular value of $T(s)$ is the measure of the tracking performance, which is positive in the frequency bands of $\omega < 5.62$ radians/second with the full 17-th order controller and $\omega < 6.34$ radians/second with the reduced 11-th order controller. Therefore the command following performance is achieved with the reduced order controller in the above low frequency region of interest.

In Fig. 4, the singular values of the sensitivity TFM, $\sigma[S(j\omega)]$, are compared to examine the performance of disturbance rejection. The maximum singular value is negative in the frequency bands of $\omega < 5.62$ radians/second with the full order controller and $\omega < 6.15$ radians/second with the reduced order controller. The performance of disturbance rejection is attained in the frequency range lower than 5 radians/second, where the disturbance caused by load variations are usually imposed.

In Fig. 5, the singular values of the closed loop TFM, $\sigma[C(j\omega)]$, are compared to check the performance of the noise insensitivity. The maximum singular value is close to zero in the frequency bands of $\omega < 10$ radians/second with the full order controller and $\omega < 14$ radians/second with the reduced order controller. The minimum singular value is also close to zero up to 13 radians/second for both controllers. The closed loop TFM singular value plots decrease sharply as the frequency band exceeds 10 or more radians/second, illustrating that the designed LQG/LTR controllers preserve immunity to the sensor noise with a high frequency bandwidth.

V. SIMULATION

The above LQG/LTR controller design relies on linear models, which allow a quick initial evaluation of the stability robustness of the closed-loop system. However the initial evaluation is not sufficient for confirming whether the designed controller will behave satisfactorily when implemented in a real plant that is nonlinear and uncertain. The reduced order LQG/LTR controller is applied to the full order nonlinear model of the cogeneration plant to verify numerically.

We normalized some values in simulation. The bases of power frequency, steam pressure, electric load, steam load are 60Hz, 43.2kg/cm$^2$, 1 SMW, 56t/h, respectively.

In our study, the load changes act as disturbances. So we want to make variations of frequency and steam pressure small in the presence of abrupt load changes.

A. Case 1: Step Change in Electric Load

This simulates the case when a step change in electric load, from 66.7% to 100%, is applied at $t = 0.5$ second to the cogeneration plant embedded with classical PID controllers. Fig. 6 shows the time response of variations in frequency and steam pressure for the case with and without the auxiliary
LQG/LTR controller. The maximum variation in the frequency is reduced from -0.65 Hz without the auxiliary controller to -0.3 Hz with the auxiliary controller, which gives a noticeable improvement of 0.35 Hz in frequency variation. It should be emphasized that this improvement can significantly reduce the nuisance shedding of essential loads and unnecessary interruptions in the process industry.

Meanwhile the maximum variation in extraction steam pressure is made smaller from +9.6% without the controller to +0.9% with the controller. An improvement of 8.7% is obtained in the extraction steam pressure variation.

B. Case 2: Step Change in Steam Load

This simulates the case for a step change in steam load from 56.4% to 100% applied at \( t = 0.5 \) second. As shown in Fig. 7, the maximum frequency deviation without and with the LQG/LTR controller is -0.020 Hz and -0.014 Hz, respectively, and the extraction steam pressure deviation is -3.8% and -1.0%, respectively. An improvement of 2.8% is obtained in the steam pressure, while the frequency improvement is negligible.

C. Case 3: Cyclic Variation in Electric and Steam Load

In this test, cyclic variations of \( 0.667 + 0.3 \sin(4t) \) p.u. in electric load and \( 0.564 + 0.3 \sin(t) \) p.u. in steam load are applied at \( t = 0.5 \) second on the cogeneration plant which are usual frequencies in cogeneration plant. As indicated in Fig. 8, the LQG/LTR controller reduces the system frequency range from -0.59/+0.74 Hz to -0.28 Hz/+0.32 Hz and the steam pressure range from -2.5%/+2.5% to -0.7%/+0.7%. The disturbance rejection is better performed at the lower frequency. Therefore the steam load disturbance of 1 rad/s is better rejected than electric load of 4 rad/s. So it seems that there is no oscillation of 1 rad/s in Fig. 8.

VI. CONCLUSION

A multi-input multi-output (MIMO) robust controller is designed for a cogeneration plant using the LQG/LTR methodology. Its objective is to improve the frequency and extraction steam pressure response, which is the prime concern for engineers who are involved in the design and operation of the cogeneration plant. As an important step in the LQG/LTR design, the natural frequency of generator output is analyzed in order to determine the frequency band of disturbances due to variation in load and to set up the design specification for the LQG/LTR controller.

A robust LQG/LTR controller is designed as an auxiliary controller for a cogeneration plant embedded with classical PID controllers. The full order controller is reduced to a low order controller using the Hankel-norm approximation. For nonlinear verification, the reduced order controller is applied to the full order nonlinear cogeneration plant model. Simulation results demonstrate that the LQG/LTR controller significantly improves the performance of frequency and extraction steam pressure in the cogeneration plant against abrupt changes in loads.
**VII. REFERENCES**


**VIII. ACKNOWLEDGMENT**

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**IX. APPENDIX**

The full order nonlinear plant model as well as the constants and parameters of the study system used in the simulation will be available upon request.

**X. BIOGRAPHY**

**Young-Moon Park** was born in Masan, Korea on Aug. 20, 1933. He received his B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University in 1956, 1959 and 1971, respectively. His research interests include power system operation and control, and artificial intelligence applications to power systems. Since 1959, he has been a faculty of Seoul National University where he is currently Professor of Electrical Engineering. He is also serving as the president of the Electrical Engineering and Science Research Institute. Dr. Park is a senior member of IEEE.

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