LQG/LTR Robust Control of Nuclear Reactors with Improved Temperature Performance

Adel Ben-Abdennour, Robert M. Edwards, and Kwang Y. Lee

Abstract—**Controller robustness is always a major concern. A controller that meets certain performance design objectives cannot be satisfactory unless it can preserve such quality in the presence of expected uncertainties. For nuclear reactors, a controller that preserves stability and performance for a wide range of operating conditions and disturbances is especially desirable. This paper presents the design of a robust controller using the linear quadratic gaussian with loop transfer recovery (LQG/LTR) for nuclear reactors with the objective of keeping a desirable performance for reactor fuel temperature and temperature of the coolant leaving the reactor for a wide range of reactor power. The obtained results are compared to an observer-based state feedback optimal reactor temperature controller. Sensitivity analysis of the dominant closed-loop eigenvalues and nonlinear simulation are used to demonstrate and compare the performance and robustness of the two controllers. The LQG/LTR approach is systematic, methodical, easy to design, and can give improved temperature performance over a wide range of reactor operation.**

I. Introduction

THE design of a control system is usually based on a nominal model of the plant to be controlled. Oftentimes, the design procedure goes through the usual simplification, such as linearization about an operating point or lumped parameter approximation, etc. The result is an approximate plant or, as often referred to, uncertain plant. The usual sources of uncertainties are due to linearization of a nonlinear system, unmodeled dynamics, sensor/actuator noise, and undesired external disturbances on different parts of the system. The designer must, therefore, be concerned about how well the controller will work with the actual plant to achieve the design objectives, and whether it is possible to design a controller that takes care not only of these given uncertainties but also of others, such as those due to component failures, changes in environmental conditions, manufacturing tolerances, and wear due to aging, etc.

Over the last decade, the above challenging questions motivated theoreticians and practitioners to develop a control methodology which is, today, known as *Robust Control*. The robust control problem is the problem of analyzing and designing an accurate control system given models with significant uncertainties [1]; synthesizing a control law which maintains system response and error signals to within prespecified tolerances despite the effects of uncertainty on the system [2]; or maintaining stability for all plant models in an expected "band of uncertainty" [3].

Many approaches have been developed for the robust control problem and yet more are under investigation. However, the linear quadratic gaussian with loop transfer recovery (LQG/LTR) got a special attention due to its effectiveness in accommodating plant uncertainty in a more systematic yet more straight forward way. The LQG/LTR robust controller has been considered for a nuclear power plant deaerator and has been shown to provide not only desirable performance in normal operation of the controlled plant, but also in fault accommodation and for good robustness to plant uncertainties [4]–[7].

This paper demonstrates the robustness of the LQG/LTR controller for improving nuclear reactor temperature response, and compares it to a robust observer-based optimal state feedback controller designed in recent years [8]–[10]. The systematic LQG/LTR controller gives improved temperature performance over a wide range of reactor operation. This paper is organized as follows. Section II presents the reactor modeling. Section III presents the LQG/LTR controller design. Simulation results and discussion are given in Section IV, and conclusions are drawn in Section V.

II. Reactor Model

The nominal pressurized water reactor (PWR) model for controller design used in this paper is point kinetics with one delayed neutron group and temperature feedback from lumped fuel and coolant temperature calcula-

Manuscript received June 10, 1992; revised August 14, 1992. This work was supported in part by a grant from the United States Department of Energy, University Grant no. DE-FG-07-89ER 12889, Intelligent Distributed Control for Nuclear Power Plants. However, any findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of U.S. D.O.E.

A. Ben-Abdennour and K. Y. Lee are with the Department of Electrical and Computer Engineering, The Pennsylvania State University, University Park, PA 16802.

R. M. Edwards is with the Department of Nuclear Engineering, The Pennsylvania State University, University Park, PA 16802.

IEEE Log Number 9203990.

tion which is summarized as follows [11]

$$\frac{dn_r}{dt} = \frac{\delta\rho - \beta}{\Lambda} n_r + \frac{\beta}{\Lambda} c_r, \tag{1a}$$

$$\frac{dc_r}{dt} = \lambda n_r - \lambda c_r, \tag{1b}$$

$$\frac{dT_f}{dt} = \frac{f_f P_{0a}}{\mu_f} n_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{2\mu_f} T_l + \frac{\Omega}{2\mu_f} T_e, \tag{1c}$$

$$\frac{dT_l}{dt} = \frac{(1-f_f)P_{0a}}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f - \frac{(2M+\Omega)}{2\mu_c} T_l + \frac{(2M-\Omega)}{2\mu_c} T_e, \tag{1d}$$

$$\frac{d\,\delta\rho_r}{dt} = G_r z_r, \tag{1e}$$

$$\delta\rho = \delta\rho_r + \alpha_f(T_f - T_{f0}) + \frac{\alpha_c(T_l - T_{l0})}{2} + \frac{\alpha_c(T_e - T_{e0})}{2}, \tag{1f}$$

where the variables are fully explained in Nomenclature. Later, the robustness of the closed loop system based on this uncertain model is demonstrated via application to a much higher order simulation. The nominal model contains five states: relative reactor power n_r, relative precursor density c_r, average fuel temperature T_f, average coolant temperature leaving the reactor T_l, and control reactivity $\delta\rho_r$. Furthermore, this low order model is also nonlinear because total reactivity $\delta\rho$, which is composed of rod reactivity and temperature feedback reactivity (1f), multiplies the reactor power state in the determination of the reactor power rate of change (see references [8], [9], and [10] for more details).

While the simulation demonstrations are obtained by application of a controller to the nonlinear system, a linearized reactor model is used for controller design and analysis. Linearizing equation (1) around an equilibrium point, the following state space representation is obtained

$$\frac{dx}{dt} = A_p x + B_p u$$
$$y = C_p x + D_p u \tag{2}$$

where

$$A_p = \begin{bmatrix} -\beta/\Lambda & \beta/\Lambda & n_{r0}\alpha_f/\Lambda & n_{r0}\alpha_c/2\Lambda & n_{r0}/\Lambda \\ \lambda & -\lambda & 0 & 0 & 0 \\ f_f P_{0a}/\mu_f & 0 & -\Omega/\mu_f & \Omega/2\mu_f & 0 \\ (1-f_f)P_{0a}/\mu_c & 0 & \Omega/\mu_c & -(2M+\Omega)/2\mu_c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ G_r \end{bmatrix},$$

$$C_p = [1\,0\,0\,0\,0], \quad D_p = [0].$$

The state, output, and control vectors are defined as $\boldsymbol{x} = [\delta n_r\ \delta c_r\ \delta T_f\ \delta T_l\ \delta\rho_r]^T$, $y = [\delta n_r]$, and $u = [z_r]$, respectively. The symbol δ indicates the deviation of a variable from an equilibrium value; e.g., $\delta n_r(t) = n_r(t) - n_{r0}$ with n_{r0} being the value of n_r at an equilibrium condition. Note that a linear model (2) is defined at each equilibrium power and the presence of n_{r0} in the linear model is a result of the linearization process. Besides this explicit dependency on n_{r0}, other parameters, α_f, α_c, μ_c, Ω, and M, are also dependent on n_{r0} implicitly [12]:

$$\alpha_f(n_{r0}) = (n_{r0} - 4.24) \times 10^{-5}\left(\frac{\delta k}{k}/\,^\circ C\right)$$

$$\alpha_c(n_{r0}) = (-4.0 n_{r0} - 17.3) \times 10^{-5}\left(\frac{\delta k}{k}/\,^\circ C\right)$$

$$\mu_c(n_{r0}) = \left(\frac{160}{9} n_{r0} + 54.022\right)(MWs/\,^\circ C)$$

$$\Omega(n_{r0}) = \left(\frac{5}{3} n_{r0} + 4.9333\right)(MW/\,^\circ C)$$

$$M(n_{r0}) = (28.0 n_{r0} + 74.0)(MW/\,^\circ C).$$

Thus, uncertainty is induced between the time varying linear model and the general nonlinear plant.

III. Reactor Control

A conventional output feedback reactor control uses control rod motion to regulate power output to a demand value (set point), as shown in Fig. 1. This approach is simple and direct in regulating reactor power to accomplish limited control objectives by proper selection of the single design variable, gain G_c. If improved performance and robustness are sought, state feedback design techniques provide a methodology with more flexibility to achieve system objectives through the proper selection of state feedback gains representing many design variables.

A. Conventional State Feedback Control (CSFC)

The advantage of the state feedback design techniques is that the dynamic characteristics of the nominal plant can be completely assigned. Design objectives on the response of all the model reactor states, including reactor temperatures in addition to the power output, can be simultaneously accommodated. Reference [8] contains a complete derivation and analysis of the CSFC. The CSFC configuration is diagrammed in Fig. 2. As simple or as direct as this approach may seem, systematic methodology to achieve good robustness characteristics in this configuration is not well developed.

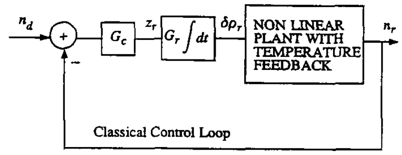


Fig. 1. Conventional output feedback reactor control.

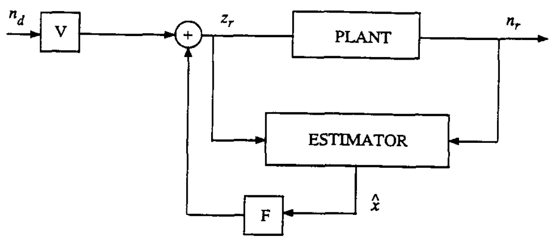


Fig. 2. Conventional state feedback control (CSFC).

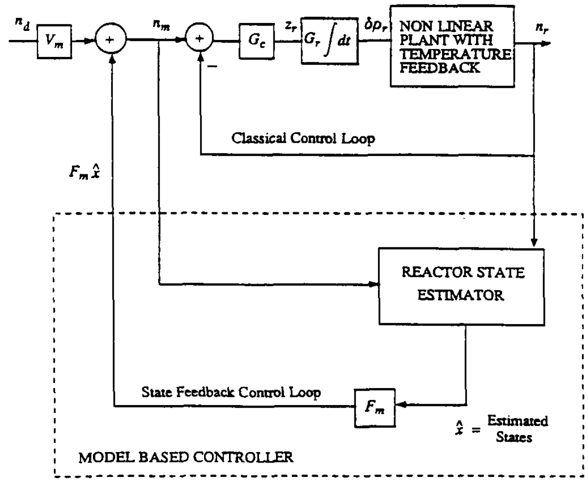


Fig. 3. State feedback assisted control (SFAC).

B. State Feedback Assisted Control (SFAC)

This improved state feedback control design method is described in [8], [9], [13]. As the name suggests, it is an approach that uses the concept of state feedback to modify the demand signal for an embedded classical output feedback controller. One objective of this approach is to facilitate implementation of advanced model-based control techniques within the established structures of power plant operations. As shown in Fig. 3, the SFAC regulates the reactor power by using both a state feedback loop and an output feedback loop. With the power demand n_d, the state feedback loop generates a modified demand signal n_m, and with the modified demand signal, the output feedback loop generates an error signal. This error is multiplied by the output feedback controller gain G_c to determine the control speed z_r. Reference [9] demonstrates that there is a unique robustness advantage of the SFAC configuration in comparison to the CSFC configuration.

C. Linear Quadratic Gaussian with Loop Transfer Recovery (LQG / LTR)

Since the robustness advantage to incorporating an embedded output feedback controller has been demonstrated, the LQG/LTR design presented in this paper is also applied to modify the demand signal to an embedded output feedback controller. The LQG/LTR method is a model-based compensator design with unity feedback gain. This approach uses the linear quadratic gaussian (LQG) and the loop transfer recovery (LTR) at the same time to methodically achieve performance and robustness objectives [14]. In the case of desired robustness at the output, this procedure consists of two steps: the first is the design of a target feedback loop (TFL) with desired loop shape via Kalman filter, i.e., use the Kalman filter to obtain a loop that serves as a target for the controlled system to converge to. The second step is to recover the target loop shape using the linear quadratic regulator (LQR). If robustness at the input is desired, i.e., assuming the uncertainty to be in the input side, which generally is not the case, the dual procedure is used which is a TFL design using LQR, and then a LTR design using the Kalman filter. In the robustness at the output case, the first step is referred to as filter design while the second step is referred to as controller design.

Since the purpose of this paper is to compare the LQG/LTR controller with the SFAC controller, our design is based on the multilayer structure with the classical unity feedback loop embedded as shown in Fig. 3. Moreover, to eliminate steady state error for step and ramp commands, integral control action is provided by appending an integrator to the plant. The augmented plant then has the form:

$$A = \begin{bmatrix} A_p - G_c B_p C_p & G_c B_p \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad (3)$$
$$C = [C_p \; 0] \qquad D = D_p,$$

where G_c is the gain for the classical output feedback control. This augmentation adds a sixth state to the reactor model for the LQG/LTR design.

A. Filter design

This step is a standard Kalman filtering problem. Consider the following model:

$$\dot{x} = Ax + Bu + \Gamma w \qquad (4)$$

$$z = Hx + v \qquad (5)$$

$$y = Cx, \qquad (6)$$

where w and v are zero mean Gaussian white-noise processes at the input and output with covariances Q and $R(= \mu)$, respectively, z are the measurements available, and y are the controlled plant outputs. The LQG/LTR procedure uses μ and Γ as design parameters in order to synthesize a compensator that would meet the desired performance and robustness specifications. We will consider $C = H$ for convenience. The Kalman filter equations for the state estimate, the error, and the gain are

$$\dot{\hat{x}} = A\hat{x} + K_F[z - H\hat{x}] \quad (7)$$

$$\dot{e} = [A - K_F H]e + \Gamma w - K_F v \quad (8)$$

$$K_F = PH^T R^{-1}, \quad (9)$$

where the error covariance matrix P is the solution of the Riccati equation

$$\dot{P} = AP + PA^T + \Gamma Q \Gamma^T - PH^T R^{-1} HP, \; P(0) = P_o. \quad (10)$$

The goal is to design a Target Feedback Loop (TFL), $G_{KF}(s)$, with desired loop shape, in terms of performance. The Kalman filter gain K_F is varied by adjusting design parameters μ and Γ, to get a desired loop shape for $G_{KF}(s)$.

B. Controller Design

This step is an optimal control problem. We need to solve for the full state feedback regulator gain matrix K_o via the optimal control technique to recover for the TFL transfer function. The optimal control performance measure is chosen as

$$J = \int_0^{\infty} \left[q x^T Q_o x + u^T R_o u \right] dt, \quad (11)$$

where T denotes the matrix transpose,

$$Q_o = Q_o^T \geq 0, \quad (12)$$

$$R_o = R_o^T > 0, \quad (13)$$

and $q > 0$ is a scalar design parameter. The optimal control law is given by

$$u = -K_o x \quad (14)$$

with

$$K_o = R_o^{-1} B^T P, \quad (15)$$

where P satisfies the algebraic Riccati equation

$$0 = PA + A^T P + qC^T Q_o C + PB^T R_o^{-1} BP.$$

The LQG/LTR robust controller as a compensator is composed of the Kalman filter dynamics defined in (7)–(9) and the optimal feedback control in equations (14)–(15). The dynamics of the robust controller $K(s)$ is shown in Fig. 4, where K_F is obtained from the filter design and K_o is obtained from the controller design.

If one is able to

- adjust K_F so that $G_{KF}(s)$ has the desired loop shape; and
- construct a K_o so that $GK_o(s) \simeq G_{KF}(s)$, where G being the plant and G_{KF} being the TFL, over the band of frequencies relevant to our concern for performance and robustness, then the $K(s)$ is a robust compensator. Once designed, the robust controller $K(s)$ is implemented as a compensator in the forward loop with unity feedback as shown in Fig. 4.

IV. Simulation Results

The same middle of cycle full power plant parameters as used in the optimal SFAC, given in Table I, are used to design a LQG/LTR controller for a complete fuel cycle of a Three Mile Island (TMI)-type Pressurized Water Reactor (PWR). For comparison with previously reported observer-based state feedback designs [8], [9], the embedded output feedback controller gain G_c is chosen to be 0.2.

Since reactor temperatures determine the structural integrity of the reactor core more so than does the instantaneous value of reactor power, our design objective is to tightly control those temperatures for a wide range of reactor operation. Since the design steps of the observer-based state feedback can be found in [8], [9], [13], we will focus only on the design of the LQG/LTR controller in this paper.

For the filter design, the process noise and measurement noise covariances for (4) and (5) are chosen, respectively, to be

$$Q = BB^T \quad R = \mu = 1.0, \quad (16)$$

where B is the augmented reactor input matrix in (3). Using (9)–(10), the Kalman filter gain was found to be

$$K_F = [1.0000 \; 0.4936 \; 0.1251 \; 64.8000 \; 3.8310 \; 0.0047]^T.$$

The open-loop transfer function of the TFL is, then, given by

$$G_{KF} = C(sI - A)^{-1} K_F$$

and the closed-loop transfer function is given by

$$[I + G_{KF}(s)]^{-1} G_{KF}(s) = C(sI - A + K_F C)^{-1} K_F. \quad (17)$$

If the pair (A, C) is detectable and (A, Γ) is stabilizable, then the stability of the system given by (17) is guaranteed [14].

For the controller design, the optimal control performance measure given by (11) is represented by

$$J = \int_0^{\infty} \left[q\left(0.5\,\delta T_f^2 + 5.0\,\delta T_l^2\right) + z_r^2 \right] dt. \quad (18)$$

Equation (17) sets the target feedback loop to which we want our system to recover, and (18) simply indicates that we want tighter control on the reactor temperatures while appropriately allowing the reactor power to vary to achieve this objective. Yet, we still have the variable q, referred to as the recovery gain, to manipulate in order to recover our filter design. Fig. 5 shows Nyquist plots of the feedback loop gain for different values of q. As can be seen from the figure, the larger q, the better the recovery. However, we do not want the recovery gain q to be larger than necessary because if we do, this would cause unnecessary noise amplification at high frequencies which in

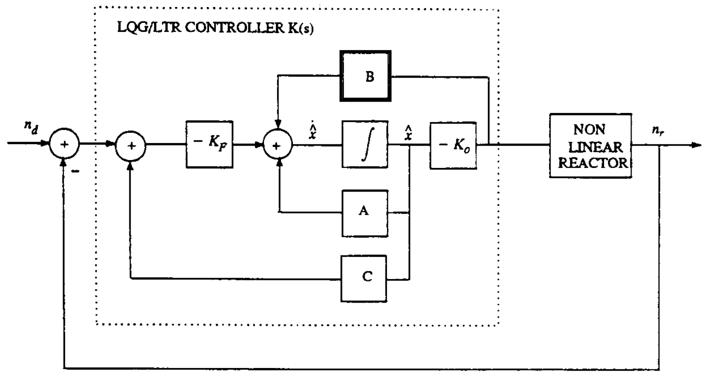


Fig. 4. Dynamics of the LQG/LTR robust controller.

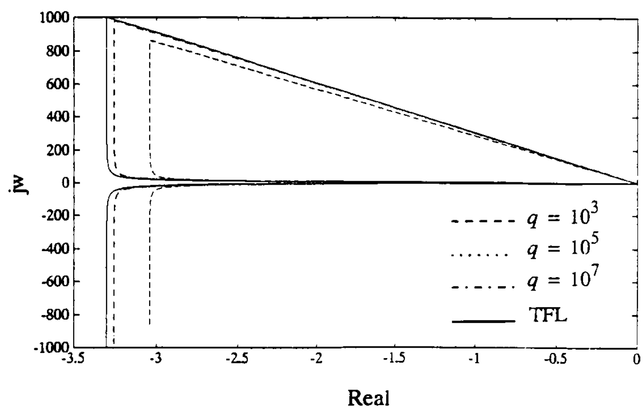


Fig. 5. Nyquist plot for different recovery gain q.

TABLE I
PARAMETERS FOR ROBUST CONTROLLER DESIGN AT THE MIDDLE OF THE FUEL CYCLE OF A TMI-TYPE PWR

β	= 0.006019
Λ	= 0.00002 s
α_f	$= -0.0000324\ \Delta k/k \cdot {}^\circ\mathrm{C}^{-1}$
P_{0a}	= 2500 MW
μ_f	= 26.3 MW · s/°C
Ω	= 6.6 MW/°C
G_r	$= 0.01450\ \Delta k/k$
λ	$= 0.150\ \mathrm{s}^{-1}$
α_c	$= -0.000213\ \Delta k/k \cdot {}^\circ\mathrm{C}^{-1}$
f_f	= 0.92
μ_c	= 71.8MW · s/°C
M	= 102.0 MW/°C

turn will reduce the robustness of the design in the face of high frequency unstructured perturbations [14]. Fig. 5 shows that full recovery was approximately achieved by choosing $q = 10^7$.

For the obtained value of q and using (15), the optimal control feedback matrix was found to be

$$K_o = [54.237\ \ 13.689\ \ 3.036 \times 10^3\ \ -16.459\ \ -51.247\ \ 5.072 \times 10^5].$$

The resulting LQG/LTR robust controller is then given by

$$K(s) = K_o(sI - A + BK_o + K_F C)^{-1} K_F, \qquad (19)$$

where the A, B, C, and D matrices are the matrices of the plant augmented with an integrator (3). Note that K_F and K_o are, respectively, 6×1 and 1×6 vectors due to the extra state introduced by the appended integrator. The LQG/LTR compensator given by (19) is guaranteed to meet the Kalman filter and the optimal control specifications as long as the plant is minimum phase. It is interesting to note that the observer and the controller gains of the SFAC [9] are considerably smaller than those of the LQG/LTR.

1) Sensitivity Analysis: In this paper, the LQG/LTR controller is compared with the observer-based state feedback controller designed for the nominal plant (100% power $\equiv n_{r0} = 1.0$). Ultimately, we want a controller that provides good stability and performance to the full range of expected uncertainties. One indicator of this desirable robustness is to have closed-loop eigenvalues whose values are insensitive or even invariant to uncertainties. Fig. 6 presents the closed-loop dominant eigenvalue sensitivity

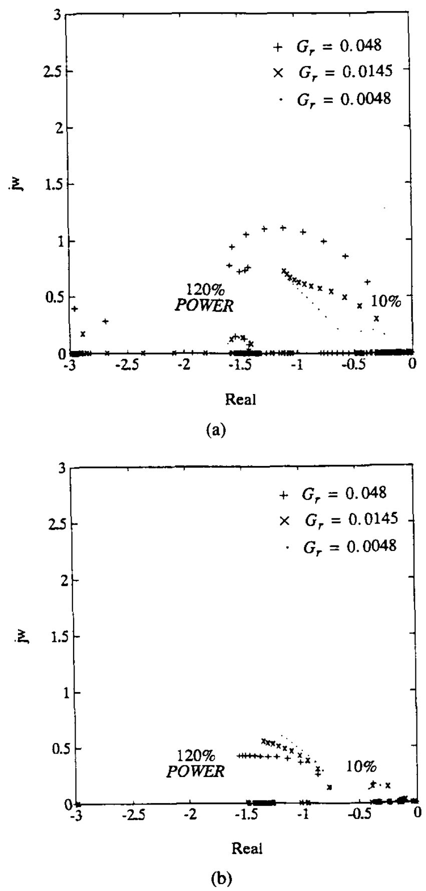


Fig. 6. Dominant eigenvalue sensitivity for power levels from 10 to 120%. $G_c = 0.2$, and $G_r = 0.0048$, 0.0145 (design), and 0.048. (a) SFAC. (b) LQG/LTR.

for the power range of 10 to 120%, control rod worth G_r from 0.0048 to 0.048, and classical control gain $G_c = 0.2$, where the reduced order controllers are applied to a 10^{th} order plant with 6 delayed neutron groups. The results of the linear analysis show that both controllers are comparable but this LQG/LTR design maintains a slightly larger damping ratio and is clearly less sensitive to variation in G_r.

2) Nonlinear Verification: Many controller designs, including the LQG/LTR, are based upon linear models because of the well-developed theory of linear systems. The linear controller design also permits rapid initial testing for whether the closed loop system might be acceptable. However, linear simulations cannot provide an accurate evaluation of how the controller will behave when implemented in the real plant, which is nonlinear (a source of uncertainties for linear model). Therefore, a controller design is incomplete unless the controller is tested with a high order nonlinear model.

Two reactor models are used for nonlinear verification. The first is an advanced continuous simulation language (ACSL) 10th order nonlinear model representing the reactor with all 6 delayed neutron groups [9]. The second is a commercial grade modular modeling system (MMS) which has a 23rd order nonlinear reactor model [9], [15]. Once the controller is designed for the nominal operating condition (100% power and average control rod worth $G_r = 0.0145$), three case studies are performed to confirm the robustness of the LQG/LTR controller:

Case A: Full power operation
1. 100% to 80% power level change, MMS model with high rod worth.
2. 100% to 80% power level change, ACSL model with nominal rod worth.

Case B: Low power operation
1. 10% to 20% power level change, MMS model with high rod worth.
2. 10% to 20% power level change, ACSL model with low rod worth.

Case C: Emergency operation
1. 100% to 10% power level change, ACSL model with high rod worth.

A. Full Power Operation:

In this test, the power demand changes in a step from 100% to 80% with classical control gain $G_c = 0.2$ (nominal value). Fig. 7 presents a comparison between the LQG/LTR and the observer-based SFAC for the MMS 23rd order model with high control rod worth $G_r = 0.04$, Fig. 8 presents the same comparison for the ACSL 10th order model and nominal rod worth. The results of this test show that the LQG/LTR and the observer-based SFAC virtually match. They provided a well damped and reasonably fast temperature response without overshoot.

B. Low Power Operation:

In this verification, a step in power from 10% to 20% is performed. Fig. 9 presents the performance of the MMS reactor model with high rod worth ($G_r = 0.04$) and Fig. 10 shows the performance of the ACSL reactor model when the control rod worth is reduced to 0.007. While Fig. 10 shows that both controllers match, Fig. 9 shows that the LQG/LTR provides a slightly faster response for the reactor temperatures.

C. Emergency Operation:

This case represents the most stressed operation. The power level for the ACSL model is changed from 100% to 10% with high control rod worth ($G_r = 0.029$) and high classical control gain ($G_c = 0.2$). As can be seen from Fig. 11, both the LQG/LTR and the observer-based SFAC controllers preserved a very desirable performance even in this stressed operation and the difference between the two controllers is not perceptible.

V. Conclusions

A robust controller using the LQG/LTR for a Three Mile Island (TMI)-type pressurized water reactor (PWR) was designed with an objective of improving reactor tem-

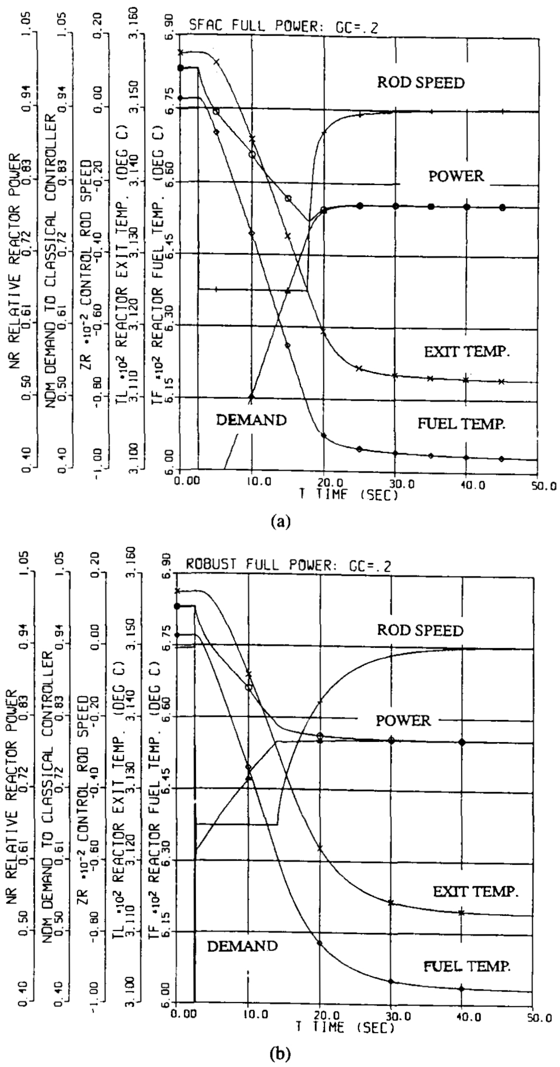


Fig. 7. Case A1: High power operation for 100% to 80% power change (MMS model with high rod worth). (a) SFAC. (b) LQG/LTR.

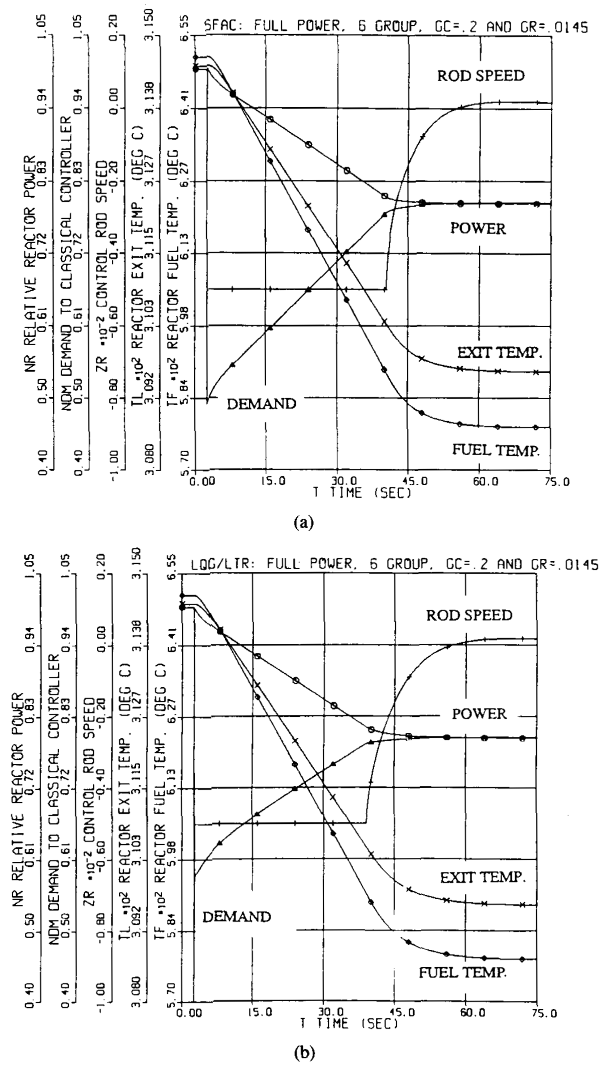


Fig. 8. Case A2: High power operation for 100% to 80% power change (ACSL model with nominal rod worth). (a) SFAC. (b) LQG/LTR.

perature response. The designed LQG/LTR controller was compared to a robust optimal observer-based state feedback assisted controller (SFAC). Both controllers were designed using a time-invariant reduced order linear reactor model with temperature feedback. A dominant closed-loop eigenvalue sensitivity analysis was conducted by applying the reduced order controllers to a 10th order plant with parameter uncertainty due to power level variation of a factor of 12 and control rod worth variation of a factor of 10.

For nonlinear verification, the ACSL 10th order and the MMS 23rd order nonlinear reactor models were used to examine both controllers in full power operation, low power operation, and emergency operation. Simulation results showed that in almost all the cases the differences between the two controllers were not significant. The results of the observer-based SFAC were obtained after extensive testing and fine-tuning of the observer and the controller. In contrast, the LQG/LTR results were obtained methodically with virtually no need for fine-tuning. Furthermore, the results presented for the observer-based SFAC, unlike the LQG/LTR, include an important correction with a nonlinear observer to reduce uncertainties between controller and plant. This indicates that the LQG/LTR results are obtained with less effort and no need for correction as is the case with the observer-based SFAC.

NOMENCLATURE

λ ≡ effective precursor radioactive decay constant (s^{-1}).
Λ ≡ effective prompt neutron lifetime (s).
β ≡ fraction of delayed fission neutrons.

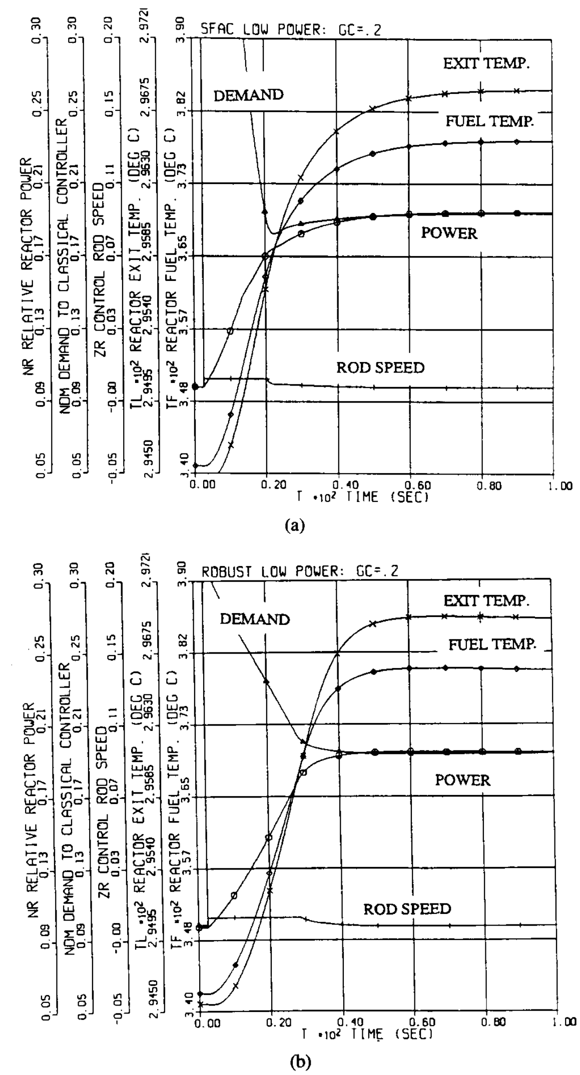


Fig. 9. Case B1: Low power operation for 10% to 20% power level change (MMS model with high rod worth). (a) SFAC. (b) LQG/LTR.

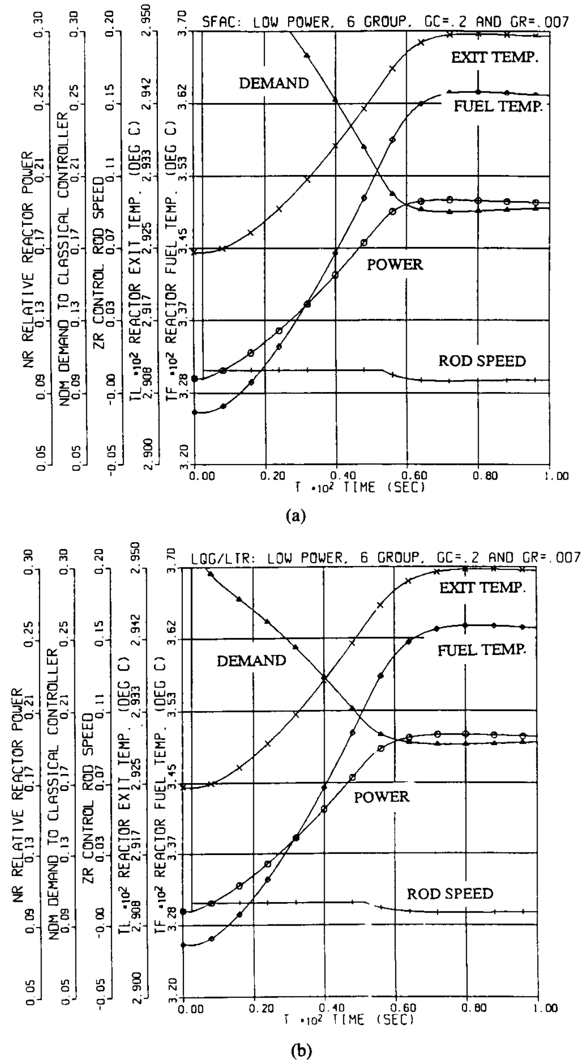
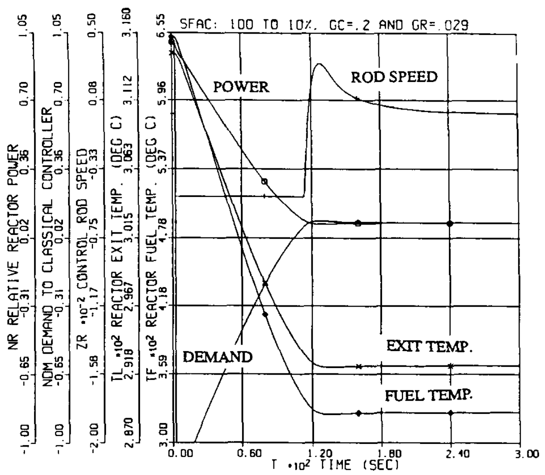
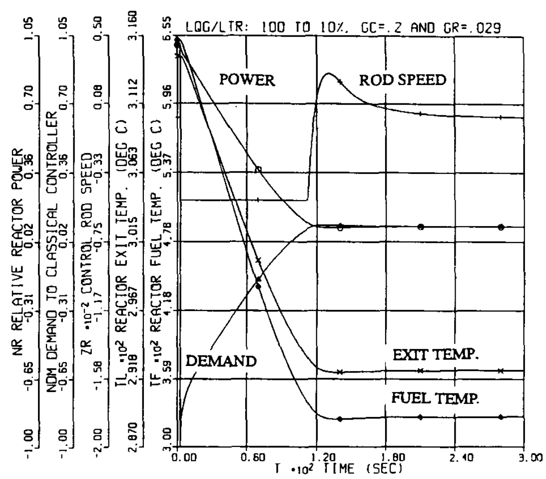


Fig. 10. Case B2: Low power operation for 10% to 20% power level change (ACSL model with low rod worth). (a) SFAC. (b) LQG/LTR.

$\delta\rho$ ≡ reactivity.
n_0 ≡ equilibrium neutron density at rated power.
c_0 ≡ equilibrium precursor density at rated power.
n_r ≡ n/n_0, neutron density relative to density at rated condition.
c_r ≡ c/c_0, precursor density relative to density at rated condition.
P_{0a} ≡ rated power level (MW).
f_f ≡ fraction of reactor power deposited in fuel.
μ_f ≡ heat capacity of the fuel (MW.s/°C).
μ_c ≡ heat capacity of the coolant (MW.s/°C).
Ω ≡ heat transfer coefficient between fuel and coolant (MW/°C).
M ≡ mass flow rate times heat capacity of the coolant (MW/°C).
$\delta\rho_r$ ≡ reactivity due to the control rod.
z_r ≡ control input, control rod speed (fraction of core length per second).
G_r ≡ total reactivity worth of the rod.
α_f ≡ fuel temperature reactivity coefficient.
α_c ≡ coolant temperature reactivity coefficient.
T_f ≡ average reactor fuel temperature (°C).
T_l ≡ temperature of the coolant leaving the reactor (°C).
T_e ≡ temperature of the coolant entering the reactor (°C).
T_{f0} ≡ equilibrium average reactor fuel temperature (°C).
T_{l0} ≡ equilibrium temperature of the coolant leaving the reactor (°C).
T_{e0} ≡ equilibrium temperature of the coolant entering the reactor (°C).



(a)



(b)

Fig. 11. Case C: Emergency operation for 100% to 10% power level change (ACSL model with high rod worth). (a) SFAC. (b) LQG/LTR.

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