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OPTIMAL OPERATION OF LARGE-SCALE POWER SYSTEMS

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Abstract - This paper presents a method for an optimal operation of large-scale power systems similar to the one utilized by the Houston Lighting and Power Company. The main objective is to minimize the system fuel costs, and maintain an acceptable system performance in terms of limits on generator real and reactive power outputs, transformer tap settings, and bus voltage levels. Minimizing the fuel costs of such large-scale systems enhances the performance of optimal real power generator allocation and of optimal power flow that results in an economic dispatch.

To handle the large-scale systems of this nature, the idea of decomposing the problem into the real power optimization problem and the reactive power optimization problem is introduced. The control variables are generator real power outputs for the real power optimization problem and generator reactive power outputs, compensating capacitors and transformer tap settings for the reactive power optimization.

The gradient projection method (GPM) is utilized to solve the optimization problems. It is an iterative numerical procedure for finding an extremum of a function of several variables that are required to satisfy various constraining relations without using penalty functions or Lagrange multipliers among other advantages. Mathematical models are developed to represent the sensitivity relationships between dependent and control variables for both real- and reactive-power optimization procedures; and thus eliminate the use of B-coefficients. Data provided by the Houston Lighting and Power Company are used to demonstrate the effectiveness of the proposed procedures.

1. INTRODUCTION

The problem of economic operation in power systems had its start from the time that two or more units were committed to take on load on a power system whose total capacities exceeded the generation required [1]. Economic dispatch then is used in real time control to allocate the total generation among the units available to take on load in interchange costing and billing.

Due to the need of large-scale power systems, the idea of optimal power operation was first introduced by Dommel and Tinney [2] and many articles have appeared in the literature on this subject [1,3,4].

The optimal power operation is equivalent to the optimal real and reactive power flow problem. An optimal power flow, is a power flow in which the fuel costs are minimized, with the ordinary load flow constraints around all buses; and the system losses are also minimized while maintaining an acceptable system performance in terms of limits on generator real and reactive power outputs, transformer tap settings, and bus voltage levels. Minimizing the fuel costs of such a large system will enhance the performance of optimal real power generator allocation and of optimal power flow that results in an economic dispatch.

To handle large-scale systems like the one utilized by the Houston Lighting and Power (*HL&P*) Company, the idea of decomposing the problem into the real power optimization problem (P-problem) and the reactive power optimization problem (Q-problem) is introduced [5]. The P-problem is defined as minimizing the production cost while maintaining the system voltage constraint, and the Q-problem is defined as minimizing the production cost while maintaining the system real power generation constraint.

Dopazo et al. [6] presented a method of minimizing the production cost by coordinating real and reactive power allocations in the system. The procedure uses the Lagrangian multipliers to determine the real power dispatch. Another approach to solve this nonlinear problem is to augment the constraints into objective function by using the Lagrange multipliers and/or penalty functions, and to minimize the augmented objective function by using one of the optimization schemes, such as the steepest descent algorithm, or the sequential unconstrained minimization technique (SUMT) [19]. Other approaches are the use of linear programming approximation to the objective function in order to apply the quadratic programming technique [19]. Due to the size of the problem as well as the large number of functional inequality constraints, improvement on computational efficiency has been the thrust of most works.

The method presented in this paper is based upon the following procedures: the P-optimization procedure, which is equivalent to the conventional economic load dispatch, optimally allocates the real power generation among generators; the Q-optimization procedure, optimally determines the reactive power output of generators and other var sources as well as transformer tap setting; and the load-flow procedure, which is used to make fine adjustments on the results of P- and Q-optimization procedures.

II. GENERAL FORMULATION

The optimal power flow problem is defined by choosing a cost function $f(u)$, and then minimizing $f(u)$ with respect to control variables, u, subject to equality constraints of the form

$$g(x,u) = 0 \tag{1}$$

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and the inequality constraints on the control variables, u, of the form

$$\underline{u} \le u \le \bar{u} \qquad (2)$$

and the inequality constraints on the state (dependent) variables of the form

$$\underline{x} \le x \le \bar{x} \qquad (3)$$

Equation (2) represents the constraints on real and reactive power generations, on transformer tap settings, and on compensating capacitors; while Eq. (3) represents the limitations on bus voltage magnitudes and on reactive power line flows. The equality constraint of Eq. (1) represents the power flow balance between generation of the state and control variables.

III. OPTIMAL REAL AND REACTIVE POWER OPERATION

The optimal real and reactive power operation is defined as Minimize

$$C = f(P_{sg}, Q_{sgc}, n) \qquad (4)$$

subject to

$$g(P_{sg}, Q_{sgc}, n) = 0 \qquad (5)$$

and

$$\underline{P_{sg}} \le P_{sg} \le \overline{P_{sg}} \qquad (6a)$$

$$\underline{Q_{sgc}} \le Q_{sgc} \le \overline{Q_{sgc}} \qquad (6b)$$

$$\underline{n} \le n \le \bar{n} \qquad (6c)$$

$$\underline{V} \le V(P_{sg}, Q_{sgc}, n) \le \overline{V} \qquad (6d)$$

$$h(V, \delta) \le \bar{h} \qquad (6e)$$

where

P_{sg} = Vector of real power generators, g, including the swing bus, s
Q_{sgc} = Vector of reactive power of generators, g, including the swing bus generator, s, and other reactive power compensating devices, c, such as compensating capacitors and reactors
n = Vector of off-nominal tap settings of tap-changing transformers (LTC)
V = Vector of bus voltage magnitudes
$g(.)$ = Real and reactive power supply and demand balance equation
$h(.)$ = Vector of transmission line flows
δ = Vector of bus voltage angles
$\overline{(.)}, \underline{(.)}$ = Upper and lower limits, respectively

The fuction $f(P_{sg}, Q_{sgc}, n)$ is the total summation of generator fuel costs. The vector V is a dependent variable depends on the control variables P_{sg}, Q_{sgc}, and n. It should be noted here that the consideration of line flow constraints is optional to avoid unnecessary increase in computational time. The optimization problem of this nature can be decompose into the following two procedures:

1. P – optimization Procedure

Minimize

$$C_p = f_p(P_{sg}) \qquad (7)$$

subject to the equality constraint

$$g(P_{sg}) = 0 \qquad (8)$$

and to the inequality constraints

$$\underline{P_{sg}} \le P_{sg} \le \overline{P_{sg}} \qquad (9a)$$

$$h(\delta) \le \bar{h} \qquad (9b)$$

where $f_p(P_{sg})$ is the total summation of genrator fuel costs expressed as a function of P_{sg}.

2. Q – optimization Procedure

Minimize

$$C_Q = f_Q(Q_{sgc,n}) \qquad (10)$$

subject to the following inequality constraints:

$$\underline{Q_{sgc}} \le Q_{sgc} \le \overline{Q_{sgc}} \qquad (11a)$$

$$\underline{n} \le n \le \bar{n} \qquad (11b)$$

$$\underline{V} \le V(Q_{sgc}, n) \le \overline{V} \qquad (11c)$$

$$h(V) \le \overline{V} \qquad (11d)$$

where $f_Q(Q_{sgc}, n)$ is the total summation of generator fuel costs expressed as a function of Q_{sgc} and n.

It is important to note that the cost functions $f_p(P_{sg})$ and $f_Q(Q_{sgc}, n)$ are derived from the same cost function. Therefore, both optimization procedures are using the total fuel cost as the objective function. It is obvious that the adoption of $f_Q(Q_{sgc}, n)$ as the objective function, in the Q-optimization procedure, would be more realistic than the use of transmission losses as in other conventional approaches [8,10,11]. Minimization of power production cost is more economical than minimization of system losses where the fuel costs required to produce the same quantity of power are different among generator units [5].

The gradient projection method is used to solve these optimization procedures. This method assumes the approximated linearized constraints so that its optimum value is not exact. Therefore, it is necessary to use a Load-Flow calculation procedure in order to make fine adjustments on the optimum values of both P- and Q-optimization procedures. This iteration is repeated until optimum values are obtained as shown in Figure 1.

Cost Function

The cost function is given by the total summation of generator fuel costs which is normally expressed as the quadratic function [1] of generating power P_k for all $k \in G$:

$$C(P_{sg}) = \sum_{k \in G} (a_k + b_k P_k + c_k P_k^2) \qquad (12)$$

where G is a set of indices of generator buses including the swing bus. The cost function of Eq. (12) is approximated in the 2_{nd} order Taylor series expansion as

$$C(P_{sg} + \Delta P_{sg}) = \sum_{k \in G} [(a_k + b_k P_k + c_k P_k^2)$$
$$+ (b_k + 2c_k P_k)\Delta P_k + c_k \Delta P_k^2] \qquad (13)$$

Subtracting Eq. (12) from Eq. (13), one can get an incremental cost as

$$\Delta C(\Delta P_{sg}) = \sum_{k \in G} [(b_k + 2c_k P_k)\Delta P_k + c_k \Delta P_k^2] \qquad (14)$$

or, in matrix form

$$\Delta C(\Delta P_{sg}) = \beta_P \Delta P_{sg} + \Delta P_{sg}^T \gamma_p \Delta P_{sg} \qquad (15)$$

where

$$\beta_P \triangleq [b_1 + 2c_1P_1, b_2 + 2c_2P_2, \cdots, b_m + 2c_mP_m]$$

$$\gamma_p \triangleq \begin{bmatrix} c_1 & 0 & 0 & \cdots & 0 \\ 0 & c_2 & 0 & \cdots & 0 \\ 0 & 0 & c_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c_m \end{bmatrix}$$

and

m = Total number of generator buses

ΔP_{sg} = Vector of changes in real power generations P_{sg}

It is important to note that Eq. (15) can be used directly in the P- optimization procedure since ΔP_{sg} itself is the decision variable. In the Q-optimization procedure, however, ΔP_{sg} should be expressed as a function of the Q-optimization control variables ΔQ_{sgc} and Δn.

IV. P – OPTIMIZATION PROCEDURE

The sensitivity relationships between the changes in real and reactive powers and the changes of bus voltages and angles are defined as

$$\begin{bmatrix} \Delta P \\ \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \delta \\ \\ \Delta V \end{bmatrix}$$

where the Jacobian J is partitioned as

$$\begin{bmatrix} \Delta P_s \\ \cdots \\ \Delta P_g \\ \Delta P_l \\ \cdots \\ \Delta Q_s \\ \cdots \\ \Delta Q_g \\ \Delta Q_c \\ \cdots \\ \Delta Q_{l'} \end{bmatrix} = \begin{bmatrix} \cdots & \vdots & J_{11} & \vdots & \cdots \\ \cdot\cdot & & \cdots & & \cdot\cdot \\ \cdots & \vdots & J_{12} & \vdots & \cdots \\ \cdot\cdot & & \cdots & & \cdot\cdot \\ \cdots & \vdots & \cdots & \vdots & \cdots \\ \cdot\cdot & & \cdots & & \cdot\cdot \\ \cdots & \vdots & J_{13} & \vdots & \cdots \\ \cdot\cdot & & \cdots & & \cdot\cdot \\ \cdots & \vdots & \cdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} \Delta \delta_s \\ \cdots \\ \\ \Delta \delta_g \\ \Delta \delta_l \\ \cdots \\ \\ \Delta V_s \\ \Delta V_g \\ \Delta V_c \\ \Delta V_{l'} \end{bmatrix} \tag{16}$$

where

s, g, c = indices for swing bus, other generator buses, and reactive power compensating device buses, respectively.

l, l' = indices for all load buses, and the load buses which do not have reactive power compensating devices, respectively.

Exact decomposition can be realized by setting

$$\Delta P_l = \Delta V = \Delta \delta_s = \Delta Q_{l'} = 0 \tag{17}$$

The condition $\Delta Q_{l'}$ destroys the sparsity property of Jacobian matrices. For that reason and considering the fact that the calculated values of $\Delta Q_{l'}$ in the P-optimization procedure are close to zero, the condition $\Delta Q_{l'} = 0$ is relaxed.

Using the conditions $\Delta V = \Delta \delta_s = 0$ of Eq. (17),the real powers in Eq. (16) can be expressed in terms of power angles as

$$\Delta P_s \triangleq J_A \Delta P_g \tag{18}$$

where J_A is defined in Appendix A.

Consequently, the P-optimization procedure can be summarized as follows:

Minimize

$$\Delta C_p = \beta_P \Delta P_{sg} + \Delta P_{sg}^T \gamma_p \Delta P_{sg} \tag{19}$$

subject to

$$\begin{bmatrix} 1 & \vdots & -J_A \end{bmatrix} \Delta P_{sg} = 0 \tag{20}$$

and

$$\underline{\Delta P_{sg}} \leq \Delta P_{sg} \leq \overline{\Delta P_{sg}} \tag{21a}$$

where

$$\underline{\Delta P_{sg}} \triangleq \underline{P_{sg}} - P_{sg} \tag{21b}$$

and

$$\overline{\Delta P_{sg}} \triangleq \overline{P_{sg}} - P_{sg} \tag{21c}$$

The dependent variables ΔQ_{gc} for the P-optimization procedure can be derived as

$$\Delta Q_{gc} \triangleq J_B \Delta P_g \tag{22}$$

where J_B is defined in Appendix A.

The optimization problem defined in Eq. (19) is used for incremental variables. Therefore, after applying the P-optimization procedure, both the real and reactive powers are updated from P_g and Q_{gc} to $P_g + \Delta P_g$ and $Q_{gc} + \Delta Q_{gc}$, respectively.

V. THE Q – OPTIMIZATION PROCEDURE

In this procedure, the Jacobian matrix J is augmented to include the sensitivity coefficients representing the changes in real and reactive power with respect to the changes in off-nominal tap settings of the LTC as

$$\begin{bmatrix} \Delta P \\ \\ \Delta Q \end{bmatrix} = \tilde{J} \begin{bmatrix} \Delta \delta \\ \Delta V \\ \Delta n \end{bmatrix} \tag{23}$$

where

$$\tilde{J} \triangleq \begin{bmatrix} J & \vdots & J_n \end{bmatrix}$$

The sensitivity matrix with respect to off-nominal tap settings, J_n, can be obtained by differentiating the nodal power equations with respect to the off-nominal tap setting values, n. The matrix $\tilde{J}$ is partitioned so that the formulation of the Q-optimization procedure can be obtained as

$$\begin{bmatrix} \Delta P_s \\ \Delta P_g \\ \\ \cdots \\ \Delta P_l \\ \cdots \\ \Delta Q_{sgc} \\ \\ \\ \Delta Q_{l'} \end{bmatrix} = \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \vdots & J_{21} & \vdots & J_{22} \\ \cdot\cdot & & \cdots & & \cdots \\ \cdots & \vdots & \cdots & \vdots & \cdots \\ \cdot\cdot & & \cdots & & \cdots \\ & \vdots & & \vdots & \\ \cdots & \vdots & J_{23} & \vdots & J_{24} \\ & \vdots & & \vdots & \end{bmatrix} \begin{bmatrix} \Delta \delta_s \\ \\ \Delta \delta_g \\ \\ \Delta \delta_l \\ \cdots \\ \\ \Delta V \\ \cdots \\ \\ \Delta n \end{bmatrix} \tag{24}$$

Although more exact decomposition can be realized by setting

$$\Delta Q_{l'} = \Delta \delta_s = \Delta \delta_g = \Delta \delta_l (or\, \Delta P_l) = 0 \tag{25}$$

the condition $\Delta \delta_l = 0$ or $\Delta P_l = 0$ is relaxed in order to preserve the sparsity property of the Jacobian matrices. Therefore, we can solve Eq. (24) for the dependent variables of the Q-optimization procedure, ΔP_s, ΔP_g and ΔV, in terms of the control variables ΔQ_{sgc} and Δn as shown in the following equations:

$$\Delta V \triangleq J_D \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{26}$$

The substitution of Eq. (32) into Eq. (30) yields to

$$\Delta P_{sg} \triangleq J_C \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{27}$$

where J_C and J_D are defined in Appendix A.

Consequently, the Q-optimization procedure in Eqs. (2.49) and (2.50) can be summarized as

Minimize

$$\Delta C_Q = \beta_Q \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} + \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix}^T \gamma_Q \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{28}$$

subject to

$$\underline{\Delta Q_{sgc}} \leq \Delta Q_{sgc} \leq \overline{\Delta Q_{sgc}} \tag{29a}$$

$$\underline{\Delta n} \leq \Delta n \leq \overline{\Delta n} \tag{29b}$$

$$\underline{\Delta V} \leq J_D \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \leq \overline{\Delta V} \tag{29c}$$

where

$$\beta_Q \triangleq \beta_P J_C \tag{29d}$$

$$\gamma_Q \triangleq J_C^T \gamma_p J_C \tag{29e}$$

The optimization problem, defined by Eqs. (28) and (29), is for the incremental variables. Therefore, we need to update the real and reactive powers and the tap settings from P_g, Q_{gc}, and n to $P_g + \Delta P_g$, $Q_{gc} + \Delta Q_{gc}$, and $n + \Delta n$, respectively. It is important to note that the reactive power of the swing bus, Q_s, is not updated since the Load-Flow procedure, that immediately follows, provides the exact updated value. It is also important to note the benefit of the Q-optimization procedure that computes the real power adjustment ΔP_{sg} using Eq. (27).

VI. LOAD – FLOW PROCEDURE

The P- and Q-optimization procedures are solved using the gradient projection method that assumes the approximated linearized constraints as given in Eqs. (2.59), (2.60), and (2.72). Therefore, it is necessary to use a load-flow procedure in order to make fine adjustments on the optimum values obtained from the P- and Q-optimization procedures.

In this procedure, P-Q values are assigned to the generator buses except for the swing bus in which the bus voltage and angle are assigned. The Newton-Raphson method is used to solve the load-flow, where all the bus equations are included with the exception of the swing bus.

VI. APPLICATIONS

The algorithm was tested using the 6-bus model of Figure 2 [19], the modified IEEE 30-bus system [19], and the large-scale system currently utilized by the Houston Lighting and Power Company.

1. Efficiency test of small – scale systems

The efficiency test was performed in the previous paper [5] with the 6-bus system to show the flexibility and convenience of the new algorithm, and how it compares with other conventional methods. It was concluded that the further reduction in fuel cost was achieved due to the unique use of one cost function for both P- and Q-optimization procedures. Computationally, it took 0.173 sec. of C.P.U time per iteration and the solution converged at the 5th. iteration with the AS/9000N computer system at the University of Houston, which is comparable to IBM 370.

The efficiency test was also performed in the previous paper [5] with the IEEE 30-bus system. The system consists of 41 lines, 6 generators, 4 tap-changing transformers, and shunt capacitor banks located at 9 buses.

Two different studies were performed. In the first study the system is optimized using the P- and Q-optimization algorithms developed. The resulting cost and power loss are presented in Table 1. To compare these results with conventional methods, the system is optimized in the second study using the same P-optimization procedure, but using a Q-optimization with the line loss objective function instead of Eq. (28).

The results obtained show that the method presented in this paper using the same generation cost objective function for both P- and Q-optimization procedures gives much better results than the other method. The difference in generation cost between these two studies (804.853 $\$/hr$ compared to 823.629 $\$/hr$) clearly shows the advantage of this method. Also, it is important to point out that with this method and the use of the gradient projection method, the real and reactive power dispatch problem has considerably faster convergence compared with conventional methods. The time per single iteration for this system was approximately 2.162 seconds, and it converged in 2 iterations.

TABLE 1. Efficiency Test for The 30-bus system.

Variable	Limits		Initial	Final States	
	Lower	Upper	State	First Study	Second Study
P_1(MW)	50.	200.	99.211	187.219	166.554
P_2(MW)	20.	80.	80.000	53.781	80.000
P_5(MW)	15.	50.	50.000	16.955	15.000
P_8(MW)	10.	35.	20.000	11.288	10.000
P_{11}(MW)	10.	30.	20.000	11.287	10.000
P_{13}(MW)	12.	40.	20.000	13.355	12.000
Q_1(MVAR)	-20.	200.	5.335	19.342	18.353
Q_2(MVAR)	-20.	100.	27.657	27.423	27.439
Q_5(MVAR)	-15.	80.	21.544	21.205	21.223
Q_8(MVAR)	-15.	60.	22.933	23.086	23.104
Q_{11}(MVAR)	-10.	50.	38.583	38.438	38.453
Q_{13}(MVAR)	-15.	60.	40.345	40.423	40.355
V_1(p.u.)	0.95	1.10	$1.05\angle 0.00°$	$1.10\angle 0.00°$	$1.10\angle 0.00°$
V_2(p.u.)	0.95	1.10	$1.04\angle -1.77°$	$1.08\angle -3.38°$	$1.08\angle -2.87°$
V_5(p.u.)	0.95	1.10	$1.01\angle -6.50°$	$1.03\angle -10.02°$	$1.03\angle -9.71°$
V_8(p.u.)	0.95	1.10	$1.01\angle -5.64°$	$1.04\angle -8.11°$	$1.04\angle -7.87°$
V_{11}(p.u.)	0.95	1.10	$1.05\angle -4.66°$	$1.08\angle -8.72°$	$1.09\angle -8.72°$
V_{13}(p.u.)	0.95	1.10	$1.05\angle -7.24°$	$1.08\angle -10.43°$	$1.08\angle -10.41°$
n_{11}	0.900	1.100	1.078	1.072	1.072
n_{12}	0.900	1.100	1.069	1.070	1.069
n_{15}	0.900	1.100	1.032	1.032	1.032
n_{36}	0.900	1.100	1.068	1.068	1.068
$Q_{c_{10}}$(MVAR)	0.00	5.0	0.00	0.692	0.711
$Q_{c_{12}}$(MVAR)	0.00	5.0	0.00	0.046	0.045
$Q_{c_{15}}$(MVAR)	0.00	5.0	0.00	0.285	0.304
$Q_{c_{17}}$(MVAR)	0.00	5.0	0.00	0.287	0.306
$Q_{c_{20}}$(MVAR)	0.00	5.0	0.00	0.206	0.129
$Q_{c_{21}}$(MVAR)	0.00	5.0	0.00	0.000	0.000
$Q_{c_{23}}$(MVAR)	0.00	5.0	0.00	0.330	0.350
$Q_{c_{24}}$(MVAR)	0.00	5.0	0.00	0.938	0.957
$Q_{c_{29}}$(MVAR)	0.00	5.0	0.00	0.269	0.290

Generation Cost ($/hr)	901.918	804.853	823.629
Real Power Loss (MW)	5.812	10.486	10.154

The security constraints are also checked for voltage magnitudes and angles. The voltage magnitudes are from the minimum of 0.926 p.u. to maximum of 1.10 p.u., and the angles are from the minimum of −14.61° to the maximum of 0.0°. No load bus was at the lower limit of 0.9 p.u.. Table 1 also shows that power factor corrections are made all but at one location (bus 21). Finally, it should be pointed out that the first study shows a slightly higher transmission loss but much lower generation cost compared to the second study. This fact illustrates that the choice of fuel cost as the cost function for the Q-optimization further reduces the operation cost compared to the conventional loss minimization approach.

2. Efficiency test of large – scale systems

The optimal operation of large-scale power systems is tested using the system utilized by the *HL&P* Company. An equivalent system consists of the following items: 147 power buses, 369 transmission lines, 53 power generators, 90 compensating capacitors, and 35 transformers.

The *HL&P* Company controls only 18 of the 53 power generators. Therefore, it was necessary to find the appropriate generation cost coefficients for those 18 generators. The performance data of generating units that are represented by a polynomial of the form

$$Y = A + B\,X + D\,X^3$$

describes the relation between the net electric power output X and the thermal fuel flow input Y. This polynomial is termed the units input–output curve. The coefficients A, B, and D are derived such that the Y variable has units of $MBTU/hr$, and the X variable has units of Mwatts (MW). The equivalent generators that are used by the *HL&P* Company are those obtained by combining these generating units.

Since our cost function is in the form

$$f(x) = A + B\,X + C\,X^2$$

therefore, it is necessary to find the coeffiecints A, B, and C using the total thermal fuel flow input Y for each generator. A Least Squares Approximation is performed on the combined generating units in order to find the the equivalent cost coefficients A, B, and C for each generator. The bus numbers of those generators and their corresponding cost coefficients are shown in Table 2. The cost coefficients a_k, b_k, and c_k in Eqs. (19) and (28) are set to those shown in Table 2.

TABLE 2 Cost Coefficients

Generator Bus	Cost Coefficients		
	A	B	C
546	1420.69	7.63	0.102505E-02
53	435.57	7.42	0.427086E-02
55	559.74	7.60	0.243384E-02
111	1732.34	6.97	0.118060E-02
112	698.33	7.35	0.181606E-02
176	142.19	8.69	0.654662E-02
274	230.15	9.95	0.293905E-02
275	219.40	9.00	0.794294E-02
276	649.98	12.10	-0.450847E-06
278	421.39	6.87	0.494409E-02
487	2314.17	6.44	0.113435E-02
488	3078.21	5.10	0.158735E-02
547	1511.49	5.36	0.290358E-02
725	377.70	6.50	0.139257E-01
726	447.73	7.82	0.236094E-02
735	76.56	9.66	0.191366E-02
736	1038.41	7.09	0.148330E-02
737	649.98	12.10	-0.450847E-06

Four different examples were performed using the large-scale system utilized by the *HL&P* Company. In each example, the security constraints of Eqs. (20)–(21a), of the P-optimization procedure, and Eqs. (29a)–(29c), of the Q-optimization procedure, are checked for optimal real powers, reactive powers, reactive compensations, transformer tap settings, and for voltage magnitudes and angles.

In the first example, the nominal real and reactive power load are used. It is important to note that both the generation cost and the transmission loss are decreased from 112,663.62 $\$/hr$ and 969.53 MW, obtained from the initial load flow, to 108,703.31 $\$/hr$ and 726.42 MW after optimization, respectively. The computer results for all system variables for this example are summarized in Table 3.

TABLE 3 Summary of Results–Nominal Load

Variable	Limits		Initial State	Final State
	Lower	Upper		
P_{546}(MW)	690.0	1355.0	690.0	882.34
P_{53}(MW)	120.0	366.0	333.9	364.5
P_{55}(MW)	180.0	490.0	441.4	489.6
P_{111}(MW)	800.0	1540.0	1439.3	1487.2
P_{112}(MW)	400.0	770.0	719.7	768.4
P_{176}(MW)	60.0	183.0	160.2	169.0
P_{274}(MW)	80.0	150.0	134.3	142.9
P_{275}(MW)	80.0	236.0	214.9	122.5
P_{276}(MW)	60.0	360.0	353.1	60.4
P_{278}(MW)	125.0	400.0	381.9	392.9
P_{487}(MW)	845.0	1836.0	1761.8	1800.9
P_{488}(MW)	900.0	1755.0	1698.4	1688.8
P_{547}(MW)	500.0	900.0	846.3	878.9
P_{725}(MW)	90.0	236.0	209.2	152.8
P_{726}(MW)	180.0	390.0	359.8	389.2
P_{735}(MW)	30.0	75.0	68.1	75.0
P_{736}(MW)	185.0	845.0	674.6	845.0
P_{737}(MW)	60.0	360.0	333.9	60.2
Q_{546}(MVAR)	0.0	424.0	220.1	87.2
Q_{53}(MVAR)	0.0	152.0	8.0	35.4
Q_{55}(MVAR)	0.0	158.0	158.0	158.0
Q_{111}(MVAR)	0.0	314.0	314.0	314.0
Q_{112}(MVAR)	0.0	201.0	0.0	0.0
Q_{176}(MVAR)	0.0	66.0	66.0	66.0
Q_{274}(MVAR)	0.0	63.0	32.4	8.5
Q_{275}(MVAR)	0.0	57.0	18.1	6.8
Q_{276}(MVAR)	0.0	76.8	76.8	76.8
Q_{278}(MVAR)	0.0	156.0	156.0	176.0
Q_{487}(MVAR)	0.0	734.0	577.5	586.7
Q_{488}(MVAR)	0.0	625.0	473.7	392.0
Q_{547}(MVAR)	0.0	276.0	145.3	151.1
Q_{725}(MVAR)	0.0	35.0	0.0	0.0
Q_{726}(MVAR)	0.0	31.0	31.0	31.0
Q_{735}(MVAR)	0.0	29.0	18.3	20.2
Q_{736}(MVAR)	0.0	296.0	296.0	296.0
Q_{737}(MVAR)	0.0	90.0	0.0	0.0
V_{546}(p.u.)	1.000	1.050	$1.050\angle 0.00^\circ$	$1.032\angle 0.00^\circ$
V_{53}(p.u.)	1.000	1.029	$1.026\angle 6.00^\circ$	$1.018\angle -1.69^\circ$
V_{55}(p.u.)	1.000	1.036	$1.036\angle 4.79^\circ$	$1.024\angle -2.70^\circ$
V_{111}(p.u.)	1.000	1.050	$1.050\angle 8.52^\circ$	$1.037\angle 1.32^\circ$
V_{112}(p.u.)	1.000	1.035	$1.035\angle 8.79^\circ$	$1.021\angle 2.41^\circ$
V_{176}(p.u.)	1.000	1.014	$1.012\angle 3.34^\circ$	$1.006\angle -5.06^\circ$
V_{274}(p.u.)	1.000	1.014	$1.014\angle 4.99^\circ$	$1.005\angle -4.04^\circ$
V_{275}(p.u.)	1.000	1.014	$1.014\angle 3.43^\circ$	$1.005\angle -7.09^\circ$
V_{276}(p.u.)	1.000	1.036	$1.032\angle 2.50^\circ$	$1.027\angle -6.42^\circ$
V_{278}(p.u.)	1.000	1.036	$1.036\angle 5.18^\circ$	$1.025\angle -2.91^\circ$
V_{487}(p.u.)	1.000	1.050	$1.050\angle 9.64^\circ$	$1.045\angle 2.04^\circ$
V_{488}(p.u.)	1.000	1.050	$1.028\angle 10.25^\circ$	$1.004\angle 2.80^\circ$
V_{547}(p.u.)	1.000	1.050	$1.05\angle 9.31^\circ$	$1.035\angle 1.76^\circ$
V_{725}(p.u.)	1.000	1.014	$1.001\angle 4.29^\circ$	$1.001\angle -1.90^\circ$
V_{726}(p.u.)	1.000	1.036	$1.036\angle 3.11^\circ$	$1.025\angle -1.88^\circ$
V_{735}(p.u.)	1.000	1.014	$1.014\angle 0.97^\circ$	$1.010\angle -6.38^\circ$
V_{736}(p.u.)	1.000	1.036	$1.036\angle 2.68^\circ$	$1.030\angle -4.71^\circ$
V_{737}(p.u.)	1.000	1.014	$1.014\angle 6.22^\circ$	$1.002\angle -2.03^\circ$
Q_{c_1}(MVAR)	0.00	8.75	0.00	0.32
$Q_{c_{44}}$(MVAR)	0.00	14.40	0.00	0.17
$Q_{c_{50}}$(MVAR)	0.00	97.20	0.00	0.69
$Q_{c_{64}}$(MVAR)	0.00	10.30	0.00	0.59
$Q_{c_{83}}$(MVAR)	0.00	14.40	0.00	0.31
$Q_{c_{84}}$(MVAR)	0.00	33.29	0.00	0.34
$Q_{c_{138}}$(MVAR)	0.00	14.84	0.00	1.49
$Q_{c_{139}}$(MVAR)	0.00	7.50	0.00	0.17
$Q_{c_{141}}$(MVAR)	0.00	64.80	0.00	0.51
$Q_{c_{183}}$(MVAR)	0.00	14.40	0.00	0.49
$Q_{c_{199}}$(MVAR)	0.00	14.40	0.00	0.61
$Q_{c_{209}}$(MVAR)	0.00	6.23	0.00	0.61
$Q_{c_{245}}$(MVAR)	0.00	14.40	0.00	0.43
$Q_{c_{246}}$(MVAR)	0.00	10.43	0.00	0.43
$Q_{c_{249}}$(MVAR)	0.00	64.80	0.00	0.43
$Q_{c_{259}}$(MVAR)	0.00	46.33	0.00	0.44
$Q_{c_{366}}$(MVAR)	0.00	64.80	0.00	0.52
$Q_{c_{371}}$(MVAR)	0.00	36.30	0.00	0.88
$Q_{c_{391}}$(MVAR)	0.00	64.80	0.00	0.33
$Q_{c_{394}}$(MVAR)	0.00	4.41	0.00	1.03
$Q_{c_{307}}$(MVAR)	0.00	3.97	0.00	0.29
$Q_{c_{323}}$(MVAR)	0.00	20.40	0.00	0.16
$Q_{c_{356}}$(MVAR)	0.00	42.94	0.00	0.78
$Q_{c_{372}}$(MVAR)	0.00	1.56	0.00	0.76
$Q_{c_{378}}$(MVAR)	0.00	64.80	0.00	0.66
$Q_{c_{389}}$(MVAR)	0.00	1.23	0.00	1.23
$Q_{c_{467}}$(MVAR)	0.00	20.40	0.00	0.21
$Q_{c_{515}}$(MVAR)	0.00	73.62	0.00	0.61
$Q_{c_{614}}$(MVAR)	0.00	64.80	0.00	0.17
$Q_{c_{620}}$(MVAR)	0.00	8.78	0.00	1.00
$Q_{c_{675}}$(MVAR)	0.00	85.20	0.00	0.20
$Q_{c_{699}}$(MVAR)	0.00	26.40	0.00	0.58
$Q_{c_{700}}$(MVAR)	0.00	5.58	0.00	0.51
$Q_{c_{713}}$(MVAR)	0.00	44.40	0.00	0.53
$Q_{c_{714}}$(MVAR)	0.00	20.69	0.00	0.52
$Q_{c_{737}}$(MVAR)	0.00	7.50	0.00	0.56
$Q_{c_{738}}$(MVAR)	0.00	72.91	0.00	0.58
$Q_{c_{739}}$(MVAR)	0.00	14.40	0.00	0.17
$Q_{c_{731}}$(MVAR)	0.00	28.80	0.00	1.23
$Q_{c_{732}}$(MVAR)	0.00	16.62	0.00	1.09
$Q_{c_{741}}$(MVAR)	0.00	15.00	0.00	0.43
$Q_{c_{742}}$(MVAR)	0.00	75.00	0.00	0.36

n_1	0.900	1.100	1.097	1.086
n_9	0.900	1.100	0.970	0.962
n_{34}	1.000	1.100	1.087	1.100
n_{35}	1.000	1.100	1.087	1.100
n_{36}	1.000	1.100	1.087	1.100
n_{67}	0.900	1.100	0.979	0.974
n_{74}	0.900	1.100	0.974	0.952
n_{98}	0.900	1.100	0.980	0.977
n_{109}	0.900	1.100	0.980	0.979
n_{110}	0.900	1.100	0.980	0.979
n_{111}	0.900	1.100	0.985	0.993
n_{112}	0.900	1.100	0.985	0.993
n_{121}	0.900	1.100	0.996	0.996
n_{127}	0.900	1.100	0.995	0.997
n_{130}	0.900	1.100	0.993	1.011
n_{135}	0.900	1.100	0.987	0.972
n_{136}	0.900	1.100	0.977	1.100
n_{163}	0.900	1.100	1.002	1.003
n_{168}	0.900	1.100	0.979	1.001
n_{179}	0.900	1.100	0.972	0.975
n_{180}	0.900	1.100	0.981	0.976
n_{186}	0.900	1.100	0.988	1.079
n_{194}	0.900	1.100	1.014	1.012
n_{203}	0.900	1.100	0.956	0.968
n_{204}	0.900	1.100	0.989	0.990
n_{205}	0.900	1.100	0.989	0.990
n_{210}	0.925	1.025	0.994	1.007
n_{215}	0.900	1.100	0.998	1.022
n_{216}	0.900	1.100	0.964	0.964
n_{217}	0.900	1.100	0.964	0.964
n_{219}	0.900	1.100	1.014	1.034
n_{226}	0.900	1.100	0.964	0.963
n_{227}	0.900	1.100	0.993	1.025
n_{228}	0.900	1.100	0.993	1.025
n_{236}	0.925	1.125	1.014	1.061
Generation Cost ($/hr)		-	112663.62	108703.31
Real Power Loss (MW)		-	969.53	726.42

In the second example, half the nominal load are used. The results obtained from this example show a decrease in both the generation cost and the transmission loss from 63,773.90 $\$/hr$ and 590.75 MW to 62,587.17 $\$/hr$ and 565.47 MW, respectively. Also, it should be noted that the reactive power compensation of the capacitor banks are lower than those obtained for the nominal load due to the low power load demand.

In the third example, the nominal load as well as the real and reactive power generation are increased by 20%. The initial cost and transmission loss (138,863.17 $\$/hr$ and 1,493.92 MW) are higher than those obtained in the nominal load due to the higher load. However, the cost and transmission loss are also decreased to 128,770.21 $\$/hr$ and 1,016.50 MW, respectively. Here, we observe that the generator voltages and the reactive power compensation are higher than those obtained for the nominal load example. The higher values of those reactive power compensation are necessary to compensate for the high load demand.

The fourth example consists of adding two more generators, bus numbers 715 and 755, and increasing the load in Dallas, bus number 1032, by 25%. Bus 715 represents a remote lime stone unit, and bus 755 represents the cogeneration at Dow chemical in Freeport. Since the new generators provide real powers of 2,020 MW with no load, the higher transmission loss of 2,738.7 MW was expected. However, after optimization the transmission loss is decreased to 2,072.6 MW. It is important to note that the angles of the generator voltages are increased due to the increase of real powers of those new generators. Also, the reactive power compensation of the capacitor banks are higher than those obtained for the nominal load to compensate for the higher load in bus 1032.

It is important to indicate the advantage of using Eq. (27) that redistribute the real power generation resulting in reduction of both the transmission line loss and the generation cost.

VII. *CONCLUSIONS*

An optimal operation of large-scale power systems is developed for the following objectives: minimize the system fuel costs, minimize the system loses, and maintain an acceptable system performance in terms of limits on generator real and reactive power outputs, transformer tap settings, and bus voltage levels.

Unlike the conventional power optimization, the method presented here utilizes the same fuel costs for both P- and Q-optimization procedures. This approach unifies the two procedures into one reference frame work and avoids the switching of objective functions from one to another as in other methods. Moreover, it is known fact that minimizing the power production cost is more economical than minimizing system loss if the fuel costs required to produce the same quantity of power are different.

It is important to note that the Q-optimization procedure presented here optimally reallocate all generator real powers because of the performance measure being defined as the total fuel cost. Also, the swing bus is optimally determined, along with any other bus voltages, rather than being fixed as in the conventional algorithm.

Another important advantage is in the computational aspect. The Load-flow procedure uses an optimally ordered triangular factorization technique which allows for handling of matrices operations of large-scale systems with faster computation. Also, the Gradient Projection Method, used to solve the optimization procedures, provides faster convergence in the optimal power of large-scale systems than other conventional methods. The GPM generally converged in only few iterations [17].

VIII. *ACKNOWLEDGEMENT*

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APPENDIX A

P – optimization Procedure

Using the conditions $\Delta V = \Delta\delta_s = 0$ of Eq. (17), the real powers in Eq. (16) can be expressed in terms of power angles as

$$\Delta P_s = J_{11}\begin{bmatrix}\Delta\delta_g\\ \\ \Delta\delta_l\end{bmatrix} \tag{A1}$$

$$\begin{bmatrix}\Delta P_g\\ \\ \Delta P_l\end{bmatrix} = J_{12}\begin{bmatrix}\Delta\delta_g\\ \\ \Delta\delta_l\end{bmatrix} \tag{A2}$$

From Eqs. (A1) and (A2), ΔP_s can be obtained as

$$\Delta P_s = \begin{bmatrix} J_{11} & J_{12}^{-1}\end{bmatrix}\begin{bmatrix}\Delta P_g\\ \\ \Delta P_l\end{bmatrix} \triangleq J_A \Delta P_g \tag{A3}$$

or

$$\begin{bmatrix} 1 & \vdots & -J_A\end{bmatrix}\begin{bmatrix}\Delta P_s\\ \\ \Delta P_g\end{bmatrix} = 0 \tag{A4}$$

where J_A is the vector of the first $(m - l)$ elements of the matrix product in Eq. (A3).

The dependent variable ΔQ_{gc} for the P-optimization procedure can be derived as

$$\Delta Q_{gc} = \begin{bmatrix}\Delta Q_g\\ \\ \Delta Q_c\end{bmatrix} = \begin{bmatrix} J_{13} & J_{12}^{-1}\end{bmatrix}\begin{bmatrix}\Delta P_g\\ \\ \Delta P_l\end{bmatrix} \triangleq J_B \Delta P_g \tag{A5}$$

where J_B is the vector of first $(m - l)$ elements of the matrix product in Eq. (A5).

Q – optimization Procedure

Using the conditions $\Delta\delta_s = \Delta\delta_g = \Delta\delta_l = 0$ of Eq. (25), the real and reactive power generations in Eq. (24) can be expressed as

$$\Delta P_{sg} = \begin{bmatrix}\Delta P_s\\ \\ \Delta P_g\end{bmatrix} = J_{21}\Delta V + J_{22}\Delta n \tag{A7}$$

$$\begin{bmatrix}\Delta Q_{sg}\\ \\ \Delta Q_{l'}\end{bmatrix} = J_{23}\Delta V + J_{24}\Delta n \tag{A8}$$

Accordingly, the dependent variable for the Q-optimization procedure, ΔV, can be expressed in terms of the control variables ΔQ_{sgc} and Δn as

$$\Delta V = J_{23}^{-1}\begin{bmatrix}\Delta Q_{sgc}\\ \\ \Delta Q_{l'}\end{bmatrix} - J_{23}^{-1}J_{24}\Delta n \tag{A9}$$

or

$$\Delta V \triangleq J_E \Delta Q_{sgc} - J_{23}^{-1}J_{24}\Delta n \tag{A10}$$

where J_E is the matrix of the first $(m + l - l')$ columns of J_{23}^{-1}. Rearranging Eq. (A10) yields the following:

$$\Delta V = \begin{bmatrix} & \vdots & & \\ J_E & \vdots & -J_{23}^{-1} & J_{24} \\ & \vdots & & \end{bmatrix}\begin{bmatrix}\Delta Q_{sgc}\\ \\ \Delta n\end{bmatrix} \triangleq J_D \begin{bmatrix}\Delta Q_{sgc}\\ \\ \Delta n\end{bmatrix} \tag{A11}$$

The substitution of Eq. (A10) into Eq. (A7) yields to

$$\Delta P_{sg} = J_{21}J_E\Delta Q_{sgc} + [J_{22} - J_{21}J_{23}^{-1}J_{24}]\Delta n$$

$$= \begin{bmatrix} & \vdots & \\ J_{21}J_E & \vdots & J_{22} - J_{21}J_{23}^{-1}J_{24} \\ & \vdots & \end{bmatrix}\begin{bmatrix}\Delta Q_{sgc}\\ \\ \Delta n\end{bmatrix}$$

or

$$\triangleq J_C \begin{bmatrix}\Delta Q_{sgc}\\ \\ \Delta n\end{bmatrix} \tag{A12}$$

Discussion

Norton Savage (US Department of Energy, Washington, DC): One comment I have on this paper relates to the statement on page 5, that "The HL&P Company controls only 18 of the 53 power generators." Are the other 35 generators controlled by other utilities, or by cogenerating entities? Is the power output of these units negligible with respect to the power output of the generators controlled by HL&P, such that the application of the method of the paper can be applied to the 18 generators only, and the side-effects of the other 35 units can be disregarded?

Another comment relates to the reason for introduction of the polynomial $Y = A + BX + DX^3$ and the switch to the second-degree form of input–output function $f(y) = A + BX + CX^2$. I do not see where the cubic is used in the development and use of the method described. Could the authors enlighten me? Another point I do not understand is the statement that a least squares approximation is performed "on the combined generating units" to find the A, B, and C for each generator. It is my understanding that an input–output curve is individual to a generator, under specified conditions, so the reference to "combined generating units" puzzles me. Could this point be reviewed?

A third comment relates to interconnections with other systems. How does the operational method described take account of power inputs from other utilities? Instead of speculating on how this might be done, I think it would be more useful for the authors to provide their views.

In passing, I note that a statement on p. 5 puts the coefficients of the input–output equation in (19) and (28). The B and the C of the cost function do appear in the β_p and γ_p of eq. (19), evidently. Do they appear in the same form in the β_Q and γ_Q of eq. (28)? Where does the A coefficient appear?

These comments are those of the author and do not necessarily represent the official views of the Department of Energy.

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K. Y. Lee, M. A. Mohtadi, J. L. Ortiz, and **Y. M. Park**: We would like to thank the discusser for his interest in the paper and his thorough review. The discusser's comments are well founded and we hope that the following statements will clarify some of the comments.

1) As the discusser pointed out, only 18 of the 53 generators are controlled by the HL & P Company and the other 35 units belong to other neighboring utilities in the Texas Interconnected System (TIS). Consequently, the effect of these 35 units cannot be neglected. Since the HL & P has no control over these units, they are treated as loads (negative loads represented by ΔP_l in eqs. (16) and (24)) and the optimal economic dispatch is sought among the 18 generators controlled by the HL & P.
2) The cost function used in our development is the usual quadratic function as seen in (12). However, the HL & P Company historically has been using the cubic function in its economic dispatch algorithm (see Ref. [13]). Therefore a new set of A, B, and C coefficients for the quadratic cost function had to be estimated from the data given for the cubic function.
3) In reality, there are several units connected to a generator bus and these units need to be grouped into one equivalent generator. When the individual units are not identical, their cost functions (the cubic ones) can be used to generate a net cost curve following the concept of equal incremental costs. From this net cost curve, the cost coefficients for the quadratic form are estimated using the least squares method.
4) As stated above, the 147-bus system contains 53 generator buses, of which 35 units belong to other utilities in the TIS. Since the HL & P has no control over these units, they are treated as loads. Each time the economic real and reactive power dispatch is made, the procedure will be repeated with the same assumption that the generation of these units is known.
5) The cost coefficients B and C do appear in the incremental cost (15) or (19) in the form of β_p and γ_p for the p-optimization problem. Similarly, they also appear in the incremental cost (28) in the form of β_Q and γ_Q because of (29d) and (29e). Since the economic dispatch is based on the concept of incremental cost, the A coefficients do not appear in the cost functions for p- and Q-optimization problems.

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