Low-Order Robust Damping Controller Design for Large-scale PV Power Plants

Rakibuzzaman Shah*, N. Mithulananthan
School of Information Technology and Electrical Engineering
The University of Queensland
Brisbane, Australia
e-mail: md.shah@uq.edu.au; mithulan@uq.edu.au

Kwang. Y. Lee
Department of Electrical and Computer Engineering
Baylor University
Waco, TX, USA
e-mail: Kwang_Y_Lee@baylor.edu

***Abstract*— The paper presents a method of designing a low order robust wide-area damping controller (WADC) for large-scale photovoltaic (PV) power plants. The controller design procedure is focused on designing a second-order wide-area damping controller by convex optimization method. A Hankel singular value (HSV) based hybrid signal selection method is also proposed for optimal selection of the control signal. The performance of the synthesized controller is investigated on a 16-machine 68-bus system, typically used for power system oscillation studies.**

***Index Terms*—Convex optimization, inter-area mode, low-order controller, PV generator.**

I. Introduction

The paradigms that are faced by the power system worldwide is more complicated than ever before due to the proliferation of renewable energy sources, specially wind and solar photovoltaic (PV). The world wide installation of centralized PV generator has increased significantly in last ten years [1]. According to International Energy Agency Photovoltaic Power System Programme report, among total installed capacity, 33% are centralized grid connected PV systems [2]. High PV penetration levels could significantly affects the stability of the power system due to their distinct characteristics that differ from those of the conventional generations. With the increased levels of PV in power system, a significant amount of synchronous generators would be re-dispatched, affecting the electromagnetic torque providing capability which could lead to the power system instability [3]-[6].

The interconnected power system inherently exhibits the electromechanical oscillation of 0.1-2 Hz when subjected to small disturbance. One of the important oscillations in the range of 0.1-1 Hz involves many generators in the system, known as inter-area oscillations [7]. Often the damping associated with these oscillations is poor and depends on: level of generations, demand, tie-line power flows, network strength and topology. With the increased penetrations of intermittent and volatile generators like PV, the power systems become more prone to such oscillations. Therefore, the increased penetrations of PV plants should also contribute to the network support through damping control of inter-area mode, which could add the additional technical value of PV to the system. The research effort in our recent work [8] proposed a damping controller based on minimax-LQG method. The proposed controller is efficient and robust enough; however, the structure of the controller is complex and high in order. Such controllers are rarely practical from the user's point of view. The power utilities are not inclined to use these higher order complex controllers for the sake of system security. Thus, there is a research scope for lower order robust controller design for PV using typical transfer function similar to those that are commonly used in power system control.

Kamwa *et al.* [9] proposed multiple fixed order structure PSS design using a non-linear constraints optimization technique. However, the design is based on a single operating condition, which is the worst scenario in terms of required phase compensation. Research efforts in [10] have proposed a V-K iteration based technique to obtain a fixed order robust damping controller. This involves the solution of BMI problem as a series of LMI problems, where the matrix inequalities are iteratively solved by either fixing the Lyapunov function V or controller gain K, and the solution of the inequalities depends on the selection of initial conditions of variables. Kim and others [11] have proposed an LMI-based control synthesis method for robust low-order PSS design in a single machine infinite bus system, where the non-convex control synthesis problem was formulated as the penalty function based rank-constrained LMI problem. The limitation of the above mentioned approach is that it introduces a number of Lyapunov variables which grows quadratically with the system's size. Therefore, the optimization problem involves a large number of control variables requiring high computational efforts. Furthermore, there is no guarantee of global convergence of the solution in these methods.

With this assertion, this paper has proposed a lower order control synthesis method for a wide-area damping controller (WADC) with PV based on convex optimization. The method used the strictly positive realness (SPR) of a selected transfer function to decouple the controller parameters and the Lyapunov matrices, and represent the stability and performance criteria as a set of LMIs. The polytopic approach has been used to consider the system uncertainty in controller design.

The remainder of the paper is as follows: Section II illustrates the mathematical background behind the convex optimization based controller design. Application example is shown in Section III. In Section IV, conclusions are duly drawn.

978-1-4799-6415-4/14/$31.00

II. Mathmetical Background

A. Convex Optimization for Controller Design

The general control configuration of a fixed order control for polytopic system with output signal delay is illustrated in Fig. 1. Polytopic modeling has certain advantages as it allows considering the span of model uncertainty in the design stage. That is, instead of designing a robust controller and testing it in a variety of situations, polytopic model includes the test cases. In the figure the plant is represented by $P(\Omega)$, which is the polytope in the matrix space.

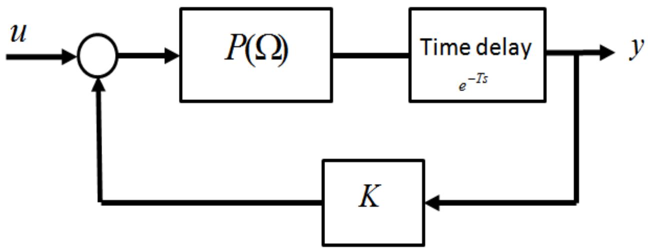

Fig. 1. Polytopic system with a fixed order compensator at feedback loop.

The state-space realization of linear time-invariant (LTI) polytopic system can be expressed as follows:

$$\begin{aligned} \dot{x}_g(t) &= A_g x_g(t) + B_g u(t) \\ y(t) &= C_g x_g(t) \end{aligned} \tag{1}$$

where x_g, u and y are the states, input, and output of the system, respectively. Let's, assume that the matrices A_g and C_g have the following polytopic uncertainty:

$$\begin{aligned} A_g(\lambda) &= \sum_i^q \lambda_i A_i \\ C_g(\lambda) &= \sum_i^q \lambda_i C_i \end{aligned} \tag{2}$$

where $\lambda_i \geq 0$, $\sum_{i=1}^q \lambda_i = 1$, and $(A_i, B_g, C_i, 0)$ represent the vertex of the "polytopic" system. Considering a fixed order feedback controller of the following form

$$K(s) = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \tag{3}$$

where A_k, B_k, C_k, and D_k are the matrices with appropriate dimension. The closed-loop model of the system can be obtained by combining the open loop system in (1) with (3), and can be written as follows:

$$A_j(\lambda) = \begin{bmatrix} A_g - B_g D_k C_g & B_g C_k \\ -B_k C_g & A_k \end{bmatrix} \tag{4}$$

The basic idea of designing a fixed order controller for a single-input-single-output (SISO) system has been illustrated in [12]. According to [12], if $c_i(s), i = 1, \ldots, q$, are characteristics polynomials of closed-loop system for the i^{th} vertex, then the polytopic system is stable if and only if $\frac{c_i(s)}{d(s)}$ is strictly positive real (SPR). In this paper, the similar idea for fixed order controller design as refer to [12] is used. The main idea associated with the controller design is presented in the following Lemma and Theorem [13].

Lemma 1 [13]:

$H(s) = \begin{bmatrix} A & B \\ C & I \end{bmatrix}$ and $H^{-1}(s) = \begin{bmatrix} A - BC & B \\ -C & I \end{bmatrix}$ are equivalent if $H(s)$ and $H^{-1}(s)$ are SPR. According to KYP *Lemma*, $H(s)$ is equivalent to the existence of $P = P^T > 0$, such that

$$A^T P + PA + \frac{1}{2}(PB - C^T)(B^T P - C) < 0 \tag{5}$$

By rearranging the inequality, the following expression can be obtained

$$(A - BC)^T P + P(A - BC) + \frac{1}{2}(PB + C^T)(B^T P + C) < 0 \tag{6}$$

which is equivalent to

$$\begin{bmatrix} (A - BC)^T P + P(A - BC) & PB + C^T \\ B^T P + C & -2I \end{bmatrix} < 0 \tag{7}$$

Hence, $H^{-1}(s)$ is SPR. It is worth noting that both A and $A - BC$ are stable with a common Lyapunov matrix P. Now, the two matrices M and A $(M \in R^{n\times n}, A \in R^{n\times n})$ are called SPR pair, if and only if $H(s) = \begin{bmatrix} M & I \\ M - A & I \end{bmatrix}$ is SPR. According to Lemma 1, it is evident that if M and A are SPR-pair then A and M are also SPR pair, and stable with common Lyapunov matrix. Therefore, the following two LMIs are equivalent:

$$\begin{bmatrix} M^T P + PM & P - M^T + A^T \\ P - M + A & -2I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} A^T P + PA & P - A^T + M^T \\ P - A + M & -2I \end{bmatrix} < 0 \tag{9}$$

Theorem 1 [13]: A fixed order controller as defined in (3) stabilizes the polytopic system in (1) if and only if the given stable matrix M makes a SPR pair with closed state matrix $A_j^i (i = 1, \ldots, q)$ of the i^{th} vertex, which is expressed as:

$$A_j^i = \begin{bmatrix} A_i - B_g D_k C_i & B_g C_k \\ -B_k C_i & A_k \end{bmatrix} \tag{10}$$

Thus, a convex set of stabilizing controller for the underlying system can be obtained by the following set of LMIs:

$$\begin{bmatrix} M^T P_i + P_i M & P_i - M^T + (A_j^i)^T \\ P_i - M + A_j^i & -2I \end{bmatrix} < 0 \qquad (11)$$

In equation (11), the variables are the controller parameters and symmetric matrices P_i. The solution and quality of the inner convex approximation (local optimization method to solve a class of nonconvex semidefinite programming problem) based control synthesis depends on the selection of central state matrix M. The central matrix M can be obtained by the feasible solution of the following LMIs:

$$\begin{bmatrix} (A_j^i)^T P_i + P_i A_j^i & P_i - (A_j^i)^T + M^T \\ P_i - A_j^i + M & -2I \end{bmatrix} < 0 \qquad (12)$$

B. *Design Algorithm*

The step by step design algorithm of convex optimization based fixed order power oscillation damper (POD) for PV plant is given next:

Step 1: Form a volume of operating conditions; linearize the power system for various operating conditions.

Step 2: Form the state-space model of the system with time delays.

Step 3: Design a lead-lag based POD for each vertex of the polytope.

Step 4: Obtain central matrix M from equation (12).

Step 5: Check if there is any feasible fixed order controller for underlying polytopic system with central matrix M from previous step.

Step 6: If there is no feasible solution for (11), then go to *step 1* and reduce the size of the operating volume, and follow the subsequent steps to obtain the feasible solution of (11); otherwise stop, and evaluate the controller performance.

III. Application Example

A. *Test System*

A single-line diagram of 16-machine 68-bus test system is depicted in Fig. 2. The dynamic system data and the nominal power transfer between the areas are available in [14]. The PV generation systems are considered at buses 16, 30, 38, and 48 of the system. The generic model of PV system has been considered for simulation. The detail description of the model can be found in [15]. The 6% PV penetration is considered for the analysis. The load sharing of the conventional generators is changed in the same proportion to accommodate the PV generator to the system. It is assumed that PV plants are operating at 60% of their rated capacity in the nominal system operating condition. Modal analysis has been performed to assess the low-frequency oscillatory stability of the system with the large-scale PV plants. There are two lightly damped inter-area modes of the system. Between the two modes, mode 1 is more critical as the damping of the mode is very low (% ζ = 2.34), suggesting a remedial measure needs to be taken to enhance the damping of the mode. Damping of the other mode is greater than 5%, settles within 15 s.

B. *Feedback Signal Selection*

A step by step methodology as shown in the flowchart of Fig. 3 has been used for the feedback signal selection. A hybrid signal selection method for time delayed system is used in terms of Hankel singular value (HSV) based modal controllability/ observability [16], right-half-plane-zeros (RHP-zeros), and mode to loop interaction (interaction index). The algorithm can be organized in to two separate sections: 1) Pre-screening of the candidate signals; 2) final selection of the signal. The proposed signal selection method reveals that the power flow between buses 36-37 is optimal signal for PV-WADC design.

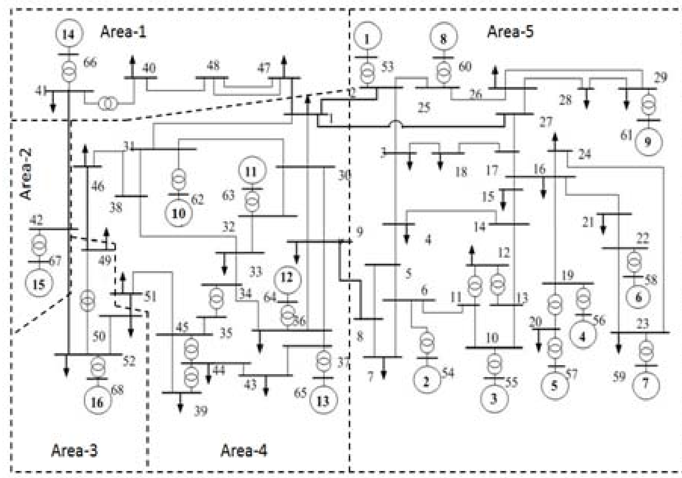

Fig. 2. Single-line diagram of 16-machine 68-bus test system.

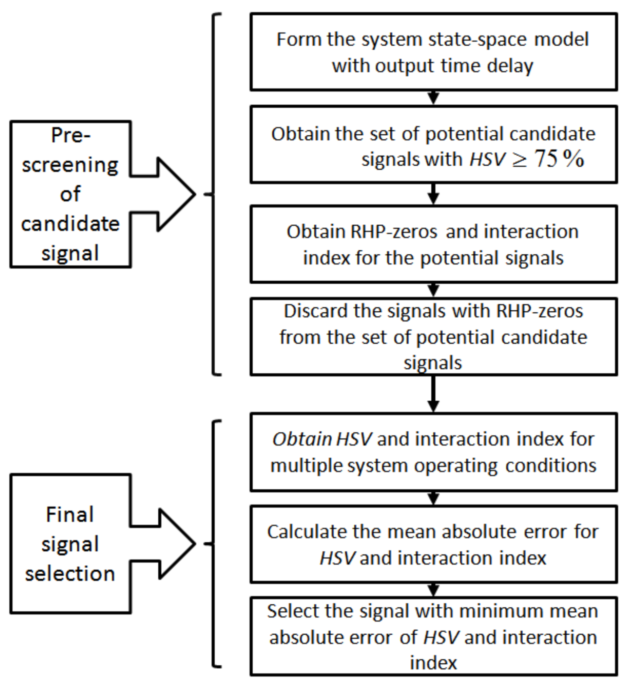

Fig. 3. Flowchart for feedback signal selection.

C. *Controller Design*

The reactive power modulation at the PV converter is used here for inter-area oscillation damping. In light of the algorithm depicted in the previous section, the lower order

POD for PV has been designed. As per the control design algorithm, the first step is to form the volume of system operating conditions to construct the polytope. The total sixteen operating conditions have been selected based on the PV output variation and load uncertainty. To obtain the central matrix M, sixteen lead-lag PODs corresponding to the operating conditions were designed. The controllers were designed by means of classical control technique, i.e., residue analysis. If a feasible solution of (12) is obtained with the designed controllers, then the final robust controller is evaluated by equation (11). The LMIs associated with (11) and (12) has been solved in MatLab, *YALMIP* as the interface and the *SDPT3* as the solver [17], [18]. The obtained second order robust controller is given by

$$K(s) = 3.58 \frac{10s}{1+10s} \frac{(1+0.2113s)^2}{(1+0.394s)^2}. \qquad (13)$$

D. Performance Evluation

In this section, the robust performance of the designed controller is evaluated in detail. The closed-loop poles of the system corresponding to the electromechanical (EM) modes are given in Fig. 4 for three different load conditions, low, medium and heavy. From the figure it is evident that when the system is installed with the proposed controller; the target inter-area mode is damped effectively with relatively higher damping ratio. From Fig. 4, it is worth noting that the designed controller does not have any adverse effect on other EM modes of the system. Fig. 5 shows the settling time of the closed-loop system as compared to the open-loop system (without POD at PV) for heavy load condition. From the figure it can be seen that the settling time for the target mode (mode 1) is about 12 s or less. Moreover, the figure also shows the settling time of other inter-area modes of the closed-loop system is comparable to the open-loop system. From the figure it is evident that the settling time of other modes are well within the acceptable range.

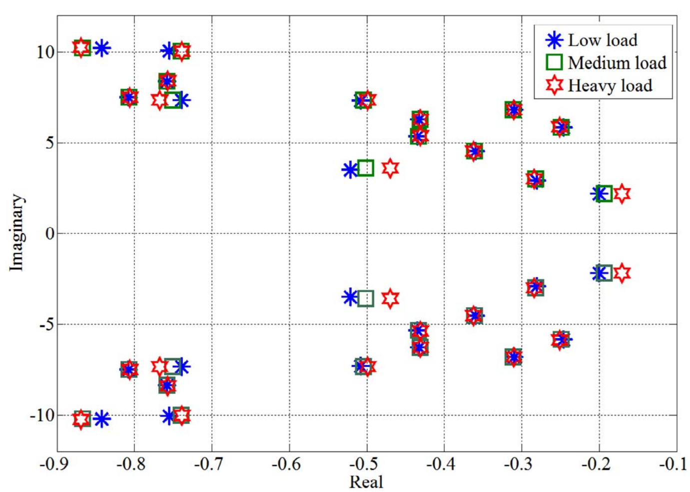

Fig. 4. EM modes of the closed-loop system for various load conditions.

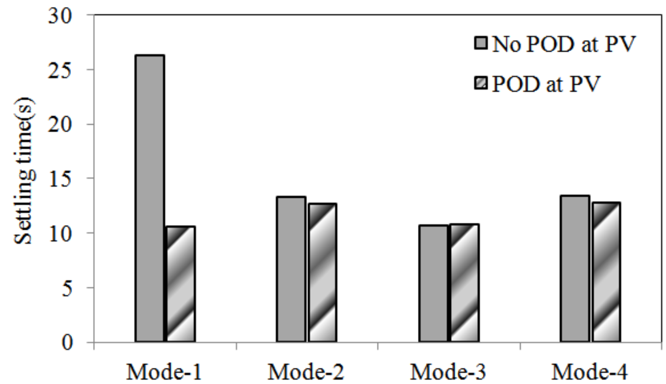

Fig. 5. Comparison of settling time of four inter-area modes (with and without POD).

Furthermore, the linear analysis has performed in this section to evaluate the robustness of the designed controller for different load compositions and system topologies. Table I shows the damping of target inter-area mode for different load characteristics. From the table, it can be seen that the proposed POD at PV can provide sufficient damping to the target mode for different load characteristics. The results in Table I depict that the designed controller provides the best damping to the target mode for the load composition with 50 % constant impedance and 50% constant current for both active and reactive component of the load. The table shows that when using load composition with constant power active component and constant current reactive component, the damping of the target mode is the lowest among all the scenarios considered for the analysis. Moreover, Table II shows the simulation results concerning the effect of different line outages and generation loss on the target inter-area mode of the system. It can be observed that the proposed controller can provide adequate damping even in severe system conditions.

TABLE I. DAMPING RATIOS AND FREQUENCIES OF CRITICAL INTER-AREA MODE

Type of load	Low-order controller	
	% ζ	Freq. (Hz)
Z	12.62	0.598
50% Z, 50% I	13.07	0.588
Real: I; Reactive: P	13.03	0.588
Real: Z; Reactive: I	11.30	0.589
Real: I; Reactive: Z	12.18	0.589
Real: Z; Reactive: P	11.30	0.589
Real: P; Reactive: I	10.90	0.583
Dynamic	12.30	0.584

TABLE II. DAMPING RATIOS AND FREQUENCIES OF CRITICAL INTER-AREA MODE

Case	% ζ	Freq. (Hz)
Outage of line 1-2	10.43	0.49
Outage of line 8-9	11.04	0.501
Outage of line1-27	12.2	0.55
Outage of line 9-30	9.86	0.56
25 % loss of generation of G_1	14.01	0.58

The performance of the controller is also assessed for variation of time delay from the value considered at controller synthesis stage, i.e., 100 ms for one way. The effect of feedback signal delay on the damping of inter-area modes is given in Fig. 6. In the figure, damping of all four inter-area modes deteriorates significantly with the increment of signal delay. It is also observed that up to 150 ms the change of mode 1 damping is minimal while mode 4 experiences sharp degradation of its damping with the increment of time delays up to 150 ms.

Fig. 7 shows the dynamic responses of tie-line between buses 1-31 power flow for 5% increased load condition from base case. The fault is applied near to the load bus 49 and disconnects the load from the system. Looking at the results in Fig. 7 it can be seen that implementation of WADC at PV can effectively damped the inter-area mode of the system for heavy loading condition in the presence of non-linearity.

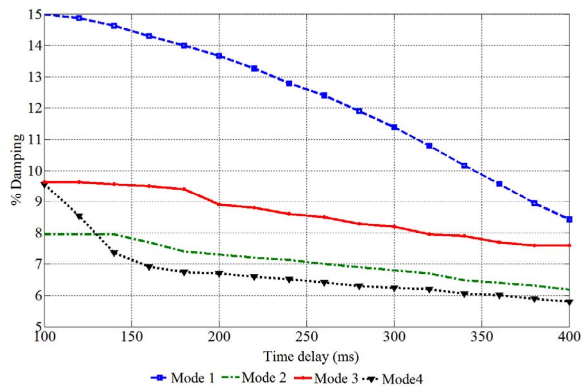

Fig. 6. Effect of feedback signal time delay on the damping of inter-area modes.

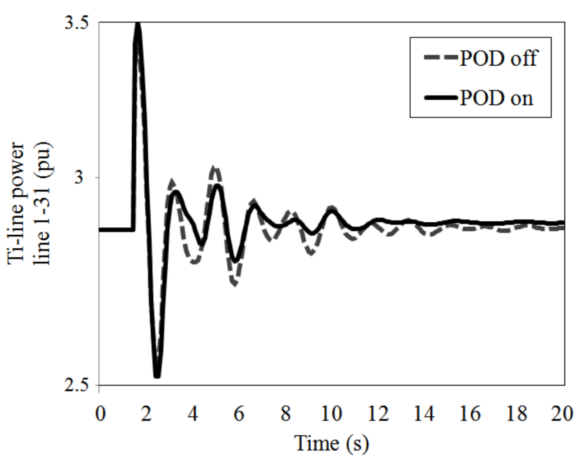

Fig. 7. Dynamic response of the power flow in the tie-line for 5% increased load level.

IV. Conclusions

This paper has demonstrated the effectiveness of the convex optimization in obtaining lower order robust damping controller for inter-area oscillation damping. The wide-area damping controller is considered at one of the PV plant of the system. For robust controller design, the uncertainties associated with the system are confined by system matrices which are the affine function belonging to the convex polytopic region. A suitable central matrix is evaluated by combining the fixed structured controller for each vertex of polytope. Then, the robust controller is obtained by convex optimization method which is based on the new definition of SPR pairing that can decouple the Lyapunov matrix variables from the controller variables. The lower order robust PV-WADC was implemented and tested in 16-machine 68-bus test system. The small-and large-disturbance performance of the controller were evaluated in different system operating conditions and topologies. The small signal and time domain simulation show that the proposed controller was successful in damping the target inter-area mode.

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