Incorporated Multi-Stage Nash Equilibriums for the Generation Allocation Problem Considering Ramp Rate Effects

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Abstract--This paper presents a novel method to find the profit-maximizing Nash Equilibriums in allocating generation amounts with consideration of ramp-rates under competitive market environment. In order to find the Nash equilibriums it is necessary to search all the feasible combinations of generators' outputs which satisfy various constraints. The procedure to eliminate the dominated strategies can be formulated using Bellman's optimality principle of dynamic programming problem and hence the backward or forward search algorithm of dynamic programming can be easily applied. Therefore, the Nash equilibriums are found using dynamic programming method and we found that there exist several Nash Equilibriums in the generation allocation problem. Individual generators participate in a game to maximize its profit through competitions and play a game with bidding strategies of its generation quantities in a spot market. The ramp-rate physically or technically limits generators to increase or decrease outputs in its range and restricts the number of bidding strategies of each generator. We suggest the Dynamic Programming to find the Nash Equilibriums while removing the dominated strategies in each stage (or each time). In the case studies, we analyzed the generation allocation game for a 12-hour multi-stage and compared it with the results of dynamic economic dispatch. Both of the two cases were considered generator's ramp-rate effects.

***Index Terms*--Generation allocation, game theory, competitive market, Ramp-rate, Nash equilibrium, Dynamic Programming.**

I. INTRODUCTION

Generation allocation problem is to find the best possible combination of generators' outputs which satisfies various constraints such as demand-and-supply condition and ramp rates, etc. Before the competitive markets have been introduced in electricity industry, generation allocation problem was regarded as economic dispatch problem [1]. This method is to minimize the total system costs and is focused on how effectively distribute the resources. More extended researches to consider reserve margins and generators' ramp-rates can be found in various literatures such as [3]-[6].

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However, ever since the competitive market has been introduced to the industry, strategic biddings of the market participants are prevailing to maximize the profits using their intrinsic market powers in the oligopolistic environment. Therefore, new analysis techniques are needed to consider the strategic biddings of the generators. One of the most typical methods for this analysis is to utilize the game theories.

To name but a few, Haurie *et al.* described a two-player game theory to solve the cogeneration problem where demand elasticity was not considered [7]. Ferrero *et al.* modeled the power transaction as a static and complete information game where the cost information of each participant is shared among players and bidding prices are linked with generation output [8]. Park *et al.* analyzed a market in a continuous strategy space and proposed an approach covering a 2-dimensional graphical and an analytical method to determine the equilibriums [9]. Limitation of these studies is that the electricity trading is assumed to be one-shot or non-repeated game to find the Nash Equilibriums. However, it is more natural to model the electricity market as repeated game. Recently, Jung modeled the electricity market as a dynamic bidding game and presented a method to find the sub-game perfect Nash Equilibriums through the backward induction approach considering ramp-rate [10].

In this paper, we will propose a novel method to model the generation allocation problem with consideration of the ramp-rate effects using dynamic programming to find the multi-stage Nash Equilibriums. At first, each Genco will establish pure strategies to find its outputs meeting timely total demand and formed combinations of strategy with passage of time. And then Nash Equilibriums are found in the last time stage. It will be shown that there exist many dominated strategies at a certain time stage. They still remained as dominated ones in a next time stage, even these dominated strategies created strategy combination. This explains exactly same as Bellman's optimality principle of dynamic programming method. Thus these dominated strategies will be eliminated before moving to next time stage.

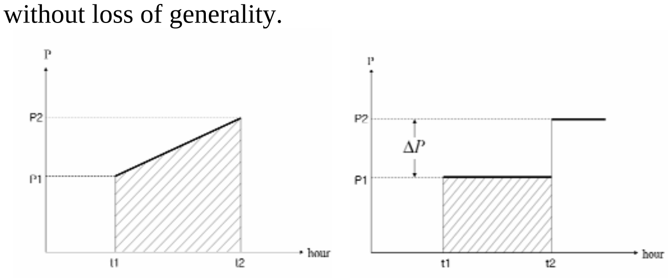
Still the limitation of game theory to model generation allocation problem is that it is very complicated to apply the theory to the problem where more than two players exist. Thus we generally analyze through two player game model and expand the number of players. Likewise, in this paper, we apply the method to the generation allocation problem with two generators.

This paper is organized as follows. Section II describes the dynamic ED with the ramp rate constraints. Section III represents the generation allocation game considering the ramp rates. In Section IV, a method to apply the dynamic programming to the game is suggested. Section IV presents a simple numerical example of the approach and finds the Nash Equilibriums. Finally, the conclusion is provided in Section V.

II. Dynamic ED Considering the Ramp-rate

In the unit commitment and economic dispatch problems, the ramp-rate constraint is contained to find more exact solutions [3]-[6]. The ramp rate is one of the most typical physical characteristics of the generators. By including these constraints in the problem, the solutions of economic dispatch problem become more realistic.

Generally, we assume the ramp-rate with linear characteristics as shown in Fig. 1(a). In this paper we assume that the ramp rate is discrete characteristic described in the Fig. 1(b) to make the problem simple. So ramp-up and ramp-down rates are simply denoted as $P_{Gi}^{Ramp-up}$ and $P_{Gi}^{Ramp-down}$, respectively. In most cases, two parameters have same values, we can denote the ramp rates of generator i as ΔP_{Gi}^{ramp} without loss of generality.



(a) Continuous characteristic (b) Discrete characteristics

Fig. 1 Ramp-rate of a generator

Dynamic ED is formulated as the following optimization problem, where we should find the optimal generation outputs $P_{G_i}^t$ to minimize the total production cost:

$$Min\sum_{t=1}^{T}\sum_{i=1}^{N}C_i^t = Min\sum_{t=1}^{T}\sum_{i=1}^{N}\left\{\alpha_i + \beta_i P_{Gi}^t + \gamma_i {P_{Gi}^t}^2\right\} \quad (1)$$

subject to,

$$\sum_{i=1}^{N}P_{Gi}^t = P_L^t \quad (2)$$

$$\underline{P_{Gi}} \le P_{Gi} \le \overline{P_{Gi}} \quad (3)$$

$$\left|P_{Gi}^t - P_{Gi}^{t-1}\right| \le \Delta P_{Gi} \quad (4)$$

where,

C_i^t : Production cost of the generator i at time t (\$/h).

$\alpha_i, \beta_i, \gamma_i$: Coefficients of the generator i's cost function.

P_{Gi}^t : Output of the generator i at time t (MW).

P_L^t : Total demand at time t (MW).

ΔP_{Gi} : Ramp-rate of the generator i (MW/h).

$\underline{P_{Gi}}$: Minimum output of the generator i (MW).

$\overline{P_{Gi}}$: Maximum output of the generator i (MW).

Equation (4) is an inequality constraint for ramp rates of generators meaning that generator i can increase or decrease its output at time t within ΔP_{Gi}^{ramp} from output P_{Gi}^{t-1} at time $t-1$.

III. Generation Allocation Game in the Market

A. Market Rules and Assumptions

Since the bidding strategy of a generation company is significantly influenced by the market type and rules, we assume the followings:

1. There exists only one electricity market and all the generators should participate in the market. It is also assumed that there is no bilateral contract between generation companies and customers.
2. The market price and the generation allocation are determined by the ISO in consideration of the submitted bids by the generation companies.
3. The price elasticity of demand is ignored.
4. Information for all generators is opened.
5. Each generation company owns only one generator.

B. Generation Allocation Game

In a competitive electricity market, the generation allocation problem can be interpreted as the problem for maximization of total generation companies' profits. Therefore, the objective function of generation allocation game considering ramp-rate of generator is defined as the following equations:

$$Max\sum_{t=1}^{T}\text{Profit}_i^t = Max\sum_{t=1}^{T}\left\{\rho^t P_{Gi}^t - C_i^t\right\}, \quad \text{for } \forall i \quad (5)$$

subject to,

$$\sum_{i=1}^{N}P_{Gi}^t \ge P_L^t \quad (6)$$

$$\underline{P_{Gi}} \le P_{Gi} \le \overline{P_{Gi}} \quad (7)$$

$$\left|P_{Gi}^t - P_{Gi}^{t-1}\right| \le \Delta P_{Gi} \quad (8)$$

where, Profit_i^t is the profit of generator i at time t and ρ^t is the market clearing price at time t

The eq. (6) means that the sum of submitted bids of generation amounts should be more than P_L^t. If it is not satisfied with this condition for each time, the payoff is assumed that will be not paid. The eq. (8) is the generator's ramp-rate constraint as one of the physical and technical limits at the time from $t-1$ to t. Eq. (6) and (8) can be effectively used to eliminate the infeasible solutions when we make the feasible combinations of bidding strategies at time t. Thus, overall selectable combinations of bidding strategies are much less than k^2.

When N generators participate in the market, each generator's bidding strategies can be established as follows.

- The number of generator: N
- Total demand for each time stage: P_L^t, $(t = 1,2,\cdots,T)$
- The set of bidding strategies of generator i
  $S_i = \{P_{Gi,1}^t, P_{Gi,2}^t, \cdots, P_{Gi,k}^t\}$
- The number of incorporated bidding strategy combinations of generator i : k^T

Even though each generator bids with their strategies into the market, individual outputs are determined by market mechanism. Therefore each generator's profit depends on 'allocated generation' and the objective function (5) of the generation allocation problem in competitive market exchanges as following equation (9).

$$Max\sum_{t=1}^{T}\text{Profit}_i^t \quad , \text{ for } \forall i \tag{9}$$

$$= Max\sum_{t=1}^{T}\left\{\rho^t \times P_{Gi}^{t,allocated} - \left(\alpha_i + \beta_i P_{Gi}^{t,allocated} + \gamma_i P_{Gi}^{t,allocated\,2}\right)\right\}.$$

where, $P_{Gi}^{t,allocated}$ is the allocated generation amount for generator i at time t.

IV. Applying Dynamic Programming

If the game is progressed to the next stage with pure strategies, the dominated strategies are eliminated while searching the Nash equilibriums after making up the payoff matrix [2].

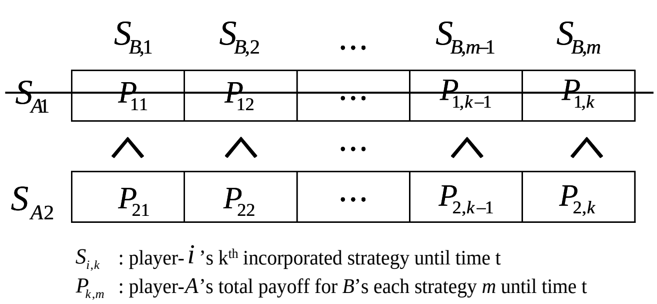


Fig. 2 Player-A's dominated strategy at time-t

In analyzing the multi-stage game, for player-B(opponent)'s every strategy, if player A's sum of payoff due to s_{A1}^t (player A's incorporated strategy combination up to time-t) at each time stage is inferior than s_{A2}^t, s_{A1}^t become a dominated strategy as describing in Fig.2. The elimination of the dominated strategies can be interpreted exactly with the Bellman's optimality principle of the dynamic programming theory. Thus the dynamic programming algorithm can be applied to the optimization problem described above to eliminate the dominated strategies, or the dominated strategy s_{A1}^t is able to be eliminated before moving to next time stage [1].

The procedure of the generation allocation game using the dynamic programming is shown in Fig. 3. First, each generator makes up the combinations of the possible bidding strategies for each time stage. Then the strategies which do not satisfy the ramp rate constraint will be eliminated. Secondary, the payoff matrix will be calculated, which implies former strategies and profits. At the last step, the dominated strategies are to be eliminated. These processes are repeated until time T and we can find the Nash Equilibriums in the final payoff matrix.

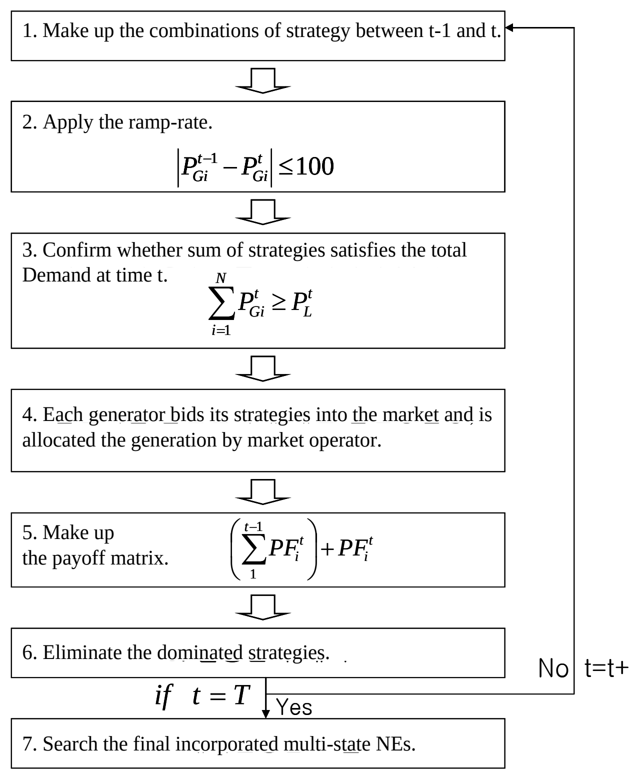


Fig. 3 The procedure of a game using Dynamic Programming

V. Numerical Examples

A. Case 1: Generation allocation game Using Dynamic Programming

Here the simulation results for the generation bidding game between two generators during 12 hour periods will be explained.

TABLE I
GENERATOR'S INPUT DATA

Unit	Fuel cost coefficients			Generation limits		Ramp-rate
	α_i	β_i	γ_i	Min [MW]	Max [MW]	[MW/h]
G1	200	7.92	0.001562	150	600	100
G2	100	7.85	0.001940	100	500	100

The conditions of two generators for this game are specified in table I. There were constraints for maximum and minimum output for generators and both generators' ramp-rate are 100 MW per hour. Each generator's bidding strategies can be chosen within each output limits and are decided by 50 MW. Therefore, G1 and G2 generators had ten and nine strategies by time stage, respectively.

Sum of each generator's strategies should satisfy total demand. In addition, it was eliminated strategies for G1 and G2, which were not able to supply total demand.

TABLE II
TOTAL DEMAND AT EACH HOUR FOR 12 HOURS

Hour [h]	08	09	10	11	12	13	14	15	16	17	18	19
Demand[MW]	800	860	900	920	910	830	880	900	880	870	850	860

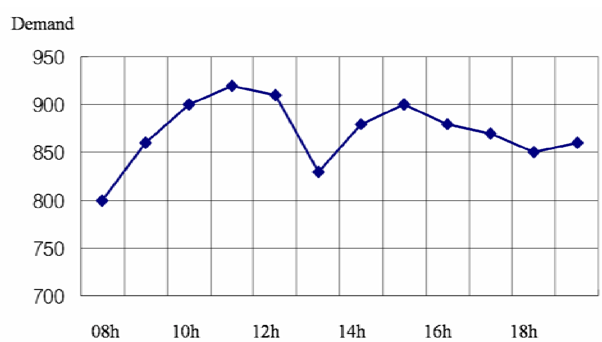


Fig. 4 demand of each hour in spring (08h-19h)

Fig. 4 shows daily demand pattern during the spring in Korea, though it is scaled down for two generator problem [11]. We performed a simulation for the generation allocation game during the period between 8AM to 7PM of the day where demand changes severely. Nash equilibriums, results of the game, are described in Table III.

TABLE III
NASH EQUILIBRIUMS OF THE GENERATION ALLOCATION GAME

(A) BIDDING STRATEGIES AT EACH HOUR [MW]

Time		08h	09h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h
Nash 01	G1	300	400	500	450	450	350	400	500	400	400	350	400
	G2	500	500	400	500	500	500	500	400	500	500	500	500
Nash 02	G1	450	450	400	450	450	350	400	500	600	600	600	600
	G2	350	450	500	500	500	500	500	400	300	300	250	300
Nash 03	G1	300	400	400	450	450	350	400	500	400	400	350	400
	G2	500	500	500	500	500	500	500	400	500	500	500	500
Nash 04	G1	300	400	400	450	450	350	400	400	400	400	350	400
	G2	500	500	500	500	500	500	500	500	500	500	500	500
Nash 05	G1	450	450	550	600	600	600	600	600	600	600	600	600
	G2	350	450	350	350	350	250	300	300	300	300	250	300
Nash 06	G1	300	400	500	450	550	550	600	600	600	600	600	600
	G2	500	500	400	500	400	300	300	300	300	300	250	300
Nash 07	G1	600	600	600	600	600	600	600	600	600	600	600	600
	G2	250	300	300	350	350	250	300	300	300	300	250	300
Nash 08	G1	450	450	400	450	550	550	600	600	600	600	600	600
	G2	350	450	500	500	400	300	300	300	300	300	250	300
Nash 09	G1	300	400	400	450	450	350	400	400	500	400	350	400
	G2	500	500	500	500	500	500	500	500	400	500	500	500
Nash 10	G1	300	400	500	450	450	350	400	400	500	400	350	400
	G2	500	500	400	500	500	500	500	500	400	500	500	500
Nash 11	G1	450	450	400	450	450	350	400	500	400	400	350	400
	G2	350	450	500	500	500	500	500	400	500	500	500	500
Nash 12	G1	450	450	400	450	450	350	400	400	500	600	600	600
	G2	350	450	500	500	500	500	500	500	400	300	250	300
Nash 13	G1	300	400	500	600	600	600	600	600	600	600	600	600
	G2	500	500	400	350	350	250	300	300	300	300	250	300
Nash 14	G1	450	450	400	450	450	350	400	400	400	400	350	400
	G2	350	450	500	500	500	500	500	500	500	500	500	500
Nash 15	G1	300	400	400	450	450	350	400	400	500	600	600	600
	G2	500	500	500	500	500	500	500	500	400	300	250	300
Nash 16	G1	300	400	500	450	450	350	400	500	600	600	600	600
	G2	500	500	400	500	500	500	500	400	300	300	250	300
Nash 17	G1	300	400	400	450	550	550	600	600	600	600	600	600
	G2	500	500	500	500	400	300	300	300	300	300	250	300
Nash 18	G1	300	400	500	450	450	350	400	400	500	600	600	600
	G2	500	500	400	500	500	500	500	500	400	300	250	300
Nash 19	G1	300	400	400	450	450	350	400	500	600	600	600	600
	G2	500	500	500	500	500	500	500	400	300	300	250	300
Nash 20	G1	450	450	400	450	450	350	400	400	500	400	350	400
	G2	350	450	500	500	500	500	500	500	400	500	500	500

(B) ALLOCATED GENERATION AT EACH HOUR[MW]

Time		08h	09h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h
Nash 01	G1	300	400	500	450	450	350	400	500	400	400	350	400
	G2	500	460	400	470	460	480	480	400	480	470	500	460
Nash 02	G1	450	450	400	450	450	350	400	500	580	570	600	560
	G2	350	410	500	470	460	480	480	400	300	300	250	300
Nash 03	G1	300	400	400	450	450	350	400	500	400	400	350	400
	G2	500	460	500	470	460	480	480	400	480	470	500	460
Nash 04	G1	300	400	400	450	450	350	400	400	400	400	350	400
	G2	500	460	500	470	460	480	480	500	480	470	500	460
Nash 05	G1	450	450	550	570	560	580	580	600	580	570	600	560
	G2	350	410	350	350	350	250	300	300	300	300	250	300
Nash 06	G1	300	400	500	450	510	530	580	600	580	570	600	560
	G2	500	460	400	470	400	300	300	300	300	300	250	300
Nash 07	G1	550	560	600	570	560	580	580	600	580	570	600	560
	G2	250	300	300	350	350	250	300	300	300	300	250	300
Nash 08	G1	450	450	400	450	510	530	580	600	580	570	600	560
	G2	350	410	500	470	400	300	300	300	300	300	250	300
Nash 09	G1	300	400	400	450	450	350	400	400	480	400	350	400
	G2	500	460	500	470	460	480	480	500	400	470	500	460
Nash 10	G1	300	400	500	450	450	350	400	400	480	400	350	400
	G2	500	460	400	470	460	480	480	500	400	470	500	460
Nash 11	G1	450	450	400	450	450	350	400	500	400	400	350	400
	G2	350	410	500	470	460	480	480	400	480	470	500	460
Nash 12	G1	450	450	400	450	450	350	400	400	480	570	600	560
	G2	350	410	500	470	460	480	480	500	400	300	250	300
Nash 13	G1	300	400	500	570	560	580	580	600	580	570	600	560
	G2	500	460	400	350	350	250	300	300	300	300	250	300
Nash 14	G1	450	450	400	450	450	350	400	400	400	400	350	400
	G2	350	410	500	470	460	480	480	500	480	470	500	460
Nash	G1	300	400	400	450	450	350	400	400	480	570	600	560

15	G2	500	460	500	470	460	480	480	500	400	300	250	300
Nash 16	G1	300	400	500	450	450	350	400	500	580	570	600	560
	G2	500	460	400	470	460	480	480	400	300	300	250	300
Nash 17	G1	300	400	400	450	510	530	580	600	580	570	600	560
	G2	500	460	500	470	400	300	300	300	300	300	250	300
Nash 18	G1	300	400	500	450	450	350	400	400	480	570	600	560
	G2	500	460	400	470	460	480	480	500	400	300	250	300
Nash 19	G1	300	400	400	450	450	350	400	500	580	570	600	560
	G2	500	460	500	470	460	480	480	400	300	300	250	300
Nash 20	G1	450	450	400	450	450	350	400	400	480	400	350	400
	G2	350	410	500	470	460	480	480	500	400	470	500	460

(C) MARKET PRICE [$/MW]

Time	08h	09h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h
Nash 01	9.79	9.63	9.48	9.67	9.63	9.71	9.71	9.48	9.71	9.67	9.79	9.63
Nash 02	9.33	9.44	9.79	9.67	9.63	9.71	9.71	9.48	9.73	9.70	9.79	9.67
Nash 03	9.79	9.63	9.79	9.67	9.63	9.71	9.71	9.48	9.71	9.67	9.79	9.63
Nash 04	9.79	9.63	9.79	9.67	9.63	9.71	9.71	9.79	9.71	9.67	9.79	9.63
Nash 05	9.33	9.44	9.64	9.70	9.67	9.73	9.73	9.79	9.73	9.70	9.79	9.67
Nash 06	9.79	9.63	9.48	9.67	9.51	9.58	9.73	9.79	9.73	9.70	9.79	9.67
Nash 07	9.64	9.67	9.79	9.70	9.67	9.73	9.73	9.79	9.73	9.70	9.79	9.67
Nash 08	9.33	9.44	9.79	9.67	9.51	9.58	9.73	9.79	9.73	9.70	9.79	9.67
Nash 09	9.79	9.63	9.79	9.67	9.63	9.71	9.71	9.79	9.42	9.67	9.79	9.63
Nash 10	9.79	9.63	9.48	9.67	9.63	9.71	9.71	9.79	9.42	9.67	9.79	9.63
Nash 11	9.33	9.44	9.79	9.67	9.63	9.71	9.71	9.48	9.71	9.67	9.79	9.63
Nash 12	9.33	9.44	9.79	9.67	9.63	9.71	9.71	9.79	9.42	9.70	9.79	9.67
Nash 13	9.79	9.63	9.48	9.70	9.67	9.73	9.73	9.79	9.73	9.70	9.79	9.67
Nash 14	9.33	9.44	9.79	9.67	9.63	9.71	9.71	9.79	9.71	9.67	9.79	9.63
Nash 15	9.79	9.63	9.79	9.67	9.63	9.71	9.71	9.79	9.42	9.70	9.79	9.67
Nash 16	9.79	9.63	9.48	9.67	9.63	9.71	9.71	9.48	9.73	9.70	9.79	9.67
Nash 17	9.79	9.63	9.79	9.67	9.51	9.58	9.73	9.79	9.73	9.70	9.79	9.67
Nash 18	9.79	9.63	9.48	9.67	9.63	9.71	9.71	9.79	9.42	9.70	9.79	9.67
Nash 19	9.79	9.63	9.79	9.67	9.63	9.71	9.71	9.48	9.73	9.70	9.79	9.67
Nash 20	9.33	9.44	9.79	9.67	9.63	9.71	9.71	9.79	9.42	9.67	9.79	9.63

(D) REVENUE, COST AND PROFIT FOR EACH NEs [$/MW]

	Nash No.	01	02	03	04	05	06	07	08	09	10
revenue	G1	47,281	55,541	46,456	45,631	64,321	59,799	67,168	60,628	46,267	47,092
	G2	53,751	45,304	54,853	55,955	36,758	41,382	34,498	40,292	55,061	53,959
	Sum	101,032	100,845	101,309	101,586	101,079	101,180	101,666	100,919	101,328	101,051
	Nash No.	11	12	13	14	15	16	17	18	19	20
revenue	G1	48,110	53,592	62,107	47,285	51,938	54,711	58,974	52,763	53,886	47,921
	G2	52,661	47,255	39,369	53,763	49,447	46,394	42,484	48,345	47,497	52,869
	Sum	100,771	100,847	101,477	101,048	101,385	101,106	101,457	101,108	101,383	100,790

	Nash No.	01	02	03	04	05	06	07	08	09	10
Cost	G1	44,395	52,451	43,462	42,529	60,869	56,461	63,348	57,354	43,273	44,206
	G2	49,866	41,832	50,826	51,786	33,501	37,912	31,129	36,970	51,021	50,061
	Sum	94,261	94,283	94,288	94,315	94,370	94,373	94,476	94,324	94,294	94,267
	Nash No.	11	12	13	14	15	16	17	18	19	20
Cost	G1	45,288	50,561	58,565	44,355	48,735	51,558	55,528	49,668	50,625	45,099
	G2	48,924	43,713	35,868	49,884	45,614	42,774	38,871	44,655	43,734	49,119
	Sum	94,212	94,274	94,432	94,239	94,349	94,332	94,400	94,322	94,359	94,218

	Nash No.	01	02	03	04	05	06	07	08	09	10
Pro	G1	**2,886**	**3,089**	**2,994**	**3,102**	**3,453**	**3,338**	**3,820**	**3,273**	**2,994**	**2,887**
	G2	**3,884**	**3,472**	**4,027**	**4,170**	**3,257**	**3,470**	**3,370**	**3,322**	**4,040**	**3,897**
	Sum	**6,771**	**6,561**	**7,021**	**7,271**	**6,710**	**6,808**	**7,190**	**6,595**	**7,034**	**6,784**
	Nash No.	11	12	13	14	15	16	17	18	19	20
Profit	G1	**2,886**	**3,089**	**2,994**	**3,102**	**3,453**	**3,338**	**3,820**	**3,273**	**2,994**	**2,887**
	G2	**3,884**	**3,472**	**4,027**	**4,170**	**3,257**	**3,470**	**3,370**	**3,322**	**4,040**	**3,897**
	Sum	**6,771**	**6,561**	**7,021**	**7,271**	**6,710**	**6,808**	**7,190**	**6,595**	**7,034**	**6,784**

B. *Case 2: Dynamic ED*

Next, we performed a simulation for the dynamic economic dispatch to compare with the results of a bidding game. Table IV shows the results of the dynamic economic dispatch for the exactly same input values of the previous case.

TABLE IV
THE RESULTS OF ECONOMIC DISPATCH CONSIDERING THE RAMP-RATE

time	P_L	Price	Generation [MW]		Revenue [$/MW]		Cost [$/MW]		Profit [$/MW]	
			G1	G2	G1	G2	G1	G2	G1	G2
08h	800	9.27	433.2	366.8	4,017.0	3,401.6	3,923.9	3,240.6	93.1	161.0
09h	860	9.38	466.4	393.6	4,373.7	3,690.6	4,233.8	3,490.1	139.8	200.5
10h	900	9.45	488.6	411.4	4,615.3	3,886.4	4,442.4	3,658.0	172.9	228.4
11h	920	9.48	499.7	420.3	4,737.2	3,985.2	4,547.3	3,742.5	190.0	242.8
12h	910	9.46	494.1	415.9	4,676.1	3,935.8	4,494.8	3,700.2	181.4	235.5
13h	830	9.33	449.8	380.2	4,194.5	3,545.4	4,078.4	3,365.0	116.0	180.4
14h	880	9.41	477.5	402.5	4,494.1	3,788.2	4,337.9	3,573.9	156.1	214.3
15h	900	9.45	488.6	411.4	4,615.3	3,886.4	4,442.4	3,658.0	172.9	228.4
16h	880	9.41	477.5	402.5	4,494.1	3,788.2	4,337.9	3,573.9	156.1	214.3
17h	870	9.39	472.0	398.0	4,433.8	3,739.4	4,285.8	3,532.0	147.9	207.4
18h	850	9.36	460.9	389.1	4,313.7	3,642.1	4,181.9	3,448.3	131.8	193.7
19h	860	9.38	466.4	393.6	4,373.7	3,690.6	4,233.8	3,490.1	139.8	200.5
Sum					53,338.5	44,979.9	51,540.3	42,472.6	1,797.8	2,507.2
Total					98,318.4		94,012.9		4,305.1	

Generation:[MW], Price:[$/MWh]

In this case, twenty Nash equilibriums were derived for twelve hours. The order of Nash equilibriums in table IV is not meaningful because the initial value is acquired at random while searching Nash equilibriums. It should be noted that the Nash equilibriums acquired by the above procedure are equally qualified for the optimal solution and it is necessary to set up an additional study to find which equilibrium is superior and the optimal solution. This paper is focus on finding Nash equilibriums derived from the game.

Because the objective functions used in the proposed method and dynamic ED are different, it is impossible to compare the results of both methods directly. However, considering the benefit of each generator or generation company, the cost for ED is much lower than Nash equilibrium strategy. It means that the generation companies can increase their profits significantly by gaming.

TABLE V
COMPARISON OF THE RESULT BETWEEN THE GAME AND DYNAMIC ED

	Total Revenue [$/h]	Total Cost [$/h]	Total Profit [$/h]
11^{th} Nash	100,770.5	94,206.8	6,558.3
Dynamic ED	98,318.4	94,012.9	4,305.1

In the table V, we compared the 11^{th} Nash equilibrium strategy among twenty Nash equilibriums with the results of dynamic ED. The cost of 11^{th} Nash equilibrium is $194.00 per hour greater than that of ED but the profit of Nash equilibrium is $2,253 per hour larger than that of ED.

VI. Conclusions

In this study, generation allocation problem in a competitive electricity market was analyzed using game theory and dynamic programming method. The Nash equilibriums were derived, which maximize the profit of individual generation company considering ramp-rate of generators by time stage under the competitive power market. For analyzing the solution of the game, dynamic programming was used to eliminate the dominated strategies and to find the Nash equilibriums effectively.

In this paper, two-generator problem was handled for simplicity. Dynamic programming is more efficient than brute-force to find the solution. However, it still takes long time to analyze the entire power system. Therefore, it is necessary to introduce evolutionary algorithm, which provide the efficient game in consideration of the whole generators in the actual power system.

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