

Applications of MATLAB Symbolic and Optimization Toolboxes in Static Voltage Stability in Power Systems

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Abstract: This paper presents a simple simulation approach to study static voltage stability in power systems. Symbolic and Optimization toolboxes in MATLAB are used to simulate load flow, continuation power flow and optimization technique in static voltage stability study. Flexible AC Transmission System devices including SVC, TCSC, STATCOM, SSSC and UPFC are used to validate the proposed technique. Various voltage stability parameters including PV curves, loading margin and simulation times are investigated. Three-bus and IEEE fourteen-bus test systems are used to demonstrate the proposed technique. The proposed approach may be of interest to utilities and researchers who wish to study voltage stability of medium size power systems in a simple way.

Keywords: Symbolic toolbox, Optimization toolbox, static voltage stability, MATLAB, FACTS.

1. INTRODUCTION

Modern electric power utilities are facing many challenges due to ever-increasing complexity in their operation and structure. In the recent years, one of the problems that receive wide attention is the voltage instability. With an open-access market, poorly scheduled generation for the competitive bidding is one of many reasons for voltage instability problem in the deregulated electricity environment. Thus, in order to relieve or at least minimize the system from the voltage instability problem, many electric utilities and researchers have devoted a great deal of efforts in system studies related to static voltage stability.

In static voltage stability study, Continuation Power Flow (CPF) and optimization methods are the main analysis techniques that are used to find voltage stability margin or loading margin (LM) of the system (Sode-Yome, *et al.*, 2004, 2006). Based on these techniques, utilities and researchers may require developing and devoting a great deal of effort to create a program. Adding to this, they may face a difficulty to ensure the correct answers. Moreover, with the use of Flexible AC Transmission system (FACTS) to enhance static voltage stability, simulation effort may be increased.

According to above observation, attention is drawn in this paper to propose a new simulation approach for voltage stability study using Symbolic and Optimization toolboxes in MATLAB. The paper simulates load flow, CPF and optimization technique in static voltage stability study. The proposed technique is applied to system with FACTS devices. Moreover, simulation times are studied and compared in test systems. This may be useful for utilities to

find voltage stability assessment of medium-size power systems without devoting a great deal of effort.

Rest of the paper is organized as follows: Section 2 presents static voltage stability. FACTS devices are introduced in Section 3. A new simulation approach is proposed in Section 4 for load flow, CPF and optimization technique. Test systems and analysis tools are briefly stated in Section 5. In Section 6, numerical results are presented. Finally, in Section 7, major contributions and conclusions are presented.

2. STATIC VOLTAGE STABILITY

Voltage stability is the ability of power system to maintain adequate voltage magnitude so that when the system nominal load is increased, the actual power transferred to that load will increase (IEEE/PES Power System Stability Subcommittee, 2003). It is mainly associated with reactive power imbalance. Slowly developing changes in the power system occur that eventually lead to a shortage of reactive power and, hence, declining voltage. Maximum load that the system can be supplied before entering to the collapse point is called loading margin (LM) or static voltage stability margin of the system. Continuation Power Flow and optimization technique (or direct) methods are two analysis techniques used for voltage stability study.

2.1 Continuation Power Flow Method

Continuation Power Flow (CPF) presents a way to plot complete PV curves by automatically changing the value of Loading Factor (LF or λ). It involves predictor and corrector

steps to guarantee a well behaved numerical solution of the related equations.

2.2 Optimization Technique Method

Static voltage stability study can be carried out by formulating the problem as an optimization problem. Thus, distance to collapse can be maximized as follows:

$$\begin{aligned} &\text{Minimize} && -\lambda \\ &\text{subject to} && F(z, \lambda) = 0 \end{aligned}$$

where F are load flow equations with state variables z . Necessary conditions are used to find state variables including bus angles, bus voltages and loading margin. Other power system limits such as voltage, thermal and generator limits can be easily introduced as inequality constraints.

The CPF method is the promising approach, since it is based on power flow calculation and provides completed PV curves as well as voltages at every bus at various loading factors. It can be used as an indicator to detect the proximity to voltage collapse and the weakest bus of the system. Optimization technique provides a more accurate solution and it is able to handle power system constraints in a simple way. However, it gives only the solution at the optimal point, which may not be useful in the operation of an intermediate loading point, between base case and collapse point.

Although CPF and optimization methods are promising methods, it may not be easy for utilities to implement. Moreover, if the FACTS devices are used in the power system, it may make the study more complicated. In the following section, FACTS devices are introduced. Then, a simple approach based on Symbolic and Optimization toolboxes is proposed for finding the solutions for load flow, CPF and optimization technique methods in a simple way.

3. FLEXIBLE AC TRANSMISSION SYSTEM DEVICES

The development of FACTS devices in power transmission system has led to many applications of these controllers not only improve various stability issues but also provide operating flexibility to power systems. There are various types of FACTS devices available for this purpose, namely, Static Var Compensator (SVC), Static Synchronous Compensator (STATCOM), Thyristor-Controlled Series Capacitor (TCSC), Static Synchronous Series Compensator (SSSC) and Unified Power Flow Controller (UPFC). In the following subsections, concept of these well-known FACTS devices is presented.

3.1 SVC

The SVC is a shunt connected static Var generator/load whose output is adjusted to exchange capacitive or inductive current so as to maintain or control specific power system variables. It is similar to a synchronous condenser in that it is used to supply or absorb reactive power but without rotating

part. It contains the equivalent of automatic voltage regulator system to set and maintain a target voltage level.

3.2 STATCOM

The STATCOM is the voltage-source converter, which converts a DC input voltage into AC output voltage in order to compensate the active and reactive needed by the system. It could be viewed as superior to SVC, as STATCOM provides better terminal characteristics compared to diminishing characteristics at low terminal voltages by SVC.

3.3 TCSC

The TCSC controllers use Thyristor Control Reactor (TCR) in parallel with segments of series capacitor bank. The combination of TCR and capacitor allow the capacitive reactance to be smoothly controlled over a wide range and switched upon command to a condition where the bi-directional thyristor pairs conduct continuously and insert appropriate reactance into the line. The total susceptance of the line is controlled by controlling the firing angle of the thyristor.

3.4 SSSC

The SSSC can be used for series compensation of transmission lines. It is similar to the STATCOM, as it is based on a DC capacitor fed Voltage Source Inverter (VSI) that generates a three-phase voltage at fundamental frequency, which is then injected in a transmission line through a transformer connected in series with line.

3.5 UPFC

The UPFC consists of two identical voltage-source inverters, namely shunt inverter and series inverter which operate via a common DC link with a DC storage capacitor, allow UPFC to independently control active and reactive power flows on the line as well as the bus voltage. Active power can freely flow in either direction between the AC terminals of the two inverters through the DC link.

Appropriate model including AC and DC equations of FACTS devices can be incorporated in static voltage stability study by adding FACTS equations in the load flow equations.

4. PROPOSED SIMULATION APPROACH

The proposed simulation approach is applied to load flow, CPF and optimization techniques. Load flow is simulated first. Then, the method is applied to the CPF and optimization process.

4.1 Load Flow

Power flow or load flow consists of solving real and reactive power balance equations of all buses in power systems to obtain all state variables when control variables are specified. According to this, Symbolic toolbox in MATLAB can be used to create power flow equations when the system data and control variables are known. Then, a simple command called "lsqnonlin" in Optimization toolbox is used to find the

solution for all state variables. The steps behind the proposed method can be summarized as shown in the Fig. 1.

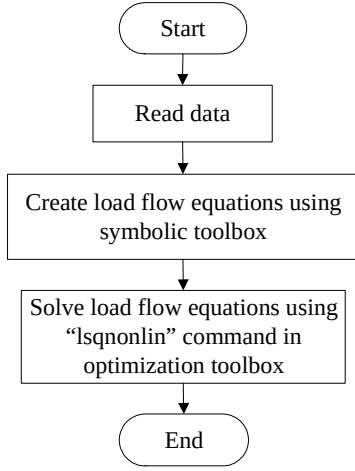


Fig. 1 Load flow calculation using Symbolic and Optimization toolboxes.

From Fig. 1, system parameters are read from input data to create power balance equations using Symbolic toolbox. The solution is then found by using a single “lsqnonlin” command. It can be noticed that Jacobian is not required to compute in the formulation process since it is already embedded in Optimization toolbox. The solution is found in a simple way, as only one command is used.

4.2 Continuation Power Flow

Continuation Power Flow is basically a series of load flow calculation with predictor and corrector steps. The formation of CPF is complicated and it requires good programming skill. However, with the help of Symbolic and Optimization toolboxes, the formulation is much easier. Figure 2 illustrates steps behind the CPF process with Symbolic and Optimization toolboxes. From Fig. 2, the system data is read first, then Symbolic toolbox is introduced to create power flow equations. The power flow calculation is performed to find the load flow Jacobian for the following predictor step.

In the predictor step, state variables are predicted from the current status of load flow Jacobian to predict the bus angles and voltages at higher LF. In the corrector step, the actual value of state variables are computed from load flow equations and initial condition obtained from the predictor step. Prior to the collapse point, parameterization step is performed to avoid convergence difficulty of CPF process by switching state variable from LF to the voltage at the weakest bus, which is found from a bus having highest voltage decrease. The process is repeated until the PV curve is completed. For simplicity, in the paper, parameterization is introduced after the last LF that allows load flow solution to converge.

4.3 Optimization Technique

Optimization technique can be solved with the help of Symbolic and Optimization toolboxes as well. Fig. 3 shows various steps of optimization technique including reading input data, creating power flow equations, creating necessary conditions and solving necessary conditions for optimal solutions. From the proposed method, Lagrangian function and necessary conditions are automatically created by using Symbolic toolbox. The optimal solution is then found from the necessary conditions by using a single command, “lsqnonlin”.

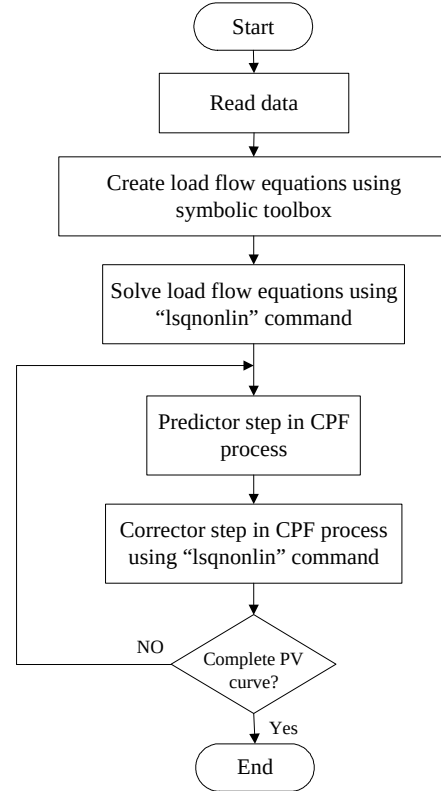


Fig. 2 CPF using Symbolic and Optimization Toolboxes.

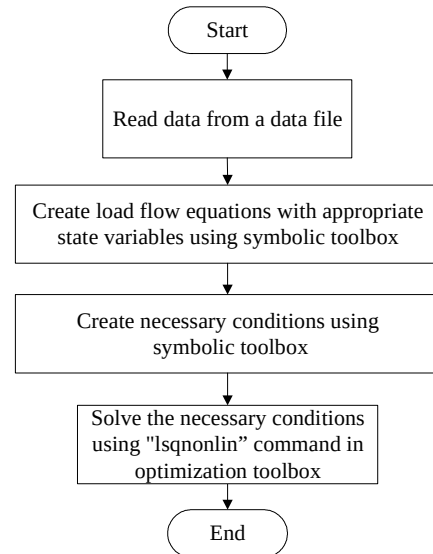


Fig. 3 Optimization technique using Symbolic and Optimization Toolboxes.

FACTS devices are applied in the process by adding equations including AC and DC representation in load flow equation. The proposed simulation method is validated in two test systems. In the following section, these test systems along with analysis tools are presented.

5. TEST SYSTEMS AND ANALYSIS TOOLS

The simple 3-bus test system and the modified IEEE 14-bus test systems are used to validate the proposed approach. Single line diagrams of the 3-bus and modified IEEE 14-bus test systems are illustrated in Figs. 4 and 5, respectively. The modification from the original IEEE 14-bus test system is that generators located at buses 6 and 8 were changed from synchronous compensators to generators.

The developed tool is tested and validated with the help of a standard CPF program, UWPFLOW. UWPFLOW is a research tool that has been designed to calculate maximum loading margin of the power system.

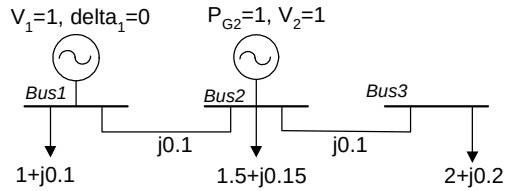


Fig. 4 Single line diagram of the 3-bus system.

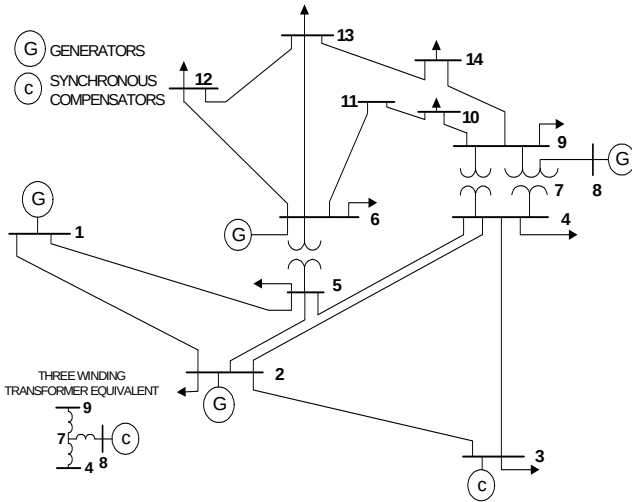


Fig. 5 Single line diagram of the modified IEEE 14-bus system.

6. NUMERICAL RESULTS

The proposed simulation approach is simulated for load flow, CPF and Optimization techniques. Load flow is tested in the first step, as it is the first part of CPF process. Optimization

method is tested in the last step in order to compare the result with that of CPF method. Finally, FACTS devices are applied in the modified IEEE 14-bus test system.

6.1 Load Flow

a) The Three-bus Test System

In order to solve power flow equation in a simple way, state variables are selected to be equal to two times number of buses plus 1. The last state variable is LF, which is introduced in the load flow for CPF in the following step. In the three-bus test system, there are 7 state variables: 3 for bus angles, 3 for bus voltages and 1 for LF, which is zero for load flow. After reading the input data, Symbolic toolbox is introduced to create the following power flow equations:

$$\begin{aligned} \delta_1 &= 0 \\ P_{G2} - (1 + \lambda)P_{D2} - 10V_2V_1 \sin(\delta_2 - \delta_1) - 10V_2V_3 \sin(\delta_2 - \delta_3) &= 0 \\ -(1 + \lambda)P_{D3} - 10V_3V_2 \sin(\delta_3 - \delta_2) &= 0 \\ V_1 &= 1 \\ V_2 &= 1 \\ -(1 + \lambda)Q_{D3} + 10V_3V_2 \cos(\delta_3 - \delta_2) - 10V_3^2 &= 0 \\ \lambda &= 0 \end{aligned}$$

From the above equations, there are 7 equations with 7 state variables. State variables namely angle at bus 1, voltages at bus 1 and 2, and LF are control variables as they are specified. A single MATLAB command “lsqnonlin” is then used to obtain the solution in a simple way. The solution of load flow is therefore $x = [\delta_1, \delta_2, \delta_3, V_1, V_2, V_3, \lambda]^T = [0.0000, -0.2527, -0.4632, 1.0000, 1.0000, 0.9570, 0.0000]^T$.

b) The Modified IEEE 14-bus Test System

In the 14-bus test system, there are 28+1 state variables: 14 for bus angles, 14 for bus voltages and 1 for LF. There are 28 equations, which are created by using Symbolic toolbox. The solution is then found from the help of “lsqnonlin” command in Optimization toolbox. The solution of load flow is, therefore,

$$[0.0000, -0.0437, -0.1563, -0.0988, -0.0808, -0.1017, -0.0876, -0.0307, -0.1188, -0.1201, -0.1127, 0.1159, -0.1172, -0.1343, 1.0600, 1.0750, 1.0612, 1.0600, 1.0603, 1.1246, 1.0956, 1.1312, 1.0813, 1.0816, 1.0993, 1.1084, 1.1015, 1.0727, 0.0000] \text{ for state variables } [\delta_1, \dots, \delta_{14}, V_1, \dots, V_{14}, \lambda].$$

Load flow solution is used in CPF process, which is presented in the next subsection.

6.2 Continuation Power Flow

a) The Three-bus Test System

Jacobian of the first load flow is used in the predictor step to predict state variables for the next LF. At the next LF and predicted state variables, the corrected state variables can be found in the corrector step. When the load flow solution is diverged, parameterization step is activated at the last converged LF.

PV curves of the test systems based on CPF with the help of Symbolic and Optimization toolboxes are shown in Fig. 6. From PV curve, LM of the system is 1.2625 p.u. Numerical results of all state variables during predictor, corrector and parameterization steps are tabulated in Tables 1 and 2. In this study, step sizes of LF in the predictor step and voltage in parameterization step are specified at 0.1 and 0.02 p.u., respectively. These values are important and they should be carefully selected.

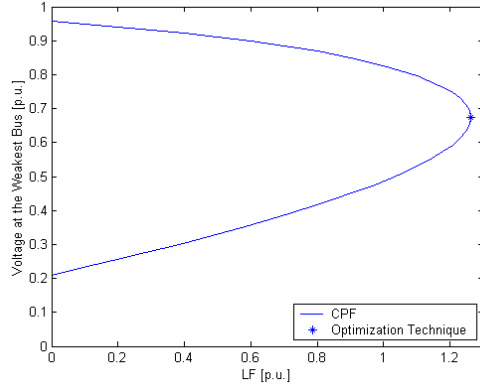


Fig. 6 PV curve of three bus test system.

Table 1 State Variables during Corrector and Parameterization Steps at some LFs of the Three-bus System

	Corrector Steps			Parameterization	
δ_1	0.0000	0.0000	0.0000	0.0000	0.0000
δ_2	-0.2527	-0.2890	-0.7342	-0.7478	-0.7571
δ_3	-0.4632	-0.5228	-1.3580	-1.4012	-1.4392
V_1	1.0000	1.0000	1.0000	1.0000	1.0000
V_2	1.0000	1.0000	1.0000	1.0000	1.0000
V_3	0.9570	0.9496	0.7532	0.7332	0.7132
λ	-0.0000	0.1000	1.2000	1.2286	1.2481

Table 2 State Variables during Predictor Step at some LFs of the Three-bus System

	Predictor Step				
δ_1	0.0000	0.0000	0.0000	0.0000	0.0000
δ_2	-0.2888	-0.3255	-0.3627	-0.4004	-0.4386
δ_3	-0.5223	-0.5829	-0.6448	-0.7082	-0.7734
V_1	1.0000	1.0000	1.0000	1.0000	1.0000
V_2	1.0000	1.0000	1.0000	1.0000	1.0000
V_3	0.9500	0.9419	0.9329	0.9230	0.9119
λ	0.1000	0.2000	0.3000	0.4000	0.5000

b) The Modified IEEE 14-bus Test System

The proposed simulation approach is tested in the modified IEEE 14-bus test system. PV curve at the weakest bus of the system is illustrated in Fig. 7. From Fig. 7, LM of the system is 0.9278 p.u.

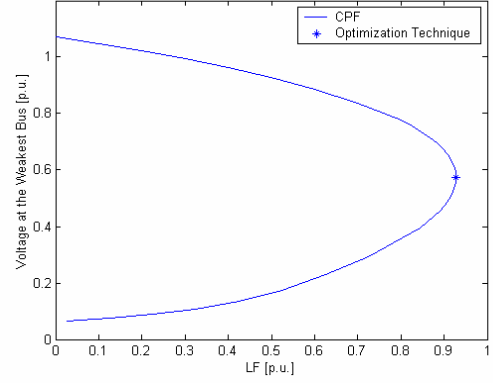


Fig. 7 PV curve of 14-bus test system.

Optimization Technique

a) The Three-bus Test System

Symbolic and Optimization toolboxes can be used to find loading margin of the system based on optimization technique. All state variables and LM at the collapse point of the 3-bus test systems are $x = [\delta_1, \delta_2, \delta_3, V_1, V_2, V_3, \lambda]^T = [0.0000, -0.6334, -1.3689, 1.0000, 1.0000, 0.6744, 1.2625]^T$. The result is compared with CPF in Fig. 6. From Fig. 6, it is noticed that optimization technique provides only single solution, the maximum point. CPF, on the other hand, provides a complete PV curve and voltages at various loading factors. Moreover, CPF can provide the information about the weakest location in the system.

b) The Modified IEEE 14-bus Test System

Symbolic and Optimization toolboxes is further applied to the modified IEEE 14-bus test system. LM of the system is 0.9278 p.u. and the voltage at the weakest bus, bus 14, is 0.5749 p.u. The LM obtained from the optimization technique is plotted with the complete PV curve obtained from CPF method in Fig. 7. Again, the optimization technique provides only single solution, the maximum solution.

The necessary equation of the optimization process is shown as follows:

$$\begin{aligned}
\delta_1 &= 0 \\
P_{G2} - (1 + \lambda)P_{D2} + 10V_1V_2 \sin(\delta_1 - \delta_2) - 10V_2V_3 \sin(\delta_2 - \delta_3) &= 0 \\
-(1 + \lambda)P_{D3} + 10V_2V_3 \sin(\delta_2 - \delta_3) &= 0 \\
V_1 &= 1 \\
V_2 &= 1 \\
-(1 + \lambda)Q_{D3} + 10V_2V_3 \cos(\delta_2 - \delta_3) - 10V_3^2 &= 0 \\
w_1 + 10w_2V_1V_2 \cos(\delta_1 - \delta_2) &= 0 \\
w_2 [-10V_1V_2 \cos(\delta_1 - \delta_2) - 10V_2V_3 \cos(\delta_2 - \delta_3)] \\
+ 10w_3V_2V_3 \cos(\delta_2 - \delta_3) - 10w_6V_2V_3 \sin(\delta_2 - \delta_3) &= 0 \\
10w_2V_2V_3 \cos(\delta_2 - \delta_3) - 10w_3V_2V_3 \cos(\delta_2 - \delta_3) \\
+ 10w_6V_2V_3 \sin(\delta_2 - \delta_3) &= 0 \\
10w_2V_2 \sin(\delta_1 - \delta_2) + w_4 &= 0 \\
w_2 [10V_1 \sin(\delta_1 - \delta_2)] - 10V_3 \sin(\delta_2 - \delta_3) \\
+ 10w_3V_3 \sin(\delta_2 - \delta_3) + w_5 + 10w_6V_3 \cos(\delta_2 - \delta_3) &= 0 \\
-10w_2V_2 \sin(\delta_2 - \delta_3) + 10w_3V_2 \sin(\delta_2 - \delta_3) \\
+ w_6 [10V_2 \cos(\delta_2 - \delta_3) - 20V_3] &= 0 \\
-1 - 1.5w_2 - 2w_3 - w_6Q_{D3} &= 0
\end{aligned}$$

6.3 Simulation Times

Simulation times used to simulate load flow, CPF and optimization technique are shown in Table 3 for the test systems. From Table, it can be observed that the proposed approach takes about 6 and 32 seconds to find the load flow solutions for the 3-bus and 14-bus test systems, respectively. The proposed approach uses about 2 and 17 minutes to plot complete PV curves based on CPF process. Simulation time for optimization technique is close to that of load flow in the test systems. It is noticed that the proposed approach is simulated by using AMD 1.3 MHz notebook computer.

Table 3. Simulation time for the test systems

Case	Simulation Time for	
	Three Bus	Fourteen Bus
Load Flow	5.588 sec	32.056
CPF	2 min 0.634 sec	17 min 6.353 sec
Optimization	6.349 sec	26.478 sec

6.4 Voltage Stability with FACTS Devices

Voltage stability with various FACTS controllers are compared in the same system. Figure 8 shows PV curves of base case and system with FACTS devices for the modified IEEE 14-bus test system. From the figure, UPFC gives the highest LM improvement followed by shunt FACTS devices and series FACTS devices. The IEEE 14-bus test system requires reactive power compensation at the distribution level, thus installing shunt reactive devices could provide higher LM than series devices. On the other hand, UPFC is composed of both shunt and series devices. Introducing UPFC can provide reactive power both at the distribution level and at the line, thus making the device the most

effective one in the terms of LM improvement in this test system.

From the simulation results, it may be concluded that the proposed method can be used to investigate voltage stability in an easy way. Simulation time used to find the solution is acceptable, not too high. This may be useful for utilities, researchers and students to do simulation on voltage stability study in an easy way.

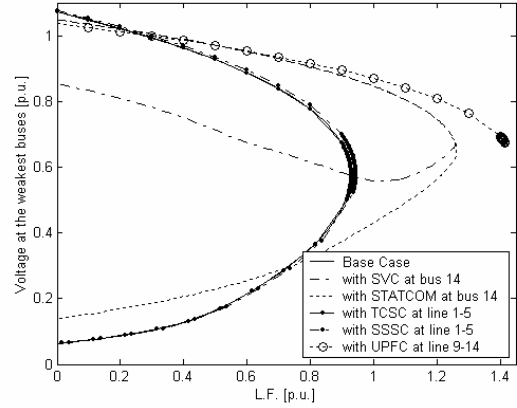


Fig. 8 PV curves of base case, and with FACTS devices.

6. CONCLUSIONS

This paper proposes a new simulation approach for static voltage stability study based on Symbolic and Optimization toolboxes. Load flow, CPF and optimization techniques are used to analyze the static voltage stability study based on the proposed technique. Equations are created by using Symbolic toolbox and the solutions can be found by using a single command in Optimization toolbox. The proposed method is applied in the power system with FACTS devices. The simulation time is acceptable, not too high. This may be useful for utilities or researchers to find voltage stability assessment of medium size power systems in a simple way.

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