

Spinning Reserve Pricing via Security Instruments in Competitive Electricity Markets
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Abstract—In new environment of restructured electric power markets, ancillary services should be procured via risk hedged mechanisms. Spinning reserve is an ancillary service that may be needed, or may not, for the purpose of reliability in power systems. Option contracts are such security instruments which can guarantee the risks for ancillary services contracts. Competition may imply electricity spot price spike, in the case of supply shortages and high variations in demand. To facilitate perfect competition, exotic option contracts in ancillary service markets will consequently induce an optimal level of reliability in power systems. This paper focuses non-standard or exotic contracts; the so-called reliability spread options that may present a basic challenge to the quantitative risk management in ancillary service markets.

Index Terms—Ancillary services, risk management, electricity contracts, deregulation, restructuring, option pricing.

I. INTRODUCTION

The ongoing deregulation of the wholesale electricity markets is forcing electric utilities to change their management strategies towards electrical energy trading. Since deregulation increases price volatility, it may result in exposing new financial risks to market participants [1]. From the application point of view, standard financial instruments can be used to emulate various flexible electricity contracts, which serve many purposes such as risk management and improving economic efficiency [2]. It can be argued that, electrical energy may require non-standard contracts. The fact that makes electrical energy and especially ancillary services (AS) so different from money markets can be found in the type of financial contracts required by the end user of derivative securities [3,4]. In electric power and ancillary service markets the derivative contracts are typically for delivery of electricity with regards to individual specifications. Examples of such specifications are a desired level of reliability and the location of delivering electricity [2]. This paper focuses on pricing and hedging of such contracts by using proposed non-standard or exotic options.

II. WHY OPTION CONTRACTS ARE NEEDED FOR AS?

There has been a tremendous increase in the awareness and activities of financial derivatives in commodity markets in recent years. Option markets are the most popular part of financial institutions. To explore the markets well and improve investment, pricing of options has attracted researchers’ challenges for years. Modeling and predicting option prices is very important in the sense of market efficiency. The contingent claim valuation of physical assets and derivative securities depends drastically on the characteristic of the stochastic process, which may make price’s trend so volatile [3,5]. Energy commodity markets grew up rapidly as deregulation and restructuring of electricity industry were happening in the UK, USA and all around the world. It can be said that, the global trend of electricity restructuring may expose the portfolios of utilities’ assets through the risk management and asset valuation [2,6]. In fact, accurate modeling of electrical energy spot prices is needed for financial risk management issues. Furthermore, to prevent the electrical energy supply from collapsing, electricity has to be balanced instantaneously. Since there is no inventory for electricity to smooth the supply-demand imbalance, therefore, the electricity spot price is so volatile. On the other hand, among different types of derivative security contracts, option contracts respect the price volatility as a major issue considerably [5].

Electricity derivatives have payoffs dependent on the spot price of electricity, which is volatile, therefore, the electrical energy and ancillary services may require exotic option contracts rather than the standard ones. In fact, exotic options are such derivative securities with more complicated payoff than the standard ones, either European or American call and/or put options. Most exotic options, which are traded in every market, should be designed to meet particular needs with regards to the market efficiency [6]. On the other hand, competition may imply electricity spot price spike, in the case of supply shortages and high variations in demand. To facilitate perfect competition, exotic option contracts in electricity markets will consequently induce an optimal level of reliability in power systems [3]. Therefore, new non-standard or exotic options may present a basic challenge to the quantitative risk management in electricity industry.

III. OPTION STRUCTURE AND OPTION EVALUATION

The holder of an option will pay the option premium for the right the option provides. Whether exercising the right or not,
the option will be expired after expiration date. The relative value of the underlying price at the time of the option exercise to the option’s strike price will specify whether the option will be exercised or not. The difference between the underlying price and the strike price is called option’s parity value, which represents the value of the option at the option expiration date. A call option and a put option are represented with the following mathematical statements, respectively.

\[ CPV: \quad \text{Call Parity Value} = \max(0, SP - Xc) \]

\[ PPV: \quad \text{Put Parity Value} = \max(0, Xp - SP) \]

Up on the amount of parity value, an option can be identified in three different situations: Out of the Money, At the Money, and In the Money, which are schematically illustrated as both call and put options, in Fig. 1.

![Figure 1. Call/put parity values.](image)

**IV. RESERVE CONTRACTS VIA EXOTIC OPTIONS**

For option evaluation, the basic formulation proposed by Black-Scholes can be used as a foundation of most option evaluation techniques [10]. Evaluation of exotic options can be done through a wide range of methods such as Modified Black-Scholes (MBS) equations, Binomial Tree (BT), Geometric Brownian Motion (GBM) price process, and Mean Reverting (MR) price process. In addition, some advanced intelligent techniques can be applied for modeling and evaluating electricity options [9]. In the following section some of these techniques are briefly discussed.

**A. Exotic Reliability Spread Option for Electricity and Reserve**

Based on the definition, a spread option is an option where the pay-off is dependent on the difference between two market variables. In other words a spread option is obtainable by holding a long position and a short position in two related contracts to capture profit from a changing price relationship [7]. In fact, in electricity industry, electrical energy and reserve are such commodities where the markets for reserve and electric power are tightly dependent. On the other hand, due to the non-storable nature of electricity, the traditional storage-based, no-arbitrage methods of valuing commodity derivatives are not achievable [8]. By assuming different reliability levels in power systems, the price of real power as well as the price of ancillary services may change widely. For example, different System Reserve Requirement (SRR) can provide different level of reliability, while it makes different commodities from economic point of view [9,10]. It can be said that a Reliability Spread is an option where the pay-off is dependent on the difference between energy market and reserve market. In the next section, such Reliability Spread is introduced considering different SRR. Different pricing mechanisms are considered to evaluate the value of Reliability Spread option for electricity industry.

**B. The Black-Scholes Option Pricing Formula and Drift**

The fundamental insight of the option pricing models of Black and Scholes is the existence of a dynamic investment strategy involving the underlying assets that replicates the option’s payoff exactly [7]. In particular, if the underlying asset’s price \( P(t) \) satisfies the following stochastic differential equation:

\[
\begin{align*}
\ln( (P(t)) dP(t) &= \mu dt + \sigma dW(t) \\
\end{align*}
\]

Where:

\( \sigma \) is diffusion coefficient,

\( \mu \) is drift coefficient,

\( W(t) \) is standard Wiener process. Under the no-arbitrage condition associated with instantaneous risk-free rate of return, \( r \), the following result on call option price is obtained:

\[
\begin{align*}
&\frac{1}{2} \sigma^2 P(t)^2 \frac{\partial^2 C}{\partial P^2} + rP(t) \frac{\partial C}{\partial P} + \frac{\partial C}{\partial t} - rC = 0
\end{align*}
\]

Given the two boundary conditions for call option,

\[
C(D(T), T) = \max[P(T) - K, 0]
\]

and \( C(0,t) = 0 \), there exists a unique solution to the partial differential equation (3) as the following:

\[
C_{bs}(P(t), t; K, T, r, \sigma) = P(t)\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)
\]

Where:

\[
\begin{align*}
d_1 &= \frac{\ln(P(t)/K) + (r + \frac{1}{2} \sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= d_1 - \sigma\sqrt{T-t}
\end{align*}
\]

and \( \Phi(\cdot) \) is the standard normal cumulative distribution function.
\[ \frac{dF_E}{F_E} = \mu_E dt + \sigma_E dB_1 \quad \text{and} \quad \frac{dF_R}{F_R} = \mu_R dt + \sigma_R dB_2 \]

Where $B_1$ and $B_2$ are two Wiener processes with instantaneous correlation $\rho$ [4,5,7].

1) **Valuation of Reliability Spread Options for GBM.**

The time value of a Reliability Spread call option at maturity time, $T$, can be denoted by $V(x,y,t) = C_{GBM}(F_{E,T}^t, F_{R,T}^t, K_{RE}, T-t)$, where $F_{E,T}^t$ and $F_{R,T}^t$ represent the price of electrical energy (£/MWh) and capacity reservation (£/MW) at time $t$, respectively, for future contract with maturity date $T$ [10]. The value of a reliability spread call option that can be calculated from the closed-form solution for $C_{GBM}$ is:

\[ C_{GBM}(F_{E,T}^t, F_{R,T}^t, K_{RE}, T-t) = e^{-r(T-t)}[F_{E,T}^t N(d_1) - K_{RE} F_{R,T}^t N(d_2)] \]

Where:

\[ d_1 = \frac{\ln \left( \frac{F_{E,T}^t}{F_{R,T}^t} \right) + \frac{\nu^2}{2} (T-t) }{\nu \sqrt{T-t}} \]

\[ d_2 = d_1 - \nu \sqrt{T-t} \]

\[ \nu^2 = \sigma_E^2 - 2\rho \sigma_E \sigma_R + \sigma_R^2 \]

D. **Mean Reversion (MR) Price Process**

If the futures price processes of electricity and contingency capacity reservation can be assumed to follow the mean reversion processes, then we can write:

\[ \begin{cases} dF_E = \alpha_E (\mu_E(t) - \ln F_E)F_E dt + \sigma_E(t)F_E dB_1 \\ dF_R = \alpha_R (\mu_R(t) - \ln F_R)F_R dt + \sigma_R(t)F_R dB_2 \end{cases} \]

Where $\mu_E(t)$ and $\mu_R(t)$ are the long-term means, $\alpha_E$ and $\alpha_R$ are the mean reversion coefficients, and $B_1$ and $B_2$ are two Wiener processes with instantaneous correlation $\rho$ [10,11].

1) **Valuation of Reliability Spread Options for MR**

The time value of a reliability spread call option at maturity time, $T$, can be denoted by

\[ V(x,y,t) = C_{MR}(F_{E,T}^t, F_{R,T}^t, K_{RE}, T-t) \]

where $F_{E,T}^t$ and $F_{R,T}^t$ represent the price of electrical energy (£/MWh) and capacity reservation (£/MW) at time $t$, respectively, for future contracts with maturity date $T$. The value of a reliability spread call option that can be calculated from the closed-form solution for $C_{MR}$ is:

\[ C_{MR}(F_{E,T}^t, F_{R,T}^t, K_{RE}, T-t) = e^{-r(T-t)}[F_{E,T}^t N(d_1) - K_{RE} F_{R,T}^t N(d_2)] \]

Where:

\[ \begin{cases} d_1 = \frac{\ln \left( \frac{F_{E,T}^t}{F_{R,T}^t} \right) + \frac{\nu^2}{2} (T-t) }{\nu \sqrt{T-t}} \\ d_2 = d_1 - \nu \sqrt{T-t} \end{cases} \]

\[ \nu^2 = \sigma_E^2 - 2\rho \sigma_E \sigma_R + \sigma_R^2 \]

V. **Case Study and Results Analysis**

In the following sections two different case studies are presented to show the effectiveness of the proposed exotic options for contingency reserve option contracts. To carry out the value of call option for contingency reserve, the proposed Reliability Spread is applied to the IEEE 30-bus system. Firstly, the spot price of real power and reserve for different level of reliability (SRR) are derived through flexible joint dispatch model [13], which is shown in Fig. 2. The statistical data of spot price of real power and reserve, which are extracted from the second phase of the three-stage algorithm [14,15], are shown in Table I.
contracts via introducing **Reliability Spread** option as an efficient trading mechanism is presented. Valuation of reliability spread option through different criteria such as Geometric Brownian Motion model and Mean Reverting model is derived. To show the effectiveness of proposed methodology, these evaluation techniques are applied to a case that shows the MR model has better result than GBM model.

**VII. References**


**VIII. Biographies**

Ali Akbar Gharaveisi received his BSc., MSc., and PhD. in 1990, 1994, and 2000 respectively in Elec. Eng. from Ferdoussi University of Mashad, Iran. His research interests are in Nonlinear control, Power system stability studies, Fuzzy systems, Artificial intelligence. He is now assistant professor in Shahid Bahonar of Kerman University of Iran.
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