Application of Particle Swarm Optimization to Economic Dispatch Problem: Advantages and Disadvantages

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Abstract—This paper summarizes the state-of-art particle swarm optimization (PSO) applications for resolving the economic dispatch (ED) problem, which is considered as one of the complex problems to be tackled. The PSO techniques have drawn much attention from the power system community and been successfully applied in many complex optimization problems in power systems. This paper focuses on the application of PSO techniques to the ED problems and describes their advantages and disadvantages in resolving the ED problems.

Index Terms—Particle swarm optimization, economic dispatch, advantages and disadvantages of PSO.

I. INTRODUCTION

PARTICLE swarm optimization (PSO) is one of the modern heuristic algorithms, which can be used to solve nonlinear and non-continuous optimization problems [1]. It is a population-based search algorithm and searches in parallel using a group of particles similar to other AI-based heuristic optimization techniques. The original PSO suggested by Kennedy and Eberhart is based on the analogy of swarm of bird and school of fish [2]. Each particle in PSO makes its decision using its own experience and its neighbor’s experiences for evolution. That is, particles approach to the optimum through its present velocity, previous experience, and the best experience of its neighbors [3]. The main advantages of the PSO algorithm are summarized as: simple concept, easy implementation, robustness to control parameters, and computational efficiency when compared with mathematical algorithm and other heuristic optimization techniques.

In a physical n-dimensional search space, the position and velocity of particle-i are represented as the vectors $X_i = (x_{i1}, \ldots, x_{in})$ and $V_i = (v_{i1}, \ldots, v_{in})$ in the PSO algorithm. Let $P_{besti} = (v_{i1}^{Pbest}, \ldots, v_{in}^{Pbest})$ and $G_{best} = (v_{1}^{Gbest}, \ldots, v_{n}^{Gbest})$ be the best position of particle $i$ and its neighbors’ best position so far, respectively. The modified velocity and position of each particle can be calculated using the current velocity and the distance from $P_{besti}$ to $G_{best}$ as follows:

$$V_i^{k+1} = \omega \cdot V_i^k + c_1 \cdot r_1 \cdot (P_{besti}^k - X_i^k) + c_2 \cdot r_2 \cdot (G_{best}^k - X_i^k)$$

(1)

$$X_i^{k+1} = X_i^k + V_i^{k+1}$$

(2)

where,$\quad V_i^k$ velocity of particle $i$ at iteration $k$,
$\omega$ inertia weight factor,
$c_1, c_2$ acceleration coefficients,
$r_1, r_2$ random numbers between 0 and 1,
$X_i^k$ position of particle $i$ at iteration $k$,
$P_{besti}^k$ best position of particle $i$ until iteration $k$,
$G_{best}^k$ best position of the group until iteration $k$.

In this velocity updating process, the values of parameters such as $\omega$, $c_1$, and $c_2$ should be determined in advance. In general, the weight $\omega$ is set according to the following equation [1]:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{Iter_{max}} \times Iter$$

(3)

where,$\quad \omega_{max}, \omega_{min}$ initial, final weights,
$Iter_{max}$ maximum iteration number,
$Iter$ current iteration number.

Recently, PSO has been successfully applied to various fields of power system optimization. Most of power system optimization problems including economic dispatch (ED) have complex and nonlinear characteristics with heavy equality and inequality constraints. The primary objective of the ED problem is to determine the optimal combination of power outputs of all generating units so as to meet the required load demand at minimum operating cost while satisfying system equality and inequality constraints. In the traditional ED problem, the cost function for each generator has been approximately represented by a single quadratic function and is solved using mathematical programming based on the optimization techniques such as lambda-iteration method, gradient-based method, etc. [4]. These mathematical methods require incremental or marginal fuel cost curves which should be monotonically increasing to find global optimal solution. Unfortunately, the input-output
characteristics of generating units are inherently highly nonlinear because of prohibited operating zones, valve-point loadings, and multiple effects, etc. Thus, the practical ED problem is represented as a non-smooth optimization problem with equality and inequality constraints, which cannot be solved by the traditional mathematical methods. Dynamic programming (DP) method [5] can solve such types of problems, but it suffers from so-called the curse of dimensionality. Over the past few years, in order to solve these problems, many salient methods have been developed such as genetic algorithm (GA) [6], evolutionary programming (EP) [7], [8], Tabu search [9], neural network approaches [10], and PSO based approaches [11]-[15].

Among these methods, the focus of this paper is to survey and summarize PSO applications to the ED problems. We hope that this paper will serve as a good starting point for those interested in learning about the development of PSO and its applications in ED problems.

II. ED PROBLEM FORMULATIONS

A. Objective Function

The objective of the economic dispatch problem is to minimize the total fuel cost of thermal power plants subjected to the operating constraints of a power system. The simplified cost function of each generator can be represented as a quadratic function as described in (5).

\[ F_T = \sum_{i=1}^{N} F_i(P) \]

\[ F_i(P) = a_i + b_i P_i + c_i P_i^2 \]

where,

- \( F_T \) = total generation cost,
- \( F_i \) = function of generator \( i \),
- \( a_i, b_i, c_i \) = cost coefficients of generator \( i \),
- \( P_i \) = power of generator \( i \),
- \( N \) = number of generators.

1) Cost Function Considering Valve-Point Effects

The generating units with multi-valve steam turbines exhibit a greater variation in the fuel-cost functions. Since the valve point results in the ripples as shown in Fig. 1, a cost function contains higher order nonlinearity [11]. To take account for the valve-point effect, sinusoidal functions are added to the quadratic cost functions. Therefore, equation (5) should be replaced as (6) as follows:

\[ F_i(P) = a_i + b_i P_i + c_i P_i^2 + e_i \times \sin(f_i \times (P_{\min} - P_i)) \]

where \( e_i \) and \( f_i \) are the coefficients of unit \( i \) reflecting valve-point effects.

2) Cost Function with Multiple Fuels

Since the dispatching units are practically supplied with multi-fuel sources, each unit should be represented with several piecewise quadratic functions reflecting the effects of fuel type changes [11] as shown in Fig. 2. In general, a piecewise quadratic function can be used to represent the input-output curve of a generator with multiple fuels and described as (7).

\[ F_i(P) = \begin{cases} 
  a_{i1} + b_{i1} P_i + c_{i1} P_i^2 & \text{if } P_{\text{min}} \leq P_i \leq P_{i1} \\
  a_{i2} + b_{i2} P_i + c_{i2} P_i^2 & \text{if } P_{i1} \leq P_i \leq P_{i2} \\
  \vdots & \vdots \\
  a_{ip} + b_{ip} P_i + c_{ip} P_i^2 & \text{if } P_{ip-1} \leq P_i \leq P_{ip} \\
  \end{cases} \]

where \( a_{ip}, b_{ip}, c_{ip} \) are the cost coefficients of generator \( i \) for the \( p \)-th power level.

B. Equality and Inequality Constraints

1) Active power balance equation

For power balance, an equality constraint should be satisfied. The total generated power should be the same as total load demand plus the total line loss

\[ \sum_{i=1}^{N} P_i = P_D + P_{\text{loss}} \]

where \( P_D \) is the total system demand and \( P_{\text{loss}} \) is the total line loss.
2) **Minimum and maximum power limits**

Generation output of each generator should be laid between maximum and minimum limits. The corresponding inequality constraints for each generator are

\[
P_{i,\text{min}} \leq P_i \leq P_{i,\text{max}}
\]  

(9)

where \( P_{i,\text{min}} \) and \( P_{i,\text{max}} \) are the minimum and maximum output of generator \( i \), respectively.

3) **Generator ramp rate limits**

The generator output cannot be raised or lowered to any value instantaneously. The operating range of all online units is restricted by their ramp rate limits as follows:

\[
P_i - P_i^0 \leq DR_i \quad \text{and} \quad P_i - P_i^0 \leq UR_i
\]  

(10)

where \( P_i^0 \) is the previous output power of unit \( i \). \( DR_i \) and \( UR_i \) are the down ramp and up ramp limits, respectively.

4) **System spinning reserve requirements**

The system spinning reserve constraint for securing power system security is summarized as follows:

\[
\sum_{i=1}^{N_G} \left[ \min(P_{i,\text{max}} - P_i, UR_i) \right] \geq S_R
\]  

(11)

where \( S_R \) is the system spinning reserve requirement in MW.

5) **Generator prohibited operating zones**

The operating zone of a generating unit may not be available always for power generation due to limitations in practical operating constraints as shown in Fig. 3. The constraint is described in (12);

\[
P_i \in \begin{cases} 
P_{i,\text{min}} \leq P_i \leq P_{i,k}^l, & k = 2,3,\ldots,N_{PZ,i} \\
P_{i,k-1}^l \leq P_i \leq P_{i,k}^u, & k = 2,3,\ldots,N_{PZ,i} \\
P_{i,N_{PZ,i}} \leq P_i \leq P_{i,\text{max}} 
\end{cases}
\]  

(12)

where \( P_{i,k}^l \) and \( P_{i,k}^u \) are the lower and upper boundary of prohibited operating zone of unit \( i \), respectively. \( N_{PZ,i} \) is the number of prohibited zones of unit \( i \).

6) **Line flow constraints**

\[
\left| P_{ij,k} \right| \leq P_{ij,k}^{\text{max}}, \quad k = 1,2,\ldots,L
\]  

(13)

where \( P_{ij,k} \) is the real power of line \( k \) and \( L \) is the number of transmission lines.

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**III. SURVEY OF PSO APPLICATIONS TO ED PROBLEMS**

The practical ED problem must consider not only the cost function with valve-point and multi-fuel effects but also equality and inequality constraints such as power balance, power generation limits, system spinning reserve, generator ramp rate limits, and generator prohibited operating zones, etc.

Park et al. [11] suggested a modified PSO technique to solve the ED problems with non-smooth cost functions incorporating dynamic search space reduction strategy. In this paper, the objective function is formulated as a combination of piecewise quadratic cost functions instead of having a single convex function for each generating unit in order to consider practical operating conditions like valve-point and multi-fuel effects as described in (6) and (7), respectively. In order to implement the PSO algorithm for solving the ED with valve-point effect, the parameters were set as follows; the population size is set as 20, initial weight (i.e., \( \omega_{\text{max}} \)) is 1.0, final weight (i.e., \( \omega_{\text{min}} \)) is 0.5, and acceleration coefficients are set as 2.

And PSO algorithm for solving the ED with multiple fuels assigned the parameters as follows; the population size is 30, initial weight (i.e., \( \omega_{\text{max}} \)) is 0.5, final weight (i.e., \( \omega_{\text{min}} \)) is 0.1, and acceleration coefficients are set as 2. Also, in the paper, a technique to treat equality and inequality constraints in a PSO mechanism is suggested.

El-Gallad et al. [12] added new constraints to the ED problems by considering system spinning reserve and generator prohibited operating zones as given in (11) and (12), respectively. In this paper, the conventional PSO technique is applied to solve non-smooth ED problems. Gaing [13] considered the generator ramp rate limits (10) in the same problem treated in reference [12]. Also, the paper adopts the original PSO technique while the fitness function is mapped into \([0,1] \). In the paper, the parameters were used as follows; the population size is 100, the maximum iteration number is 200, final weight (i.e., \( \omega_{\text{max}} \)) is 0.9, initial weight (i.e., \( \omega_{\text{min}} \)) is 0.4, and acceleration coefficients are 2.

Gaing [14] carried out the dynamic ED that must satisfy not only the system load demand and the spinning reserve capacity but also some practical operation constraints that include the ramp rate limits (10), the prohibited operating zones (12), and the line flow limits (13). Victoire and Jeyakumar [15] also implemented the dynamic ED problem using PSO combined with sequential quadratic programming (SQP). This hybrid method integrates PSO algorithm as the main optimizer with SQP as the local optimizer to fine-tune the solution region. In order to implement the PSO algorithm, the parameters were set as follows: the population size is 100, the maximum iteration number is 30000, initial weight (i.e., \( \omega_{\text{max}} \)) is 1.3, final weight (i.e., \( \omega_{\text{min}} \)) is 0.7, and acceleration coefficients are 2.

**IV. ADVANTAGES AND DISADVANTAGES OF PSO**

A PSO is considered as one of the most powerful methods for resolving the non-smooth global optimization problems.
and has many key advantages as follows:

- PSO is a derivative-free technique just like as other heuristic optimization techniques.
- PSO is easy in its concept and coding implementation compared to other heuristic optimization techniques.
- PSO is less sensitivity to the nature of the objective function compared to the conventional mathematical approaches and other heuristic methods.
- PSO has limited number of parameters including only inertia weight factor and two acceleration coefficients in comparison with other competing heuristic optimization methods. Also, the impact of parameters to the solutions is considered to be less sensitive compared to other heuristic algorithms [16].
- PSO seems to be somewhat less dependent of a set of initial points compared to other evolutionary methods, implying that convergence algorithm is robust.
- PSO techniques can generate high-quality solutions within shorter calculation time and stable convergence characteristics than other stochastic methods [13].

The major drawback of PSO, like in other heuristic optimization techniques, is that it lacks somewhat a solid mathematical foundation for analysis to be overcome in the future development of relevant theories. Also, it can have some limitations for real-time ED applications such as 5-minute dispatch considering network constraints since the PSO is also a variant of stochastic optimization techniques requiring relatively a longer computation time than mathematical approaches. However, it is believed that the PSO-based approach can be applied in the off-line real-world ED problems such as day-ahead electricity markets. Also, the PSO-based approach is believed that it has less negative impact on the solutions than other heuristic-based approaches. However, it still has the problems of dependency on initial point and parameters, difficulty in finding their optimal design parameters, and the stochastic characteristic of the final outputs.

V. CONCLUSION

The main focus of this discussion is to survey and summarize the applications of PSO for solving the ED problems including the advantages and disadvantages of PSO-based approaches. The PSO algorithm has been getting much attention in various applications including power system optimization problems. Especially, in solving the ED problems, it is believed that the PSO-based application to non-smooth ED problems outperforms other state-of-the-art heuristic or mathematical algorithms. However, PSO algorithms still need further research and development to improve its performance and to obtain the robustness. Also, for the application in the real-world ED problems, it is necessary to combine the conventional mathematical approach with the PSO methods based on their own merits.

VI. REFERENCES


VII. BIOGRAPHIES

Kwang Y. Lee received the B.S. degree in Electrical Engineering from Seoul National University, Seoul, Korea, in 1964, the M.S. degree in Electrical Engineering from North Dakota State University, Fargo, ND, in 1967, and the Ph. D. degree in Systems Science from Michigan State University, East Lansing, in 1971. He is currently a Professor of Electrical Engineering at the Pennsylvania State University, University Park, PA. His research interests include control theory, computational intelligence and their application to power systems. Dr. Lee is currently the Director of Power Systems Control Laboratory at Penn State. He is a Fellow of IEEE, an Associate Editor of IEEE Transactions on Neural Networks, and Editor of IEEE Transactions on Energy Conversion.
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