Allocation of TCSCs for Mitigating Low-frequency Oscillation on a Tie-line in An Interconnected Power Systems

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Abstract—Low-frequency oscillations observed on trunk transmission systems have been the subject of studies in many areas of operation, control, and devices. While stabilizing controllers such as excitation controllers and speed governors have been widely applied to power systems, power electronics-based control devices have been the focus of research and development in recent years in improving transient stability. Such control devices are utilized in controlling real and reactive power flows in a transmission line or in a power system. However, there is a limit to the number of such devices to install; therefore, a proper allocation of these controllers is an important and practical issue. This paper proposes an approach to determine the number and location of Thyristor-Controlled Series Capacitors (TCSCs) to mitigate low-frequency oscillation in an interconnected power system.

Index Terms—Low-frequency oscillation, FACTS, TCSC, inter-area mode, Japanese power system model.

I. INTRODUCTION

As the trend of restructuring power utility industry continues world wide, power system stability has become an important issue, and maintaining the stability has become a social concern. Recently, the combined effect of various factors has made it more difficult to maintain system stability. These factors include the increased use of heavy-loaded long-distance transmission lines from power sources in remote and distant locations, an imbalance in power stations in different areas due to environmental constraints and construction cost, and the difficulty of securing the right of way for transmission lines. In addition to the increased complication of the system, the societal requirement to maintain and supply high quality power is heightened and maintaining the stability has become a significant issue. Also, under the present system in Japan, where only conventional stabilization control devices are being used to damp oscillation, there is a growing concern on the occurrence of low-frequency oscillation phenomena and developing countermeasures is viewed as an urgent task.

A typical low-frequency oscillation is observed in a tie-line between two large power systems. Such low-frequency oscillations are due to inter-area modes. The inter-area mode oscillation has a long history. It has been observed in the tie-line connecting the large Pacific Southwest and the Pacific Northwest in the United States. It has been reported on the tie-line connecting the northern Midwest and Canada [1][2]. Low-frequency oscillations have been also observed in a longitudinal power system in Taiwan [3][4][5]. In the Japanese power system, low-frequency oscillation has been observed at 2.3 seconds per cycle on a tie-line connecting two utilities [6]. As for improving the inter-area modes, power system stabilizers (PSSs) have been effective controllers, since the controllers can increase damping torque by simply introducing additional signal into the automatic voltage regulator (AVR) [7][8].

To improve the transient stability, various power electronics-based control devices have been the focus of research and development in recent years. Flexible AC Transmission System (FACTS) devices, including the Thyristor Controlled Series Capacitor (TCSC), have been implemented in practical applications. The TCSC has a capacitor and a thyristor-controlled reactance in parallel, and its net capacitive reactance can be controlled in a fast and continuous fashion.

There have been several of these FACTS devices implemented in real power systems in the United States and other places [9]. A TCSC was installed at Kayenta substation in Arizona to compensate its 300 km, 230 kV transmission line. This TCSC was used to evaluate the role of power flow control in transmission line capacity enhancement, power flow control, and damping of sub-synchronous resonance [10]. A similar system has been installed in Slatt substation in Oregon [11]. In China, a TCSC is installed at Fengtun station for the 500 kV Yimin-Fengtun line [12]. The TCSCs are still under evaluation for power system stabilization and their impact on reducing capital investment for future transmission lines [13]. More recently, a unified power flow controller (UPFC) that
compensates both active and reactive power flows is under development [14][15]. These devices are utilized for the control of power flow in transmission lines or reactive power in a power system. For reactive power compensation, an optimal allocation of capacitors has been studied [16]. The research provides an appropriate placement of compensation devices to satisfy suitable reactive constraints while minimizing the cost of compensation. A proper allocation of FACTS devices as well as the placement of compensation devices is an important and practical issue.

In recent years, various advanced control theory have been introduced to design controllers to improve power system stability. The H∞ control theory enables the design of a robust controller against various operating conditions. The Linear Quadratic Regulator (LQR) guaranteeing the system stability is utilized in the control system design, and the H∞ control ensures the robustness against changing operating conditions.

This paper proposes an approach to determine the number and location of the TCSCs in an interconnected power system. An appropriate allocation result is determined to mitigate low-frequency oscillation in a tie-line. If low-frequency oscillation is damped by installing the least number of control devices at appropriate locations, significant economic gain can be expected. For this reason, it is very important to have a method for determining the location for TCSCs on a realistic power system model. In the proposed approach, the H∞ norm is utilized as an index for an appropriate allocation. The proposed method is applied to the public domain IEEJ EAST10-machine system model, which is a standard model of the Japanese trunk power system [17][18].

II. OPTIMAL ALLOCATION ASSESSMENT OF TCSCS

The FACTS devices, such as TCSC and UPFC, are installed at a generator terminal, and utilized to control Subsynchronous resonance (SSR) in the generator [26]. They can be also installed in a substation, and stabilize a power system by changing the impedance of transmission lines. In other words, they can be installed anywhere in a trunk power system. On the other hand, there is a limit to the total number of FACTS devices to be installed, since their cost is very expensive; therefore, a proper allocation of controllers is an important and practical issue. This paper determines the optimal allocation of the TCSCs, which mitigate the low-frequency oscillation by enhancing power system stability.

Low-frequency oscillation occurs in a very large system of interconnected, multi-machine systems. Therefore, an aggregate linear model of a multi-machine power system has been used, which does not involve detailed analysis of a multi-machine power system but is effective in developing the transmission line system structure [19].

A. Eigenvalue Analysis of Multi-Machine Power System

Eigenvalue analysis captures the characteristics of the system dynamics without a time domain simulation; therefore, it is effective in evaluating the system stability for a multi-machine power system [20][21].

The swing equation of a generator, which indicates the energy balance between mechanical input and electrical output, is expressed as follows:

\[ M_i \frac{d^2}{dt^2} \delta_i = P_{mi} - P_{ei} - D_i \omega_i \]  \hspace{1cm} (1)

The electrical output of the generator is calculated as:

\[ P_{ei} = \sum_{j=1}^{n} E_j E_j' \cos(\theta_{ij} - \delta_j + \delta_i) \]  \hspace{1cm} (2)

Also, a damping constant of each generator is expressed as follows [22]:

\[ D = e^\omega_{b} \left\{ \left( X_d ' - X_q ' \right) T_{se} ' \sin^2 \delta_0 + \left( X_q ' - X_q ' \right) T_{se} ' \cos^2 \delta_0 \right\} \]  \hspace{1cm} (3)

The parameters in (1) to (3) are listed in Table A1 in Appendix.

In the case of stability analysis, the damping constants are important parameters since the parameters affect the system eigenvalues. Network reduction has been performed for a multi-machine power system, which includes transmission lines, transformers, loads, and other control equipments. The system eigenvalues evaluated for the reduced system thus reflect the effects of the components and structures of the original power system. A condition, that all eigenvalues are in the negative real half of the complex plane, has been well known for a stable system. Also, an eigenvalue close to the imaginary axis, influences the system stability severely. Moreover, the imaginary parts of the system eigenvalues dominate the system oscillation frequency in the time domain.

B. Calculation of Participation Rate

The power system dynamics can be evaluated by analyzing the system eigenvalues. In this evaluation, a dominant root located near the imaginary axis, can be recognized. In the case of stabilizing the dominant root by applying an appropriate control action, the power system stability can be enhanced. This paper proposes an approach to determine which generators should be included in order to enhance the system stability by using participation factors [23]. The participation factor is effective in finding state variables affecting the dominant root. The factor is derived from eigenvectors. First, eigenvector \( \phi_i \) is calculated with an eigenvalue \( \lambda_i \) of the system to form a matrix:

\[ \Phi = [\phi_1 | \phi_2 | \cdots | \phi_n] \]  \hspace{1cm} (4)

Next, a vector \( \psi_i \) is defined as below:

\[ \Psi = \left[ \Phi^{-1} \right]^T = [\psi_1 | \psi_2 | \cdots | \psi_n] \]  \hspace{1cm} (5)

In this case, a participation factor \( p_{ij} \) is defined as follows:

\[ p_{ij} = \phi_{ij} * \psi_{ij} \]  \hspace{1cm} (6)

Here, the matrices \( \Phi \) and \( \Psi \) are related as \( \Phi * \Psi^T = I \) and \( \Psi^T * \Phi = I \); therefore:

\[ \sum_{j=1}^{n} p_{ij} = \sum_{j=1}^{n} \phi_{ij} * \psi_{ij} = 1.0 + j0.0 \]  \hspace{1cm} and \[ \sum_{j=1}^{n} p_{ij} = 1.0 + j0.0 \]  \hspace{1cm} (7)

The participation factor \( p_{ij} \) expresses the influence or
sensitivity of the \( i \)-th state variable on the eigenvalue \( \lambda_j \). Equation (7) imply that the numerical values are normalized. In this paper, the state variables affecting the dominant root is determined by the evaluation of the participation factors. Thus, controllers which can stabilize the dominant root has been determined and stability is assured with the least number of the controllers.

C. Sensitivity Analysis and Sensitivity Matrix

Sensitivity analysis determines the influences on each variable due to the changes in the system. Sensitivity can be classified into two types. One is the steady-state sensitivity, and the other is the dynamic sensitivity. Steady-state sensitivity is evaluated with the power flow equation. On the other hand, dynamic sensitivity is evaluated with the dynamic system models including generator dynamics.

Power system is treated as a steady-state system in power flow calculations. A steady-state sensitivity matrix can be calculated with deviations from a nominal steady state:

\[
F(X_0 + \Delta X, U_0 + \Delta U, P_f + \Delta P) \approx F(X_0, U_0, P_0)
+ F_x \Delta X + F_u \Delta U + F_p \Delta P = 0
\]  
(8)

where, \( F \) is a vector function \( F=[f_1, f_2, \ldots, f_n]^T \), \( X \) is a vector of state variables \( X=[x_1, x_2, \ldots, x_n]^T \), \( U \) is a vector of control variables \( U=[u_1, u_2, \ldots, u_m]^T \), and \( P \) is a vector of parameter variables \( P=[p_1, p_2, \ldots, p_n]^T \). Subscript \( i \) indicates a nominal steady state and matrices \( F_x, F_u, \) and \( F_p \) are Jacobian matrices. From the definition of the nominal state, (8) implies

\[
F_x \Delta X + F_u \Delta U + F_p \Delta P = 0
\]  
(9)

Hence, the sensitivity matrices of state variables is derived as:

\[
\Delta X = -F_x^{-1}F_u \Delta U - F_x^{-1}F_p \Delta P
\]  
(10)

D. Robust Stability Index for Installing Controllers

In power system planning, reactive power equipments such as capacitors or reactors are installed to minimize the cost subject to voltage constraints. However, installation of controllers needs to satisfy the stability requirement as well as minimizing the cost. This paper proposes a robust stability index for installing TCSCs to maximize the power system stability using the \( H_\infty \) control theory. The purpose of the \( H_\infty \) control is to design a robust controller by minimizing the \( H_\infty \) norm of the closed-loop system given in (11) to (14):

\[
\dot{x}(t) = Ax(t) + B_u u(t) + B_p p(t)
\]  
(11)

\[
y(t) = C_x x(t)
\]  
(12)

\[
z(t) = C_z x(t)
\]  
(13)

\[
\begin{bmatrix}
\dot{x} \\
z
\end{bmatrix} =
\begin{bmatrix}
A - B_u F C_x & B_p \\
C_z & 0
\end{bmatrix}
\begin{bmatrix}
x \\
w
\end{bmatrix}
\]  
(14)

where, \( x \) is the state variable, \( w \) is a disturbance into the controlled system, and \( z \) is the output from the system. In the closed-loop system, the \( H_\infty \) control designs a feedback control gain \( F \), which mitigates disturbances from a disturbance parameter \( w \) to an output parameter \( z \). An influence of disturbances against the system is measured by the \( H_\infty \) norm.

The \( H_\infty \) control realizes a robust controller, but it does not consider stabilizing eigenvalues; therefore, the proposed approach makes use of the concept of \( H_\infty \) control only as a stability index, but determines the controller by using the Linear Quadratic Regulator (LQR) [24]. The LQR solves the algebraic Riccati equation, which is defined by:

\[
A^T X + XA + Q - XB R^{-1} B^T X = 0
\]  
(15)

where, matrices \( Q \) and \( R \) are weighting matrices in a quadratic performance index as follows:

\[
J = \int_0^\infty [x^T Q x + u^T R u] dt
\]  
(16)

A feedback gain based on the LQR is given by:

\[
F = -R^{-1} B^T X
\]  
(17)

The LQR regulates and stabilizes eigenvalues of the controlled system. Hence, when the control gain \( F \) in LQR satisfies the \( H_\infty \) norm index, it is guaranteed to stabilize the system and thus, allocation of controllers is determined.

E. \( H \infty \) Norm

In the case of a single-input single-output control system (SISO), the \( H_\infty \) norm is represented by a maximum singular value of the transfer function (14) in a Bode diagram. Therefore, the \( H_\infty \) norm means a gain, which amplifies the disturbance through the system. In the case of a multi-input multi-output system, the \( H_\infty \) norm is defined by [25]:

\[
\gamma = \|G(s)\|_{\infty} = \sup_{\omega} \sigma_{\max} [G(j \omega)]
\]  
(18)

where, \( \sigma_{\max} [G(j \omega)] \) is a maximum singular value of a matrix \( G(j \omega) \), and \( \sup \) means search for \( \omega \) from zero to infinity in a radian frequency.

F. Sensitivity Matrix of Power Flow on A Tie-line

Under deregulated power systems, it is important for a power system operator to improve power flows in transmission lines or voltages at nodes by means of adjusting control equipments. Power flow between nodes is expressed by a vector of state variables and a vector of control variables:

\[
\Delta F_L(X, U) = F_L(X + \Delta X, U + \Delta U) - F_L(X, U)
\]  
(19)

where \( F_L \) denotes power flows in transmission lines.

By using the sensitivity (10), the sensitivity of power flows is obtained as follows:

\[
\Delta F_L(X, U) = \left( -F_{x^{-1}} C_u - F_{x^{-1}} C_w \right) \Delta U
\]  
(20)

where, \( F_{x^{-1}} C_u \) means a state variable sensitivity due to adjusting control equipments.

III. SYSTEM MODEL INCLUDING TCSCS

Power system includes many generators, substations, transmission lines, loads, and controllers. These components and elements have various characteristics and affect the power system operation. A linearized power system model is used in the analysis and simulation of the power system.

A. Generator Model

The dynamic equations of a generator are expressed as follows:
\[ \frac{d}{dt} \delta(t) = \omega(t) - \omega_0 \]  

(21)

\[ \frac{d}{dt} \omega(t) = \frac{\omega_0}{2H} \left[ T_m(t) - T_e(t) - D(\omega(t) - \omega_0) \right] \]  

(22)

where, \( T_m \) expresses mechanical torque, \( T_e \) electrical torque, and \( H \) and \( D \) are inertia and damping constants, respectively.

The electrical torque is derived from voltages and currents in the \( d-q \) frame of the generator:

\[ T_e(t) = v_d(t)i_d(t) + v_q(t)i_q(t) + R \left( i_d(t)^2 + i_q(t)^2 \right) \]  

(23)

The Park’s model for generator is expressed as follows:

\[ \frac{d}{dt} e_q'(t) = \frac{1}{T_{dq}} \left[ e_f(t) - \frac{(L_d - L_q')(L_d' - L_q')}{(L_d' - L_q)^2} e_q''(t) - \omega(t) \frac{(L_d' - L_q')(L_d'' - L_q) - i_d(t)}{L_d' - L_q} \right] \]  

(24)

\[ \frac{d}{dt} e_d''(t) = \frac{1}{T_{eq}} \left[ \frac{(L_d - L_q')(L_d' - L_q')}{(L_d' - L_{dq})^2} K_d e_d''(t) - \omega(t) \frac{(L_d'(L_d'' - L_q) - i_d(t)}{L_d' - L_{dq}} \right] \]  

(25)

\[ \frac{d}{dt} e_q''(t) = \frac{1}{T_{dq}} \left[ \frac{1}{K_q} e_q''(t) - \frac{1}{K_q} e_q'(t) - \omega(t) \frac{(L_q' - L_{dq}) L_q'' - L_q}{L_q' - L_{dq}} i_q(t) \right] \]  

(26)

\[ K_d = 1 + \frac{(L_d' - L_q)(L_d'' - L_q)}{(L_d' - L_{dq})(L_d'' - L_q)} \]  

(27)

\[ K_q = 1 + \frac{(L_q' - L_{dq})(L_q'' - L_q)}{(L_q' - L_{dq})(L_q'' - L_q)} \]  

(28)

The parameters mentioned above are listed in Table A1 in Appendix.

**B. Thyristor Controlled Series Capacitor**

A TCSC controller is a capacitive reactance compensator, which consists of a series capacitor bank shunted by Thyristor-controlled reactors as illustrated in Fig. 1. It can change the impedance of transmission line in continuum by switching thyristors. A TCSC can be installed in a transmission line that does not have enough margin in power flow; therefore, it can enhance the power system stability by controlling the power flow in the line. A TCSC controller has upper and lower limits on its output values. These limits are respectively set to ±30% of the value of the transmission line susceptance. Therefore, performance of the controller is considered to be in a linear range.

The effect of TCSCs on generators is evaluated in advance by using sensitivity analysis. The sensitivity analysis is performed by changing the impedance of transmission lines in small amount. Sensitivity is evaluated for the active power outputs of all generators. Hence, a transmission line, which has the most efficient performance to the dominant generator, is selected as a candidate line in installing a TCSC. When the system stability is not improved sufficiently with one line, the same procedure is repeated again for a new dominant pole or generator.

IV. OPTIMAL ALLOCATION OF TCSCS IN AN INTERCONNECTED POWER SYSTEM

In this section, the optimal allocation of TCSCs is determined for a test system based on the approach proposed in Section II. The allocation and designed controllers for TCSCs have been verified in a Japanese power system model, IEEJ EAST10-machine model. This test system is created as a benchmark model based on the trunk transmission systems in the eastern area of Japan, and reflects characteristic features found in the actual power systems. All generators in the test system are equipped with an AVR and a speed governor recommended in the IEEJ Power System Standard Models [17].

In this paper, the peak load condition is used in allocating TCSCs. First, the paper identifies inter-area modes of the test system. Next, the location of TCSCs is determined by identifying transmission lines, which mostly affect the selected generator modes. For this purpose, the sensitivity analysis is performed by analyzing the sensitivity of generators against the changes in transmission line admittances. In the case of a fault in a transmission line, such as a three-phase fault, the configuration of power system changes significantly. Power system needs to be robust to keep its stability against such large disturbances. The \( H_\infty \) norm measures the degree of effects of an input disturbance on the system outputs. In the test system, active power flow in the tie-line is set as an output variable. The TCSCs are installed following the proposed methodology until the robust stability index is satisfied.

_A. Spectrum Analysis of Power Flow in A Tie-line_

Inter-area modes have been identified by two methods in this paper. One is the spectrum analysis, and the other is the eigenvalue analysis. Inter-area modes give rise to low-frequency oscillations in the power system. The oscillation is observed in a tie-line between large power systems. In the spectrum analysis of active power in the tie-line, the power system has two inter-area modes. The oscillation and the result of the spectrum analysis are shown in Fig. 2. Fig. 2(b) shows two major spectrums, i.e., inter-area modes No. 1 and No. 2. The cycles in the time domain in Fig. 2(a) are shown to be about 4.4 and 2.2 sec., respectively.

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**Fig. 1. Model of Thyristor Controlled Series Capacitor.**
B. Eigenvalue Analysis of Multi-machine Power System

Next, the eigenvalue analysis was applied to the test power system model expressed by (1) - (3). The eigenvalues of the controlled system are derived from these expressions with the aggregate of the power system network. The obtained eigenvalues are shown in Fig. 3. The eigenvalue analysis takes notice of the conjugate eigenvalues; therefore, only the eigenvalues in the upper half-plane are numbered in Fig. 3. A slant line area in the figure shows that an eigenvalue in this area has a peculiar oscillation between 2 and 4 seconds per cycle.

The results of the eigenvalue analysis indicate that Mode 2 and Mode 5 dominate the low-frequency oscillation in the IEEJ EAST10-machine model, and Mode 5 affects the power system stability. When these modes have been stabilized, the low-frequency oscillation is mitigated and the power system stability is enhanced.

C. Determination of the Dominant Generator by Utilizing Participation Rate

The modes dominating the low-frequency oscillation and the power system stability are selected in the eigenvalue analysis. In the case when a state variable affecting each dominant mode is identified, a dominant generator is determined. In this paper, a participation factor is utilized to determine the dominant generator. The factor is expressed in complex numbers. However, the proposed approach deals with the real part of the factor and ignores the imaginary part, since the summation of the participation factor in row or column is 1.0 + j0.0 by normalization, as shown in (7). The factors, ignoring the imaginary parts, are called participation rates in distinction from the participation factors. The participation rates for IEEJ EAST10-machine model are shown in Fig. 4. Participation rates are presented for all state variables numbered as [1: \( \Delta \omega_1 \), ..., 10: \( \Delta \omega_{10} \), 11: \( \Delta \delta_{2,1} \), ..., 19: \( \Delta \delta_{10,1} \)], and for each system mode as numbered in Fig. 3. In Fig. 4, if a state variable taking the highest participation rate is selected for the dominant root, then the corresponding dominant generator can also be determined.

D. Determination of TCSC Locations by Evaluating Generator Sensitivity

The proposed approach analyzes generator sensitivities. Generator sensitivity has been calculated in (10), which is based on the power injection equation. Therefore, the active power and reactive power of each generator are expressed as:

\[
P_g = \sum_{j=1}^{n} (G_g e_j e_j f_j - B_g e_j f_j + B_g e_j f_j)
\]

\[
Q_g = \sum_{j=1}^{n} (-B_g e_j e_j f_j + G_g e_j f_j - B_g e_j f_j)
\]
Following the procedure given in the approach, the candidate transmission lines satisfying the index are determined as lines \{16-35\}, \{13-25\}, \{14-15\}, \{12-13\}, \{36-37\}, \{17-31\}, \{20-37\}, and \{11-21\}, which are illustrated in Fig. 5. As for eigenvalue analysis, the more the TCSCs installed the more the stability enhanced by the LQR. The minimum value of $H_\infty$ norm is 0.123; therefore, it satisfies the robust stability index. The value of $H_\infty$ norm by installing different number of TCSCs is shown in Fig. 6, where the TCSCs are added cumulatively according to their numbers indexed in the figure, from TCSC 1 to TCSC 9. It should be noted that not until all nine TCSCs are added, the stability index is not satisfied. There is, however, a tradeoff of sacrificing the stability index slightly and installing only 7 TCSCs. It is clear that at least 7 TCSCs must be installed.

where, $G_{ij}$ and $B_{ij}$ are respectively conductance and susceptance between nodes $i$ and $j$, $e_i$ and $f_i$ are respectively real and imaginary expressions of voltage at node $i$. In these equations, the control variables are the admittance values of transmission lines. By analyzing the participation rates and the generator sensitivities, a candidate transmission line has been determined.

### E. Allocation of TCSCs Satisfying the $H_\infty$ Norm Index

Design of the TCSC controller is based on the LQR, which stabilizes the system eigenvalues. The obtained feedback gain $F$ in (17) is evaluated to see if it has satisfied the $H_\infty$ norm index or not. The index is calculated in (18) for the closed-loop system expressed in (14). The disturbance matrix $B_w$ in (11) is viewed as an error of the system matrix due to disturbance (i.e., $B_w=\Delta A$). In the test system, the error is defined as the change of the system matrix due to a contingency; single-line opening at line \{17-36\} in Fig. 5. Moreover, the calculation requires the control output matrix $C_z$ in (13) to be set. The approach aims at mitigating the low-frequency oscillation in the tie-line; therefore, output variable $z$ is set to the active power on the tie-line.

Active power flow in a branch in a power system is obtained from terminal voltages and line impedances. Hence, control output variable is expressed as follow:

$$P_y = \frac{r_{ij}\{e_i(e_i-e_j)+f_i(f_i-f_j)\}-x_{ij}\{f_i(e_i-e_j)-e_i(f_i-f_j)\}}{r_{ij}^2+x_{ij}^2}$$

(32)

where, $r_{ij}$ and $x_{ij}$ are respectively the real and imaginary parts of the impedance between nodes $i$ and $j$.

In the case when the termination condition is not satisfied, the approach repeats the selection of transmission lines. A transmission line already equipped with a controller is excluded from the selection. The process is repeated until the index is satisfied (i.e., $H_\infty$ norm < 1.0).

### F. Verification of the Allocation of TCSCs

The proposed approach is applied to a case of single-line
contingency, and the enhancement of transient stability is verified in time domain. In the simulation, a single-line fault is applied at line 7. The fault occurs at 1.0sec.; line opens at 1.07sec.; and recloses at 2.07sec. The generator angles measured from the slack generator: \( \delta_i \) and the active power on the tie-line are shown in Figs. 7 and 8, respectively. Fig. 7 illustrates angles of typical generators (Generators 1, 6, 7 and 9) for both with and without controllers. For each machine, light line is for the case without control and heavy line for the case with control. The TCSCs allocated at appropriate transmission lines in the test system were shown to stabilize generator angles. Comparing with Fig. 7, Fig. 8 shows that the low-frequency oscillation in the tie-line is damped much faster utilizing eigenvalues, participation rates, active power sensitivity, and Linear Quadratic Regulator. Moreover, the approach makes use of the \( H_{\infty} \) norm as a stability index for installing controllers.

The proposed method is applied to damp the low-frequency oscillation in a tie-line in a Japanese power system model. It has been demonstrated that the proposed approach can effectively allocate TCSC controllers in a realistic power system. This approach can be extended to installing UPFC or other FACTS devices.

This work provides a tool to determine the number and location of TCSCs in a power system for stability. However, since FACTS devices are expensive, economic constraints should also be considered for capital investment. In a future research, allocation of TCSCs to include not only the control system design, but also the power system expansion planning aspect will be considered.

VI. APPENDIX

Table A1. List of Parameters in Eqs. (1) to (3) and (21) to (29).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_i )</td>
<td>inertial constant [p.u.MWsec./( \tau )]</td>
</tr>
<tr>
<td>( \delta_i )</td>
<td>rotor angle [rad.]</td>
</tr>
<tr>
<td>( P_{m_i} )</td>
<td>mechanical input [p.u.MW]</td>
</tr>
<tr>
<td>( P_{e_i} )</td>
<td>electrical output [p.u.MW]</td>
</tr>
<tr>
<td>( D_j )</td>
<td>damping constant [p.u.MWsec./( \tau )]</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>deviation from reference speed [rad./sec.]</td>
</tr>
<tr>
<td>( E_{ij} )</td>
<td>magnitude of internal voltage [p.u.]</td>
</tr>
<tr>
<td>( Y_{ij} )</td>
<td>magnitude of admittance between generators i and j [p.u.]</td>
</tr>
<tr>
<td>( \theta_{ij} )</td>
<td>phase angle of admittance between generators i and j [p.u.]</td>
</tr>
<tr>
<td>( e_{ij} )</td>
<td>voltage of infinite bus [p.u.]</td>
</tr>
<tr>
<td>( X_{ei} )</td>
<td>external reactance [p.u.]</td>
</tr>
<tr>
<td>( \delta_{ii} )</td>
<td>initial rotor angle of infinite bus voltage [rad.]</td>
</tr>
<tr>
<td>( \omega_{ii} )</td>
<td>rotation speed of generator in steady-state [rad./sec.]</td>
</tr>
<tr>
<td>( H )</td>
<td>inertial constant expressed in seconds [sec.]</td>
</tr>
<tr>
<td>( X_{ii} )</td>
<td>d-axis transient reactance [p.u.]</td>
</tr>
<tr>
<td>( X_{qi} )</td>
<td>q-axis transient reactance [p.u.]</td>
</tr>
<tr>
<td>( X_{qi}'' )</td>
<td>d-axis transient reactance [p.u.]</td>
</tr>
<tr>
<td>( X_{qi}''' )</td>
<td>q-axis transient reactance [p.u.]</td>
</tr>
<tr>
<td>( T_{xu} )</td>
<td>d-axis transient open-circuit time constant [sec.]</td>
</tr>
<tr>
<td>( T_{qo} )</td>
<td>q-axis transient open-circuit time constant [sec.]</td>
</tr>
<tr>
<td>( T_m )</td>
<td>mechanical torque [p.u.]</td>
</tr>
<tr>
<td>( T_e )</td>
<td>electrical torque [p.u.]</td>
</tr>
<tr>
<td>( \delta_i )</td>
<td>d-axis current [p.u.]</td>
</tr>
<tr>
<td>( \delta_e )</td>
<td>q-axis current [p.u.]</td>
</tr>
<tr>
<td>( e_f )</td>
<td>exciter voltage [p.u.]</td>
</tr>
<tr>
<td>( e_{FD} )</td>
<td>flux linkage in first dumper on q-axis [p.u.]</td>
</tr>
<tr>
<td>( e_{SD} )</td>
<td>flux linkage in second dumper on q-axis [p.u.]</td>
</tr>
<tr>
<td>( e_{ID} )</td>
<td>flux linkage in field circuit [p.u.]</td>
</tr>
<tr>
<td>( L_{d} )</td>
<td>d-axis transient inductance [p.u.]</td>
</tr>
<tr>
<td>( L_{q} )</td>
<td>q-axis transient inductance [p.u.]</td>
</tr>
<tr>
<td>( L_{di} )</td>
<td>d-axis subtransient inductance [p.u.]</td>
</tr>
<tr>
<td>( L_{qi} )</td>
<td>q-axis subtransient inductance [p.u.]</td>
</tr>
<tr>
<td>( L_{qi} )</td>
<td>d-axis inductance of stator winding [p.u.]</td>
</tr>
<tr>
<td>( R )</td>
<td>resistance of winding [p.u.]</td>
</tr>
</tbody>
</table>

VII. REFERENCES


VIII. Biographies

Chun Liu received his B.S. degree in Electrical Engineering Department of North China Electric Power University, Baoding China, in 1995, and M.S. degree from Tokyo Metropolitan University, Tokyo, Japan, in 2002 respectively. Currently he is a Ph.D candidate student in system control engineering department of electrical engineering of Tokyo Metropolitan University, Japan. He is a student member of IEEE of Japan.

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