Lecture Series on

Intelligent Control

Lecture 18

Fuzzy Logic Control

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Fuzzy Control

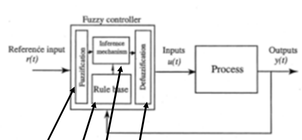

Figure 5.1: Fuzzy controller.

1. *A rule base* (a set of If-Then rules), which contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control.
2. *An inference mechanism* (also called an "inference engine" or "fuzzy inference" module), which emulates the expert's decision-making in interpreting and applying knowledge about how best to control the plant.
3. *A fuzzification interface*, which converts controller inputs into information that the inference mechanism can easily use to activate and apply rules.
4. *A defuzzification interface*, which converts the conclusions of the inference mechanism into actual inputs for the process.

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Inputs and Outputs for Fuzzy Controller

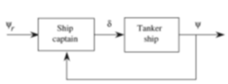

Figure 5.2: Human controlling a tanker ship.

$$e(t) = \psi_r(t) - \psi(t)$$

$$\frac{de(t)}{dt} = \dot{e}(t)$$

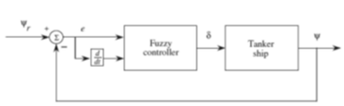

Figure 5.3: Fuzzy controller for a tanker ship steering problem.

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Rule Base

Linguistic Descriptions

"error" describes $e(t)$

"change-in-error" describes $\frac{de(t)}{dt}$

"rudder-input" describes $\delta(t)$

Just as $e(t)$ takes on a value of, for example, 0.1 at $t = 2$ ($e(2) = 0.1$), linguistic variables assume "linguistic values." That is, the values that linguistic variables take on over time change dynamically. Suppose for the tanker ship example that "error," "change-in-error," and "rudder-input" take on the following values:

-5 for	"neghuge"
-4 for	"neglarge"
-3 for	"negbig"
-2 for	"negmed"
-1 for	"negsmall"
0 for	"zero"
1 for	"possmall"
2 for	"posmed"
3 for	"posbig"
4 for	"poslarge"
5 for	"poshuge"

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Rules

Ψ_r = desired ship heading is 45 deg., the dotted lines

Ψ = ship heading, thin solid lines with arrow at end indicating direction of ship travel

Gray arrows indicate angular direction the ship is moving

Rudder angles shown are approximate

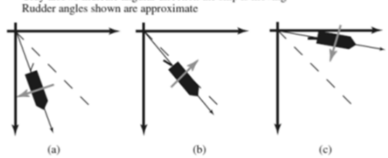

Figure 5.4: Tanker ship in various positions.

1. **If** error is negsmall **and** change-in-error is negsmall **Then** rudder-input is posmed
2. **If** error is zero **and** change-in-error is possmall **Then** rudder-input is negsmall
3. **If** error is possmall **and** change-in-error is negsmall **Then** rudder-input is zero

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Rule Bases

Table 5.1: Rule Table for the Tanker Ship

δ		$\dot{e}$										
		−5	−4	−3	−2	−1	0	1	2	3	4	5
e	−5	5	5	5	5	5	5	4	3	2	1	0
	−4	5	5	5	5	5	4	3	2	1	0	−1
	−3	5	5	5	5	4	3	2	1	0	−1	−2
	−2	5	5	5	4	3	2	1	0	−1	−2	−3
	−1	5	5	4	3	2	1	0	−1	−2	−3	−4
	0	5	4	3	2	1	0	−1	−2	−3	−4	−5
	1	4	3	2	1	0	−1	−2	−3	−4	−5	−5
	2	3	2	1	0	−1	−2	−3	−4	−5	−5	−5
	3	2	1	0	−1	−2	−3	−4	−5	−5	−5	−5
	4	1	0	−1	−2	−3	−4	−5	−5	−5	−5	−5
	5	0	−1	−2	−3	−4	−5	−5	−5	−5	−5	−5

Then, for instance, the $(+1, -1)$ position (where the "+1" represents the row having "+1" for a numeric-linguistic value and the "−1" represents the column having "−1" for a numeric-linguistic value) has a 0 ("zero") in the body of the table and represents the rule

If error is possmall **and** change-in-error is negsmall **Then** rudder-input is zero

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Fuzzy Quantification of Knowledge

Membership Function

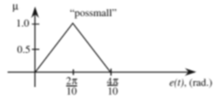

Figure 5.5: Membership function for linguistic value "possmall."

A "fuzzy set": a set *A* of values that is described by a membership function.

e.g., $\frac{2\pi}{10}$ is an element of a fuzzy set *A* with absolute certainty, but less certain that $\frac{4\pi}{10}$ is an element of *A*.

Membership in the set is *fuzzy*

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Fuzzy Quantification of Knowledge

Membership Function

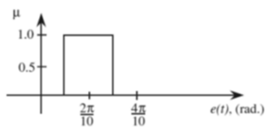

Figure 5.7: Membership function for a crisp set.

"Crisp" membership function:

normal interval $\frac{\pi}{10} \leq e(t) \leq \frac{3\pi}{10}$

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Membership Function Shapes

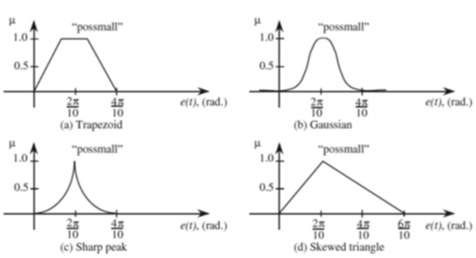

Figure 5.6: Some example membership function choices for representing "error is possmall."

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Input and Output Membership Functions

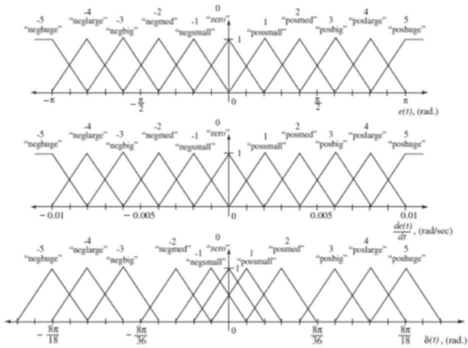

Figure 5.8: Membership functions for a ship steering example.

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Rule Table

Table 5.2: Rule Table for the Tanker Ship (body of table holds the output membership function centers where each element should be multiplied by $8\pi/18$).

		$\dot{e}$										
		−5	−4	−3	−2	−1	0	1	2	3	4	5
e	−5	1	1	1	1	1	1	.8	.6	.3	.1	0
	−4	1	1	1	1	1	.8	.6	.3	.1	0	−.1
	−3	1	1	1	1	.8	.6	.3	.1	0	−.1	−.3
	−2	1	1	1	.8	.6	.3	.1	0	−.1	−.3	−.6
	−1	1	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8
	0	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1
	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1
	2	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1
	3	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1
	4	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1	−1
	5	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1	−1	−1

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Matching: Determining Which Rules to Use

Premise Quantification via Fuzzy Logic

To perform inference we must first quantify each of the rules with fuzzy logic. To do this, we first quantify the meaning of the premises of the rules that are composed of several terms, each of which involves a fuzzy controller input. Consider Figure 5.9, where we list two terms from the premise of the rule

If error is zero **and** change-in-error is possmall **Then** rudder-input is negsmall

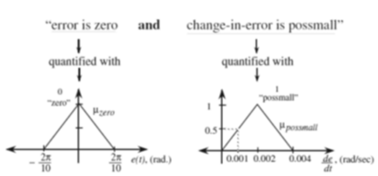

Figure 5.9: Membership functions of premise terms.

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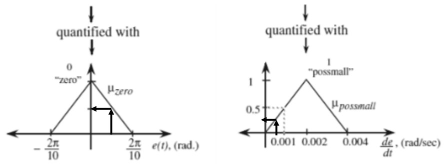
"And" Operation

To see how to quantify the "and" operation, begin by supposing that $e(t) = \pi/10$ and $\dot{e}(t) = 0.0005$, so that using Figure 5.8 (or Figure 5.9) we see that

$$\mu_{zero}(e(t)) = 0.5$$

and

$$\mu_{possmall}(\dot{e}(t)) = 0.25$$



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"And" Operation

$$\mu_{zero}(e(t)) = 0.5$$

$$\mu_{possmall}(\dot{e}(t)) = 0.25$$

What, for these values of $e(t)$ and $\dot{e}(t)$, is the certainty of the statement

"error is zero and change-in-error is possmall"

that is the premise from the above rule? We will denote this certainty by $\mu_{premise}$. There are actually several ways to define it:

- *Minimum:* Define $\mu_{premise} = \min\{0.5, 0.25\} = 0.25$, that is, using the minimum of the two membership values.
- *Product:* Define $\mu_{premise} = (0.5)(0.25) = 0.125$, that is, using the product of the two membership values.

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Minimum Operation

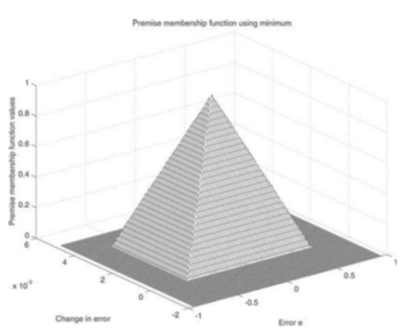

Figure 5.10: Membership function of the premise for a single rule using minimum to represent the conjunction.

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Product Operation

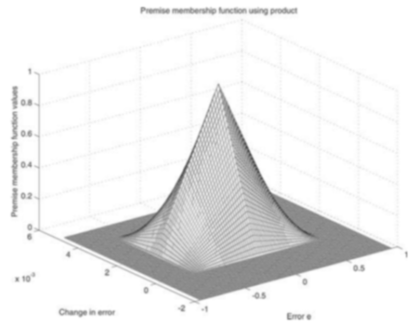

Figure 5.11: Membership function of the premise for a single rule using product to represent the conjunction.

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Active Rules

Consider, for the ship steering example, how we compute the rules that are on. Suppose that

$$e(t) = 0$$

and

$$\dot{e}(t) = 0.0015$$

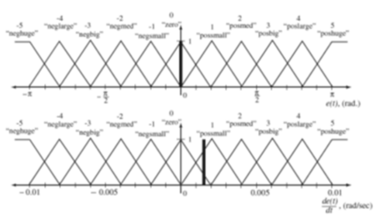

Figure 5.12: Input membership functions with input values. 17

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Active Rules

Table 5.3: Rule Table for the Tanker Ship with Rules That Are "On" (highlighted). (Body of table holds the output membership function centers where each element should be multiplied by $8\pi/18$.)

		$\dot{e}$										
		−5	−4	−3	−2	−1	0	1	2	3	4	5
e	−5	1	1	1	1	1	1	.8	.6	.3	.1	0
	−4	1	1	1	1	1	.8	.6	.3	.1	0	−.1
	−3	1	1	1	1	.8	.6	.3	.1	0	−.1	−.3
	−2	1	1	1	.8	.6	.3	.1	0	−.1	−.3	−.6
	−1	1	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8
	0	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1
	1	.8	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1
	2	.6	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1
	3	.3	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1
	4	.1	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1	−1
	5	0	−.1	−.3	−.6	−.8	−1	−1	−1	−1	−1	−1

1. If error is zero **and** change-in-error is zero **Then** rudder-input is zero
2. If error is zero **and** change-in-error is possmall **Then** rudder-input is negsmall

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Inference

Recommendation from One Rule

Consider the conclusion reached by the rule

If error is zero **and** change-in-error is zero **Then** rudder-input is zero

which for convenience we will refer to as "rule (1)." Using the minimum to represent the premise, we have

$$\mu_{premise_{(1)}} = \min\{1, 0.25\} = 0.25$$

Membership function for the consequent by rule (1):

$$\mu_{(1)}(\delta) = \min\{\mu_{premise_{(1)}}, \mu_{zero}(\delta)\}$$

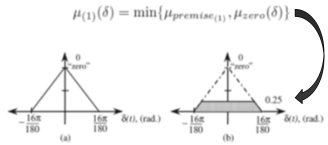

Figure 5.13: (a) Consequent membership function and (b) implied fuzzy set with membership function $\mu_{(1)}(\delta)$ for rule (1).

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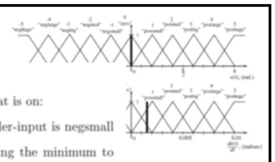
Inference



…ship functions with input values.

Recommendation from Another Rule

Next, consider the conclusion reached by the other rule that is on:

If error is zero **and** change-in-error is possmall **Then** rudder-input is negsmall

which, for convenience, we will refer to as "rule (2)." Using the minimum to represent the premise, we have

$$\mu_{premise_{(2)}} = \min\{1, 0.75\} = 0.75$$

Membership function for the consequent by rule (2):

$$\mu_{(2)}(\delta) = \min\{\mu_{premise_{(2)}}, \mu_{negsmall}(\delta)\}$$

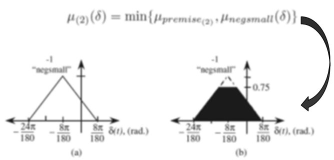


Figure 5.14: (a) Consequent membership function and (b) implied fuzzy set with membership function $\mu_{(2)}(\delta)$ for rule (2).

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Defuzzification

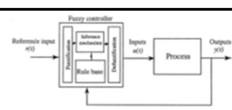

Figure 5.1: Fuzzy controller.

5.1.6 Converting Decisions into Actions

Next, we consider the defuzzification operation, which is the final component of the fuzzy controller shown in Figure 5.1 on page 156. Defuzzification operates on the implied fuzzy sets produced by the inference mechanism and combines their effects to provide the "most certain" controller output (plant input). Some think of defuzzification as "decoding" the fuzzy set information produced by the inference process (i.e., the implied fuzzy sets) into numeric fuzzy controller outputs.

To understand defuzzification, it is best to first draw all the implied fuzzy sets on one axis as shown in Figure 5.15. We want to find the one output, which we denote by "δ^{crisp}," that best represents the conclusions of the fuzzy controller that are represented with the implied fuzzy sets. There are actually many approaches to defuzzification. We will consider two here.

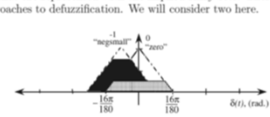

Figure 5.15: Implied fuzzy sets.

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Inference

Combining Recommendations

Due to its popularity, we will first consider the "center of gravity" (COG) defuzzification method for combining the recommendations represented by the implied fuzzy sets from all the rules. Let b_i denote the center of the membership function for the implied fuzzy set for the i^{th} rule (i.e., where the membership function for the i^{th} rule reaches its peak for our example since the output fuzzy sets are all symmetric about their peaks). For our example we have

$$b_1 = 0.0$$

and

$$b_2 = -0.1\left(\frac{8\pi}{18}\right)$$

as shown in Figure 5.15. Let

$$\int \mu_{(i)}$$

denote the area under the membership function $\mu_{(i)}$. The COG method computes δ^{crisp} to be

$$\delta^{crisp} = \frac{\sum_i b_i \int \mu_{(i)}}{\sum_i \int \mu_{(i)}} \qquad (5.1)$$

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Center of Gravity

Using Equation (5.1) with Figure 5.15, we have

$$\delta^{crisp} = \frac{(0)\left(0.25 - \frac{(0.25)^2}{2}\right) + \left(-0.1\frac{8\pi}{18}\right)\left(0.75 - \frac{(0.75)^2}{2}\right)}{\left(0.25 - \frac{(0.25)^2}{2}\right) + \left(0.75 - \frac{(0.75)^2}{2}\right)} = -0.0952$$

as the input to the ship for the given $e(t)$ and $\dot{e}(t)$.

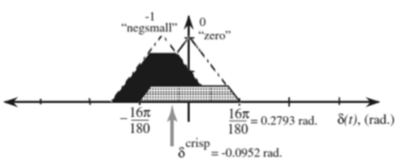

Figure 5.16: Implied fuzzy sets.

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Alternative Ways

Other Ways to Compute and Combine Recommendations

As another example, it is interesting to consider how to compute, by hand, the operations that the fuzzy controller takes when we use the product to represent the implication or the "center-average" defuzzification method.

First, consider the use of the product. Consider Figure 5.18, where we have drawn the output membership functions for "negsmall" and "zero" as dotted lines. The implied fuzzy set from rule (1) is given by the membership function

$$\mu_{(1)}(\delta) = 0.25\mu_{zero}(\delta)$$

shown in Figure 5.18 as the shaded triangle; the implied fuzzy set for rule (2) is given by the membership function

$$\mu_{(2)}(\delta) = 0.75\mu_{negsmall}(\delta)$$

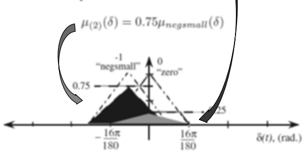

Figure 5.18: Implied fuzzy sets when the product is used to represent the implication.

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Center-Average

Next, as another example of how to combine recommendations, we will introduce the "center-average" method for defuzzification. For this method we let

$$\delta^{crisp} = \frac{\sum_i b_i \mu_{premise_{(i)}}}{\sum_i \mu_{premise_{(i)}}} \qquad (5.2)$$

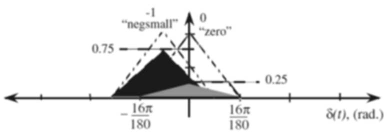

Figure 5.18: Implied fuzzy sets when the product is used to represent the implication.

$$\delta^{crisp} = \frac{(0)(0.25) + \left(-0.1\frac{8\pi}{18}\right)(0.75)}{0.25 + 0.75} = -0.1047$$

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Summary

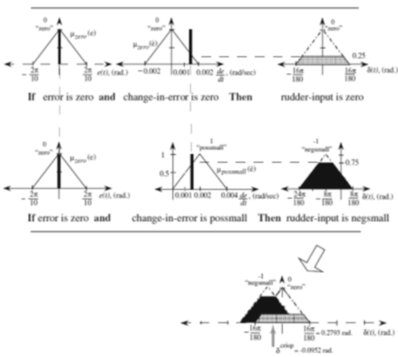

Figure 5.19: Graphical representation of fuzzy controller operations.

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