Lecture Series on

Intelligent Control

Lecture 14

Neural Networks in
Control System Applications - Examples

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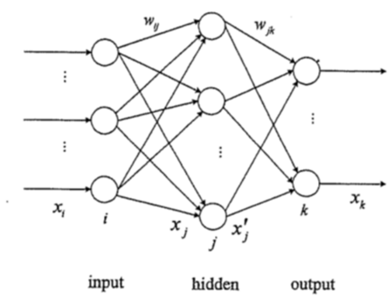
Feedforward NN



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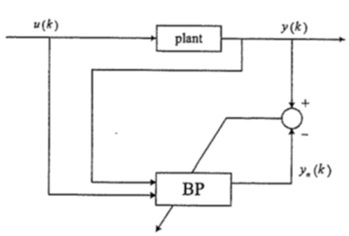
Modeling with NN



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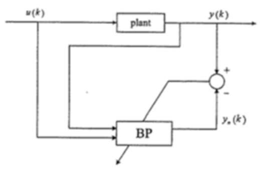
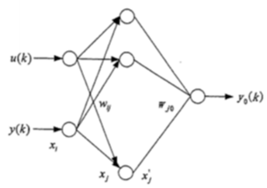
Modeling with NN





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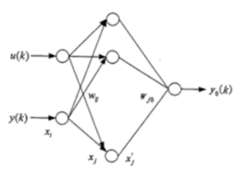
Backpropagation



(1) Feed-forward calculation

Input of hidden layer is

$$x_j = \sum_i w_{ij} x_i \tag{7.7}$$

Output of hidden layer is

$$x_j' = f(x_j) = \frac{1}{1+e^{-x_j}} \tag{7.8}$$

then

$$\frac{\partial x_j'}{\partial x_j} = x_j'(1-x_j')$$

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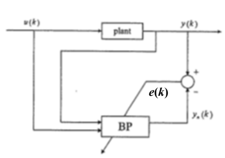

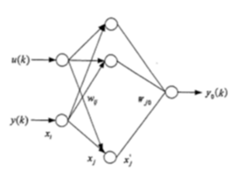

Output of output layer is

$$y_0(k) = \sum_j w_{jo} x_j' \tag{7.9}$$

Then, the approximation error is

$$e(k) = y(k) - y_n(k)$$

Error index function is designed as

$$E = \frac{1}{2} e(k)^2 \tag{7.10}$$

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Backpropagation

Output of output layer is

$$y_o(k) = \sum_j w_{jo} x_j' \qquad (7.9)$$

Then, the approximation error is

$$e(k) = y(k) - y_n(k)$$

Error index function is designed as

$$E = \frac{1}{2} e(k)^2 \qquad (7.10)$$

(2) Learning algorithm of BP

According to the steepest descent (gradient) method, the learning of weight value w_{jo} is

$$\Delta w_{jo} = -\eta \frac{\partial E}{\partial w_{jo}} = \eta \cdot e(k) \cdot \frac{\partial y_o}{\partial w_{jo}} = \eta \cdot e(k) \cdot x_j'$$

The weight value at time $k+1$ is

$$w_{jo}(k+1) = w_{jo}(k) + \Delta w_{jo}$$

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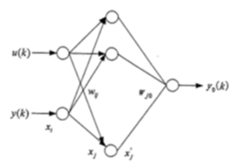
Backpropagation



Output of output layer is

$$y_o(k) = \sum_j w_{jo} x_j'$$

Then, the approximation error is

$$e(k) = y(k) - y_n(k)$$

Error index function is designed as

$$E = \frac{1}{2} e(k)^2$$

Output of hidden layer is

$$x_j' = f(x_j) = \frac{1}{1+e^{-x_j}}$$

then

$$\frac{\partial x_j'}{\partial x_j} = x_j'(1 - x_j')$$

The learning of weight value w_{ij} is

$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} = \eta \cdot e(k) \cdot \frac{\partial y_o}{\partial w_{ij}}$$

where the chain rule is used, $\frac{\partial y_o}{\partial w_{ij}} = \frac{\partial y_o}{\partial x_j'} \cdot \frac{\partial x_j'}{\partial x_j} \cdot \frac{\partial x_j}{\partial w_{ij}} = w_{jo} \cdot \frac{\partial x_j'}{\partial x_j} \cdot x_i = w_{jo} \cdot x_j'(1 - x_j') \cdot x_i$.

The weight value at time $k+1$ is

$$w_{ij}(k+1) = w_{ij}(k) + \Delta w_{ij}$$

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Backpropagation

The weight value at time $k+1$ is

$$w_{jo}(k+1) = w_{jo}(k) + \Delta w_{jo}$$

$$w_{ij}(k+1) = w_{ij}(k) + \Delta w_{ij}$$

Considering the effect of previous weight value change, the algorithm of weight value is

$$w_{jo}(k+1) = w_{jo}(k) + \Delta w_{jo} + \alpha\big(w_{jo}(k) - w_{jo}(k-1)\big) \qquad (7.11)$$

$$w_{ij}(t+1) = w_{ij}(t) + \Delta w_{ij} + \alpha(w_{ij}(t) - w_{ij}(t-1)) \qquad (7.12)$$

where η is learning rate, α is momentum factor, $\eta \in [0,1]$, $\alpha \in [0,1]$.

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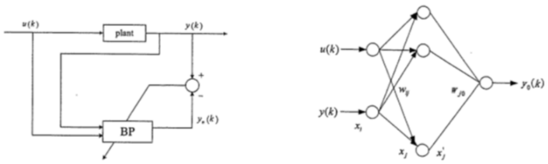
Modeling with NN



By using BP neural network approximation, Jacobian value can be calculated as follows:

$$\frac{\partial y(k)}{\partial u(k)} \approx \frac{\partial y_o(k)}{\partial u(k)} = \frac{\partial y_o(k)}{\partial x_j'} \times \frac{\partial x_j'}{\partial x_j} \times \frac{\partial x_j}{\partial x(1)} = \sum_j w_{jo} x_j' \left(1 - x_j'\right) w_{1j} \qquad (7.13)$$

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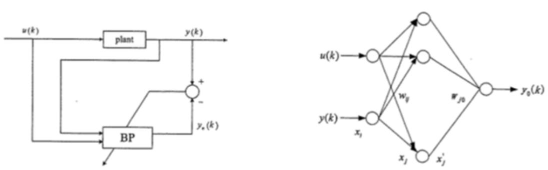
Simulation Example



The plant is as follows

$$y(k) = u(k)^3 + \frac{y(k-1)}{1 + y(k-1)^2}$$

Input signal is chosen as $u(k) = 0.5\sin(6\pi t)$, let the neural network input vector as $\boldsymbol{x} = [u(k)\ y(k)]$, NN structure is chosen as 2-6-1, the initial value of $\boldsymbol{W}_{jo}$, $\boldsymbol{W}_{ij}$ is chosen as random value in $[-1 \quad +1]$, $\eta = 0.50$, $\alpha = 0.05$.

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Simulation Example

```
%BP approximation
clear all;
close all;

xite=0.50;
alfa=0.05;

wjo=rands(6,1);
wjo_1=wjo;wjo_2=wjo_1;

wij=rands(2,6);
wij_1=wij;wij_2=wij;

dwij=0*wij;

x=[0,0]';

u_1=0;
y_1=0;

I=[0,0,0,0,0,0]';
Iout=[0,0,0,0,0,0]';
FI=[0,0,0,0,0,0]';

ts=0.001;
for k=1:1:1000

time(k)=k*ts;
u(k)=0.50*sin(3*2*pi*k*ts);
y(k)=u_1^3+y_1/(1+y_1^2);

x(1)=u(k);
x(2)=y(k);

for j=1:1:6
   I(j)=x'*wij(:,j);
   Iout(j)=1/(1+exp(-I(j)));
end
```

```
yo(k)=wjo'*Iout;      % Output of NNI networks

e(k)=y(k)-yo(k);  % Error calculation

wjo=wjo_1+(xite*e(k))*Iout+alfa*(wjo_1-wjo_2);

for j=1:1:6
  FI(j)=exp(-I(j))/(1+exp(-I(j)))^2;
end

for i=1:1:2
  for j=1:1:6
    dwij(i,j)=e(k)*xite*FI(j)*wjo(j)*x(i);
  end
End
wij=wij_1+dwij+alfa*(wij_1-wij_2);

%%%%%%%%%%%%%%Jacobian%%%%%%%%%%%%%%%%
yu=0;
for j=1:1:6
  yu=yu+wjo(j)*wij(1,j)*FI(j);
end
dyu(k)=yu;

wij_2=wij_1;wij_1=wij;
wjo_2=wjo_1;wjo_1=wjo;
u_1=u(k);
y_1=y(k);
end
figure(1);
plot(time,y,'r',time,yo,'b');
xlabel('times');ylabel('y and yo');
figure(2);
plot(time,y-yo,'r');
xlabel('times');ylabel('error');
figure(3);
plot(time,dyu);
xlabel('times');ylabel('dyu');
```

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Simulation Example

Fig. 7.8 BP approximation

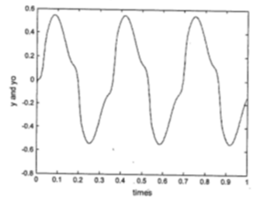
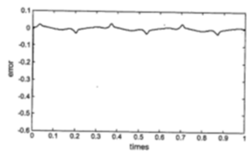


Fig. 7.9 BP approximation error



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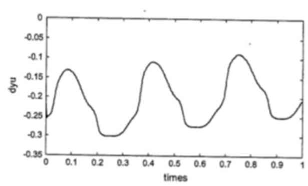
Simulation Example

Fig. 7.10 Jacobian value identification



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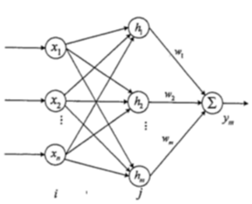
Radial Basis Function NN



In RBF neural network, $x = [x_i]^T$ is input vector. Assuming there are mth neural nets, and radial basis function vector in hidden layer of RBF is $h = [h_j]^T$, h_j is Gaussian function value for neural net j in hidden layer, and

$$h_j = \exp\left(-\frac{\|x - c_j\|^2}{2b_j^2}\right) \qquad (7.14)$$

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Radial Basis Function NN

In RBF neural network, $\boldsymbol{x} = [x_i]^{\mathrm{T}}$ is input vector. Assuming there are mth neural nets, and radial basis function vector in hidden layer of RBF is $\boldsymbol{h} = [h_j]^{\mathrm{T}}$, h_j is Gaussian function value for neural net j in hidden layer, and

$$h_j = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{c}_j\|^2}{2b_j^2}\right) \quad (7.14)$$

where $\boldsymbol{c} = [c_{ij}] = \begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \cdots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{bmatrix}$ represents the coordinate value of center point of the Gaussian function of neural net j for the ith input, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. For the vector $\boldsymbol{b} = [b_1, \ldots, b_m]^{\mathrm{T}}$, b_j represents the width value of Gaussian function for neural net j.

The weight value of RBF is

$$\boldsymbol{w} = [w_1, \ldots, w_m]^{\mathrm{T}} \quad (7.15)$$

The output of RBF neural network is

$$y(t) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{h} = w_1 h_1 + w_2 h_2 + \cdots + w_m h_m \quad (7.16)$$

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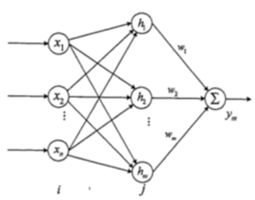
Radial Basis Function NN



The weight value of RBF is

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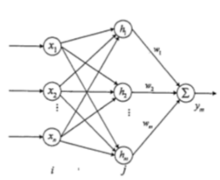
RBF Neural Network Simulation



In RBF neural network, $\boldsymbol{x} = [x_i]^{\mathrm{T}}$ is input vector. Assuming there are mth neural nets, and radial basis function vector in hidden layer of RBF is $\boldsymbol{h} = [h_j]^{\mathrm{T}}$, h_j is Gaussian function value for neural net j in hidden layer, and

$$h_j = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{c}_j\|^2}{2b_j^2}\right) \quad (7.14)$$

Consider a structure 1-5-1 RBF neural network, we have one input as $x = x_1$, and $\boldsymbol{b} = [b_1 \ b_2 \ b_3 \ b_4 \ b_5]^{\mathrm{T}}$, $\boldsymbol{c} = [c_{11} \ c_{12} \ c_{13} \ c_{14} \ c_{15}]$, $\boldsymbol{h} = [h_1 \ h_2 \ h_3 \ h_4 \ h_5]^{\mathrm{T}}$, $\boldsymbol{w} = [w_1 \ w_2 \ w_3 \ w_4 \ w_5]$, and $y(t) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{h} = w_1h_1 + w_2h_2 + w_3h_3 + w_4h_4 + w_5h_5$.

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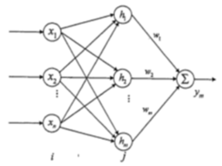
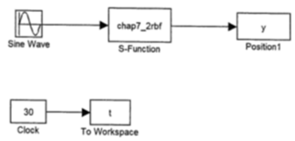
RBF Neural Network Simulation



Simulation programs:

(1) Simulink main program: chap7_2sim.mdl



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RBF Neural Network Simulation

(2) S function of RBF: chap7_2rbf.m

```
function [sys,x0,str,ts] = spacemodel(t,x,u,flag)
switch flag,
case 0,
  [sys,x0,str,ts]=mdlInitializeSizes;
case 3,
  sys=mdlOutputs(t,x,u);
case {2,4,9}
  sys=[];
otherwise
  error(['Unhandled flag = ',num2str(flag)]);
end
function [sys,x0,str,ts]=mdlInitializeSizes
sizes = simsizes;
sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 7;
sizes.NumInputs      = 1;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 0;
sys = simsizes(sizes);
x0  = [];
str = [];
ts  = [];
function sys=mdlOutputs(t,x,u)
x=u(1);  %Input Layer
%i=1
%j=1,2,3,4,5
%k=1
c=[-0.5 -0.25 0 0.25 0.5];  %cij
b=[0.2 0.2 0.2 0.2 0.2]';  %bj

W=ones(5,1);  %Wj
h=zeros(5,1);  %hj
for j=1:1:5
  h(j)=exp(-norm(x-c(:,j))^2/(2*b(j)*b(j)));  %Hidden Layer
end
y=W'*h;   %Output Layer

sys(1)=y;
sys(2)=x;
sys(3)=h(1);
sys(4)=h(2);
sys(5)=h(3);
sys(6)=h(4);
sys(7)=h(5);
```

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RBF Neural Network Simulation

(3) Plot program: chap7_2plot.m

```
close all;

% y=y(:,1);
% x=y(:,2);
% h1=y(:,3);
% h2=y(:,4);
% h3=y(:,5);
% h4=y(:,6);
% h5=y(:,7);

figure(1);
plot(t,y(:,1),'k','linewidth',2);
xlabel('time(s)');ylabel('y');

figure(2);
plot(y(:,2),y(:,3),'k','linewidth',2);
xlabel('x');ylabel('hj');
hold on;
plot(y(:,2),y(:,4),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,5),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,6),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,7),'k','linewidth',2);
```

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RBF Neural Network Simulation

Fig. 7.12 Output of RBF

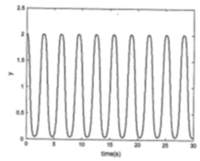
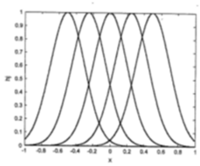


Fig. 7.13 Output of hidden neural net



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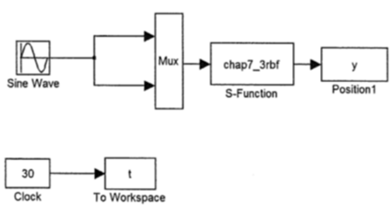
RBF Neural Network Simulation

Consider a structure 2-5-1 RBF neural network, we have $\mathbf{x} = [x_1, x_2]^{\mathrm{T}}$, $\mathbf{b} = [b_1 \quad b_2 \quad b_3 \quad b_4 \quad b_5]^{\mathrm{T}}$, $\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \end{bmatrix}$, $\mathbf{h} = [h_1 \quad h_2 \quad h_3 \quad h_4 \quad h_5]^{\mathrm{T}}$, $\mathbf{w} = [w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5]^{\mathrm{T}}$, and $y(t) = \mathbf{w}^{\mathrm{T}}\boldsymbol{h} = w_1h_1 + w_2h_2 + w_3h_3 + w_4h_4 + w_5h_5$.

(1) **Simulink main program: chap7_3sim.mdl**



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RBF Neural Network Simulation

(2) **S function of RBF: chap7_3rbf.m**

```
function [sys,x0,str,ts] = spacemodel(t,x,u,flag)
switch flag,
case 0,
  [sys,x0,str,ts]=mdlInitializeSizes;
case 3,
  sys=mdlOutputs(t,x,u);
case {2,4,9}
  sys=[];
otherwise
  error(['Unhandled flag = ',num2str(flag)]);
end
function [sys,x0,str,ts]=mdlInitializeSizes
sizes = simsizes;
sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 8;
sizes.NumInputs      = 2;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 0;
sys = simsizes(sizes);
x0  = [];
str = [];
ts  = [];
function sys=mdlOutputs(t,x,u)
x1=u(1);  %Input Layer
x2=u(2);
x=[x1 x2]';
%i=2
%j=1,2,3,4,5
%k=1
c=[-0.5 -0.25 0 0.25 0.5;
  -0.5 -0.25 0 0.25 0.5];  %cij
b=[0.2 0.2 0.2 0.2 0.2]';   %bj

W=ones(5,1);  %Wj
h=zeros(5,1); %hj
for j=1:1:5
  h(j)=exp(-norm(x-c(:,j))^2/(2*b(j)*b(j))); %Hidden
end
yout=W'*h;   %Output Layer

sys(1)=yout;
sys(2)=x1;
sys(3)=x2;
sys(4)=h(1);
sys(5)=h(2);
sys(6)=h(3);
sys(7)=h(4);
sys(8)=h(5);
```

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RBF Neural Network Simulation

(3) **Plot program: chap7_3plot.m**

```
close all;
% y=y(:,1);
% x1=y(:,2);
% x2=y(:,3);
% h1=y(:,4);
% h2=y(:,5);
% h3=y(:,6);
% h4=y(:,7);
% h5=y(:,8);

figure(1);
plot(t,y(:,1),'k','linewidth',2);
xlabel('time(s)');ylabel('y');

figure(2);
plot(y(:,2),y(:,4),'k','linewidth',2);
xlabel('x1');ylabel('hj');
hold on;
plot(y(:,2),y(:,5),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,6),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,7),'k','linewidth',2);
hold on;
plot(y(:,2),y(:,8),'k','linewidth',2);

figure(3);
plot(y(:,3),y(:,4),'k','linewidth',2);
xlabel('x2');ylabel('hj');
hold on;
plot(y(:,3),y(:,5),'k','linewidth',2);
hold on;
plot(y(:,3),y(:,6),'k','linewidth',2);
hold on;
plot(y(:,3),y(:,7),'k','linewidth',2);
hold on;
plot(y(:,3),y(:,8),'k','linewidth',2);
```

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RBF Neural Network Simulation

Fig. 7.14 Output of RBF

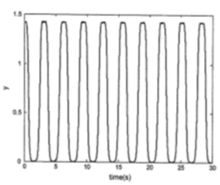
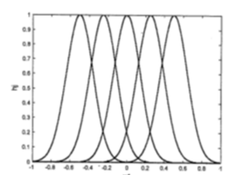


Fig. 7.15 Output of hidden neural net for first input



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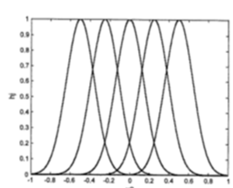
RBF Neural Network Simulation

Fig. 7.16 Output of hidden neural net for second input



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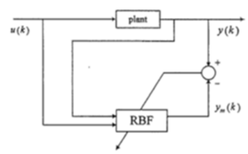
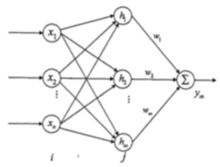
Modeling with RBF Neural Network





The output of RBF is

$$y_m(t) = w_1 h_1 + w_2 h_2 + \cdots + w_m h_m \quad (7.19)$$

The performance index function of RBF is

$$E(t) = \frac{1}{2}(y(t) - y_m(t))^2 \quad (7.20)$$

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$$E(t) = \frac{1}{2}(y(t) - y_m(t))^2 \quad (7.20)$$

According to gradient descent method, the parameters can be updated as follows:

$$\Delta w_j(t) = -\eta \frac{\partial E}{\partial w_j} = \eta (y(t) - y_m(t)) h_j$$

$$w_j(t) = w_j(t-1) + \Delta w_j(t) + \alpha \left(w_j(t-1) - w_j(t-2) \right) \quad (7.21)$$

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Modeling with RBF Neural Network

In RBF neural network, $\boldsymbol{x} = [x_i]^{\mathrm{T}}$ is input vector. Assuming there are mth neural nets, and radial basis function vector in hidden layer of RBF is $\boldsymbol{h} = [h_j]^{\mathrm{T}}$, h_j is Gaussian function value for neural net j in hidden layer, and

$$h_j = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{c}_j\|^2}{2b_j^2} \right) \quad (7.14)$$

The output of RBF is

$$y_m(t) = w_1 h_1 + w_2 h_2 + \cdots + w_m h_m \quad (7.19)$$

The performance index function of RBF is

$$E(t) = \frac{1}{2}(y(t) - y_m(t))^2 \quad (7.20)$$

$$\Delta b_j = -\eta \frac{\partial E}{\partial b_j} = \eta (y(t) - y_m(t)) w_j h_j \frac{\|\boldsymbol{x} - \boldsymbol{c}_j\|^2}{b_j^3} \quad (7.22)$$

$$b_j(t) = b_j(t-1) + \Delta b_j + \alpha \left(b_j(t-1) - b_j(t-2) \right) \quad (7.23)$$

$$\Delta c_{ji} = -\eta \frac{\partial E}{\partial c_{ji}} = \eta (y(t) - y_m(t)) w_j \frac{x_j - c_{ji}}{b_j^2} \quad (7.24)$$

$$c_{ji}(t) = c_{ji}(t-1) + \Delta c_{ji} + \alpha \left(c_{ji}(t-1) - c_{ji}(t-2) \right) \quad (7.25)$$

where $\eta \in (0,1)$ is the learning rate, $\alpha \in (0,1)$ is momentum factor.

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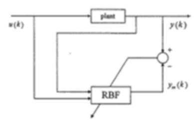
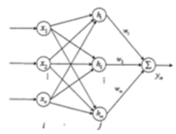
Simulation Example





First example: only update *w*

Using RBF neural network to approximate the following discrete plant

$$G(s) = \frac{133}{s^2 + 25s}$$

Consider a structure 2-5-1 RBF neural network, we choose inputs as $x(1) = u(t)$, $x(2) = y(t)$, and set $\alpha = 0.05$, $\eta = 0.5$. The initial weight value is chosen as random value between 0 and 1.

Choose the input as $u(\mathrm{t}) = \sin t$, consider the range of the first input $x(1)$ is $[0, 1]$, the range of the second input $x(2)$ is about $[0, 10]$, we choose the initial parameters of Gaussian function as $\boldsymbol{c}_j = \begin{bmatrix} -1 & -0.5 & 0 & 0.5 & 1 \\ -10 & -5 & 0 & 5 & 10 \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{b}_j = 1.5$, $j = 1, 2, 3, 4, 5$.

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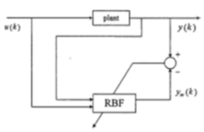
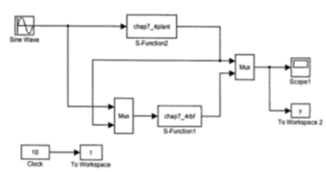
Simulation Example



(1) **Simulink main program: chap7_4sim.mdl**



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Simulation Example

(2) **S function of RBF: chap7_4rbf.m**

```
function [sys,x0,str,ts]=s_function(t,x,u,flag)
switch flag,

  [sys,x0,str,ts]=mdlInitializeSizes;
case 3,
  sys=mdlOutputs(t,x,u);
case {2, 4, 9 }
  sys = [];
otherwise
  error(['Unhandled flag = ',num2str(flag)]);
end

function [sys,x0,str,ts]=mdlInitializeSizes
sizes = simsizes;
sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 1;
sizes.NumInputs      = 2;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 0;
sys=simsizes(sizes);
x0=[];
str=[];
ts=[];
function sys=mdlOutputs(t,x,u)
persistent w w_1 w_2 b ci
alfa=0.05;
xite=0.5;
if t==0
  b=1.5;
  ci=[-1 -0.5 0 0.5 1;
     -10 -5 0 5 10];
  w=rands(5,1);
  w_1=w;w_2=w_1;
end
ut=u(1);
yout=u(2);
xi=[ut yout]';
for j=1:1:5
   h(j)=exp(-norm(xi-ci(:,j))^2/(2*b^2));
end
ymout=w'*h';

d_w=0*w;
for j=1:1:5  %Only weight value update
  d_w(j)=xite*(yout-ymout)*h(j);
end
w=w_1+d_w+alfa*(w_1-w_2);

w_2=w_1;w_1=w;
sys(1)=ymout;
```

(3) **Plot program: chap7_4plot.m**

```
close all;

close all;
figure(1);
plot(t,y(:,1),'r',t,y(:,2),'k:','linewidth',2);
xlabel('time(s)');ylabel('y and ym');
legend('ideal signal','signal approximation');
```

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Simulation Example

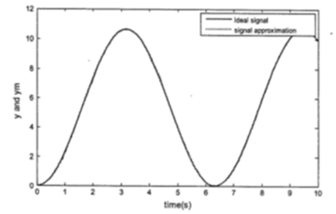
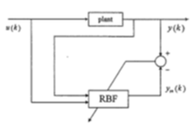
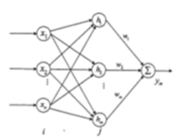


Fig. 7.18 RBF neural network approximation

Simulation Example





Second example: update w, c_j, b by gradient descent method

Using RBF neural network to approximate the following discrete plant

$$y(k) = u(k)^3 + \frac{y(k-1)}{1 + y(k-1)^2}$$

Consider a structure 2-5-1 RBF neural network, and we choose $x(1) = u(k)$, $x(2) = y(k)$, and $\alpha = 0.05$, $\eta = 0.15$. The initial weight value is chosen as random value between 0 and 1. Choose the input as $u(k) = \sin t$, $t = k \times T$, $T = 0.001$, we set the initial parameters of Gaussian function as $c_j = \begin{bmatrix} -1 & -0.5 & 0 & 0.5 & 1 \\ -1 & -0.5 & 0 & 0.5 & 1 \end{bmatrix}^T$, $b_j = 3.0$, $j = 1, 2, 3, 4, 5$.

Simulation Example

Simulation program: chap7_5.m

```
%RBF approximation
clear all;
close all;

alfa=0.05;
xite=0.15;
x=[0,1]';
b=3*ones(5,1);
c=[-1 -0.5 0 0.5 1;
  -1 -0.5 0 0.5 1];
w=rands(5,1);

w_1=w;w_2=w_1;
c_1=c;c_2=c_1;
b_1=b;b_2=b_1;
d_w=0*w;
d_b=0*b;
y_1=0;

ts=0.001;
for k=1:1:10000

time(k)=k*ts;
u(k)=sin(k*ts);

y(k)=u(k)^3+y_1/(1+y_1^2);

x(1)=u(k);
x(2)=y_1;

for j=1:1:5
  h(j)=exp(-norm(x-c(:,j))^2/(2*b(j)*b(j)));
end
ym(k)=w'*h';
em(k)=y(k)-ym(k);

M=1;
if M==1  %Only weight value update
  d_w(j)=xite*em(k)*h(j);
elseif M==2  %Update w,b,c
   for j=1:1:5
     d_w(j)=xite*em(k)*h(j);
     d_b(j)=xite*em(k)*w(j)*h(j)*(b(j)^-3)*norm(x-c(:,j))^2;
    for i=1:1:2
     d_c(i,j)=xite*em(k)*w(j)*h(j)*(x(i)-c(i,j))*(b(j)^-2);
    end
   end
   b=b_1+d_b+alfa*(b_1-b_2);
   c=c_1+d_c+alfa*(c_1-c_2);
end
w=w_1+d_w+alfa*(w_1-w_2);

y_1=y(k);

w_2=w_1;
w_1=w;

c_2=c_1;
c_1=c;

b_2=b_1;
b_1=b;
end
figure(1);
subplot(211);
plot(time,y,'r',time,ym,'k:','linewidth',2);
xlabel('time(s)');ylabel('y and ym');
legend('ideal signal','signal approximation');
subplot(212);
plot(time,y-ym,'k','linewidth',2);
xlabel('time(s)');ylabel('error');
```

Simulation Example

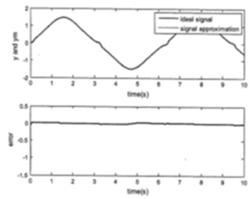


Fig. 7.19 RBF neural network approximation by only updating w (M = 1)

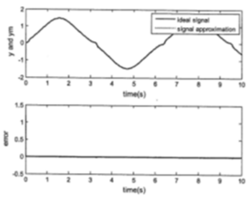


Fig. 7.20 RBF neural network approximation by updating w, b, c (M = 2)

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