2.29 The total instantaneous power absorbed by a three-phase motor (under balanced steady-state conditions) as well as a balanced three-phase impedance load is
(a) A constant (b) A function of time

2.30 Under balanced operating conditions, consider the three-phase complex power delivered by the three-phase source to the three-phase load. Match the following expressions, those on the left to those on the right.
(i) Real power, $P_{3\phi}$ (a) $(\sqrt{3}\,V_{LL}\,I_L)$VA
(ii) Reactive power, $Q_{3\phi}$ (b) $(\sqrt{3}\,V_{LL}\,I_L \sin\phi)$ var
(iii) Total apparent power, $S_{3\phi}$ (c) $(\sqrt{3}\,V_{LL}\,I_L \cos\phi)$ W
(iv) Complex power, $S_{3\phi}$ (d) $P_{3\phi} + jQ_{3\phi}$
Note that V_{LL} is the rms line-to-line voltage, I_L is the rms line current, and ϕ is the power-factor angle.

2.31 One advantage of balanced three-phase systems over separate single-phase systems is reduced capital and operating costs of transmission and distribution.
(a) True (b) False

2.32 While the instantaneous electric power delivered by a single-phase generator under balanced steady-state conditions is a function of time having two components of a constant and a double-frequency sinusoid, the total instantaneous electric power delivered by a three-phase generator under balanced steady-state conditions is a constant.
(a) True (b) False

PROBLEMS

SECTION 2.1

2.1 Given the complex numbers $A_1 = 6\underline{/30}$ and $A_2 = 4 + j5$, (a) convert A_1 to rectangular form; (b) convert A_2 to polar and exponential form; (c) calculate $A_3 = (A_1 + A_2)$, giving your answer in polar form; (d) calculate $A_4 = A_1A_2$, giving your answer in rectangular form; (e) calculate $A_5 = A_1/(A_2^*)$, giving your answer in exponential form.

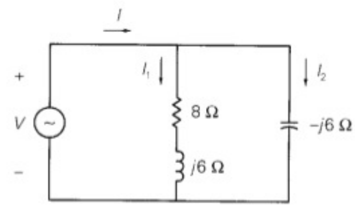
2.2 Convert the following instantaneous currents to phasors, using $\cos(\omega t)$ as the reference. Give your answers in both rectangular and polar form.
(a) $i(t) = 500\sqrt{2}\cos(\omega t - 30)$
(b) $i(t) = 4\sin(\omega t + 30)$
(c) $i(t) = 5\cos(\omega t - 15) + 4\sqrt{2}\sin(\omega t + 30)$

2.3 The instantaneous voltage across a circuit element is $v(t) = 400\sin(\omega t + 30°)$ volts, and the instantaneous current entering the positive terminal of the circuit element is $i(t) = 100\cos(\omega t + 10°)$ A. For both the current and voltage, determine (a) the maximum value, (b) the rms value, and (c) the phasor expression, using $\cos(\omega t)$ as the reference.

2.4 For the single-phase circuit shown in Figure 2.22, $I = 10\angle 0°$A. (a) Compute the phasors I_1, I_2, and V. (b) Draw a phasor diagram showing I, I_1, I_2, and V.

FIGURE 2.22

Circuit for Problem 2.4



2.5 A 60-Hz, single-phase source with $V = 277\angle 30°$ volts is applied to a circuit element. (a) Determine the instantaneous source voltage. Also determine the phasor and instantaneous currents entering the positive terminal if the circuit element is (b) a 20-Ω resistor, (c) a 10-mH inductor, and (d) a capacitor with 25-Ω reactance.

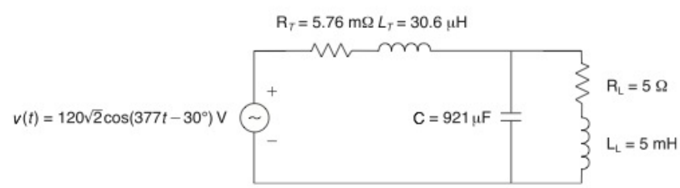
2.6 (a) Transform $v(t) = 75\cos(377t - 15°)$ to phasor form. Comment on whether $\omega = 377$ appears in your answer. (b) Transform $V = 50\angle 10°$ to instantaneous form. Assume that $\omega = 377$. (c) Add the two sinusoidal functions $a(t)$ and $b(t)$ of the same frequency given as follows: $a(t) = A\sqrt{2}\cos(\omega t + \alpha)$ and $b(t) = B\sqrt{2}\cos(\omega t + \beta)$. Use phasor methods and obtain the resultant $c(t)$. Does the resultant have the same frequency?

2.7 Let a 100-V sinusoidal source be connected to a series combination of a 3-Ω resistor, an 8-Ω inductor, and a 4-Ω capacitor. (a) Draw the circuit diagram. (b) Compute the series impedance. (c) Determine the current I delivered by the source. Is the current lagging or leading the source voltage? What is the power factor of this circuit?

2.8 Consider the circuit shown in Figure 2.23 in time domain. Convert the entire circuit into phasor domain.

FIGURE 2.23

Circuit for Problem 2.8



2.9 For the circuit shown in Figure 2.24, compute the voltage across the load terminals.

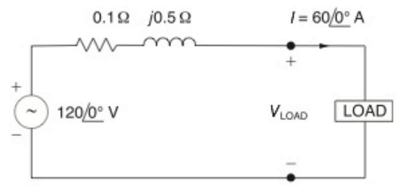


FIGURE 2.24
Circuit for Problem 2.9

SECTION 2.2

2.10 For the circuit element of Problem 2.3, calculate (a) the instantaneous power absorbed, (b) the real power (state whether it is delivered or absorbed), (c) the reactive power (state whether delivered or absorbed), (d) the power factor (state whether lagging or leading).
[*Note:* By convention the power factor $\cos(\delta - \beta)$ is positive. If $|\delta - \beta|$ is greater than 90°, then the reference direction for current may be reversed, resulting in a positive value of $\cos(\delta - \beta)$].

2.11 Referring to Problem 2.5, determine the instantaneous power, real power, and reactive power absorbed by (a) the 20-Ω resistor, (b) the 10-mH inductor, (c) the capacitor with 25-Ω reactance. Also determine the source power factor and state whether lagging or leading.

2.12 The voltage $v(t) = 359.3 \cos(\omega t)$ volts is applied to a load consisting of a 10- resistor in parallel with a capacitive reactance $X_C = 25\ \Omega$. Calculate (a) the instantaneous power absorbed by the resistor, (b) the instantaneous power absorbed by the capacitor, (c) the real power absorbed by the resistor, (d) the reactive power delivered by the capacitor, and (e) the load power factor.

2.13 Repeat Problem 2.12 if the resistor and capacitor are connected in series.

2.14 A single-phase source is applied to a two-terminal, passive circuit with equivalent impedance $Z = 3.0\angle{-45°}\ \Omega$, measured from the terminals. The source current is $i(t) = 2\sqrt{2}\cos(\omega t)$ kA. Determine the (a) instantaneous power, (b) real power, (c) reactive power delivered by the source, and (d) source power factor.

2.15 Let a voltage source $v(t) = 4\cos(\omega t + 60°)$ be connected to an impedance $Z = 2\angle 30°\ \Omega$. (a) Given the operating frequency to be 60 Hz, determine the expressions for the current and instantaneous power delivered by the source as functions of time. (b) Plot these functions along with $v(t)$ on a single graph for comparison. (c) Find the frequency and average value of the instantaneous power.

2.16 A single-phase, 120-V (rms), 60-Hz source supplies power to a series R-L circuit consisting of R = 10 Ω and L = 40 mH. (a) Determine the power factor of the circuit and state whether it is lagging or leading. (b) Determine the real and reactive power absorbed by the load. (c) Calculate the peak magnetic energy W_{int} stored in the inductor by using the expression $W_{int} = L(I_{rms})^2$ and check whether the reactive power $Q = \omega W_{int}$ is satisfied. (*Note:* The instantaneous magnetic energy storage fluctuates between zero and the peak energy. This energy must be sent twice each cycle to the load from the source by means of reactive power flows.)

SECTION 2.3

2.17 Consider a load impedance of $Z = j\omega L$ connected to a voltage and V let the current drawn be I.
(a) Develop an expression for the reactive power Q in terms of ω, L, and I, from complex power considerations.
(b) Let the instantaneous current be $i(t) = \sqrt{2}I\cos(\omega t + \theta)$. Obtain an expression for the instantaneous power $p(t)$ into L, and then express it in terms of Q.
(c) Comment on the average real power P supplied to the inductor and the instantaneous power supplied.

2.18 Let a series RLC network be connected to a source voltage V, drawing a current I.
(a) In terms of the load impedance $Z = Z\,\angle Z$, find expressions for P and Q, from complex power considerations.
(b) Express $p(t)$ in terms of P and Q, by choosing $i(t) = \sqrt{2}I\cos\omega t$.
(c) For the case of $Z = R + j\omega L + 1/j\omega C$, interpret the result of part (b) in terms of P, Q_L, and Q_C. In particular, if $\omega^2 LC = 1$, when the inductive and capacitive reactances cancel, comment on what happens.

2.19 Consider a single-phase load with an applied voltage $v(t) = 150\cos(\omega t + 10°)$ volts and load current $i(t) = 5\cos(\omega t - 50°)$ A. (a) Determine the power triangle. (b) Find the power factor and specify whether it is lagging or leading. (c) Calculate the reactive power supplied by capacitors in parallel with the load that correct the power factor to 0.9 lagging.

2.20 A circuit consists of two impedances, $Z_1 = 20\angle 30°\,\Omega$ and $Z_2 = 25\angle 60°\,\Omega$, in parallel, supplied by a source voltage $V = 100\angle 60°$ volts. Determine the power triangle for each of the impedances and for the source.

2.21 An industrial plant consisting primarily of induction motor loads absorbs 500 kW at 0.6 power factor lagging. (a) Compute the required kVA rating of a shunt capacitor to improve the power factor to 0.9 lagging. (b) Calculate the resulting power factor if a synchronous motor rated at 500 hp with 90% efficiency operating at rated load and at unity power factor is added to the plant instead of the capacitor. Assume constant voltage (1 hp = 0.746 kW).

2.22 The real power delivered by a source to two impedances, $Z_1 = 4 + j5\ \Omega$ and $Z_2 = 10\ \Omega$, connected in parallel, is 1000 W. Determine (a) the real power absorbed by each of the impedances and (b) the source current.

2.23 A single-phase source has a terminal voltage $V = 120\underline{/0^\circ}$ volts and a current $I = 15\underline{/30^\circ}$ A, which leaves the positive terminal of the source. Determine the real and reactive power, and state whether the source is delivering or absorbing each.

2.24 A source supplies power to the following three loads connected in parallel: (1) a lighting load drawing 10 kW, (2) an induction motor drawing 10 kVA at 0.90 power factor lagging, and (3) a synchronous motor operating at 10 hp, 85% efficiency and 0.95 power factor leading (1 hp = 0.746 kW). Determine the real, reactive, and apparent power delivered by the source. Also, draw the source power triangle.

2.25 Consider the series RLC circuit of Problem 2.7 and calculate the complex power absorbed by each of the R, L, and C elements, as well as the complex power absorbed by the total load. Draw the resultant power triangle. Check whether the complex power delivered by the source equals the total complex power absorbed by the load.

2.26 A small manufacturing plant is located 2 km down a transmission line, which has a series reactance of 0.5 Ω/km. The line resistance is negligible. The line voltage at the plant is $480\underline{/0^\circ}$ V (rms), and the plant consumes 120 kW at 0.85 power factor lagging. Determine the voltage and power factor at the sending end of the transmission line by using (a) a complex power approach and (b) a circuit analysis approach.

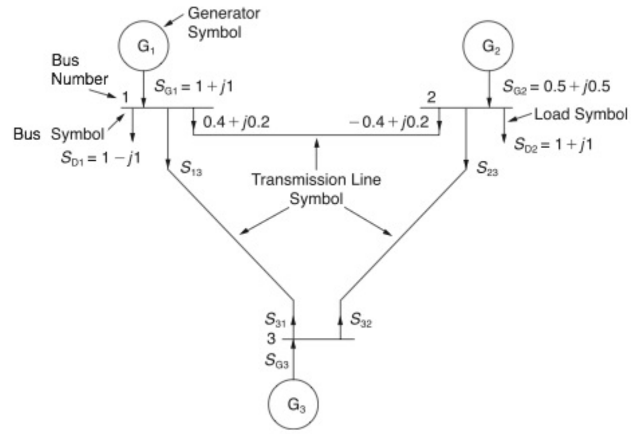
2.27 An industrial load consisting of a bank of induction motors consumes 50 kW at a power factor of 0.8 lagging from a 220-V, 60-Hz, single-phase source. By placing a bank of capacitors in parallel with the load, the resultant power factor is to be raised to 0.95 lagging. Find the net capacitance of the capacitor bank in μF that is required.

2.28 Three loads are connected in parallel across a single-phase source voltage of 240 V (RMS).
Load 1 absorbs 15 kW and 6.667 kvar;
Load 2 absorbs 3 kVA at 0.96PF leading;
Load 3 absorbs 15 kW at unity power factor.
Calculate the equivalent impedance, Z, for the three parallel loads, for two cases:
(i) Series combination of R and X, and (ii) parallel combination of R and X.

2.29 Modeling the transmission lines as inductors, with $S_{ij} = S_{ji}^*$, Compute S_{13}, S_{31}, S_{23}, S_{32}, and S_{G3} in Figure 2.25. (*Hint:* complex power balance holds good at each bus, satisfying KCL.)

FIGURE 2.25

System diagram for Problem 2.29



2.30 Figure 2.26 shows three loads connected in parallel across a 1000-V (RMS), 60-Hz single-phase source.

Load 1: Inductive load, 125 kVA, 0.28PF lagging.

Load 2: Capacitive load, 10 kW, 40 kvar.

Load 3: Resistive load, 15 kW.

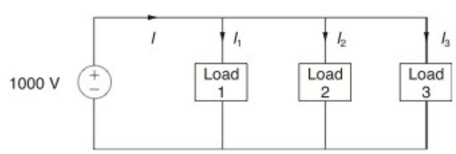
(a) Determine the total kW, kvar, kva, and supply power factor.

(b) In order to improve the power factor to 0.8 lagging, a capacitor of negligible resistance is connected in parallel with the above loads. Find the kvar rating of that capacitor and the capacitance in μF.

Comment on the magnitude of the supply current after adding the capacitor.

FIGURE 2.26

Circuit for Problem 2.30



2.31 Consider two interconnected voltage sources connected by a line of impedance $Z = jX\ \Omega$, as shown in Figure 2.27.

(a) Obtain expressions for P_{12} and Q_{12}.

(b) Determine the maximum power transfer and the condition for it to occur.

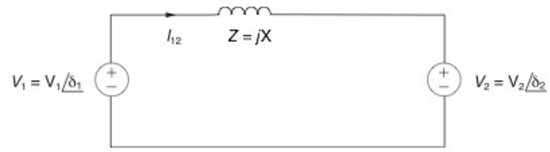


FIGURE 2.27 Circuit for Problem 2.31

2.32 In PowerWorld Simulator case Problem 2_32 (see Figure 2.28) a 8 MW and 4 Mvar load is supplied at 13.8 kV through a feeder with an impedance of $1 + j2\ \Omega$. The load is compensated with a capacitor whose output, Ω_{cap}, can be varied in 0.5 Mvar steps between 0 and 10.0 Mvars. What value of Ω_{cap} minimizes the real power line losses? What value of Ω_{cap} minimizes the MVA power flow into the feeder?

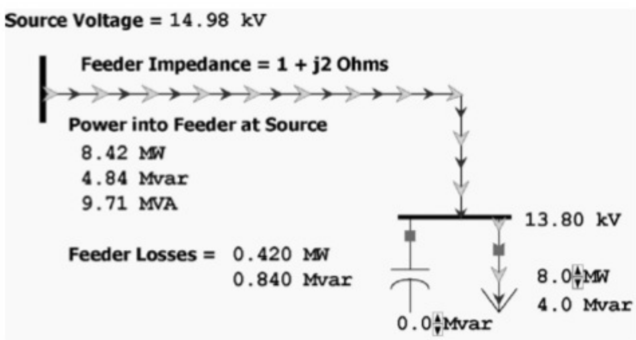


FIGURE 2.28 Screen for Problem 2.32

2.33 For the system from Problem 2.32, plot the real and reactive line losses as Ω_{cap} is varied between 0 and 10.0 Mvars.

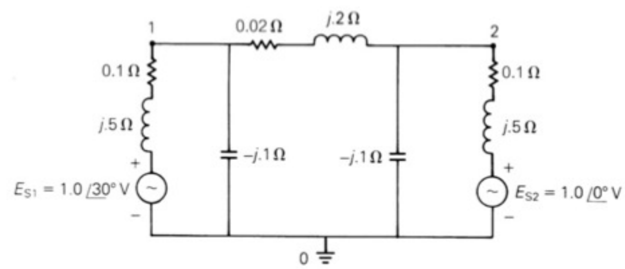
2.34 For the system from Problem 2.32, assume that half the time the load is 10 MW and 5 Mvar, and for the other half it is 20 MW and 10 Mvar. What single value of Q_{cap} would minimize the average losses? Assume that Q_{cap} can only be varied in 0.5 Mvar steps.

SECTION 2.4

2.35 For the circuit shown in Figure 2.29, convert the voltage sources to equivalent current sources and write nodal equations in matrix format using bus 0 as the reference bus. Do not solve the equations.

FIGURE 2.29

Circuit diagram for Problems 2.35 and 2.36

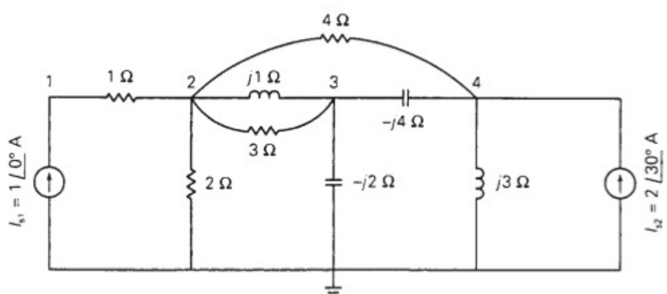


2.36 For the circuit shown in Figure 2.29, (a) determine the 2 × 2 bus admittance matrix Y_{bus}, (b) convert the voltage sources to current sources and determine the vector of source currents into buses 1 and 2.

2.37 Determine the 4 × 4 bus admittance matrix Y_{bus} and write nodal equations in matrix format for the circuit shown in Figure 2.30. Do not solve the equations.

FIGURE 2.30

Circuit for Problem 2.37



2.38 Given the impedance diagram of a simple system as shown in Figure 2.31, draw the admittance diagram for the system and develop the 4 × 4 bus admittance matrix Y_{bus} by inspection.

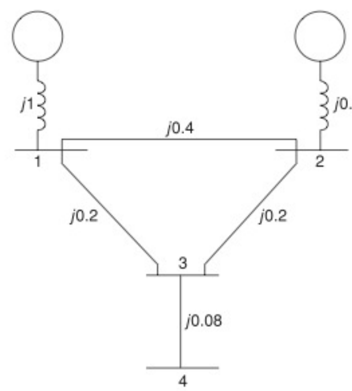


FIGURE 2.31
System diagram for Problem 2.38

2.39 (a) Given the circuit diagram in Figure 2.32 showing admittances and current sources at nodes 3 and 4, set up the nodal equations in matrix format. (b) If the parameters are given by: $Y_a = -j0.8$ S, $Y_b = -j4.0$ S, $Y_c = -j4.0$ S, $Y_d = -j8.0$ S, $Y_e = -j5.0$ S, $Y_f = -j2.5$ S, $Y_g = -j0.8$ S, $I_3 = 1.0\underline{/-90^\circ}$ A, and $I_4 = 0.62\underline{/-135^\circ}$ A, set up the nodal equations and suggest how you would go about solving for the voltages at the nodes.

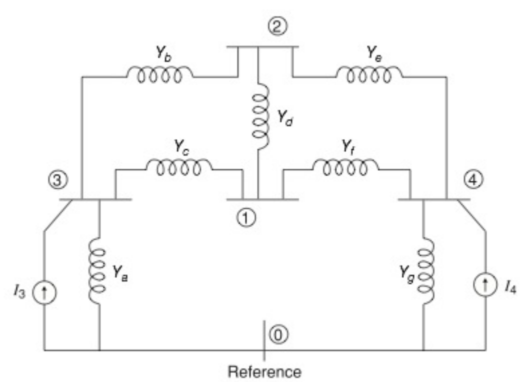


FIGURE 2.32
Circuit diagram for Problem 2.39

SECTIONS 2.5 AND 2.6

2.40 A balanced three-phase 240-V source supplies a balanced three-phase load. If the line current I_A is measured to be 15 A and is in phase with the line-to-line voltage, V_{BC}, find the per-phase load impedance if the load is (a) Y-connected, (b) Δ-connected.

2.41 A three-phase 25-kVA, 480-V, 60-Hz alternator, operating under balanced steady-state conditions, supplies a line current of 20 A per phase at a 0.8 lagging power factor and at rated voltage. Determine the power triangle for this operating condition.

2.42 A balanced Δ-connected impedance load with $(12 + j9)\ \Omega$ per phase is supplied by a balanced three-phase 60-Hz, 208-V source, (a) Calculate the line current, the total real and reactive power absorbed by the load, the load power factor, and the apparent load power, (b) Sketch a phasor diagram showing the line currents, the line-to-line source voltages, and the Δ-load currents. Use V_{ab} as the reference.

2.43 A three-phase line, which has an impedance of $(2 + j4)\ \Omega$ per phase, feeds two balanced three-phase loads that are connected in parallel. One of the loads is Y-connected with an impedance of $(30 + j40)\ \Omega$ per phase, and the other is Δ-connected with an impedance of $(60 - j45)\ \Omega$ per phase. The line is energized at the sending end from a 60-Hz, three-phase, balanced voltage source of $120\sqrt{3}$ V (rms, line-to-line). Determine (a) the current, real power, and reactive power delivered by the sending-end source; (b) the line-to-line voltage at the load; (c) the current per phase in each load; and (d) the total three-phase real and reactive powers absorbed by each load and by the line. Check that the total three-phase complex power delivered by the source equals the total three-phase power absorbed by the line and loads.

2.44 Two balanced three-phase loads that are connected in parallel are fed by a three-phase line having a series impedance of $(0.4 + j2.7)\ \Omega$ per phase. One of the loads absorbs 560 kVA at 0.707 power factor lagging, and the other 132 kW at unity power factor. The line-to-line voltage at the load end of the line is $2200\sqrt{3}$ V. Compute (a) the line-to-line voltage at the source end of the line, (b) the total real and reactive power losses in the three-phase line, and (c) the total three-phase real and reactive power supplied at the sending end of the line. Check that the total three-phase complex power delivered by the source equals the total three-phase complex power absorbed by the line and loads.

2.45 Two balanced Y-connected loads, one drawing 10 kW at 0.8 power factor lagging and the other 15 kW at 0.9 power factor leading, are connected in parallel and supplied by a balanced three-phase Y-connected, 480-V source. (a) Determine the source current. (b) If the load neutrals are connected to the source neutral by a zero-ohm neutral wire through an ammeter, what will the ammeter read?

2.46 Three identical impedances $Z_\Delta = 30\angle 30°\ \Omega$ are connected in Δ to a balanced three-phase 208-V source by three identical line conductors with