

## PROBLEMS

### SECTION 6.1

**6.1** Using Gauss elimination, solve the following linear algebraic equations:

$$-25x_1 + 10x_2 + 10x_3 + 10x_4 = 0$$

$$5x_1 - 10x_2 + 10x_3 = 2$$

$$10x_1 + 5x_2 - 10x_3 + 10x_4 = 1$$

$$10x_1 - 20x_4 = -2$$

**6.2** Using Gauss elimination and back substitution, solve

$$\begin{bmatrix} 8 & 2 & 1 \\ 4 & 6 & 2 \\ 3 & 4 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

**6.3** Rework Problem 6.2 with the value of 8 changed to 4.

**6.4** What is the difficulty in applying Gauss elimination to the following linear algebraic equations?

$$-5x_1 + 5x_2 = 5$$

$$10x_1 - 10x_2 = -5$$

**6.5** Show that, after triangularizing  $\mathbf{Ax} = \mathbf{y}$ , the back substitution method of solving  $\mathbf{A}^{(N-1)}\mathbf{x} = \mathbf{y}^{(N-1)}$  requires  $N$  divisions,  $N(N-1)/2$  multiplications, and  $N(N-1)/2$  subtractions. Assume that all the elements of  $\mathbf{A}^{(N-1)}$  and  $\mathbf{y}^{(N-1)}$  are nonzero and real.

### SECTION 6.2

**6.6** Solve Problem 6.2 using the Jacobi iterative method. Start with  $x_1(0) = x_2(0) = x_3(0) = 0$ , and continue until (6.2.2) is satisfied with  $\varepsilon = 0.01$ .

**6.7** Repeat Problem 6.6 using the Gauss-Seidel iterative method. Which method converges more rapidly?

**6.8** Express the following set of equations in the form of (6.2.6), and then solve using the Jacobi iterative method with  $\varepsilon = 0.05$  and with  $x_1(0) = 1$ , and  $x_2(0) = 1$ ,  $x_3(0) = 0$ .

$$\begin{bmatrix} 10 & -2 & -4 \\ -2 & 6 & -2 \\ -4 & -2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$$

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- 6.9** Solve for  $x_1$  and  $x_2$  in the system of equations given by

$$x_2 - 3x_1 + 1.9 = 0$$

$$x_2 + x_1^2 - 3.0 = 0$$

using the Gauss method with an initial guess of  $x_1 = 1$  and  $x_2 = 1$ .

- 6.10** Solve  $x^2 - 4x + 1 = 0$  using the Jacobi iterative method with  $x(0) = 1$ . Continue until (6.2.2) is satisfied with  $\varepsilon = 0.01$ . Check using the quadratic formula.
- 6.11** Try to solve Problem 6.2 using the Jacobi and Gauss-Seidel iterative methods with the value of  $A_{33}$  changed from 14 to 0.14 and with  $x_1(0) = x_2(0) = x_3(0) = 0$ . Show that neither method converges to the unique solution.
- 6.12** Using the Jacobi method (also known as the Gauss method), solve for  $x_1$  and  $x_2$  in the following system of equations.

$$x_2 - 3x_1 + 1.9 = 0$$

$$x_2 + x_1^2 - 1.8 = 0$$

Use an initial guess of  $x_1(0) = 1.0$  and  $x_2(0) = 1.0$ . Also, see what happens when you choose an uneducated initial guess of  $x_1(0) = x_2(0) = 100$ .

- 6.13** Use the Gauss-Seidel method to solve the following equations that contain terms that are often found in power flow equations.

$$x_1 = (1/(-20j)) * [(-1 + 0.5j)/(x_1)^* - (j10) * x_2 - (j10)]$$

$$x_2 = (1/(-20j)) * [(-3 + j)/(x_2)^* - (j10) * x_1 - (j10)]$$

Use an initial estimate of  $x_1(0) = 1$  and  $x_2(0) = 1$ , and a stopping of  $\varepsilon = 0.05$ .

- 6.14** Find a root of the following equation by using the Gauss-Seidel method: (use an initial estimate of  $x = 2$ )  $f(x) = x^3 - 6x^2 + 9x - 4 = 0$ .
- 6.15** Use the Jacobi method to find a solution to  $x^2 \cos x - x + 0.5 = 0$ . Use  $x(0) = 1$  and  $\varepsilon = 0.01$ . Experimentally determine the range of initial values that results in convergence.
- 6.16** Take the z-transform of (6.2.6) and show that  $\mathbf{X}(z) = \mathbf{G}(z)\mathbf{Y}(z)$ , where  $\mathbf{G}(z) = (z\mathbf{U} - \mathbf{M})^{-1}\mathbf{D}^{-1}$  and  $\mathbf{U}$  is the unit matrix.
- Note:*  $\mathbf{G}(z)$  is the matrix transfer function of a digital filter that represents the Jacobi or Gauss-Seidel methods. The filter poles are obtained by solving  $\det(z\mathbf{U} - \mathbf{M}) = 0$ . The filter is stable if and only if all the poles have magnitudes less than 1.
- 6.17** Determine the poles of the Jacobi and Gauss-Seidel digital filters for the general two-dimensional problem ( $N = 2$ ):

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Then determine a necessary and sufficient condition for convergence of these filters when  $N = 2$ .

## SECTION 6.3

**6.18** Use Newton-Raphson to find a solution to the polynomial equation  $f(x) = y$  where  $y = 0$  and  $f(x) = x^3 + 8x^2 + 2x - 40$ . Start with  $x(0) = 1$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.001$ .

**6.19** Repeat 6.18 using  $x(0) = -2$ .

**6.20** Use Newton-Raphson to find one solution to the polynomial equation  $f(x) = y$ , where  $y = 7$  and  $f(x) = x^4 + 3x^3 - 15x^2 - 19x + 30$ . Start with  $x(0) = 0$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.001$ .

**6.21** Repeat Problem 6.20 with an initial guess of  $x(0) = 4$ .

**6.22** For Problem 6.20, plot the function  $f(x)$  between  $x = 0$  and 4. Then provide a graphical interpretation why points close to  $x = 2.2$  would be poorer initial guesses.

**6.23** Use Newton-Raphson to find a solution to

$$\begin{bmatrix} e^{x_1 x_2} \\ \cos(x_1 + x_2) \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.5 \end{bmatrix}$$

where  $x_1$  and  $x_2$  are in radians. (a) Start with  $x_1(0) = 1.0$  and  $x_2(0) = 0.5$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.005$ . (b) Show that Newton-Raphson diverges for this example if  $x_1(0) = 1.0$  and  $x_2(0) = 2.0$ .

**6.24** Solve the following equations by the Newton-Raphson method:

$$2x_1 + x_2^2 - 8 = 0$$

$$x_1^2 - x_2^2 + x_1 x_2 - 3 = 0$$

Start with an initial guess of  $x_1 = 1$  and  $x_2 = 1$ .

**6.25** The following nonlinear equations contain terms that are often found in the power flow equations:

$$f_1(x) = 10x_1 \sin x_2 + 2 = 0$$

$$f_2(x) = 10(x_1)^2 - 10x_1 \cos x_2 + 1 = 0$$

Solve using the Newton-Raphson method starting with an initial guess of  $x_1(0) = 1$  and  $x_2(0) = 0$  radians and a stopping criteria of  $\varepsilon = 10^{-4}$ .

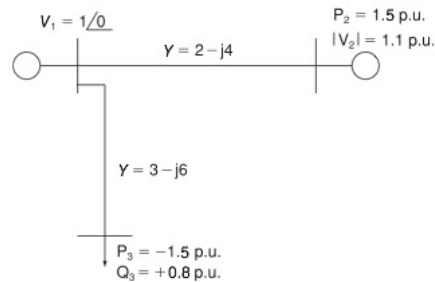
**6.26** Repeat Problem 6.25 except using  $x_1(0) = 0.25$  and  $x_2(0) = 0$  radians as an initial guess.

**6.27** For the Newton-Raphson method, the *region of attraction* (or *basin of attraction*) for a particular solution is the set of all initial guesses that converge to that solution. Usually initial guesses close to a particular solution will converge to that solution. However, for all but the simplest of multi-dimensional, nonlinear problems, the region of attraction boundary is often fractal. This makes it impossible to quantify the region of attraction and hence to guarantee convergence. Problem 6.25 has two

solutions when  $x_2$  is restricted to being between  $-\pi$  and  $\pi$ . With the  $x_2$  initial guess fixed at 0 radians, numerically determine the values of the  $x_1$  initial guesses that converge to the Problem 6.25 solution. Restrict your search to values of  $x_1$  between 0 and 1.

#### SECTION 6.4

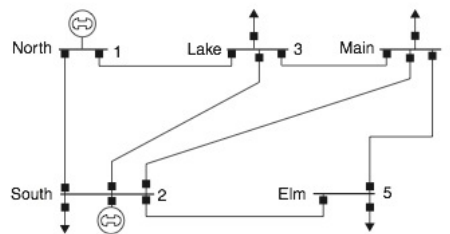
- 6.28** Consider the simplified electric power system shown in Figure 6.22 for which the power flow solution can be obtained without resorting to iterative techniques. (a) Compute the elements of the bus admittance matrix  $Y_{bus}$ . (b) Calculate the phase angle  $\delta_2$  by using the real power equation at bus 2 (voltage-controlled bus). (c) Determine  $|V_3|$  and  $\delta_3$  by using both the real and reactive power equations at bus 3 (load bus). (d) Find the real power generated at bus 1 (swing bus). (e) Evaluate the total real power losses in the system.
- 6.29** In Example 6.9, double the impedance on the line from bus 2 to bus 5. Determine the new values for the second row of  $Y_{bus}$ . Verify your result using PowerWorld Simulator case Example 6\_9.



**FIGURE 6.22**

Problem 6.28

- 6.30** Determine the bus admittance matrix ( $Y_{bus}$ ) for the three-phase power system shown in Figure 6.23 with input data given in Table 6.11 and partial results in Table 6.12. Assume a three-phase 100 MVA per unit base.



**FIGURE 6.23**

Sample System  
Diagram for  
Problem 6.30

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Bus-to-Bus	R per unit	X per unit	B per unit
1-2	0.02	0.06	0.06
1-3	0.08	0.24	0.05
2-3	0.06	0.18	0.04
2-4	0.08	0.24	0.05
2-5	0.02	0.06	0.02
3-4	0.01	0.04	0.01
4-5	0.03	0.10	0.04

**TABLE 6.11**

Bus input data for Problem 6.30

$6.25 - j18.695$	$-5.00 + j15.00$	$-1.25 + j3.75$	0	0
$-5.00 + j15.00$				

**TABLE 6.12**Partially Completed Bus Admittance Matrix ( $Y_{bus}$ ) for Problem 6.30

- 6.31** For the system from Problem 6.30, assume that a 75-Mvar shunt capacitance (three phase assuming one per unit bus voltage) is added at bus 4. Calculate the new value of  $Y_{44}$ .

**SECTION 6.5**

- 6.32** For a two-bus power system, a  $0.7 + j0.4$  per unit load at bus 2 is supplied by a generator at bus 1 through a transmission line with series impedance of  $0.05 + j0.1$  per unit. With bus 1 as the slack bus with a fixed per-unit voltage of  $1.0 \angle 0^\circ$ , use the Gauss-Seidel method to calculate the voltage at bus 2 after three iterations.
- 6.33** Repeat Problem 6.32 with the slack bus voltage changed to  $1.0 \angle 30^\circ$  per unit.
- 6.34** For the three-bus system whose  $Y_{bus}$  is given, calculate the second iteration value of  $V_3$  using the Gauss-Seidel method. Assume bus 1 as the slack (with  $V_1 = 1.0 \angle 0^\circ$ ), and buses 2 and 3 are load buses with a per-unit load of  $S_2 = 1 + j0.5$  and  $S_3 = 1.5 + j0.75$ . Use voltage guesses of  $1.0 \angle 0^\circ$  at both buses 2 and 3. The bus admittance matrix for a three-bus system is

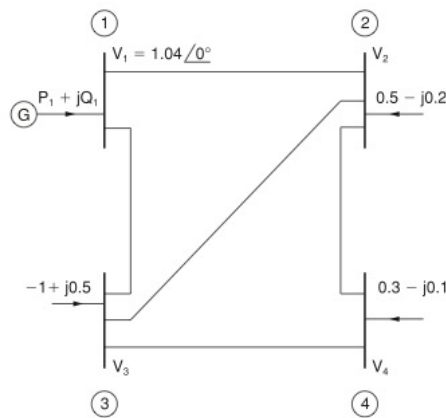
$$Y_{bus} = \begin{bmatrix} -j10 & j5 & j5 \\ j5 & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

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- 6.35** Repeat Problem 6.34 except assume the bus 1 (slack bus) voltage of  $V_1 = 1.05 \angle 0^\circ$ .
- 6.36** The bus admittance matrix for the power system shown in Figure 6.24 is given by

$$Y_{\text{bus}} = \begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 & 0 \\ -2 + j6 & 3.666 - j11 & -0.666 + j2 & -1 + j3 \\ -1 + j3 & -0.666 + j2 & 3.666 - j11 & -2 + j6 \\ 0 & -1 + j3 & -2 + j6 & 3 - j9 \end{bmatrix} \text{ per unit}$$

With the complex powers on load buses 2, 3, and 4 as shown in Figure 6.24, determine the value for  $V_2$  that is produced by the first and second iterations of the Gauss-Seidel procedure. Choose the initial guess  $V_2(0) = V_3(0) = V_4(0) = 1.0 \angle 0^\circ$  per unit.

**FIGURE 6.24**

Problem 6.36

- 6.37** The bus admittance matrix of a three-bus power system is given by

$$Y_{\text{bus}} = -j \begin{bmatrix} 7 & -2 & -5 \\ -2 & 6 & -4 \\ -5 & -4 & 9 \end{bmatrix} \text{ per unit}$$

with  $V_1 = 1.0 \angle 0^\circ$  per unit;  $V_2 = 1.0$  per unit;  $P_2 = 60$  MW;  $P_3 = -80$  MW;  $Q_3 = -60$  Mvar (lagging) as a part of the power flow solution of the system. Find  $V_2$  and  $V_3$  within a tolerance of 0.01 per unit by using the Gauss-Seidel iteration method. Start with  $\delta_2 = 0$ ,  $V_3 = 1.0$  per unit, and  $\delta_3 = 0$ .

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## SECTION 6.6

- 6.38** A generator bus (with a 1.0 per unit voltage) supplies a 180 MW, 60 Mvar load through a lossless transmission line with per unit (100 MVA base) impedance of  $j0.1$  and no line charging. Starting with an initial voltage guess of  $1.0 \angle 0^\circ$ , iterate until converged using the Newton-Raphson power flow method. For convergence criteria, use a maximum power flow mismatch of 0.1 MVA.
- 6.39** Repeat Problem 6.38 except use an initial voltage guess of  $1.0 \angle 30^\circ$ .
- 6.40** Repeat Problem 6.38 except use an initial voltage guess of  $0.25 \angle 0^\circ$ .
- 6.41** Determine the initial Jacobian matrix for the power system described in Problem 6.34.
- 6.42** Use the Newton-Raphson power flow to solve the power system described in Problem 6.34. For convergence criteria, use a maximum power flow mismatch of 0.1 MVA.
- 6.43** For a three-bus power system, assume bus 1 is the slack with a per unit voltage of  $1.0 \angle 0^\circ$ , bus 2 is a PQ bus with a per unit load of  $2.0 + j0.5$ , and bus 3 is a PV bus with 1.0 per unit generation and a 1.0 voltage setpoint. The per unit line impedances are  $j0.1$  between buses 1 and 2,  $j0.4$  between buses 1 and 3, and  $j0.2$  between buses 2 and 3. Using a flat start, use the Newton-Raphson approach to determine the first iteration phasor voltages at buses 2 and 3.
- 6.44** Repeat Problem 6.43 except with the bus 2 real power load changed to 1.0 per unit.
- PW 6.45** Load PowerWorld Simulator case Example 6\_11; this case is set to perform a single iteration of the Newton-Raphson power flow each time **Single Solution** is selected. Verify that initially the Jacobian element  $J_{33}$  is 104.41. Then, give and verify the value of this element after each of the next three iterations (until the case converges).
- PW 6.46** Load PowerWorld Simulator case Problem 6\_46. Using a 100 MVA base, each of the three transmission lines have an impedance of  $0.05 + j0.1$  p.u. There is a single 180 MW load at bus 3, while bus 2 is a PV bus with generation of 80 MW and a voltage setpoint of 1.0 p.u. Bus 1 is the system slack with a voltage setpoint of 1.0 p.u. Manually solve this case using the Newton-Raphson approach with a convergence criteria of 0.1 MVA. Show all your work. Then verify your solution by solving the case with PowerWorld Simulator.
- PW 6.47** As was mentioned in Section 6.4, if a generator's reactive power output reaches its limit, then it is modeled as though it were a PQ bus. Repeat Problem 6.46, except assume the generator at bus 2 is operating with its reactive power limited to a maximum of 50 Mvar. Then verify your solution by solving the case with PowerWorld Simulator. To increase the reactive power output of the bus 2 generator, select **Tools, Play** to

begin the power flow simulation, then click on the up arrow on the bus 2 magenta voltage setpoint field until the reactive power output reaches its maximum.

- PW 6.48** Load PowerWorld Simulator case Problem 6\_46. Plot the reactive power output of the generator at bus 2 as a function of its voltage setpoint value in 0.005 p.u. voltage steps over the range between its lower limit of  $-50$  Mvar and its upper limit of  $50$  Mvar. To change the generator 2 voltage set point first select **Tools, Play** to begin the power flow simulation, and then click on the up/down arrows on the bus 2 magenta voltage setpoint field.

## SECTION 6.7

- PW 6.49** Open PowerWorld Simulator case Problem 6\_49. This case is identical to Example 6.9, except that the transformer between buses 1 and 5 is now a tap-changing transformer with a tap range between 0.9 and 1.1 and a tap step size of 0.00625. The tap is on the high side of the transformer. As the tap is varied between 0.975 and 1.1, show the variation in the reactive power output of generator 1,  $V_5$ ,  $V_2$ , and the total real power losses.
- PW 6.50** Use PowerWorld Simulator to determine the Mvar rating of the shunt capacitor bank in the Example 6\_14 case that increases  $V_2$  to 1.0 per unit. Also determine the effect of this capacitor bank on line loadings and the total real power losses (shown immediately below bus 2 on the oneline). To vary the capacitor's nominal Mvar rating, right-click on the capacitor symbol to view the Switched Shunt Dialog, and then change Nominal Mvar field.
- PW 6.51** Use PowerWorld Simulator to modify the Example 6\_9 case by inserting a second line between bus 2 and bus 5. Give the new line a circuit identifier of "2" to distinguish it from the existing line. The line parameters of the added line should be identical to those of the existing lines 2 to 5. Determine the new line's effect on  $V_2$ , the line loadings, and on the total real power losses.
- PW 6.52** Open PowerWorld Simulator case Problem 6\_52. Open the 69 kV line between buses REDBUD69 and PEACH69 (shown towards the bottom of the oneline). With the line open, determine the amount of Mvar (to the nearest 1 Mvar) needed from the REDBUD69 capacitor bank to correct the REDBUD69 voltage to at least 1.0 p.u.
- PW 6.53** Open PowerWorld Simulator case Problem 6\_53. Plot the variation in the total system real power losses as the generation at bus PEAR138 is varied in 20 MW blocks between 0 MW and 400 MW. What value of PEAR138 generation minimizes the total system losses?
- PW 6.54** Repeat Problem 6.53, except first remove the 69 kV line between LOCUST69 and PEAR69.

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## SECTION 6.8

- 6.55** Using the compact storage technique described in Section 6.8, determine the vectors **DIAG**, **OFFDIAG**, **COL**, and **ROW** for the following matrix:

$$S = \begin{bmatrix} 17 & -9.1 & 0 & 0 & -2.1 & -7.1 \\ -9.1 & 25 & -8.1 & -1.1 & -6.1 & 0 \\ 0 & -8.1 & 9 & 0 & 0 & 0 \\ 0 & -1.1 & 0 & 2 & 0 & 0 \\ -2.1 & -6.1 & 0 & 0 & 14 & -5.1 \\ -7.1 & 0 & 0 & 0 & -5.1 & 15 \end{bmatrix}$$

- 6.56** For the triangular factorization of the corresponding  $Y_{bus}$ , number the nodes of the graph shown in Figure 6.9 in an optimal order.

## SECTION 6.10

- 6.57** Compare the angles and line flows between the Example 6\_17 case and results shown in Tables 6.6, 6.7, and 6.8.
- 6.58** Redo Example 6.17 with the assumption that the per-unit reactance on the line between buses 2 and 5 is changed from 0.05 to 0.03.
- PW 6.59** Open PowerWorld Simulator case Problem 6\_59, which models a seven-bus system using the dc power flow approximation. Bus 7 is the system slack. The real power generation/load at each bus is as shown, while the per-unit reactance of each of the lines (on a 100 MVA base) is as shown in yellow on the oneline. (a) Determine the six-by-six **B** matrix for this system and the **P** vector. (b) Use a matrix package such as Matlab to verify the angles as shown on the oneline.
- PW 6.60** Using the PowerWorld Simulator case from Problem 6.59, if the rating on the line between buses 1 and 2 is 150 MW, the current flow is 101 MW (from bus 1 to bus 3), and the bus 1 generation is 160 MW, analytically determine the amount this generation can increase until this line reaches 100% flow. Assume any change in the bus 1 generation is absorbed at the system slack.

## SECTION 6.11

- PW 6.61** PowerWorld Simulator cases Problem 6\_61\_PQ and 6\_61\_PV model a seven-bus power system in which the generation at bus 4 is modeled as a Type 1 or 2 wind turbine in the first case and as a Type 3 or 4 wind turbine in the second. A shunt capacitor is used to make the net reactive power injection at the bus the same in both cases. Compare the bus 4 voltage between the two cases for a contingency in which the line between buses 2 and 4 is opened. What is an advantage of a Type 3 or 4 wind turbine with respect to voltage regulation following a contingency? What is the variation in the Mvar output of a shunt capacitor with respect to bus voltage magnitude?

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**SECTION 6.12**

- 6.62** The fuel-cost curves for two generators are given as follows:

$$C_1(P_1) = 600 + 18 \cdot P_1 + 0.04 \cdot (P_1)^2$$

$$C_2(P_2) = 700 + 20 \cdot P_2 + 0.03 \cdot (P_2)^2$$

Assuming the system is lossless, calculate the optimal dispatch values of  $P_1$  and  $P_2$  for a total load of 1000 MW, the incremental operating cost, and the total operating cost.

- 6.63** Rework Problem 6.62, except assume that the limit outputs are subject to the following inequality constraints:

$$200 \leq P_1 \leq 800 \text{ MW}$$

$$100 \leq P_2 \leq 400 \text{ MW}$$

- 6.64** Rework Problem 6.62, except assume the 1000 MW value also includes losses, and the penalty factor for the first unit is 1.0 and for the second unit 0.95.

- 6.65** The fuel-cost curves for a two-generator power system are given as follows:

$$C_1(P_1) = 600 + 15 \cdot P_1 + 0.05 \cdot (P_1)^2$$

$$C_2(P_2) = 700 + 20 \cdot P_2 + 0.04 \cdot (P_2)^2$$

while the system losses can be approximated as

$$P_L = 2 \times 10^{-4}(P_1)^2 + 3 \times 10^{-4}(P_2)^2 - 4 \times 10^{-4}P_1P_2 \text{ MW}$$

If the system is operating with a marginal cost ( $\lambda$ ) of \$60/hr, determine the output of each unit, the total transmission losses, the total load demand, and the total operating cost.

- 6.66** Expand the summations in (6.12.14) for  $N = 2$ , and verify the formula for  $\partial P_L / \partial P_i$ , given by (6.12.15). Assume  $B_{ii} = B_{ii}$ .

- 6.67** Given two generating units with their respective variable operating costs as

$$C_1 = 0.01P_{G1}^2 + 2P_{G1} + 100 \text{ \$/hr} \quad \text{for } 25 \leq P_{G1} \leq 150 \text{ MW}$$

$$C_2 = 0.004P_{G2}^2 + 2.6P_{G2} + 80 \text{ \$/hr} \quad \text{for } 30 \leq P_{G2} \leq 200 \text{ MW}$$

determine the economically optimum division of generation for  $55 \leq P_L \leq 350$  MW. In particular, for  $P_L = 282$  MW, compute  $P_{G1}$  and  $P_{G2}$ . Neglect transmission losses.

- PW 6.68** Resolve Example 6.20, except with the generation at bus 2 set to a fixed value (i.e., modeled as off of Automatic Generation Control). Plot the variation in the total hourly cost as the generation at bus 2 is varied between 1000 and 200 MW in 5 MW steps, resolving the economic dispatch at each step. What is the relationship between bus 2 generation at the minimum point on this plot and the value from economic dispatch in Example 6.20? Assume a Load Scalar of 1.0.

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- PW 6.69** Using PowerWorld case Example 6\_22 with the Load Scalar equal to 1.0, determine the generation dispatch that minimizes system losses (*Hint: Manually vary the generation at buses 2 and 4 until their loss sensitivity values are zero*). Compare the operating cost between this solution and the Example 6\_22 economic dispatch result. Which is better?
- PW 6.70** Repeat Problem 6.69, except with the Load Scalar equal to 1.4.

### SECTION 6.13

- PW 6.71** Using LP OPF with PowerWorld Simulator case Example 6\_23, plot the variation in the bus 5 marginal price as the Load Scalar is increased from 1.0 in steps of 0.02. What is the maximum possible load scalar without overloading any transmission line? Why is it impossible to operate without violations above this value?
- PW 6.72** Load PowerWorld Simulator case Problem 6\_72. This case models a slightly modified, lossless version of the 37-bus case from Example 6.13 with generator cost information, but also with the transformer between buses PEAR138 and PEAR69 open. When the case is loaded, the “Total Cost” field shows the economic dispatch solution, which results in an overload on several lines. Before solving the case, select **Add-Ons, OPF Case Info, OPF Buses** to view the bus LMPs, noting that they are all identical. Then Select **Add-Ons, Primal LP** to solve the case using the OPF, and again view the bus LMPs. Verify the LMP at the PECAN69 bus by manually changing the load at the bus by one MW, and then noting the change in the Total Cost field. Repeat for the LOCUST69 bus. *Note:* Because of solution convergence tolerances, the manually calculated results may not exactly match the OPF calculated bus LMPs.

## CASE STUDY QUESTIONS

- What are the operational impacts on fossil-fueled power plants due to high penetrations of wind and solar generation into a power grid?
- Do high penetrations of wind and solar generation increase the wear and tear costs of fossil-fueled generation? Why?
- Which has more forecast uncertainty, wind generation or solar generation? Why?

## DESIGN PROJECT I: NEW LOAD

As a result of the low electric rates from the local utility, Metropolis Light and Power (MLP), several large server farms and a new factory are going to be built in the eastern portion of the MLP service territory (see Figure 6.25). With an anticipated peak load of about 75 MW and 20 Mvar, this new load also brings additional revenue to MLP. However, in order to supply this additional load, the new TULIP substation will need to be constructed. While they would like to receive electricity at the

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