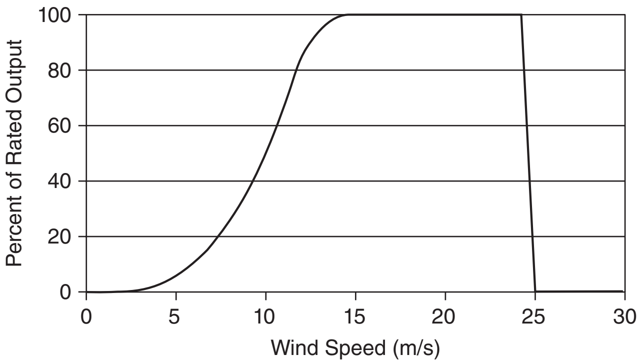
FIGURE 6.16

Typical wind speed versus power curve



controlled, typically these machines are modeled as a constant power factor PQ bus. By themselves these machines have under-excited (consuming reactive power) power factors of between 0.85 and 0.9, but banks of switched capacitors are often used to correct the wind farm power factor. Type 2 WTGs are wound rotor induction machines in which the rotor resistance can be controlled. The advantages of this approach are discussed in Chapter 11; from a power flow perspective, they perform like Type 1 WTGs.

Most of the installed wind capacity and almost all new WTGs are either Type 3 or Type 4. Type 3 wind turbines are used to represent doubly-fed asynchronous generators (DFAGs), also sometimes referred to as doubly-fed induction generators (DFIGs). This type models induction machines in which the rotor circuit is also connected to the ac network through an ac-dc-ac converter, allowing for much greater control of the WTG. Type 4 wind turbines are fully asynchronous machines in which the full power output of the machine is coupled to the ac network through an ac-dc-ac converter. From a power flow perspective, both types are capable of full voltage control like a traditional bus generator with reactive power control between a power factor of up to ± 0.9. However, like traditional synchronous generators, how their reactive power is actually controlled depends on commercial considerations, with many generator owners desiring to operate at unity power factor to maximize their real power outputs.

6.12 ECONOMIC DISPATCH

This section describes how the real power output of a controlled generating unit is selected to meet a given load and to minimize the total operating costs. This is the *economic dispatch* problem [16]. In interconnected power systems, economic dispatch is often solved for smaller portions of the system, known as *areas*, in which the total generation in each area is controlled to match the total area load; further details are provided in Chapter 12.

This section begins by considering only fossil-fuel generating units, with no constraints on maximum and minimum generator outputs, and with no transmission losses. The economic dispatch problem is first solved for this idealized case. Then it is expanded to include inequality constraints on generator outputs and to consider the impact of transmission losses. Finally, the dispatch of other types of units including solar and wind, nuclear, pumped-storage hydro, and hydro units is briefly discussed.

FOSSIL-FUEL UNITS, NO INEQUALITY CONSTRAINTS, NO TRANSMISSION LOSSES

Figure 6.17 shows the operating cost C_i, of a fossil-fuel generating unit versus its real power output P_i. Fuel cost is the major portion of the variable cost of operation, although other variable costs, such as maintenance, could have been included in the figure. Fixed costs, such as the capital cost of installing the unit, are not included. Only those costs that are a function of unit power output—that is, those costs that can be controlled by operating strategy—enter into the economic dispatch formulation.

In practice, C_i is constructed of piecewise continuous functions valid for ranges of output P_i based on empirical data. The discontinuities in Figure 6.17 may be due to the firing of equipment such as additional boilers or condensers as power output is increased. It is often convenient to express C_i in terms of BTU/hr, which is relatively constant over the lifetime of the unit, rather than \$/hr, which can change monthly or daily. C_i can be converted to \$/hr by multiplying the fuel input in BTU/hr by the cost of fuel in \$/BTU.

Figure 6.18 shows the unit incremental operating cost dC_i/dP_i versus unit output P_i, which is the slope or derivative of the C_i versus P_i curve in Figure 6.17. When C_i consists of only fuel costs, dC_i/dP_i is the ratio of the incremental fuel energy input in BTU to incremental energy output in kWh, which is called the incremental *heat rate.* Note that the reciprocal of the heat rate, which is the ratio of output energy to input energy, gives a measure of fuel efficiency for the unit. For the unit shown in Figure 6.17, maximum efficiency occurs at $P_i = 600$ MW, where the heat rate is $C_i/P_i = 5.4 \times 10^9/600 \times 10^3 = 9000$ BTU/kWh. The efficiency at this output is

$$\text{percentage efficency} = \left(\frac{1}{9000}\,\frac{\text{kWh}}{\text{BTU}}\right)\left(3413\,\frac{\text{BTU}}{\text{kWh}}\right) \times 100 = 37.92\%$$

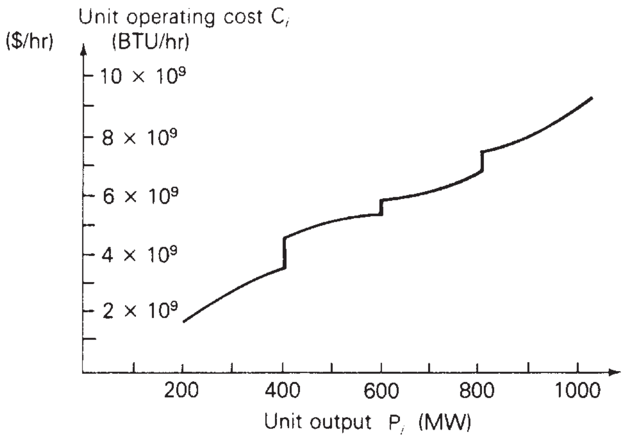
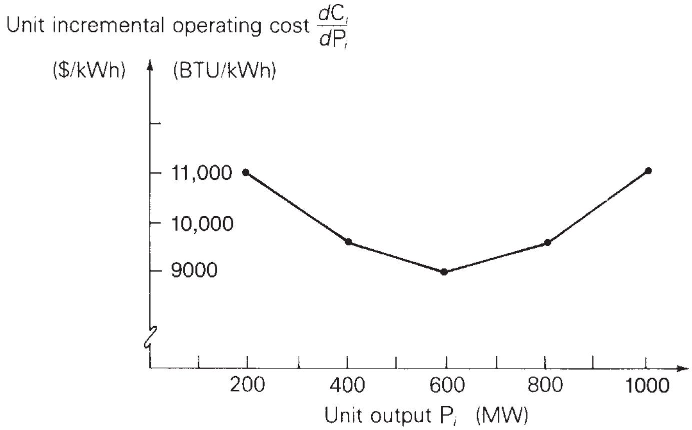


FIGURE 6.17

Unit operating cost versus real power output—fossil-fuel generating unit

FIGURE 6.18 Unit incremental operating cost versus real power output—fossil-fuel generating unit



The dC_i/dP_i curve in Figure 6.18 is also represented by piecewise continuous functions valid for ranges of output P_i. For analytical work, the actual curves are often approximated by straight lines. The ratio dC_i/dP_i can also be converted to $/kWh by multiplying the incremental heat rate in BTU/kWh by the cost of fuel in $/BTU.

For the area of an interconnected power system consisting of N units operating on economic dispatch, the total variable cost C_T of operating these units is

$$
\begin{aligned}
C_T &= \sum_{i=i}^{N} C_i \\
&= C_1(P_1) + C_2(P_2) + \cdots + C_N(P_N) \quad \$/\text{hr}
\end{aligned}
\tag{6.12.1}
$$

where C_i, expressed in $/hr, includes fuel cost as well as any other variable costs of unit i. Let P_T equal the total load demand in the area. Neglecting transmission losses.

$$
P_1 + P_2 + \cdots + P_N = P_T \tag{6.12.2}
$$

Due to relatively slow changes in load demand, P_T may be considered constant for periods of 2 to 10 minutes. The economic dispatch problem can be stated as follows:

> Find the values of unit outputs $P_1, P_2, \ldots, P_N$ that minimize C_T given by (6.12.1), subject to the equality constraint given by (6.12.2).

A criterion for the solution to this problem is: All units on economic dispatch should operate at equal incremental operating cost. That is,

$$
\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \cdots = \frac{dC_N}{dP_N} \tag{6.12.3}
$$

An intuitive explanation of this criterion is the following. Suppose one unit is operating at a higher incremental operating cost than the other units. If the output power of that unit is reduced and transferred to units with lower incremental operating costs, then the total operating cost C_T decreases. That is, reducing the output of the unit with the *higher* incremental cost results in a *greater cost decrease* than the cost increase of adding that same output reduction to units with lower incremental costs. Therefore, all units must operate at the same incremental operating cost (the economic dispatch criterion).

A mathematical solution to the economic dispatch problem also can be given. The minimum value of C_T occurs when the total differential dC_T is zero. That is,

$$dC_T = \frac{\partial C_T}{\partial P_1} dP_1 + \frac{\partial C_T}{\partial P_2} dP_2 + \cdots + \frac{\partial C_T}{\partial P_N} dP_N = 0 \tag{6.12.4}$$

Using (6.12.1), (6.12.4) becomes

$$dC_T = \frac{dC_1}{dP_1} dP_1 + \frac{dC_2}{dP_2} dP_2 + \cdots + \frac{dC_N}{dP_N} dP_N = 0 \tag{6.12.5}$$

Also, assuming P_T is constant, the differential of (6.12.2) is

$$dP_1 + dP_2 + \cdots + dP_N = 0 \tag{6.12.6}$$

Multiplying (6.12.6) by λ and subtracting the resulting equation from (6.12.5),

$$\left(\frac{dC_1}{dP_1} - \lambda\right) dP_1 + \left(\frac{dC_2}{dP_2} - \lambda\right) dP_2 + \cdots + \left(\frac{dC_N}{dP_N} - \lambda\right) dP_N = 0 \tag{6.12.7}$$

Equation (6.12.7) is satisfied when each term in parentheses equals zero. That is,

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \cdots = \frac{dC_N}{dP_N} = \lambda \tag{6.12.8}$$

Therefore, all units have the same incremental operating cost, denoted here by λ, in order to minimize the total operating cost C_T.

EXAMPLE 6.18

Economic dispatch solution neglecting generator limits and line losses

An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by

$$C_1 = 10P_1 + 8 \times 10^{-3} P_1^2 \quad \$/\text{hr}$$

$$C_2 = 8P_2 + 9 \times 10^{-3} P_2^2 \quad \$/\text{hr}$$

where P_1 and P_2 are in megawatts. Determine the power output of each unit, the incremental operating cost, and the total operating cost C_T that minimizes C_T as the total load demand P_T varies from 500 to 1500 MW. Generating unit inequality constraints and transmission losses are neglected.

(*Continued*)

SOLUTION

The incremental operating costs of the units are

$$\frac{dC_1}{dP_1} = 10 + 16 \times 10^{-3}P_1 \quad \$/\text{MWh}$$

$$\frac{dC_2}{dP_2} = 8 + 18 \times 10^{-3}P_2 \quad \$/\text{MWh}$$

Using (6.12.8), the minimum total operating cost occurs when

$$\frac{dC_1}{dP_1} = 10 + 16 \times 10^{-3}P_1 = \frac{dC_2}{dP_2} = 8 + 18 \times 10^{-3}P_2$$

Using $P_2 = P_T - P_1$, the preceding equation becomes

$$10 + 16 \times 10^{-3}P_1 = 8 + 18 \times 10^{-3}(P_T - P_1)$$

Solving for P_1,

$$P_1 = \frac{18 \times 10^{-3}P_T - 2}{34 \times 10^{-3}} = 0.5294P_T - 58.82 \quad \text{MW}$$

Also, the incremental operating cost when C_T is minimized is

$$\frac{dC_2}{dP_2} = \frac{dC_1}{dP_1} = 10 + 16 \times 10^{-3}P_1 = 10 + 16 \times 10^{-3}(0.5294P_T - 58.82)$$

$$= 9.0589 + 8.4704 \times 10^{-3}P_T \quad \$/\text{MWh}$$

P_T MW	P_1 MW	P_2 MW	dC_i/dP_1 \$/MWh	C_T \$/hr
500	206	294	13.29	5529
600	259	341	14.14	6901
700	312	388	14.99	8358
800	365	435	15.84	9899
900	418	482	16.68	11,525
1000	471	529	17.53	13,235
1100	524	576	18.38	15,030
1200	576	624	19.22	16,910
1300	629	671	20.07	18,875
1400	682	718	20.92	20,924
1500	735	765	21.76	23,058

TABLE 6.9

Economic dispatch solution for Example 6.18

and the minimum total operating cost is

$$C_T = C_1 + C_2 = (10P_1 + 8 \times 10^{-3}P_1^2) + (8P_2 + 9 \times 10^{-3}P_2^2) \quad \$/\text{hr}$$

The economic dispatch solution is shown in Table 6.9 for values of P_T from 500 to 1500 MW.

EFFECT OF INEQUALITY CONSTRAINTS

Each generating unit must not operate above its rating or below some minimum value. That is.

$$P_{i\min} < P_i < P_{i\max} \qquad i = 1, 2, \ldots, N \tag{6.12.9}$$

Other inequality constraints also may be included in the economic dispatch problem. For example, some unit outputs may be restricted so that certain transmission lines or other equipment are not overloaded. Also, under adverse weather conditions, generation at some units may be limited to reduce emissions.

When inequality constraints are included, modify the economic dispatch solution as follows. If one or more units reach their limit values, then these units are held at their limits, and the remaining units operate at equal incremental operating cost λ. The incremental operating cost of the area equals the common λ for the units that are not at their limits.

EXAMPLE 6.19

Economic dispatch solution including generator limits

Rework Example 6.18 if the units are subject to the following inequality constraints:

$$100 \leqq P_1 \leqq 600 \quad \text{MW}$$

$$400 \leqq P_2 \leqq 1000 \quad \text{MW}$$

SOLUTION

At light loads, unit 2 operates at its lower limit of 400 MW, where its incremental operating cost is $dC_2/dP_2 = 15.2$ \$/MWh. Additional load comes from unit 1 until $dC_1/dP_1 = 15.2$ \$/MWh, or

$$\frac{dC_1}{dP_1} = 10 + 16 \times 10^{-3}P_1 = 15.2$$

$$P_1 = 325 \quad \text{MW}$$

(Continued)

For P_T less than 725 MW, where P_1 is less than 325 MW, the incremental operating cost of the area is determined by unit 1 alone.

At heavy loads, unit 1 operates at its upper limit of 600 MW, where its incremental operating cost is $dC_1/dP_1 = 19.60$ \$/MWh. Additional load comes from unit 2 for all values of dC_2/dP_2 greater than 19.60 \$/MWh. At $dC_2/dP_2 =$ 19.60 \$/MWh,

$$\frac{dC_2}{dP_2} = 8 + 18 \times 10^{-3} P_2 = 19.60$$

$$P_2 = 644 \quad \text{MW}$$

For P_T greater than 1244 MW, where P_2 is greater than 644 MW, the incremental operating cost of the area is determined by unit 2 alone.

For $725 < P_T < 1244$ MW, neither unit has reached a limit value, and the economic dispatch solution is the same as that given in Table 6.9.

The solution to this example is summarized in Table 6.10 for values of P_T from 500 to 1500 MW.

P_T MW	P_1 MW	P_2 MW		dC/dP \$/MWh	C_T \$/hr
500	100	400	$\frac{dC_1}{dP_1}$	11.60	5720
600	200	400		13.20	6960
700	300	400		14.80	8360
725	325	400		15.20	8735
800	365	435		15.84	9899
900	418	482		16.68	11,525
1000	471	529		17.53	13,235
1100	524	576		18.38	15,030
1200	576	624		19.22	16,910
1244	600	644	$\frac{dC_2}{dP_2}$	19.60	17,765
1300	600	700		20.60	18,890
1400	600	800		22.40	21,040
1500	600	900		24.20	23,370

TABLE 6.10

Economic dispatch solution for Example 6.19

EXAMPLE 6.20

PowerWorld Simulator–economic dispatch, including generator limits

PowerWorld Simulator case Example 6_20 uses a five-bus, three-generator lossless case to show the interaction between economic dispatch and the transmission system (see Figure 6.19). The variable operating costs for each of the units are given by

$$C_1 = 10P_1 + 0.016P_1^2 \quad \$/\text{hr}$$

$$C_2 = 8P_2 + 0.018P_2^2 \quad \$/\text{hr}$$

$$C_4 = 12P_4 + 0.018P_4^2 \quad \$/\text{hr}$$

where P_1, P_2, and P_4 are the generator outputs in megawatts. Each generator has minimum/maximum limits of

$$100 \le P_1 \le 400 \quad \text{MW}$$

$$150 \le P_2 \le 500 \quad \text{MW}$$

$$50 \le P_4 \le 300 \quad \text{MW}$$

In addition to solving the power flow equations, PowerWorld Simulator can simultaneously solve the economic dispatch problem to optimally allocate the generation in an area. To turn on this option, select **Case Information, Aggregation, Areas…** to view a list of each of the control areas in a case (just one in this example). Then toggle the AGC Status field to ED. Now anytime the power flow equations are solved, the generator outputs are also changed using the economic dispatch.

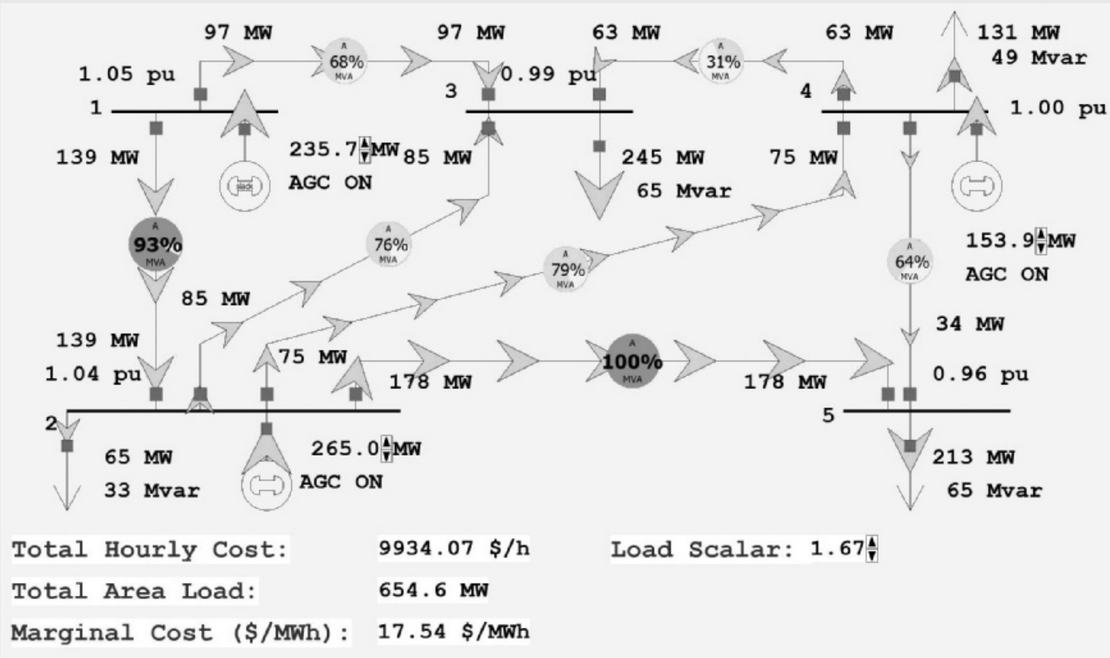


FIGURE 6.19

Example 6.20 with maximum economic loading

(Continued)

Initially, the case has a total load of 392 MW with an economic dispatch of $P_1 = 141$ MW, $P_2 = 181$, and $P_4 = 70$, and an incremental operating cost, λ, of 14.52 \$/MWh. To view a graph showing the incremental cost curves for all of the area generators, right-click on any generator to display the generator's local menu, and then select All Area Gen IC Curves (right-click on the graph's axes to change their scaling).

To see how changing the load impacts the economic dispatch and power flow solutions, first select **Tools, Play** to begin the simulation. Then, on the oneline, click on the up/down arrows next to the Load Scalar field. This field is used to scale the load at each bus in the system. Notice that the change in the Total Hourly Cost field is well approximated by the change in the load multiplied by the incremental operating cost.

Determine the maximum amount of load this system can supply without overloading any transmission line with the generators dispatched using economic dispatch.

SOLUTION

The maximum system economic loading is determined numerically to be 655 MW (which occurs with a Load Scalar of 1.67) with the line from bus 2 to bus 5 being the critical element.

EFFECT OF TRANSMISSION LOSSES

Although one unit may be very efficient with a low incremental operating cost, it also may be located far from the load center. The transmission losses associated with this unit may be so high that the economic dispatch solution requires the unit to decrease its output, while other units with higher incremental operating costs but lower transmission losses increase their outputs.

When transmission losses are included in the economic dispatch problem, (6.12.2) becomes

$$P_1 + P_2 + \cdots + P_N - P_L = P_T \tag{6.12.10}$$

where P_T is the total load demand and P_L is the total transmission loss in the area. In general, P_L is not constant but depends on the unit outputs $P_1, P_2, \ldots, P_N$, The total differential of (6.12.10) is

$$(dP_1 + dP_2 + \cdots + dP_N) - \left(\frac{\partial P_L}{\partial P_1} dP_1 + \frac{\partial P_L}{\partial P_2} dP_2 + \cdots + \frac{\partial P_L}{\partial P_N} dP_N\right) = 0 \tag{6.12.11}$$

Multiplying (6.12.11) by λ and subtracting the resulting equation from (6.12.5),

$$\left(\frac{dC_1}{dP_1} + \lambda\frac{\partial P_L}{\partial P_1} - \lambda\right) dP_1 + \left(\frac{dC_2}{dP_2} + \lambda\frac{\partial P_L}{\partial P_2} - \lambda\right) dP_2$$

$$+ \cdots + \left(\frac{dC_N}{dP_N} + \lambda \frac{\partial P_L}{\partial P_N} - \lambda \right) dP_N = 0 \tag{6.12.12}$$

Equation (6.12.12) is satisfied when each term in parentheses equals zero. That is.

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} - \lambda = 0$$

or

$$\lambda = \frac{dC_i}{dP_i}(L_i) = \frac{dC_i}{dP_i}\left(\frac{1}{1 - \dfrac{\partial P_L}{\partial P_i}} \right) \qquad i = 1, 2, \ldots, N \tag{6.12.13}$$

Equation (6.12.13) gives the economic dispatch criteria, including transmission losses. Each unit that is not at a limit value operates such that its incremental operating cost dC_i/dP_i multiplied by the *penalty factor* L_i is the same. Note that when transmission losses are negligible $\partial P_L/\partial P_i = 0$, $L_i = 1$, and (6.12.13) reduces to (6.12.8).

EXAMPLE 6.21

Economic dispatch solution including generator limits and line losses

Total transmission losses for the power system area given in Example 6.18 are given by

$$P_L = 1.5 \times 10^{-4} P_1^2 + 2 \times 10^{-5} P_1 P_2 + 3 \times 10^{-5} P_2^2 \quad \text{MW}$$

where P_1 and P_2 are given in megawatts. Determine the output of each unit, total transmission losses, total load demand, and total operating cost C_T when the area $\lambda = 16.00$ \$/MWh.

SOLUTION

Using the incremental operating costs from Example 6.18 in (6.12.13),

$$\frac{dC_1}{dP_1}\left(\frac{1}{1 - \dfrac{\partial P_L}{\partial P_1}} \right) = \frac{10 + 16 \times 10^{-3} P_1}{1 - (3 \times 10^{-4} P_1 + 2 \times 10^{-5} P_2)} = 16.00$$

$$\frac{dC_2}{dP_2}\left(\frac{1}{1 - \dfrac{\partial P_L}{\partial P_2}} \right) = \frac{8 + 18 \times 10^{-3} P_2}{1 - (6 \times 10^{-5} P_2 + 2 \times 10^{-5} P_1)} = 16.00$$

(*Continued*)

Rearranging the two equations,

$$20.8 \times 10^{-3}P_1 + 32 \times 10^{-5}P_2 = 6.00$$
$$32 \times 10^{-5}P_1 + 18.96 \times 10^{-3}P_2 = 8.00$$

Solving,

$$P_1 = 282 \quad \text{MW} \qquad\qquad P_2 = 417 \quad \text{MW}$$

Using the equation for total transmission losses,

$$\begin{aligned} P_L &= 1.5 \times 10^{-4}(282)^2 + 2 \times 10^{-5}(282)(417) + 3 \times 10^{-5}(417)^2 \\ &= 19.5 \quad \text{MW} \end{aligned}$$

From (6.12.10), the total load demand is

$$P_T = P_1 + P_2\, P_L = 282 + 417 - 19.5 = 679.5 \quad \text{MW}$$

Also, using the cost formulas given in Example 6.18, the total operating cost is

$$\begin{aligned} C_T = C_1 + C_2 &= 10(282) + 8 \times 10^{-3}(282)^2 + 8(417) + 9 \times 10^{-3}(417)^2 \\ &= 8357 \quad \$/\text{h} \end{aligned}$$

Note that when transmission losses are included, λ given by (6.12.13) is no longer the incremental operating cost of the area. Instead, λ is the unit incremental operating cost dC_i/dP_i multiplied by the unit penalty factor L_i.

EXAMPLE 6.22

PowerWorld Simulator—economic dispatch, including generator limits and line losses

This example repeats the Example 6.19 power system, except that now losses are included with each transmission line modeled with an R/X ratio of 1/3 (see Figure 6.20). The current value of each generator's loss sensitivity, $\partial P_L/\partial P_i$, is shown immediately below the generator's MW output field. Calculate the penalty factors L_i, and verify that the economic dispatch shown in the figure is optimal. Assume a Load Scalar of 1.0.

SOLUTION

From (6.12.13), the condition for optimal dispatch is

$$\lambda = dC_i/dP_i(1/(1 - \partial P_L/\partial P_i) = dC_i/dP_i L_i \qquad i = 1, 2, \ldots, N$$

with

$$L_i = 1/(1 - \partial P_L/\partial P_i)$$

Therefore, $L_1 = 1.0$, $L_2 = 0.9733$, and $L_4 = 0.9238$.

$$\begin{aligned} \text{with } P_1 = 130.1 \text{ MW}, \quad dC_1/dP_1 * L_1 &= (10 + 0.032 * 130.1) * 1.0 \\ &= 14.16 \quad \$/\text{MWh} \end{aligned}$$

$$\text{With } P_2 = 181.8 \text{ MW}, \quad dC_2/dP_2 * L_2 = (8 + 0.036 * 181.8) * 0.9733$$
$$= 14.16 \quad \$/\text{MWh}$$
$$\text{With } P_4 = 92.4 \text{ MW}, \quad dC_4/dP_4 * L_4 = (12 + 0.036 * 92.4) * 0.9238$$
$$= 14.16 \quad \$/\text{MWh}$$

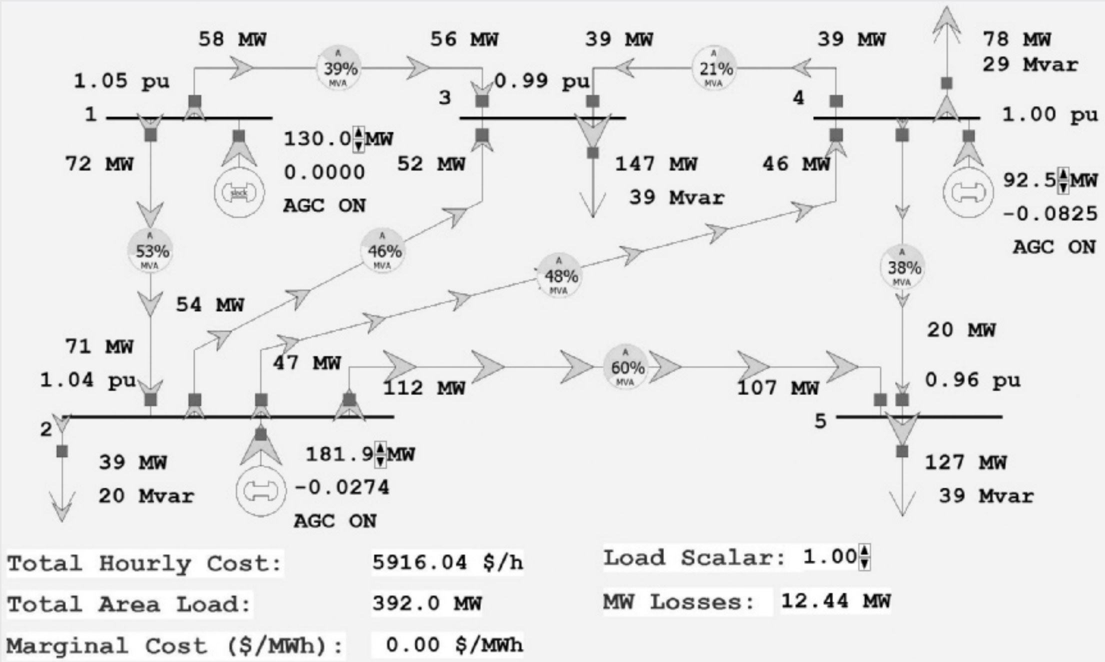


FIGURE 6.20

Example 6.22 five-bus case with transmission line losses

In Example 6.21, total transmission losses are expressed as a quadratic function of unit output powers. For an area with N units, this formula generalizes to

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j \tag{6.12.14}$$

where the B_{ij} terms are called *loss coefficients* or B *coefficients*. The B coefficients are not truly constant but vary with unit loadings. However, the B coefficients are often assumed constant in practice since the calculation of $\partial P_L / \partial P_i$ is thereby simplified. Using (6.12.14),

$$\frac{\partial P_L}{\partial P_i} = 2 \sum_{j=1}^{N} B_{ij} P_j \tag{6.12.15}$$

This equation can be used to compute the penalty factor L_i, in (6.12.13).

Various methods of evaluating B coefficients from power flow studies are available [17]. In practice, more than one set of B coefficients may be used during the daily load cycle.

When the unit incremental cost curves are linear, an analytic solution to the economic dispatch problem is possible, as illustrated by Examples 6.18 through 6.20. However, in practice, the incremental cost curves are nonlinear and contain discontinuities. In this case, an iterative solution can be obtained. Given the load demand P_T, the unit incremental cost curves, generator limits, and B coefficients, such an iterative solution can be obtained by the following nine steps. Assume that the incremental cost curves are stored in tabular form, such that a unique value of P_i can be read for each dC_i/dP_i.

STEP 1 Set iteration index $m = 1$.

STEP 2 Estimate mth value of λ.

STEP 3 Skip this step for all $m > 1$. Determine initial unit outputs P_i, ($i = 1, 2, \ldots, N$). Use $dC_i/dP_i = \lambda$ and read P_i from each incremental operating cost table. Transmission losses are neglected here.

STEP 4 Compute $\partial P_L/\partial P_i$ from (6.12.15) ($i = 1, 2, \ldots, N$).

STEP 5 Compute dC_i/dP_i from (6.12.13) ($i = 1, 2, \ldots, N$).

STEP 6 Determine updated values of unit output P_i ($i = 1, 2, \ldots, N$). Read P_i from each incremental operating cost table. If P_i exceeds a limit value, set P_i to the limit value.

STEP 7 Compare P_i determined in Step 6 with the previous value ($i = 1, 2, \ldots, N$). If the change in each unit output is less than a specified tolerance ε_1, go to Step 8. Otherwise, return to Step 4.

STEP 8 Compute P_L from (6.12.14).

STEP 9 If $\left|\left(\sum_{i=1}^{N} P_i\right) - P_L - P_T\right|$ is less than a specified tolerance ε_2, stop. Otherwise, set $m = m + 1$ and return to Step 2.

Instead of having their values stored in tabular form for this procedure, the incremental cost curves instead could be represented by nonlinear functions such as polynomials. Then, in Step 3 and Step 5, each unit output P_i would be computed from the nonlinear functions instead of being read from a table. Note that this procedure assumes that the total load demand P_T is constant. In practice, this economic dispatch program is executed every few minutes with updated values of P_T.

OTHER TYPES OF UNITS

The economic dispatch criterion has been derived for a power system area consisting of fossil-fuel generating units. In practice, however, an area has a mix of different types of units including fossil-fuel, nuclear, pumped-storage hydro, hydro, wind, and other types.

Wind and solar generation, which have no fuel costs, are represented with very low or negative cost. As such, they are preferred sources for economic dispatch and are used by system operators whenever possible, unless there are generator operating limits or transmission constraints.

Although the fixed costs of a nuclear unit may be high, their operating costs are low due to inexpensive nuclear fuel. As such, nuclear units are normally base-loaded at their rated outputs. That is, the reference power settings of turbine-governors for nuclear units are held constant at rated output; therefore, these units do not participate in economic dispatch.

Pumped-storage hydro is a form of energy storage. During off-peak hours, these units are operated as synchronous motors to pump water to a higher elevation. Then during peak-load hours the water is released, and the units are operated as synchronous generators to supply power. As such, pumped-storage hydro units are used for light-load build-up and peak-load shaving. Economic operation of the area is improved by pumping during off-peak hours when the area λ is low, and by generating during peak-load hours when λ is high. Techniques are available for incorporating pumped-storage hydro units into economic dispatch of fossil-fuel units [18].

In an area consisting of hydro plants located along a river, the objective is to maximize the energy generated over the yearly water cycle rather than to minimize total operating costs. Reservoirs are used to store water during high-water or light-load periods, although some water may have to be released through spillways. Also, there are constraints on water levels due to river transportation, irrigation, or fishing requirements. Optimal strategies are available for coordinating outputs of plants along a river [19]. Economic dispatch strategies for mixed fossil-fuel/hydro systems are also available [20, 21, 22].

Techniques are also available for including reactive power flows in the economic dispatch formulation, whereby both active and reactive powers are selected to minimize total operating costs. In particular, reactive injections from generators, switched capacitor banks, and static var systems, along with transformer tap settings, can be selected to minimize transmission-line losses [22]. However, electric utility companies usually control reactive power locally. That is, the reactive power output of each generator is selected to control the generator terminal voltage, and the reactive power output of each capacitor bank or static var system located at a power system bus is selected to control the voltage magnitude at that bus. In this way, the reactive power flows on transmission lines are low, and the need for central dispatch of reactive power is eliminated.

6.13 OPTIMAL POWER FLOW

Economic dispatch has one significant shortcoming—it ignores the limits imposed by the devices in the transmission system. Each transmission line and transformer has a limit on the amount of power that can be transmitted through it, with the limits arising because of thermal, voltage, or stability considerations (Section 5.6). Traditionally, the transmission system was designed so that when the generation was dispatched economically there would be no limit violations. Hence, just solving economic dispatch was usually sufficient. However, with the worldwide trend toward deregulation of the electric utility industry, the transmission system is becoming increasingly constrained (with these constraints sometimes called congestion). For example, in the PJM power market in the eastern United States, the costs associated with active transmission line and transformer limit violations (congestion) increased from $65 million in 1999 to almost $2.1 billion in 2005 and have averaged about $1 billion per year from 2008 to 2013 [23].

The solution to the problem of optimizing the generation while enforcing the transmission lines is to combine economic dispatch with either the full ac power flow, or a dc power flow. The result is known as the optimal power flow (OPF). There are several methods for solving the OPF with [24] providing a nice summary. One common approach is sequential linear programming (LP); this is the technique used with the PowerWorld Simulator. The LP OPF solution algorithm iterates between solving the power flow to determine the flow of power in the system devices and solving an LP to economically dispatch the generation (and possibility other controls) subject to the transmission system limits. In the absence of system elements loaded to their limits, the OPF generation dispatch is identical to the economic dispatch solution, and the marginal cost of energy at each bus is identical to the system λ. However, when one or more elements are loaded to their limits, the economic dispatch becomes constrained, and the bus marginal energy prices are no longer identical. In some electricity markets, these marginal prices are known as the Locational Marginal Prices (LMPs) and are used to determine the wholesale price of electricity at various locations in the system. For example, the real-time LMPs for the Midcontinent ISO (MISO) are available online at www.misoenergy.org/MarketsOperations.

EXAMPLE 6.23

PowerWorld Simulator—optimal power flow

PowerWorld Simulator case Example 6_23 duplicates the five-bus case from Example 6.20, except that the case is solved using PowerWorld Simulator's LP OPF algorithm (see Figure 6.21). To turn on the OPF option, first select **Case Information, Aggregation, Areas…,** and toggle the AGC Status field to OPF. Then, rather than solving the case with the "Single Solution" button, select **Add-ons, Primal LP** to solve using the LP OPF. Initially the OPF solution matches the ED solution from Example 6.20 since there are no overloaded lines. The green-colored fields on the screen immediately to the right of the buses show the marginal cost of supplying electricity to each bus in the system (i.e., the bus LMPs). With the system initially unconstrained, the bus marginal prices are all identical at $14.5/MWh, with a Load Scalar of 1.0.

Now increase the Load Scalar field from 1.00 to the maximum economic loading value, determined to be 1.67 in Example 6.20, and again select **Add-ons, Primal LP**. The bus marginal prices are still all identical, now at a value of $17.5/MWh, and with the line from bus 2 to 5 just reaching its maximum value. For load scalar values above 1.67, the line from bus 2 to bus 5 becomes constrained, with a result that the bus marginal prices on the constrained side of the line become higher than those on the unconstrained side.

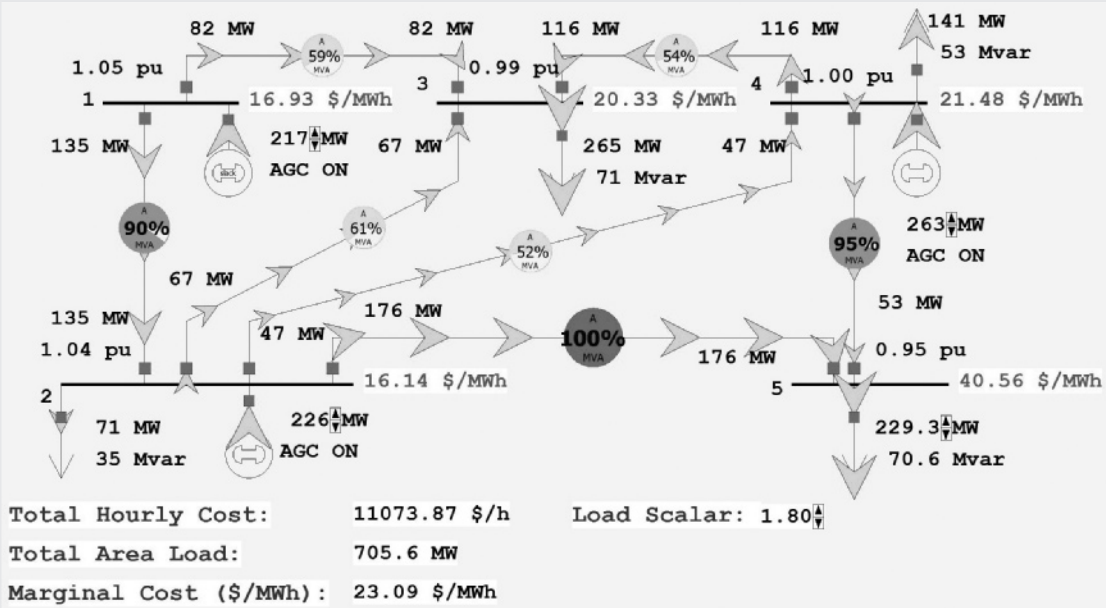


FIGURE 6.21

Example 6.23 optimal power flow solution with load multiplier = 1.80

With the load scalar equal to 1.80, numerically verify that the price of power at bus 5 is approximately \$40.60/MWh.

SOLUTION

The easiest way to numerically verify the bus 5 price is to increase the load at bus 5 by a small amount and compare the change in total system operating cost. With a load scalar of 1.80, the bus 5 MW load is 229.3 MW with a case hourly cost of \$11,073.90. Increasing the bus 5 load by 0.9 MW and resolving the LP OPF gives a new cost of \$11,110.40, which is a change of about \$40.60/MWh (note that this increase in load also increases the bus 5 price to over \$41/MWh). Because of the constraint, the price of power at bus 5 is actually more than double the incremental cost of the most expensive generator!

MULTIPLE CHOICE QUESTIONS

SECTION 6.1

6.1 For a set of linear algebraic equations in matrix format, $\mathbf{Ax} - \mathbf{y}$, for a unique solution to exist, det($\mathbf{A}$) should be ________.

6.2 For an $N \times N$ square matrix $\mathbf{A}$, in $(N - 1)$ steps, the technique of Gauss elimination can transform into an ________ matrix.