5.19 The maximum power that a lossless line can deliver, in terms of the voltage magnitudes V_S and V_R (in volts) at the ends of the line held constant, and the series reactance X' of the corresponding equivalent π circuit, is given by __________, in watts.

SECTION 5.5

5.20 The maximum power flow for a lossy line is somewhat less than that for a lossless line.
(a) True (b) False

SECTION 5.6

5.21 For short lines less than 25 km long, loadability is limited by the thermal rating of the conductors or by terminal equipment ratings, not by voltage drop or stability considerations.
(a) True (b) False

5.22 Increasing the transmission line voltage reduces the required number of lines for the same power transfer.
(a) True (b) False

5.23 Intermediate substations are often economical from the viewpoint of the number of lines required for power transfer if their costs do not outweigh the reduction in line costs.
(a) True (b) False

SECTION 5.7

5.24 Shunt reactive compensation improves transmission-line ______, whereas series capacitive compensation increases transmission-line ______.

5.25 Static-var-compensators can absorb reactive power during light loads and deliver reactive power during heavy loads.
(a) True (b) False

PROBLEMS

SECTION 5.1

5.1 A 30-km, 34.5-kV, 60-Hz, three-phase line has a positive-sequence series impedance $z = 0.19 + j0.34\ \Omega$/km. The load at the receiving end absorbs 10 MVA at 33 kV. Assuming a short line, calculate: (a) the $ABCD$ parameters, (b) the sending-end voltage for a load power factor of 0.9 lagging, and (c) the sending-end voltage for a load power factor of 0.9 leading.

5.2 A 200-km, 230-kV, 60-Hz, three-phase line has a positive-sequence series impedance $z = 0.08 + j0.48\ \Omega$/km and a positive-sequence shunt admittance $y = j3.33 \times 10^{-6}$ S/km. At full load, the line delivers 250 MW at

0.99 p.f. lagging and at 220 kV. Using the nominal π circuit, calculate: (a) the *ABCD* parameters, (b) the sending-end voltage and current, and (c) the percent voltage regulation.

5.3 Rework Problem 5.2 in per unit using 1000-MVA (three-phase) and 230-kV (line-to-line) base values. Calculate: (a) the per-unit *ABCD* parameters, (b) the per-unit sending-end voltage and current, and (c) the percent voltage regulation.

5.4 Derive the *ABCD* parameters for the two networks in series, as shown in Figure 5.4.

5.5 Derive the *ABCD* parameters for the T circuit shown in Figure 5.4.

5.6 (a) Consider a medium-length transmission line represented by a nominal π circuit shown in Figure 5.3 of the text. Draw a phasor diagram for lagging power-factor condition at the load (receiving end).

(b) Now consider a nominal T circuit of the medium-length transmission line shown in Figure 5.18.

First draw the corresponding phasor diagram for lagging power-factor load condition.

Then determine the *ABCD* parameters in terms of Y and Z for the nominal T circuit and for the nominal π circuit of part (a).

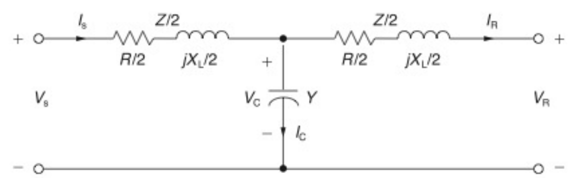


FIGURE 5.18 Nominal T-circuit for Problem 5.6

5.7 The per-phase impedance of a short three-phase transmission line is $0.5\angle 53.15^\circ\ \Omega$. The three-phase load at the receiving end is 1200 kW at 0.8 p.f. lagging. If the line-to-line sending-end voltage is 3.3 kV, determine (a) the receiving-end line-to-line voltage in kV and (b) the line current. Draw the phasor diagram with the line current I, as reference.

5.8 Reconsider Problem 5.7 and find the following: (a) sending-end power factor, (b) sending-end three-phase power, and (c) the three-phase line loss.

5.9 The 100-km, 230-kV, 60-Hz, three-phase line in Problems 4.18 and 4.39 delivers 300 MVA at 218 kV to the receiving end at full load. Using the nominal π circuit, calculate the *ABCD* parameters, sending-end voltage, and percent voltage regulation when the receiving-end power factor is (a) 0.9 lagging, (b) unity, and (c) 0.9 leading. Assume a 50°C conductor temperature to determine the resistance of this line.

5.10 The 500-kV, 60-Hz, three-phase line in Problems 4.20 and 4.41 has a 180-km length and delivers 1600 MW at 475 kV and at 0.95 power factor leading to the receiving end at full load. Using the nominal π circuit, calculate the (a) $ABCD$ parameters, (b) sending-end voltage and current, (c) sending-end power and power factor, (d) full-load line losses and efficiency, and (e) percent voltage regulation. Assume a 50°C conductor temperature to determine the resistance of this line.

5.11 A 40-km, 220-kV, 60-Hz, three-phase overhead transmission line has a per-phase resistance of 0.15 Ω/km, a per-phase inductance of 1.3263 mH/km, and negligible shunt capacitance. Using the short line model, find the sending-end voltage, voltage regulation, sending-end power, and transmission line efficiency when the line is supplying a three-phase load of (a) 381 MVA at 0.8 power factor lagging and at 220 kV and (b) 381 MVA at 0.8 power factor leading and at 220 kV.

5.12 A 60-Hz, 100-mile, three-phase overhead transmission line, constructed of ACSR conductors, has a series impedance of $(0.1826 + j0.784)$ Ω/mi per phase and a shunt capacitive reactance-to-neutral of $185.5 \times 10^3 \underline{/-90^\circ}$ Ω-mi per phase. Using the nominal π circuit for a medium-length transmission line, (a) determine the total series impedance and shunt admittance of the line; (b) compute the voltage, the current, and the real and reactive power at the sending end if the load at the receiving end draws 200 MVA at unity power factor and at a line-to-line voltage of 230 kV; and (c) find the percent voltage regulation of the line.

SECTION 5.2

5.13 Evaluate $\cosh(\gamma l)$ and $\tanh(\gamma l/2)$ for $\gamma l = 0.40\underline{/85^\circ}$ per unit.

5.14 A 500-km, 500-kV, 60-Hz, uncompensated three-phase line has a positive-sequence series impedance $z = 0.03 + j0.35$ Ω/km and a positive-sequence shunt admittance $y = j4.4 \times 10^{-6}$ S/km. Calculate: (a) Z_c, (b) (γl), and (c) the exact $ABCD$ parameters for this line.

5.15 At full load, the line in Problem 5.14 delivers 900 MW at unity power factor and at 475 kV. Calculate: (a) the sending-end voltage, (b) the sending-end current, (c) the sending-end power factor, (d) the full-load line losses, and (e) the percent voltage regulation.

5.16 The 500-kV, 60-Hz, three-phase line in Problems 4.20 and 4.41 has a 300-km length. Calculate: (a) Z_c, (b) (γl), and (c) the exact $ABCD$ parameters for this line. Assume a 50°C conductor temperature.

5.17 At full load, the line in Problem 5.16 delivers 1500 MVA at 480 kV to the receiving-end load. Calculate the sending-end voltage and percent voltage regulation when the receiving-end power factor is (a) 0.9 lagging, (b) unity, and (c) 0.9 leading.

5.18 A 60-Hz, 230-mile, three-phase overhead transmission line has a series impedance $z = 0.8431\underline{/79.04^\circ}$ Ω/mi and a shunt admittance $\gamma = 5.105 \times 10^{-6}\underline{/90^\circ}$ S/mi. The load at the receiving end is 125 MW at unity power factor and at 215 kV. Determine the voltage, current, and both real and reactive power at the sending end and the percent voltage regulation of the line. Also find the wavelength and velocity of propagation of the line.

5.19 Using per-unit calculations, rework Problem 5.18 to determine the sending-end voltage and current.

5.20 (a) The series expansions of the hyperbolic functions are given by

$$\cosh\theta = 1 + \frac{\theta^2}{2} + \frac{\theta^4}{24} + \frac{\theta^6}{720} + \cdots$$

$$\sinh\theta = 1 + \frac{\theta^2}{6} + \frac{\theta^4}{120} + \frac{\theta^6}{5040} + \cdots$$

For the $ABCD$ parameters of a long transmission line represented by an equivalent π circuit, apply the above expansion considering only the first two terms, and express the result in terms of Y and Z.

(b) For the nominal π and equivalent π circuits shown in Figures 5.3 and 5.7 of the text, show that

$$\frac{A-1}{B} = \frac{Y}{2} \quad \text{and} \quad \frac{A-1}{B} = \frac{Y'}{2}$$

hold good, respectively.

5.21 Starting with (5.1.1) of the text, show that

$$A = \frac{V_S I_S + V_R I_R}{V_R I_S + V_S I_R} \quad \text{and} \quad B = \frac{V_S^2 - V_R^2}{V_R I_S + V_S I_R}$$

5.22 Consider the A parameter of the long line given by $\cosh\theta$, where $\theta = \sqrt{ZY}$. With $x = e^{-\theta} = x_1 + jx_2$ and $A = \mathrm{A}_1 + j\mathrm{A}_2$, show that x_1 and x_2 satisfy the following:

$$x_1^2 - x_2^2 - 2(\mathrm{A}_1 x_1 - \mathrm{A}_2 x_2) + 1 = 0$$

and $x_1 x_2 - (\mathrm{A}_2 x_1 + \mathrm{A}_1 x_2) = 0.$

SECTION 5.3

5.23 Determine the equivalent π circuit for the line in Problem 5.14 and compare it with the nominal π circuit.

5.24 Determine the equivalent π circuit for the line in Problem 5.16. Compare the equivalent π circuit with the nominal π circuit.

5.25 Let the transmission line of Problem 5.12 be extended to cover a distance of 200 miles. Assume conditions at the load to be the same as in Problem 5.12. Determine the (a) sending-end voltage, (b) sending-end current, (c) sending-end real and reactive powers, and (d) percent voltage regulation.

SECTION 5.4

5.26 A 350-km, 500-kV, 60-Hz, three-phase uncompensated line has a positive-sequence series reactance $x = 0.34\ \Omega$/km and a positive-sequence shunt admittance $y = j4.5 \times 10^{-6}$ S/km. Neglecting losses, calculate: (a) Z_c, (b) γl, (c) the $ABCD$ parameters, (d) the wavelength λ of the line in kilometers, and (e) the surge impedance loading in MW.

5.27 Determine the equivalent π circuit for the line in Problem 5.26.

5.28 Rated line voltage is applied to the sending end of the line in Problem 5.26. Calculate the receiving-end voltage when the receiving end is terminated by (a) an open circuit, (b) the surge impedance of the line, and (c) one-half of the surge impedance. (d) Also calculate the theoretical maximum real power that the line can deliver when rated voltage is applied to both ends of the line.

5.29 Rework Problems 5.9 and 5.16, neglecting the conductor resistance. Compare the results with and without losses.

5.30 From (4.6.22) and (4.10.4), the series inductance and shunt capacitance of a three-phase overhead line are

$$L_a = 2 \times 10^{-7} \ln(D_{eq}/D_{SL}) = \frac{\mu_0}{2\pi} \ln(D_{eq}/D_{SL}) \quad \text{H/m}$$

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln(D_{eq}/D_{SC})} \quad \text{F/m}$$

where $\mu_0 = 4\pi \times 10^{-7}$H/m and $\varepsilon_0 = \left(\frac{1}{36\pi}\right) \times 10^{-9}$F/m.

Using these equations, determine formulas for surge impedance and velocity of propagation of an overhead lossless line. Then determine the surge impedance and velocity of propagation for the three-phase line given in Example 4.5. Assume positive-sequence operation. Neglect line losses as well as the effects of the overhead neutral wires and the earth plane.

5.31 A 500-kV, 300-km, 60-Hz, three-phase overhead transmission line, assumed to be lossless, has a series inductance of 0.97 mH/km per phase and a shunt capacitance of 0.0115 μF/km per phase. (a) Determine the phase constant β, the surge impedance Z_c, velocity of propagation v, and the wavelength λ of the line. (b) Determine the voltage, current, real and reactive power at the sending end, and the percent voltage regulation of the line if the receiving-end load is 800 MW at 0.8 power factor lagging and at 500 kV.

5.32 The following parameters are based on a preliminary line design: $V_S = 1.0$ per unit, $V_R = 0.9$ per unit, $\lambda = 5000$ km, $Z_c = 320\ \Omega$, $\delta = 36.8°$.

A three-phase power of 700 MW is to be transmitted to a substation located 315 km from the source of power. (a) Determine a nominal voltage level for the three-phase transmission line, based on the practical line-loadability equation. (b) For the voltage level obtained in part (a), determine the theoretical maximum power that can be transferred by the line.

5.33 Consider a long radial line terminated in its characteristic impedance Z_c. Determine the following:

(a) V_1/I_1, known as the driving point impedance.

(b) $|V_2|/|V_1|$, known as the voltage gain, in terms of $\alpha\ell$.

(c) $|I_2|/|I_1|$, known as the current gain, in terms of $\alpha\ell$.

(d) The complex power gain, $-S_{21}/S_{12}$, in terms of $\alpha\ell$.

(e) The real power efficiency, $(-P_{21}/P_{12}) = \eta$, terms of $\alpha\ell$.

Note: 1 refers to sending end and 2 refers to receiving end. (S_{21}) is the complex power received at 2; S_{12} is sent from 1.

5.34 For the case of a lossless line, how would the results of Problem 5.33 change?

In terms of Z_c, which is a real quantity for this case, express P_{12} in terms $|I_1|$ and $|V_1|$.

5.35 For a lossless open-circuited line, express the sending-end voltage, V_1, in terms of the receiving-end voltage, V_2, for the three cases of short-line model, medium-length line model, and long-line model. Is it true that the voltage at the open receiving end of a long line is higher than that at the sending end, for small βl?

5.36 For a short transmission line of impedance (R + jX) ohms per phase, show that the maximum power that can be transmitted over the line is

$$P_{max} = \frac{V_R^2}{Z^2}\left(\frac{ZV_S}{V_R} - R\right) \quad \text{where } Z = \sqrt{R^2 + X^2}$$

when the sending-end and receiving-end voltages are fixed, and for the condition

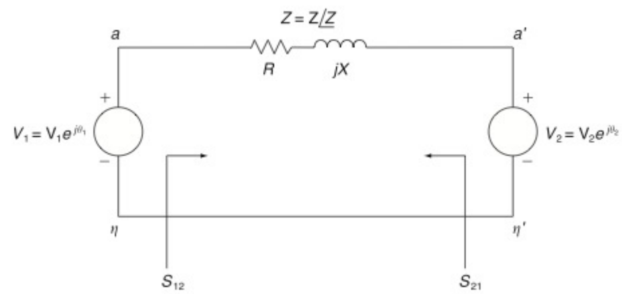
$$Q = \frac{-V_R^2 X}{R^2 + X^2} \quad \text{when } dP/dQ = 0$$

5.37 (a) Consider complex power transmission via the three-phase short line for which the per-phase circuit is shown in Figure 5.19. Express S_{12}, the complex power sent by bus 1 (or V_1), and $(-S_{21})$, the complex power received by bus 2 (or V_2), in terms of V_1, V_2, Z, $\underline{/Z}$, and $\theta_{12} = \theta_1 - \theta_2$, which is the power angle.

(b) For a balanced three-phase transmission line in per-unit notation with $Z = 1\underline{/85°}$, $\theta_{12} = 10°$, determine S_{12} and $(-S_{21})$ for

(i) $V_1 = V_2 = 1.0$
(ii) $V_1 = 1.1$ and $V_2 = 0.9$
Comment on the changes of real and reactive powers from parts (i) to (ii).

FIGURE 5.19
Per-phase circuit for Problem 5.37



SECTION 5.5

5.38 The line in Problem 5.14 has three ACSR 1113 kcmil conductors per phase. Calculate the theoretical maximum real power that this line can deliver and compare with the thermal limit of the line. Assume $V_S = V_R = 1.0$ per unit and unity power factor at the receiving end.

5.39 Repeat Problems 5.14 and 5.38 if the line length is (a) 200 km or (b) 600 km.

5.40 For the 500 kV line given in Problem 5.16, (a) calculate the theoretical maximum real power that the line can deliver to the receiving end when rated voltage is applied to both ends; (b) calculate the receiving-end reactive power and power factor at this theoretical loading.

5.41 A 230-kV, 100-km, 60-Hz, three-phase overhead transmission line with a rated current of 900 A/phase has a series impedance $z = 0.088 + j0.465$ Ω/km and a shunt admittance $y = j3.524$ μS/km. (a) Obtain the nominal π equivalent circuit in normal units and in per unit on a base of 100 MVA (three phase) and 230 kV (line-to-line). (b) Determine the three-phase rated MVA of the line. (c) Compute the *ABCD* parameters. (d) Calculate the SIL.

5.42 A three-phase power of 460 MW is transmitted to a substation located 500 km from the source of power. With $V_S = 1$ per unit, $V_R = 0.9$ per unit, $\lambda = 5000$ km, $Z_c = 500\ \Omega$, and $\delta = 36.87°$, determine a nominal voltage level for the lossless transmission line based on Eq. (5.4.29) of the text.
Using this result, find the theoretical three-phase maximum power that can be transferred by the lossless transmission line.

PW **5.43** Open PowerWorld Simulator case Example 5_4 and graph the load bus voltage as a function of load real power (assuming unity power factor at the load). What is the maximum amount of real power that can be transferred to the load at unity power factor if the load voltage always must be greater than 0.9 per unit?

PW **5.44** Repeat Problem 5.43, but now vary the load reactive power, assuming the load real power is fixed at 1499 MW.

SECTION 5.6

5.45 For the line in Problems 5.14 and 5.38, determine (a) the practical line loadability in MW, assuming $V_S = 1.0$ per unit, $V_R \approx 0.95$ per unit, and $\delta_{max} = 35°$; part (b) the full-load current at 0.99 p.f. leading, based on the above practical line loadability; (c) the exact receiving-end voltage for the full-load current in part (b); and (d) the percent voltage regulation. For this line, is loadability determined by the thermal limit, the voltage-drop limit, or steady-state stability?

5.46 Repeat Problem 5.45 for the 500 kV line given in Problem 5.10.

5.47 Determine the practical line loadability in MW and in per-unit of SIL for the line in Problem 5.14 if the line length is (a) 200 km or (b) 600 km. Assume $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, $\delta_{max} = 35°$, and 0.99 leading power factor at the receiving end.

5.48 It is desired to transmit 2000 MW from a power plant to a load center located 300 km from the plant. Determine the number of 60 Hz, three-phase, uncompensated transmission lines required to transmit this power with one line out of service for the following cases: (a) 345 kV lines, $Z_c = 300\ \Omega$, (b) 500 kV lines, $Z_c = 275\ \Omega$, (c) 765 kV lines, $Z_c = 260\ \Omega$. Assume that $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, and $\delta_{max} = 35°$.

5.49 Repeat Problem 5.48 if it is desired to transmit: (a) 3200 MW to a load center located 300 km from the plant or (b) 2000 MW to a load center located 400 km from the plant.

5.50 A three-phase power of 4000 MW is to be transmitted through four identical 60-Hz overhead transmission lines over a distance of 300 km. Based on a preliminary design, the phase constant and surge impedance of the line are $\beta = 9.46 \times 10^{-4}$ rad/km and $Z_c = 343\ \Omega$, respectively. Assuming $V_S = 1.0$ per unit, $V_R = 0.9$ per unit, and a power angle $\delta = 36.87°$, determine a suitable nominal voltage level in kV based on the practical line-loadability criteria.

5.51 The power flow at any point on a transmission line can be calculated in terms of the *ABCD* parameters. By letting $A = |A|\underline{/\alpha}$, $B = |B|\underline{/\beta}$, $V_R = |V_R|\underline{/0°}$, and $V_S = |V_S|\underline{/\delta}$, the complex power at the receiving end can be shown to be

$$P_R + jQ_R = \frac{|V_R||V_s|\underline{/\beta - \alpha}}{|B|} - \frac{|\delta||V_R^2|\underline{/\beta - \alpha}}{|B|}$$

(a) Draw a phasor diagram corresponding to the above equation. Let it be represented by a triangle O′OA with O′ as the origin and OA representing $P_R + jQ_R$.

(b) By shifting the origin from O′ to O, turn the result of part (a) into a power diagram, redrawing the phasor diagram. For a given fixed value of $|V_R|$ and a set of values for $|V_S|$, draw the loci of point A, thereby showing the so-called receiving-end circles.

(c) From the result of part (b) for a given load with a lagging power factor angle θ_R, determine the amount of reactive power that must be supplied to the receiving end to maintain a constant receiving-end voltage if the sending-end voltage magnitude decreases from $|V_{S1}|$ to $|V_{S2}|$

5.52 (a) Consider complex power transmission via the three-phase long line for which the per-phase circuit is shown in Figure 5.20. See Problem 5.37 in which the short-line case was considered. Show that

$$\text{sending-end power} = S_{12} = \frac{Y'^*}{2}V_1^2 + \frac{V_1^2}{Z'^*} - \frac{V_1V_2}{Z'^*}e^{j\theta_{12}}$$

$$\text{received power} = -S_{21} = -\frac{Y'^*}{2}V_2^2 - \frac{V_2^2}{Z'^*} + \frac{V_1V_2}{Z'^*}e^{-j\theta_{12}}$$

where $\theta_{12} = \theta_1 - \theta_2$.

(b) For a lossless line with equal voltage magnitudes at each end, show that

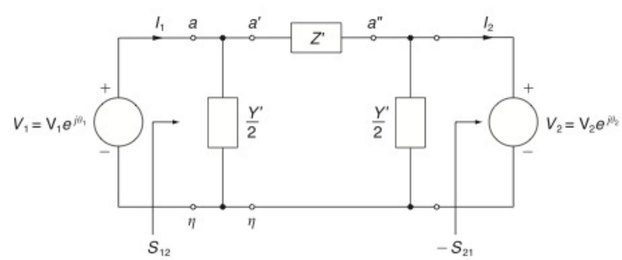
$$P_{12} = -P_{21} = \frac{V_1^2 \sin\theta_{12}}{Z_c \sin\beta\ell} = P_{SIL}\frac{\sin\theta_{12}}{\sin\beta\ell}$$

(c) For $\theta_{12} = 45°$ and $\beta = 0.002$ rad/km, find (P_{12}/P_{SIL}) as a function of line length in km, and sketch it.

(d) If a thermal limit of $(P_{12}/P_{SIL}) = 2$ is set, which limit governs for short lines and long lines?

FIGURE 5.20

Per-phase circuit for Problem 5.52



PW **5.53** Open PowerWorld Simulator case Example 5_8. If the load bus voltage is greater than or equal to 730 kV even with any line segment out of service, what is the maximum amount of real power that can be delivered to the load?

PW **5.54** Repeat Problem 5.53, but now assume any two line segments may be out of service.

SECTION 5.7

5.55 Recalculate the percent voltage regulation in Problem 5.15 when identical shunt reactors are installed at both ends of the line during light loads, providing 65% total shunt compensation. The reactors are removed at full load. Also calculate the impedance of each shunt reactor.

5.56 Rework Problem 5.17 when identical shunt reactors are installed at both ends of the line, providing 50% total shunt compensation. The reactors are removed at full load.

5.57 Identical series capacitors are installed at both ends of the line in Problem 5.14, providing 40% total series compensation. Determine the equivalent *ABCD* parameters of this compensated line. Also calculate the impedance of each series capacitor.

5.58 Identical series capacitors are installed at both ends of the line in Problem 5.16, providing 30% total series compensation. (a) Determine the equivalent *ABCD* parameters for this compensated line. (b) Determine the theoretical maximum real power that this series-compensated line can deliver when $V_S = V_R = 1.0$ per unit. Compare your result with that of Problem 5.40.

5.59 Determine the theoretical maximum real power that the series-compensated line in Problem 5.57 can deliver when $V_S = V_R = 1.0$ per unit. Compare your result with that of Problem 5.38.

5.60 What is the minimum amount of series capacitive compensation N_C in percent of the positive-sequence line reactance needed to reduce the number of 765-kV lines in Example 5.8 from five to four? Assume two intermediate substations with one line section out of service. Also, neglect line losses and assume that the series compensation is sufficiently distributed along the line so as to effectively reduce the series reactance of the equivalent π circuit to $X'(1-N_C/100)$.

5.61 Determine the equivalent *ABCD* parameters for the line in Problem 5.14 if it has 70% shunt reactive (inductors) compensation and 40% series capacitive compensation. Half of this compensation is installed at each end of the line, as in Figure 5.14.

5.62 Consider the transmission line of Problem 5.18. (a) Find the *ABCD* parameters of the line when uncompensated. (b) For a series capacitive compensation of 70% (35% at the sending end and 35% at the receiving end), determine the *ABCD* parameters. Comment on the relative change in the magnitude of the *B* parameter with respect to the relative changes in the magnitudes of the *A*, *C*, and *D* parameters. Also comment on the maximum power that can be transmitted when series compensated.

5.63 Given the uncompensated line of Problem 5.18, let a three-phase shunt reactor (inductor) that compensates for 70% of the total shunt admittance of the line be connected at the receiving end of the line during no-load conditions. Determine the effect of voltage regulation with the