PROBLEMS

SECTION 3.1

3.1 (a) An ideal single-phase two-winding transformer with turns ratio $a_t = N_1/N_2$ is connected with a series impedance Z_2 across winding 2. If one wants to replace Z_2, with a series impedance Z_1 across winding 1 and keep the terminal behavior of the two circuits to be identical, find Z_1 in terms of Z_2.

(b) Would the above result be true if instead of a series impedance there is a shunt impedance?

(c) Can one refer a ladder network on the secondary (2) side to the primary (1) side simply by multiplying every impendance bya_t^2?

3.2 An ideal transformer with $N_1 = 1000$ and $N_2 = 250$ is connected with an impedance Z_{22} across winding 2. If $V_1 = 1000 \underline{/0^\circ}$ V and $I_1 = 5 \underline{/-30^\circ}$ A, determine V_2, I_2, Z_2, and the impedance Z'_2, which is the value of Z_2 referred to the primary side of the transformer.

3.3 Consider an ideal transformer with $N_1 = 3000$ and $N_2 = 1000$ turns. Let winding 1 be connected to a source whose voltage is $e_1(t) = 100(1 - |t|)$ volts for $-1 \le t \le 1$ and $e_1(t) = 0$ for $|t| > 1$ second. A 2-farad capacitor is connected across winding 2. Sketch $e_1(t)$, $e_2(t)$, $i_1(t)$, and $i_2(t)$ versus time t.

3.4 A single-phase 100-kVA, 2400/240-volt, 60-Hz distribution transformer is used as a step-down transformer. The load, which is connected to the 240-volt secondary winding, absorbs 60 kVA at 0.8 power factor lagging and is at 230 volts. Assuming an ideal transformer, calculate the following: (a) primary voltage, (b) load impedance, (c) load impedance referred to the primary, and (d) the real and reactive power supplied to the primary winding.

3.5 Rework Problem 3.4 if the load connected to the 240-V secondary winding absorbs 110 kVA under short-term overload conditions at an 0.8 power factor leading and at 230 volts.

3.6 For a conceptual single-phase phase-shifting transformer, the primary voltage leads the secondary voltage by 30°. A load connected to the secondary winding absorbs 110 kVA at an 0.8 power factor leading and at a voltage $E_2 = 277\underline{/0^\circ}$ volts. Determine (a) the primary voltage, (b) primary and secondary currents, (c) load impedance referred to the primary winding, and (d) complex power supplied to the primary winding.

3.7 Consider a source of voltage $v(t) = 10\sqrt{2}\sin(2t)$V, with an internal resistance of 1800 Ω. A transformer that can be considered as ideal is used to couple a 50-Ω resistive load to the source. (a) Determine the transformer primary-to-secondary turns ratio required to ensure maximum power transfer by matching the load and source resistances. (b) Find the average power delivered to the load, assuming maximum power transfer.

3.8 For the circuit shown in Figure 3.31, determine $v_{out}(t)$

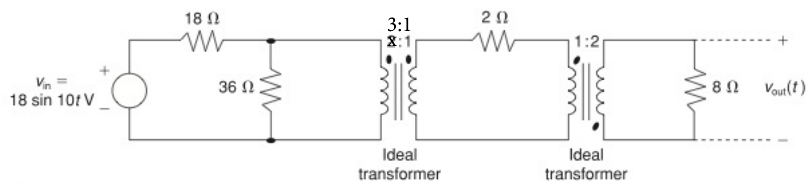


FIGURE 3.31

Problem 3.8

SECTION 3.2

3.9 A single-phase transformer has 2000 turns on the primary winding and 500 turns on the secondary. Winding resistances are $R_1 = 2\ \Omega$, and $R_2 = 0.125\ \Omega$; leakage reactances are $X_1 = 8\ \Omega$ and $X_2 = 0.5\ \Omega$. The resistance load on the secondary is 12 Ω.

(a) If the applied voltage at the terminals of the primary is 1000 V, determine V_2 at the load terminals of the transformer, neglecting magnetizing current.

(b) If the voltage regulation is defined as the difference between the voltage magnitude at the load terminals of the transformer at full load and at no load in percent of full-load voltage with input voltage held constant, compute the percent voltage regulation.

3.10 A single-phase step-down transformer is rated 13 MVA, 66 kV/11.5 kV. With the 11.5 kV winding short-circuited, rated current flows when the voltage applied to the primary is 5.5 kV. The power input is read as 100 kW. Determine R_{eq1} and X_{eq1} in ohms referred to the high-voltage winding.

3.11 For the transformer in Problem 3.10, the open-circuit test with 11.5 kV applied results in a power input of 65 kW and a current of 30 A. Compute the values for G_c and B_m in siemens referred to the high-voltage winding. Compute the efficiency of the transformer for a load of 10 MW at 0.8 p.f. lagging at rated voltage.

3.12 The following data are obtained when open-circuit and short-circuit tests are performed on a single-phase, 50-kVA, 2400/240-volt, 60-Hz distribution transformer.

	VOLTAGE (volts)	CURRENT (amperes)	POWER (watts)
Measurements on low-voltage side with high-voltage winding open	240	4.85	173
Measurements on high-voltage side with low-voltage winding shorted	52.0	20.8	650

(a) Neglecting the series impedance, determine the exciting admittance referred to the high-voltage side. (b) Neglecting the exciting admittance, determine the equivalent series impedance referred to the high-voltage side. (c) Assuming equal series impedances for the primary and referred secondary, obtain an equivalent T-circuit referred to the high-voltage side.

3.13 A single-phase 50-kVA, 2400/240-volt, 60-Hz distribution transformer has a 1-ohm equivalent leakage reactance and a 5000-ohm magnetizing reactance referred to the high-voltage side. If rated voltage is applied to the high-voltage winding, calculate the open-circuit secondary voltage. Neglect I^2R and G_c^2V losses. Assume equal series leakage reactances for the primary and the referred secondary.

3.14 A single-phase 50-kVA, 2400/240-volt, 60-Hz distribution transformer is used as a step-down transformer at the load end of a 2400-volt feeder whose series impedance is $(1.0 + j2.0)$ ohms. The equivalent series impedance of the transformer is $(1.0 + j2.5)$ ohms referred to the high-voltage (primary) side. The transformer is delivering rated load at a 0.8 power factor lagging and at a rated secondary voltage. Neglecting the transformer exciting current, determine (a) the voltage at the transformer primary terminals, (b) the voltage at the sending end of the feeder, and (c) the real and reactive power delivered to the sending end of the feeder.

3.15 Rework Problem 3.14 if the transformer is delivering rated load at rated secondary voltage and at (a) unity power factor, (b) 0.8 power factor leading. Compare the results with those of Problem 3.14.

3.16 A single-phase, 50-kVA, 2400/240-V, 60-Hz distribution transformer has the following parameters:

Resistance of the 2400-V winding: $R_1 = 0.75\ \Omega$

Resistance of the 240-V winding: $R_2 = 0.0075\ \Omega$

Leakage reactance of the 2400-V winding: $X_1 = 1.0\ \Omega$

Leakage reactance of the 240-V winding: $X_2 = 0.01\ \Omega$

Exciting admittance on the 240-V side $= 0.003 - j0.02$ S

(a) Draw the equivalent circuit referred to the high-voltage side of the transformer.

(b) Draw the equivalent circuit referred to the low-voltage side of the transformer. Show the numerical values of impedances on the equivalent circuits.

3.17 The transformer of Problem 3.16 is supplying a rated load of 50 kVA at a rated sec ondary voltage of 240 V and at 0.8 power factor lagging. Neglect the transformer exciting current. (a) Determine the input terminal voltage of the transformer on the high-voltage side. (b) Sketch the corresponding phasor diagram. (c) If the transformer is used as a step-down transformer at the load end of a feeder whose impedance is $0.5 + j2.0\ \Omega$, find the voltage V_S and the power factor at the sending end of the feeder.

SECTION 3.3

3.18 Using the transformer ratings as base quantities, work Problem 3.13 in per-unit.

3.19 Using the transformer ratings as base quantities, work Problem 3.14 in per-unit.

3.20 Using base values of 20 kVA and 115 volts in zone 3, rework Example 3.4.

3.21 Rework Example 3.5; using $S_{base3\phi} = 100$ kVA and $V_{baseLL} = 600$ volts.

3.22 A balanced Y-connected voltage source with $E_{ag} = 277\angle 0°$ volts is applied to a balanced-Y load in parallel with a balanced-Δ load where $Z_Y = 20 + j10$ and $Z\Delta = 30 - j15$ ohms. The Y load is solidly grounded. Using base values of $S_{base1\phi} = 10$ kVA and $V_{baseLN} = 277$ volts, calculate the source current I_a in per-unit and in amperes.

3.23 Figure 3.32 shows the oneline diagram of a three-phase power system. By selecting a common base of 100 MVA and 22 kV on the generator side, draw an impedance diagram showing all impedances including the load impedance in per-unit. The data are given as follows:

G:	90 MVA	22 kV	x = 0.18 per unit
T_1:	50 MVA	22/220 kV	x = 0.10 per unit
T_2:	40 MVA	220/11 kV	x = 0.06 per unit
T_3:	40 MVA	22/110 kV	x = 0.064 per unit
T_4:	40 MVA	110/11 kV	x = 0.08 per unit
M:	66.5 MVA	10.45 kV	x = 0.185 per unit

Lines 1 and 2 have series reactances of 48.4 and 65.43 Ω, respectively. At bus 4, the three-phase load absorbs 57 MVA at 10.45 kV and 0.6 power factor lagging.

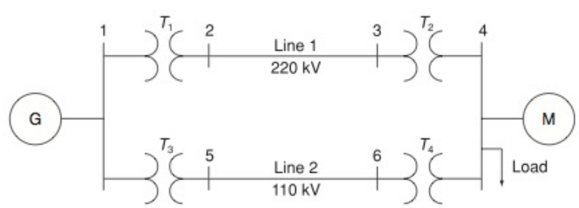


FIGURE 3.32
Problem 3.23

3.24 For Problem ~~3.18,~~ 3.23 the motor operates at full load, at 0.8 power factor leading, and at a terminal voltage of 10.45 kV. Determine (a) the voltage at bus 1, which is the generator bus, and (b) the generator and motor internal EMFs. Correction: Do for Problem 3.23.

3.25 Consider a single-phase electric system shown in Figure 3.33. Transformers are rated as follows:

X–Y 15 MVA, 13.8/138 kV, leakage reactance 10%
Y–Z 15 MVA, 138/69 kV, leakage reactance 8%

With the base in circuit Y chosen as 15 MVA, 138 kV, determine the per-unit impedance of the 500 Ω resistive load in circuit Z, referred to circuits Z, Y, and X. Neglecting magnetizing currents, transformer resistances, and line impedances, draw the impedance diagram in per unit.

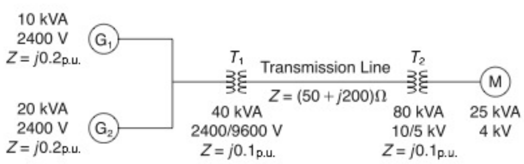
FIGURE 3.33
Single-phase electric system for Problem 3.25

3.26 A bank of three single-phase transformers, each rated 30 MVA, 38.1/3.81 kV, are connected in Y–Δ with a balanced load of three 1 Ω, Y-connected resistors. Choosing a base of 90 MVA, 66 kV for the high-voltage side of the three-phase transformer, specify the base for the low-voltage side. Compute the per-unit resistance of the load on the base for the low-voltage side. Also, determine the load resistance in ohms referred to the high-voltage side and the per-unit value on the chosen base.

3.27 A three-phase transformer is rated 1000 MVA, 220 Y/22 Δ kV. The Y-equivalent short-circuit impedance, considered equal to the leakage reactance, measured on the low-voltage side is 0.1 Ω. Compute the per-unit reactance of the transformer. In a system in which the base on the high-voltage side of the transformer is 100 MVA, 230 kV, what value of the per-unit reactance should be used to represent this transformer?

3.28 For the system shown in Figure 3.34, draw an impedance diagram in per unit by choosing 100 kVA to be the base kVA and 2400 V as the base voltage for the generators.

FIGURE 3.34
System for Problem 3.28



3.29 Consider three ideal single-phase transformers (with a voltage gain of η) put together as a Δ–Ω three-phase bank as shown in Figure 3.35. Assuming positive-sequence voltages for V_{an}, V_{bn}, and V_{cn}, find $V_{a'n'}$, $V_{b'n'}$, and $V_{C'n'}$ in terms of V_{an}, V_{bn}, and V_{cn}, respectively.

(a) Would such relationships hold for the line voltages as well?

(b) Looking into the current relationships, express I'_a, I'_b, and I'_c in terms of I_a, I_b, and I_c, respectively.

(c) Let S' and S be the per-phase complex power output and input, respectively. Find S′ in terms of S.

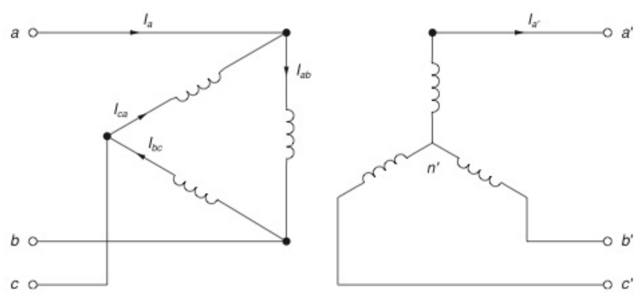


FIGURE 3.35
Δ–Y connection for Problem 3.29

3.30 Reconsider Problem 3.29. If V_{an}, V_{bn}, and V_{cn} are a negative-sequence set, how would the voltage and current relationships change?

(a) If C_1 is the complex positive-sequence voltage gain in Problem 3.29 and (b) if C_2 is the negative sequence complex voltage gain, express the relationship between C_1 and C_2

3.31 If positive-sequence voltages are assumed and the Y-Δ connection is considered, again with ideal transformers as in Problem 3.29, find the complex voltage gain C_3.

(a) What would the gain be for a negative-sequence set?

(b) Comment on the complex power gain.

(c) When terminated in a symmetric Y-connected load, find the referred impedance Z'_L, the secondary impedance Z_L referred to primary (i.e., the per-phase driving-point impedance on the primary side), in terms of Z_L and the complex voltage gain C.

SECTION 3.4

3.32 Determine the positive- and negative-sequence phase shifts for the three-phase transformers shown in Figure 3.36.

3.33 Consider the three single-phase two-winding transformers shown in Figure 3.37. The high-voltage windings are connected in Y. (a) For the low-voltage side, connect the windings in Δ, place the polarity marks, and label the terminals *a*, *b*, and *c* in accordance with the American standard. (b) Relabel the terminals a', b', and c' such that V_{AN} is 90° out of phase with $V_{a'n}$ for positive sequence.

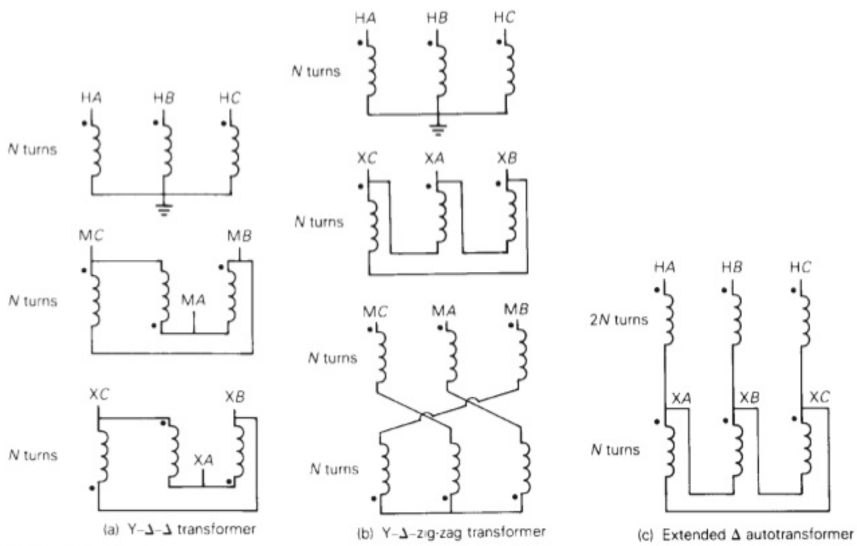
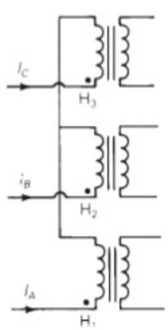


FIGURE 3.36

Problems 3.32 and 3.52 (Coils drawn on the same vertical line are on the same core)

FIGURE 3.37

Problem 3.33



3.34 Three single-phase, two-winding transformers, each rated 450 MVA, 20 kV/288.7 kV, with leakage reactance $X_{eq} = 0.10$ per unit, are connected to form a three-phase bank. The high-voltage windings are connected in Y with a solidly grounded neutral. Draw the per-unit equivalent circuit if the low-voltage windings are connected (a) in Δ with American standard phase shift or (b) in Y with an open neutral. Use the transformer ratings as base quantities. Winding resistances and exciting current are neglected.

3.35 Consider a bank of three single-phase two-winding transformers whose high-voltage terminals are connected to a three-phase, 13.8-kV feeder. The low-voltage terminals are connected to a three-phase substation load rated 2.0 MVA and 2.5 kV. Determine the required voltage, current, and MVA ratings of both windings of each transformer, when the high-voltage/low-voltage windings are connected (a) Y–Δ, (b) Δ–Y, (c) Y–Y, and (d) Δ–Δ.

3.36 Three single-phase two-winding transformers, each rated 25 MVA, 34.5/13.8 kV, are connected to form a three-phase Δ–Δ bank. Balanced positive-sequence voltages are applied to the high-voltage terminals, and a balanced, resistive Y load connected to the low-voltage terminals absorbs 75 MW at 13.8 kV. If one of the single-phase transformers is removed (resulting in an open-Δ connection) and the balanced load is simultaneously reduced to 43.3 MW (57.7% of the original value), determine (a) the load voltages V_{an}, V_{bn}, and V_{cn}; (b) load currents I_a, I_b, and I_c; and (c) the MVA supplied by each of the remaining two transformers. Are balanced voltages still applied to the load? Is the open-Δ transformer overloaded?

3.37 Three single-phase two-winding transformers, each rated 25 MVA, 54.2/5.42 kV, are connected to form a three-phase Y–Δ bank with a balanced Y-connected resistive load of 0.6 Ω per phase on the low-voltage side. By choosing a base of 75 MVA (three phase) and 94 kV (line-to-line) for the high-voltage side of the transformer bank, specify the base quantities for the low-voltage side. Determine the per-unit resistance of the load on the base for the low-voltage side. Then determine the load resistance R_L in ohms referred to the high-voltage side and the per-unit value of this load resistance on the chosen base.

3.38 Consider a three-phase generator rated 300 MVA, 23 kV, supplying a system load of 240 MVA and 0.9 power factor lagging at 230 kV through a 330 MVA, 23 Δ/ 230 Y-kV step-up transformer with a leakage reactance of 0.11 per unit. (a) Neglecting the exciting current and choosing base values at the load of 100 MVA and 230 kV, find the phasor currents I_A, I_B, and I_C supplied to the load in per unit. (b) By choosing the load terminal voltage V_A as reference, specify the proper base for the generator circuit and determine the generator voltage V as well as the phasor currents I_A, I_B, and I_C, from the generator. (*Note:* Take into account the phase shift of the transformer.) (c) Find the generator terminal voltage in kV and the real power supplied by the generator in MW. (d) By omitting the transformer phase shift altogether, check to see whether you get the same magnitude of generator terminal voltage and real power delivered by the generator.

SECTION 3.5

3.39 The leakage reactance of a three-phase, 300-MVA, 230 Y/23 Δ-kV transformer is 0.06 per unit based on its own ratings. The Y winding has a solidly grounded neutral. Draw the per-unit equivalent circuit. Neglect the exciting admittance and assume the American Standard phase shift.

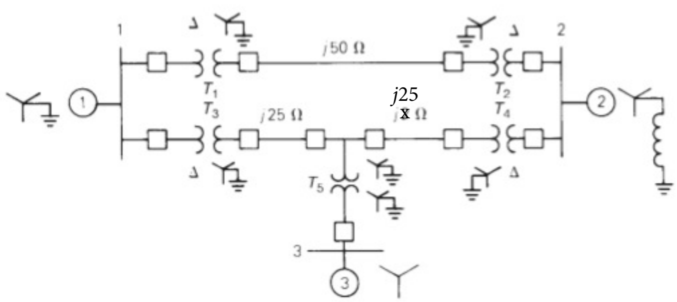
3.40 Choosing system bases to be 240/24 kV and 100 MVA, redraw the per-unit equivalent circuit for Problem 3.39.

3.41 Consider the single-line diagram of the power system shown in Figure 3.38. Equipment ratings are

Generator 1:	1000 MVA, 18 kV, X" = 0.2 per unit
Generator 2:	1000 MVA, 18 kV, X" = 0.2 p.u.
Synchronous motor 3:	1500 MVA, 20 kV, X" = 0.2 p.u.
Three-phase Δ–Y transformers T_1, T_2, T_3, T_4:	1000 MVA, 500 kV Y/20 kV Δ, X = 0.1 p.u.
Three-phase Y–Y transformer T_5:	1500 MVA, 500 kV Y/20 kV Y, X = 0.1 p.u.

Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the equivalent reactance diagram. Use a base of 100 MVA and 500 kV for the 50-ohm line. Determine the per-unit reactances.

FIGURE 3.38
Problems 3.41 and 3.42



3.42 For the power system in Problem 3.41, the synchronous motor absorbs 1500 MW at 0.8 power factor leading with the bus 3 voltage at 18 kV. Determine the bus 1 and bus 2 voltages in kV. Assume that generators 1 and 2 deliver equal real powers and equal reactive powers. Also assume a balanced three-phase system with positive-sequence sources.

3.43 Three single-phase transformers, each rated 10 MVA, 66.4/12.5 kV, 60 Hz, with an equivalent series reactance of 0.1 per unit divided equally between primary and secondary, are connected in a three-phase bank. The high-voltage windings are Y-connected and their terminals are directly connected to a 115-kV three-phase bus. The secondary terminals are all shorted together. Find the currents entering the high-voltage terminals and leaving the low-voltage terminals if the low-voltage windings are (a) Y-connected and (b) Δ-connected.

3.44 A 130-MVA, 13.2-kV three-phase generator, which has a positive-sequence reactance of 1.5 per unit on the generator base, is connected to a 135-MVA, 13.2 Δ/115 Y-kV step-up transformer with a series impedance of $(0.005 + j0.1)$ per unit on its own base. (a) Calculate the per-unit generator reactance on the transformer base. (b) The load at the transformer terminals is 15 MW at unity power factor and at 115 kV. Choosing the transformer high-side voltage as the reference phasor, draw a phasor diagram for this condition. (c) For the condition of part (b), find the transformer low-side voltage and the generator internal voltage behind its reactance. Also compute the generator output power and power factor.

3.45 Figure 3.39 shows a oneline diagram of a system in which the three-phase generator is rated 300 MVA, 20 kV with a subtransient reactance of 0.2 per unit and with its neutral grounded through a 0.4-Ω reactor. The transmission line is 64 km long with a series reactance of 0.5 Ω/km. The three-phase transformer T_1 is rated 350 MVA, 230/ 20 kV with a leakage reactance of 0.1 per unit. Transformer T_2 is composed of three single-phase transformers, each rated 100 MVA, 127/13.2 kV with a leakage reactance of 0.1 per unit. Two 13.2-kV motors M_1 and M_2 with a subtransient reactance of 0.2 per unit for each motor represent the load. M_1 has a rated input of 200 MVA with its neutral grounded through a 0.4-Ω current-limiting reactor. M_2 has a rated input of 100 MVA with its neutral not connected to ground. Neglect phase shifts associated with the transformers. Choose the generator rating as base in the generator circuit and draw the positive-sequence reactance diagram showing all reactances in per unit.

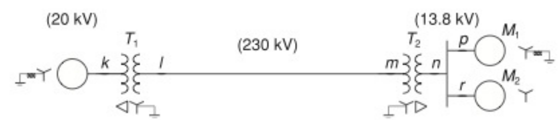


FIGURE 3.39 Problems 3.45 and 3.46

3.46 The motors M_1 and M_2 of Problem 3.45 have inputs of 120 and 60 MW, respectively, at 13.2 kV, and both operate at unity power factor. Determine the generator terminal voltage and voltage regulation of the line. Neglect transformer phase shifts.

3.47 Consider the oneline diagram shown in Figure 3.40. The three-phase transformer bank is made up of three identical single-phase transformers,