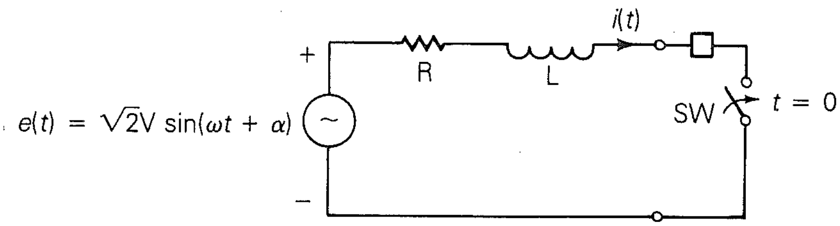
Symmetrical Faults

1. Circuit Transients



$$\frac{\mathrm{L}di(t)}{dt} + \mathrm{R}i(t) = \sqrt{2}\mathrm{V}\sin(\omega t + \alpha) \quad t \geqslant 0 \tag{7.1.1}$$

The solution to (7.1.1) is

$$i(t) = i_{\mathrm{ac}}(t) + i_{\mathrm{dc}}(t) \quad : \text{ asymmetrical fault current}$$

$$= \underbrace{\frac{\sqrt{2}\mathrm{V}}{\mathrm{Z}}[\sin(\omega t + \alpha - \theta)}_{\text{symmetrical, steady-state}} - \underbrace{\sin(\alpha - \theta)e^{-t/\mathrm{T}}]}_{\text{dc offset}} \quad \mathrm{A} \tag{7.1.2}$$

where

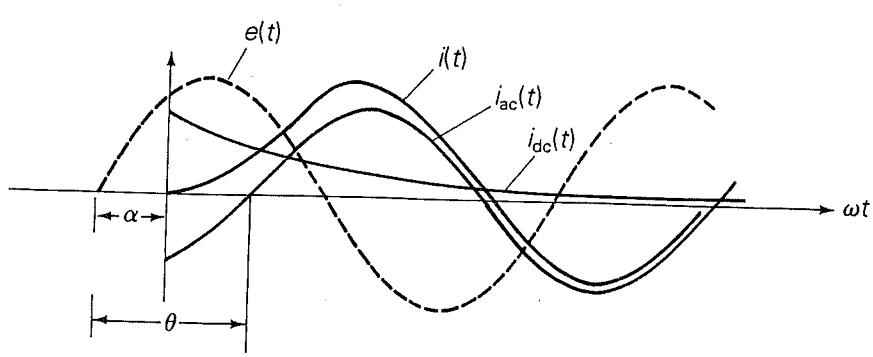
$$i_{\mathrm{ac}}(t) = \frac{\sqrt{2}\mathrm{V}}{\mathrm{Z}}\sin(\omega t + \alpha - \theta) \quad \mathrm{A} \tag{7.1.3}$$

$$i_{\mathrm{dc}}(t) = -\frac{\sqrt{2}\mathrm{V}}{\mathrm{Z}}\sin(\alpha - \theta)e^{-t/T} \quad \mathrm{A} \tag{7.1.4}$$

$$Z = \sqrt{\mathrm{R}^2 + (\omega\mathrm{L})^2} = \sqrt{\mathrm{R}^2 + \mathrm{X}^2} \quad \Omega \tag{7.1.5}$$

$$\theta = \tan^{-1}\frac{\omega\mathrm{L}}{\mathrm{R}} = \tan^{-1}\frac{\mathrm{X}}{\mathrm{R}} \tag{7.1.6}$$

$$\mathrm{T} = \frac{\mathrm{L}}{\mathrm{R}} = \frac{\mathrm{X}}{\omega\mathrm{R}} = \frac{\mathrm{X}}{2\pi f\mathrm{R}} \quad \mathrm{s} \tag{7.1.7}$$

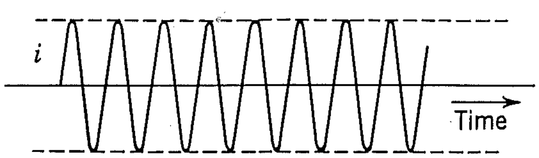


<u>Note</u>: at $t=0$, $i(t)=0$

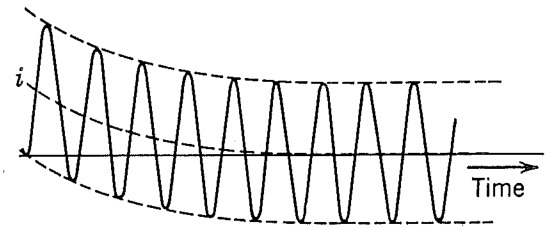
$\Rightarrow$ steady-state current = dc offset

If $\alpha-\theta = 0, \pi$, dc offset $\equiv 0$

$\alpha-\theta = \mp\frac{\pi}{2}$, dc offset $= \pm\frac{\sqrt{2}V}{Z} = \pm\frac{V_{max}}{Z}$



(a) $\alpha-\theta=0$



(b) $\alpha-\theta=-\frac{\pi}{2}$

: maximum dc offset

<u>RMS Values</u>: for $i(t)$, asymmetrical?

$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t+\alpha-\theta) \Rightarrow I_{ac}=\frac{V}{Z}, \text{ rms}$$

$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha-\theta)\, e^{-t/T}$$

: maximum at $\alpha-\theta=-\frac{\pi}{2}$

$\therefore$ The largest fault current is at $\alpha-\theta=-\frac{\pi}{2}$:

$$i(t) = \sqrt{2}\, I_{ac}\left[\sin\left(\omega t-\frac{\pi}{2}\right) + e^{-t/T}\right]$$

$$\begin{aligned} I_{rms}(t) &= \sqrt{[I_{ac}]^2 + [I_{dc}(t)]^2} \\ &= \sqrt{[I_{ac}]^2 + [\sqrt{2}I_{ac}e^{-t/T}]^2} \\ &= I_{ac}\sqrt{1+2e^{-2t/T}} \quad \text{A} \end{aligned} \tag{7.1.10}$$

It is convenient to use $T = X/(2\pi f R)$ and $t=\tau/f$, where τ is time in cycles, and write (7.1.10) as

$$I_{rms}(\tau) = K(\tau)I_{ac} \quad \text{A} \qquad K(\tau)\text{: asymmetry factor} \tag{7.1.11}$$

where

$$K(\tau) = \sqrt{1+2e^{-4\pi\tau/(X/R)}} \quad \text{per unit} \tag{7.1.12}$$

DC offset: $I_{dc} = \sqrt{2}\, I_{ac}\, e^{-t/T}$

Time constant: $T = \frac{L}{R} = \frac{2\pi f L}{2\pi f R} = \frac{X}{2\pi f R}$

Time: $t = \tau/f$, τ = cycles

Asymmetry factor:

$$K(\tau) = \sqrt{1 + 2e^{-2t/T}},$$

$$= \sqrt{1 + 2e^{-4\pi\tau/(X/R)}}$$

$$= \sqrt{3} \quad @\ \tau = 0$$

$$= 1 \quad \text{for large } \tau$$

EXAMPLE 7.1 Fault currents: R–L circuit with ac source

A bolted short circuit occurs in the series R–L circuit of Figure 7.1 with $V = 20$ kV, $X = 8\ \Omega$, $R = 0.8\ \Omega$, and with maximum dc offset. The circuit breaker opens 3 cycles after fault inception. Determine (a) the rms ac fault current, (b) the rms "momentary" current at $\tau = 0.5$ cycle, which passes through the breaker before it opens, and (c) the rms asymmetrical fault current that the breaker interrupts.

SOLUTION

a. From (7.1.9),

$$I_{ac} = \frac{20 \times 10^3}{\sqrt{(8)^2 + (0.8)^2}} = \frac{20 \times 10^3}{8.040} = 2.488 \quad \text{kA}$$

b. From (7.1.11) and (7.1.12) with $(X/R) = 8/(0.8) = 10$ and $\tau = 0.5$ cycle,

$$K(0.5 \text{ cycle}) = \sqrt{1 + 2e^{-4\pi(0.5)/10}} = 1.438$$

$$I_{momentary} = K(0.5 \text{ cycle}) I_{ac} = (1.438)(2.488) = 3.576 \quad \text{kA}$$

c. From (7.1.11) and (7.1.12) with $(X/R) = 10$ and $\tau = 3$ cycles,

$$K(3 \text{ cycles}) = \sqrt{1 + 2e^{-4\pi(3)/10}} = 1.023$$

$$I_{rms}(3 \text{ cycles}) = (1.023)(2.488) = 2.544 \quad \text{kA}$$

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2. Synchronous Machine

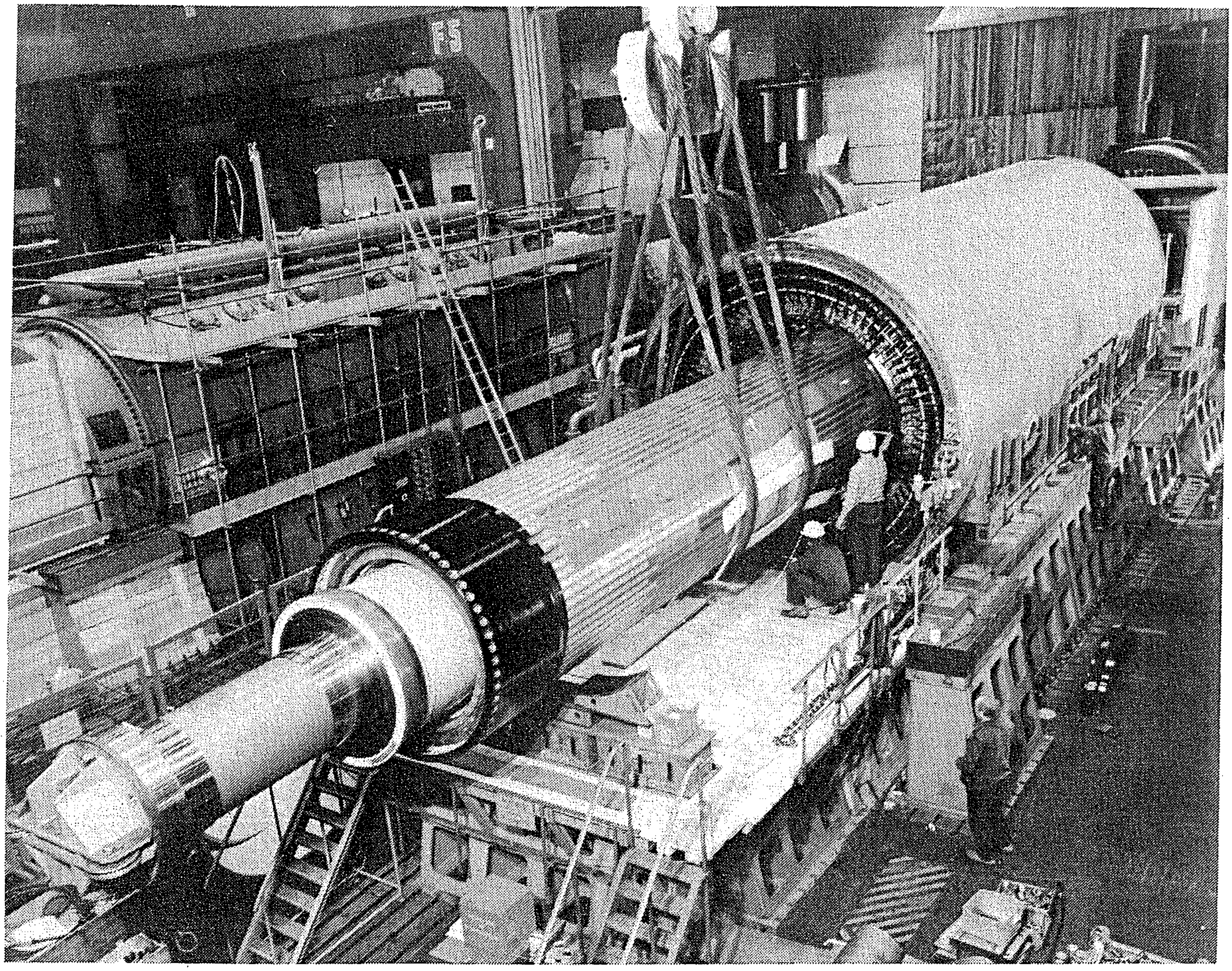
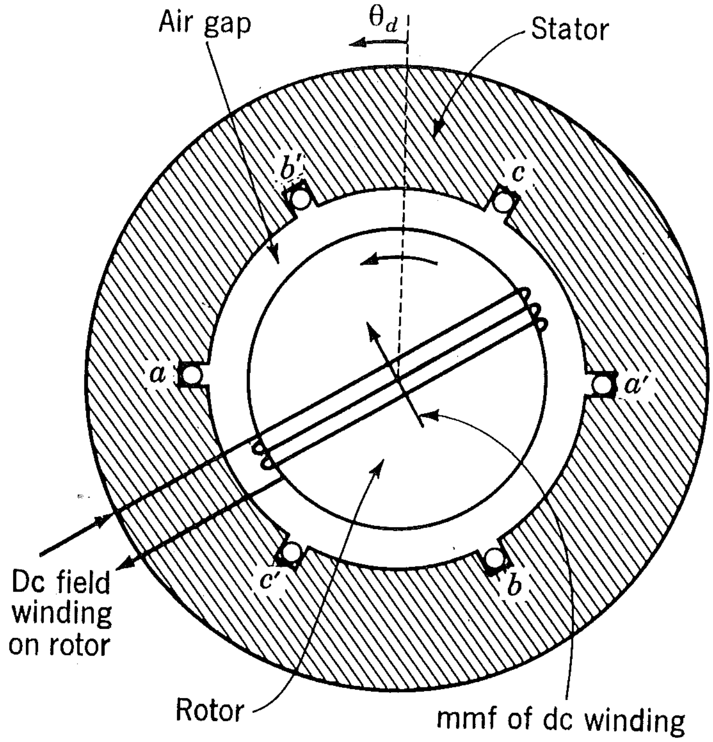
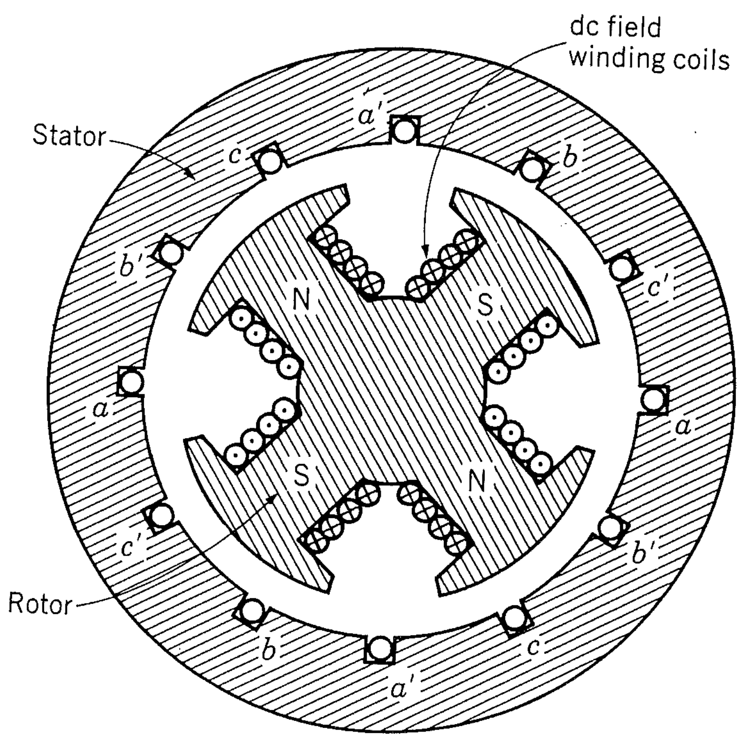


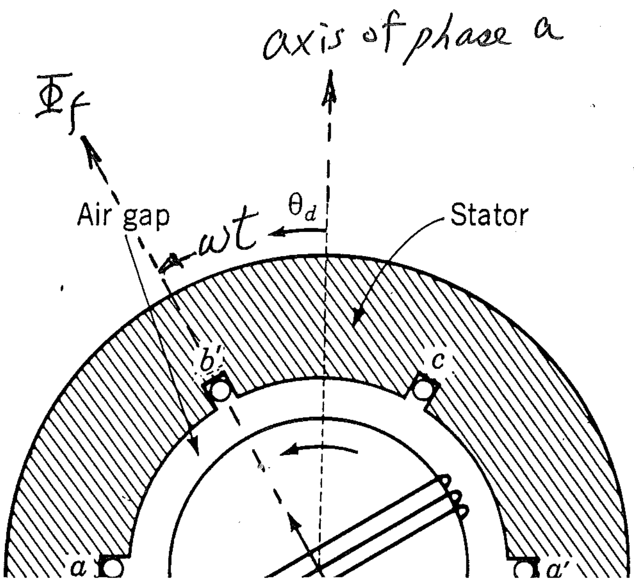
Figure 6.1 Photograph showing the threading of a four-pole cylindrical rotor into the stator of a 1525-MVA generator. (*Courtesy Utility Power Corporation, Wisconsin.*)



2-pole synchronous machine



4-pole synchronous machine



flux linkage on phase-a stator winding:

$$\phi = \Phi_f \cos \omega t$$

Faraday's law: voltage induced on coil-a:

$$e = -N \frac{d\phi}{dt}$$

$$= \omega N \Phi_f \sin \omega t$$

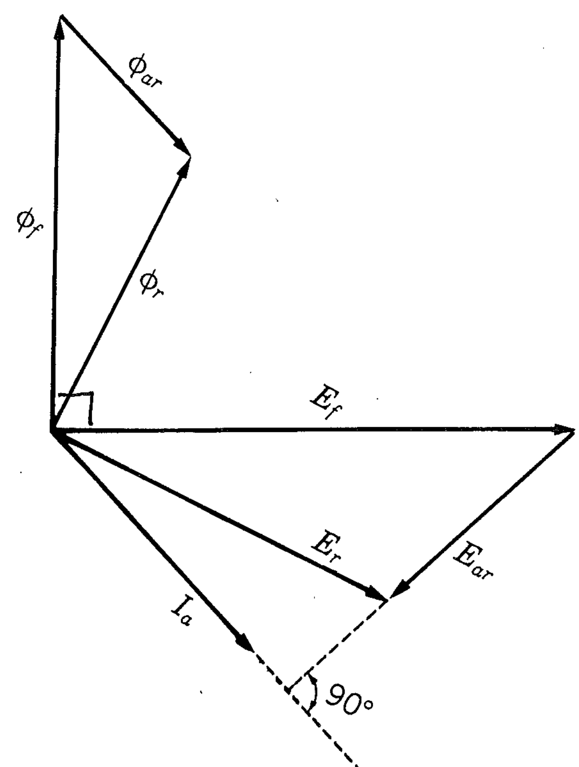
$$= \underbrace{\omega N \Phi_f}_{E_m} \cos(\omega t - 90^\circ)$$: lage by 90°

Field voltage:

$$|E_f| = \frac{E_m}{\sqrt{2}} = \frac{2\pi f N \Phi_f}{\sqrt{2}}$$

$$= 4.44\, f N \Phi_f$$

Amature Reaction:



I_a: armature current

Φ_{ar}: armature reaction flux

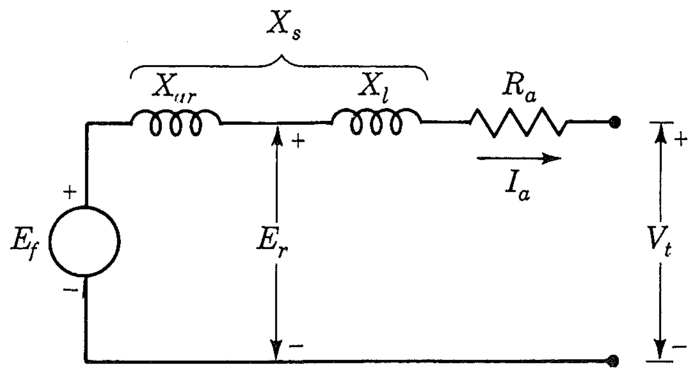
E_{ar}: armature reaction voltage

Φ_r: resultant flux

E_r: resultant voltage

Since E_{ar} lage I_a by 90°,

$$E_{ar} = -jI_a X_{ar}\text{, and } E_r = E_f + E_{ar} = E_f - jI_a X_{ar}$$



Equivalent Circuit

Terminal Voltage:

$$V_t = E_r - jI_aX_l$$

$$V_t = E_f - I_a(R_a + jX_s)$$

$X_s = X_{ar} + X_l$: synchronous reactance

R_a: armature resistance

- Effect of Armature Current:

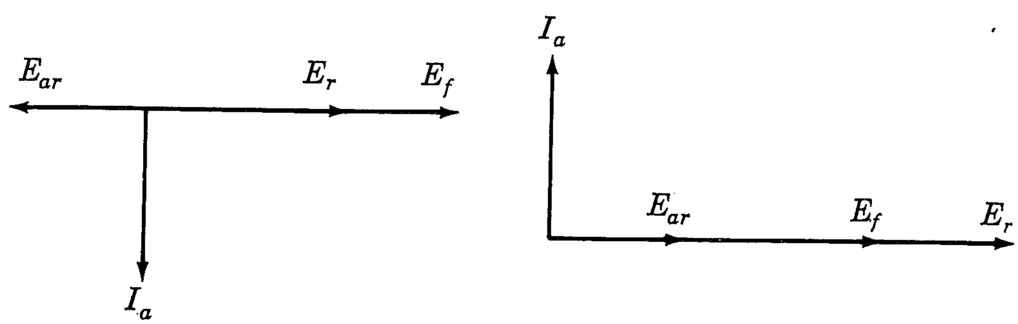


Figure 6.7 Phasor diagrams showing the relation between E_f and E_{ar} when current delivered by a generator is (*a*) lagging E_f by 90° and (*b*) leading E_f by 90°.

- Effect of Excitation:

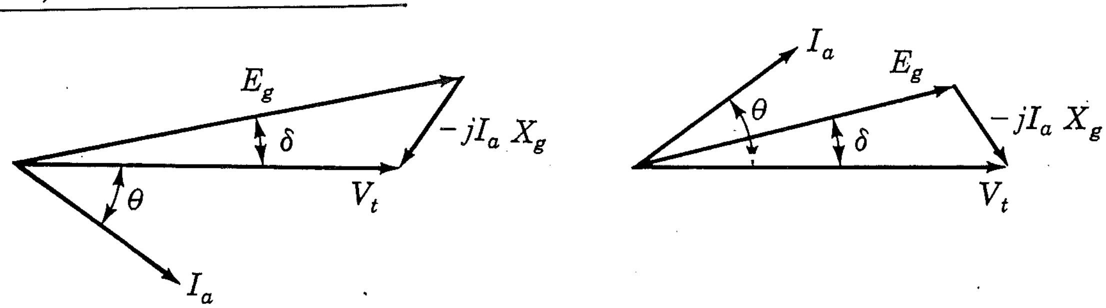


Figure 6.9 Phasor diagrams of (*a*) overexcited and (*b*) underexcited generator. I_a is current delivered by the generator.

3. Three-Phase Short Circuit

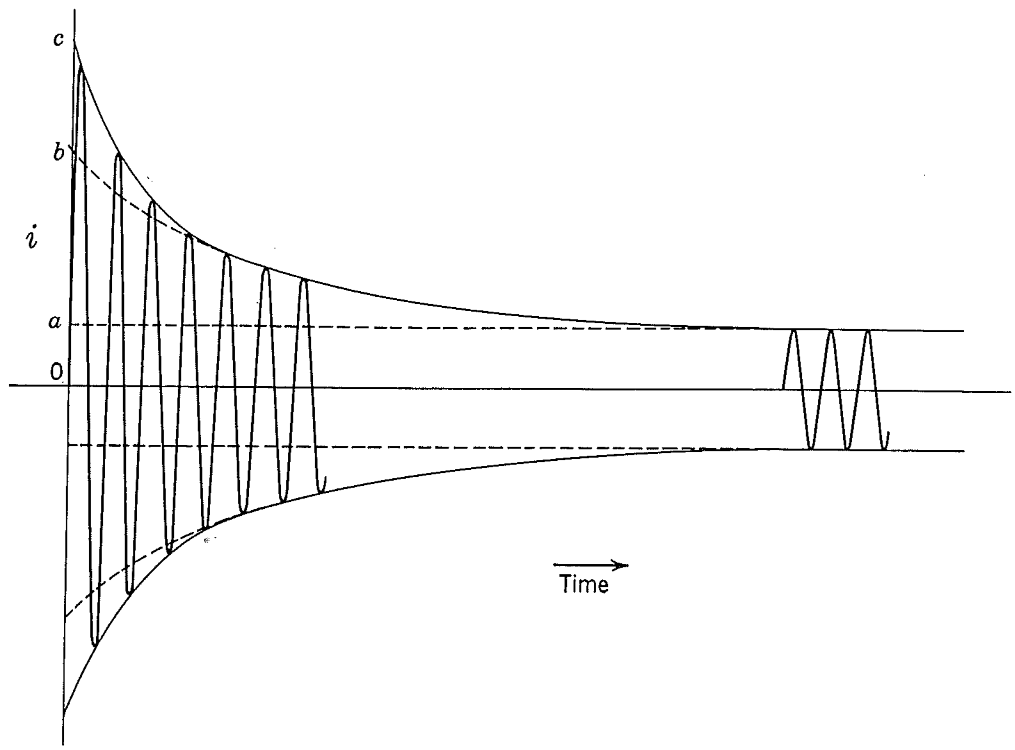


Figure 10.3 Current as a function of time for a synchronous generator short-circuited while running at no load. The unidirectional transient component of current has been eliminated in redrawing the oscillogram.

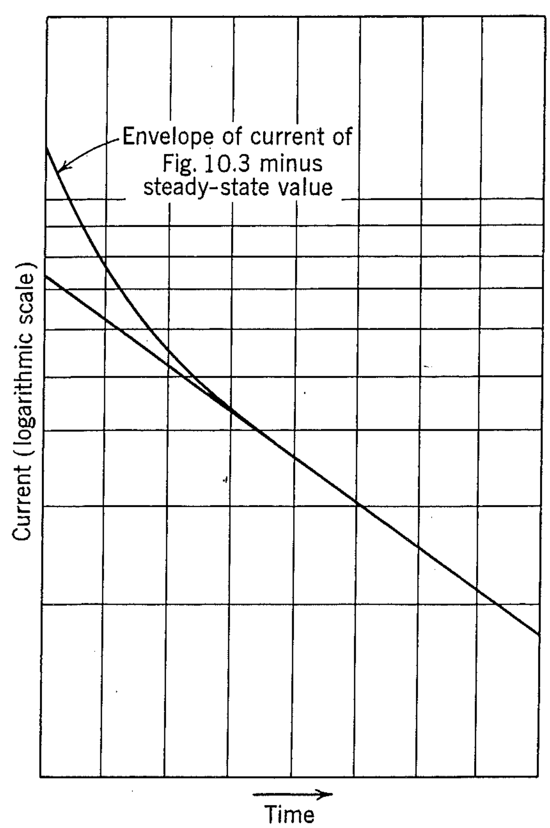
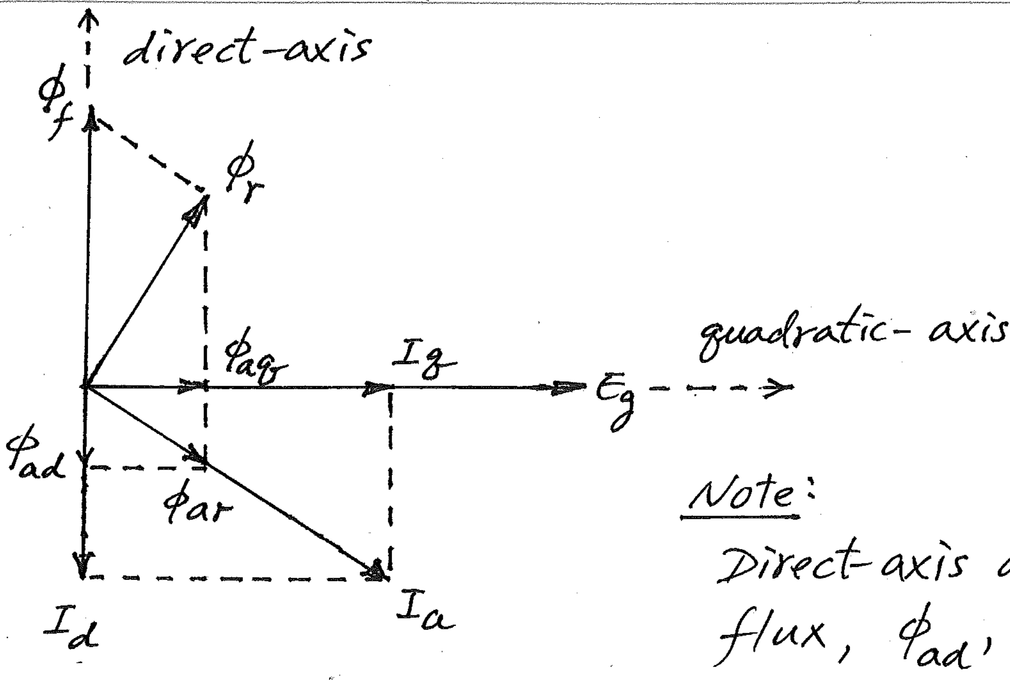


Figure 10.4 Excess of the current envelope of Fig. 10.3 over the sustained maximum current, plotted on semilogarithmic scales.



Note:
Direct-axis armature reaction flux, ϕ_{ad}, opposes the field flux, ϕ_f.

⇒ Reduces emf and current.

During fault, short circuit current is highly reactive due to negligible resistance. This tends to reduce airgap flux : Armature Reaction

⇒ Tends to reduce I_a.

$$|I| = \frac{oa}{\sqrt{2}} = \frac{|E_g|}{X_d}$$

$$|I'| = \frac{ob}{\sqrt{2}} = \frac{|E_g|}{X'_d}$$

$$|I''| = \frac{oc}{\sqrt{2}} = \frac{|E_g|}{X''_d}$$

where $|I|$ = steady-state current, rms value
$|I'|$ = transient current, rms value excluding dc component
$|I''|$ = subtransient current, rms value excluding dc component
X_d = direct-axis synchronous reactance
X'_d = direct-axis transient reactance
X''_d = direct-axis subtransient reactance
$|E_g|$ = rms voltage from one terminal to neutral at no load
oa, ob, oc = intercepts shown in Fig. 10.3.

Theorem of Constant Flux Linkages:

Flux linkage cannot change instantaneously.
Initially forced to flow through high reluctance path, i.e., low reactance path.

Reluctance: $\mathcal{R}$

Flux: $\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\mathcal{R}} \triangleq LI$

Then moves towards the lower reluctance path, i.e., high reactance path.

$\Rightarrow$ Time-varying inductance $L(t)$
or reactance $X(t) = \omega L(t)$.

Define:

$$X_d'' < X_d' < X_d$$

Instantaneous fault current:

$$i_{ac}(t) = \sqrt{2}E_g \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + \frac{1}{X_d} \right] \sin\left(\omega t + \alpha - \frac{\pi}{2} \right)$$

at $t=0$: $I_{ac}(0) = \frac{E_g}{X_d''} = I''$, subtransient current, $\theta = \frac{\pi}{2}$ for $R=0$

T_d'', d-axis subtransient current time constant

for large t, $I_{ac}' = \frac{E_g}{X_d'}$, transient current,

for much larger t, $I_{ac}(\infty) = \frac{E_g}{X_d} = I$, steady-state current

Max dc offset: @ $\alpha=0$, $i_{dc\,max}(t) = \frac{\sqrt{2}E_g}{X_d''} e^{-t/T_A} = \sqrt{2} I'' e^{-t/T_A}$
= peak of ac @ $t=0$.

T_A: armature time constant

EXAMPLE 7.2 Three-phase short-circuit currents, unloaded synchronous generator

A 500-MVA 20-kV, 60-Hz synchronous generator with reactances $X''_d = 0.15$, $X'_d = 0.24$, $X_d = 1.1$ per unit and time constants $T''_d = 0.035$, $T'_d = 2.0$, $T_A = 0.20$ s is connected to a circuit breaker. The generator is operating at 5% above rated voltage and at no-load when a bolted three-phase short circuit occurs on the load side of the breaker. The breaker interrupts the fault 3 cycles after fault inception. Determine (a) the subtransient fault current in per-unit and kA rms; (b) maximum dc offset as a function of time; and (c) rms asymmetrical fault current, which the breaker interrupts, assuming maximum dc offset.

SOLUTION

a. The no-load voltage before the fault occurs is $E_g = 1.05$ per unit. From (7.2.2), the subtransient fault current that occurs in each of the three phases is

$$I'' = \frac{1.05}{0.15} = 7.0 \quad \text{per unit}$$

The generator base current is

$$I_{\text{base}} = \frac{S_{\text{rated}}}{\sqrt{3}V_{\text{rated}}} = \frac{500}{(\sqrt{3})(20)} = 14.43 \quad \text{kA}$$

The rms subtransient fault current in kA is the per-unit value multiplied by the base current:

$$I'' = (7.0)(14.43) = 101.0 \quad \text{kA}$$

b. From (7.2.5), the maximum dc offset that may occur in any one phase is

$$i_{\text{dcmax}}(t) = \sqrt{2}(101.0)e^{-t/0.20} = 142.9e^{-t/0.20} \quad \text{kA}$$

c. From (7.2.1), the rms ac fault current at $t = 3$ cycles $= 0.05$ s is

$$\begin{aligned} I_{\text{ac}}(0.05\ \text{s}) &= 1.05\left[\left(\frac{1}{0.15} - \frac{1}{0.24}\right)e^{-0.05/0.035} + \left(\frac{1}{0.24} - \frac{1}{1.1}\right)e^{-0.05/2.0} + \frac{1}{1.1}\right] \\ &= 4.920 \quad \text{per unit} \\ &= (4.920)(14.43) = 71.01 \quad \text{kA} \end{aligned}$$

Modifying (7.1.10) to account for the time-varying symmetrical component of fault current, we obtain

$$\begin{aligned} I_{\text{rms}}(0.05) &= \sqrt{[I_{\text{ac}}(0.05)]^2 + [\sqrt{2}I''e^{-t/T_a}]^2} \\ &= I_{\text{ac}}(0.05)\sqrt{1 + 2\left[\frac{I''}{I_{\text{ac}}(0.05)}\right]^2 e^{-2t/Ta}} \\ &= (71.01)\sqrt{1 + 2\left[\frac{101}{71.01}\right]^2 e^{-2(0.05)/0.20}} \\ &= (71.01)(1.8585) \\ &= 132 \quad \text{kA} \end{aligned}$$

■

Example 10.1 Two generators are connected in parallel to the low-voltage side of a three-phase Δ-Y transformer as shown in Fig. 10.5. Generator 1 is rated 50,000 kVA, 13.8 kV. Generator 2 is rated 25,000 kVA, 13.8 kV. Each generator has a subtransient reactance of 25%. The transformer is rated 75,000 kVA, 13.8Δ/69Y kV, with a reactance of 10%. Before the fault occurs, the voltage on the high-tension side of the transformer is 66 kV. The transformer is unloaded, and there is no circulating current between the generators. Find the subtransient current in each generator when a three-phase short circuit occurs on the high-tension side of the transformer.

SOLUTION Select as base in the high-tension circuit 69 kV, 75,000 kVA. Then the base voltage on the low-tension side is 13.8 kV.

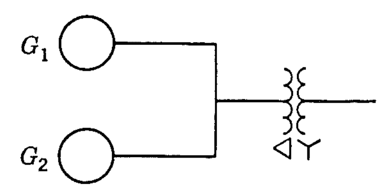


Figure 10.5 One-line diagram for Example 10.1.

Generator 1:

$$X_d'' = 0.25 \frac{75{,}000}{50{,}000} = 0.375 \text{ per unit}$$

$$E_{g1} = \frac{66}{69} = 0.957 \text{ per unit}$$

Generator 2:

$$X_d'' = 0.25 \frac{75{,}000}{25{,}000} = 0.750 \text{ per unit}$$

$$E_{g2} = \frac{66}{69} = 0.957 \text{ per unit}$$

Transformer:

$$X = 0.10 \text{ per unit}$$

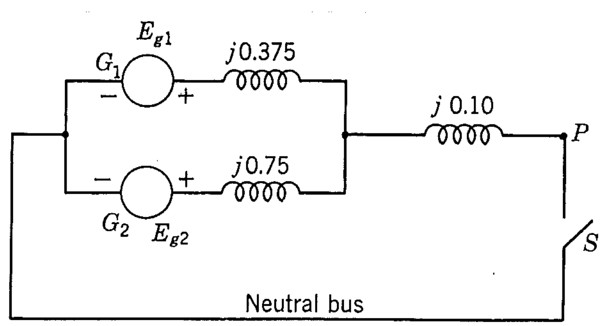


Figure 10.6 Reactance diagram for Example 10.1.

Figure 10.6 shows the reactance diagram before the fault. A three-phase fault at P is simulated by closing switch S. The internal voltages of the two machines may be considered to be in parallel since they must be identical in magnitude and phase if no circulating current flows between them. The equivalent parallel subtransient reactance is

$$\frac{0.375 \times 0.75}{0.375 + 0.75} = 0.25 \text{ per unit}$$

Therefore, as a phasor with E_g as reference, the subtransient current in the short circuit is

$$I'' = \frac{0.957}{j0.25 + j0.10} = -j2.735 \text{ per unit}$$

The voltage on the delta side of the transformer is

$$(-j2.735)(j0.10) = 0.2735 \text{ per unit}$$

and in generators 1 and 2

$$I_1'' = \frac{0.957 - 0.274}{j0.375} = -j1.823 \text{ per unit}$$

$$I_2'' = \frac{0.957 - 0.274}{j0.75} = -j0.912 \text{ per unit}$$

To find the current in amperes, the per-unit values are multiplied by the base current of the circuit:

$$|I_1''| = 1.823 \frac{75{,}000}{\sqrt{3} \times 13.8} = 5720 \text{ A}$$

$$|I_2''| = 0.912 \frac{75{,}000}{\sqrt{3} \times 13.8} = 2860 \text{ A}$$

Although machine reactances are not true constants of the machine and depend on the degree of saturation of the magnetic circuit, their values usually lie within certain limits and can be predicted for various types of machines.

4. Internal Voltages of Loaded Machines

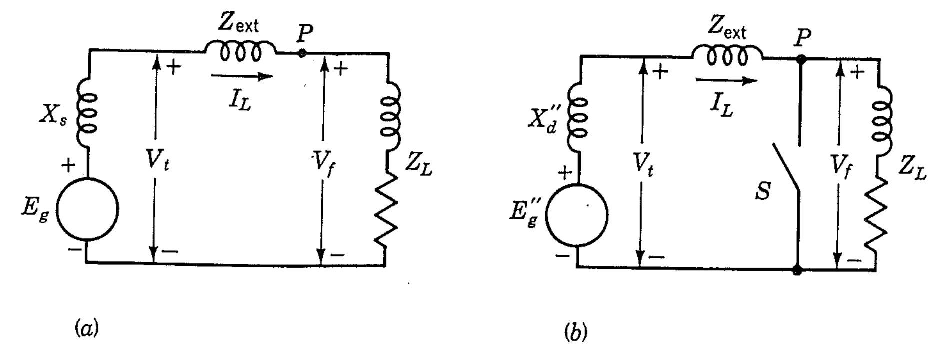


Figure 10.7 Equivalent circuits for a generator supplying a balanced three-phase load. Application of a three-phase fault at P is simulated by closing switch S. (a) Usual steady-state generator equivalent circuit with load. (b) Circuit for calculation of I''.

Generator:

$E''_g = V_t + jI_L X''_d$ — voltage behind the subtransient reactance, or subtransient internal voltage

$E'_g = V_t + jI_L X'_d$ — voltage behind the transient reactance, or transient internal voltage

Motor:

inertia of rotor and field energized. ⇒ acts as a generator

$E''_m = V_t - jI_L X''_d$

$E'_m = V_t - jI_L X'_d$

Thevenin Equivalent:

Figure 10.9 is the Thévenin equivalent of Fig. 10.7b. The impedance Z_{th} is equal to $(Z_{ext} + jX''_d)Z_L/(Z_L + Z_{ext} + jX''_d)$. Upon the occurrence of a three-phase short circuit at P, simulated by closing S, the subtransient current in the fault is

$$I'' = \frac{V_f}{Z_{th}} = \frac{V_f(Z_L + Z_{ext} + jX''_d)}{Z_L(Z_{ext} + jX''_d)} \tag{10.10}$$

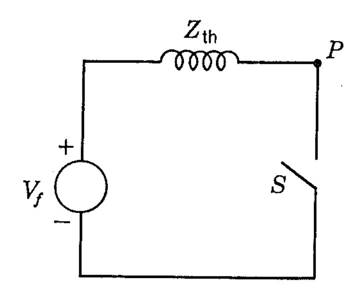


Figure 10.9 Thévenin equivalent of the circuit of Fig. 10.7b.

Systems that contain generators and motors under load may be solved either by Thévenin's theorem or by the use of transient or subtransient internal voltages, as is illustrated in the following examples.

Example 10.2 A synchronous generator and motor are rated 30,000 kVA, 13.2 kV, and both have subtransient reactances of 20%. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20,000 kW at 0.8 power factor leading and a terminal voltage of 12.8 kV when a symmetrical three-phase fault occurs at the motor terminals. Find the subtransient current in the generator, motor, and fault by using the internal voltage of the machines.

SOLUTION Choose as base 30,000 kVA, 13.2 kV.

If we use the voltage at the fault V_f as the reference phasor,

$$V_f = \frac{12.8}{13.2} = 0.97\underline{/0^\circ} \text{ per unit}$$

$$\text{Base current} = \frac{30{,}000}{\sqrt{3} \times 13.2} = 1312 \text{ A}$$

$$I_L = \frac{20{,}000\underline{/36.9^\circ}}{0.8 \times \sqrt{3} \times 12.8} = 1128\underline{/36.9^\circ} \text{ A}$$

$$= \frac{1128\underline{/36.9^\circ}}{1312} = 0.86\underline{/36.9^\circ} \text{ per unit}$$

$$= 0.86(0.8 + j0.6) = 0.69 + j0.52 \text{ per unit}$$

For the generator,

$$V_t = 0.970 + j0.1(0.69 + j0.52) = 0.918 + j0.069 \text{ per unit}$$

$$E_g'' = 0.918 + j0.069 + j0.2(0.69 + j0.52) = 0.814 + j0.207 \text{ per unit}$$

$$I_g'' = \frac{0.814 + j0.207}{j0.3} = 0.69 - j2.71 \text{ per unit}$$

$$= 1312(0.69 - j2.71) = 905 - j3550 \text{ A}$$

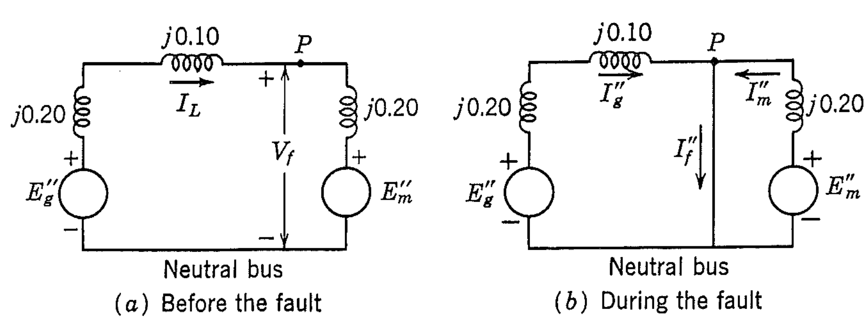


Figure 10.8 Equivalent circuits for Example 10.2.

For the motor,

$$V_t = V_f = 0.97\underline{/0^\circ} \text{ per unit}$$

$$E''_m = 0.97 + j0 - j0.2(0.69 + j0.52) = 0.97 - j0.138 + 0.104$$

$$= 1.074 - j0.138 \text{ per unit}$$

$$I''_m = \frac{1.074 - j0.138}{j0.2} = -0.69 - j5.37 \text{ per unit}$$

$$= 1312(-0.69 - j5.37) = -905 - j7050 \text{ A}$$

In the fault,

$$I''_f = I''_g + I''_m = 0.69 - j2.71 - 0.69 - j5.37 = -j8.08 \text{ per unit}$$

$$= -j8.08 \times 1312 = -j10{,}600 \text{ A}$$

Figure 10.8*b* shows the paths of I''_g, I''_m, and I''_f.

Example 10.3 Solve Example 10.2 by the use of Thévenin's theorem.

SOLUTION

$$Z_{th} = \frac{j0.3 \times j0.2}{j0.3 + j0.2} = j0.12 \text{ per unit}$$

$$V_f = 0.97\underline{/0^\circ} \text{ per unit}$$

In the fault,

$$I''_f = \frac{0.97 + j0}{j0.12} = -j8.08 \text{ per unit}$$

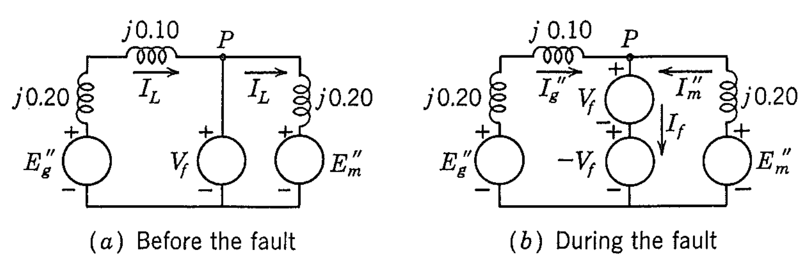


Figure 10.10 Circuits illustrating the application of the superposition theorem to determine the proportion of the fault current in each branch of the system.

$$\text{Fault current from generator} = -j8.08 \times \frac{j0.2}{j0.5} = -j3.23 \text{ per unit}$$

$$\text{Fault current from motor} = -j8.08 \times \frac{j0.3}{j0.5} = -j4.85 \text{ per unit}$$

To these currents must be added the prefault current I_L to obtain the total subtransient currents in the machines:

$$I''_g = 0.69 + j0.52 - j3.23 = 0.69 - j2.71 \text{ per unit}$$

$$I''_m = -0.69 - j0.52 - j4.85 = -0.69 - j5.37 \text{ per unit}$$

Note that I_L is in the same direction as I''_g but opposite to I''_m. The per-unit values found for I''_f, I''_g, and I''_m are the same as in Example 10.2, and so the ampere values will also be the same.

Usually load current is omitted in determining the current in each line upon the occurrence of a fault. In the Thévenin method neglect of load current means that the prefault current in each line is not added to the component of current flowing toward the fault in the line. The method of Example 10.2 neglects load current if the subtransient internal voltages of all machines are assumed equal to the voltage V_f at the fault before the fault occurs, for such is the case if no current flows anywhere in the network prior to the fault.

Neglecting load current in Example 10.3 gives

$$\text{Fault current from generator} = 3.23 \times 1312 = 4240 \text{ A}$$

$$\text{Fault current from motor} = 4.85 \times 1312 = 6360 \text{ A}$$

$$\text{Current in fault} = 8.08 \times 1312 = 10{,}600 \text{ A}$$

The current in the fault is the same whether or not load current is considered, but the contributions from the lines differ. When load current is included, we find from Example 10.2

$$\text{Fault current from generator} = |905 - j3550| = 3660 \text{ A}$$

$$\text{Fault current from motor} = |-905 - j7050| = 7200 \text{ A}$$

The arithmetic sum of the generator and motor current magnitudes does not equal the fault current because the currents from the generator and motor are not in phase.

7.3

POWER SYSTEM THREE-PHASE SHORT CIRCUITS

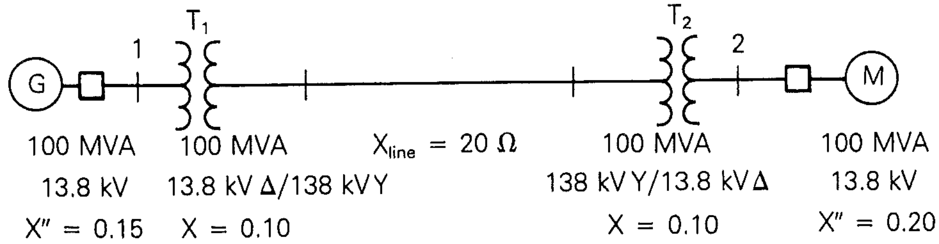
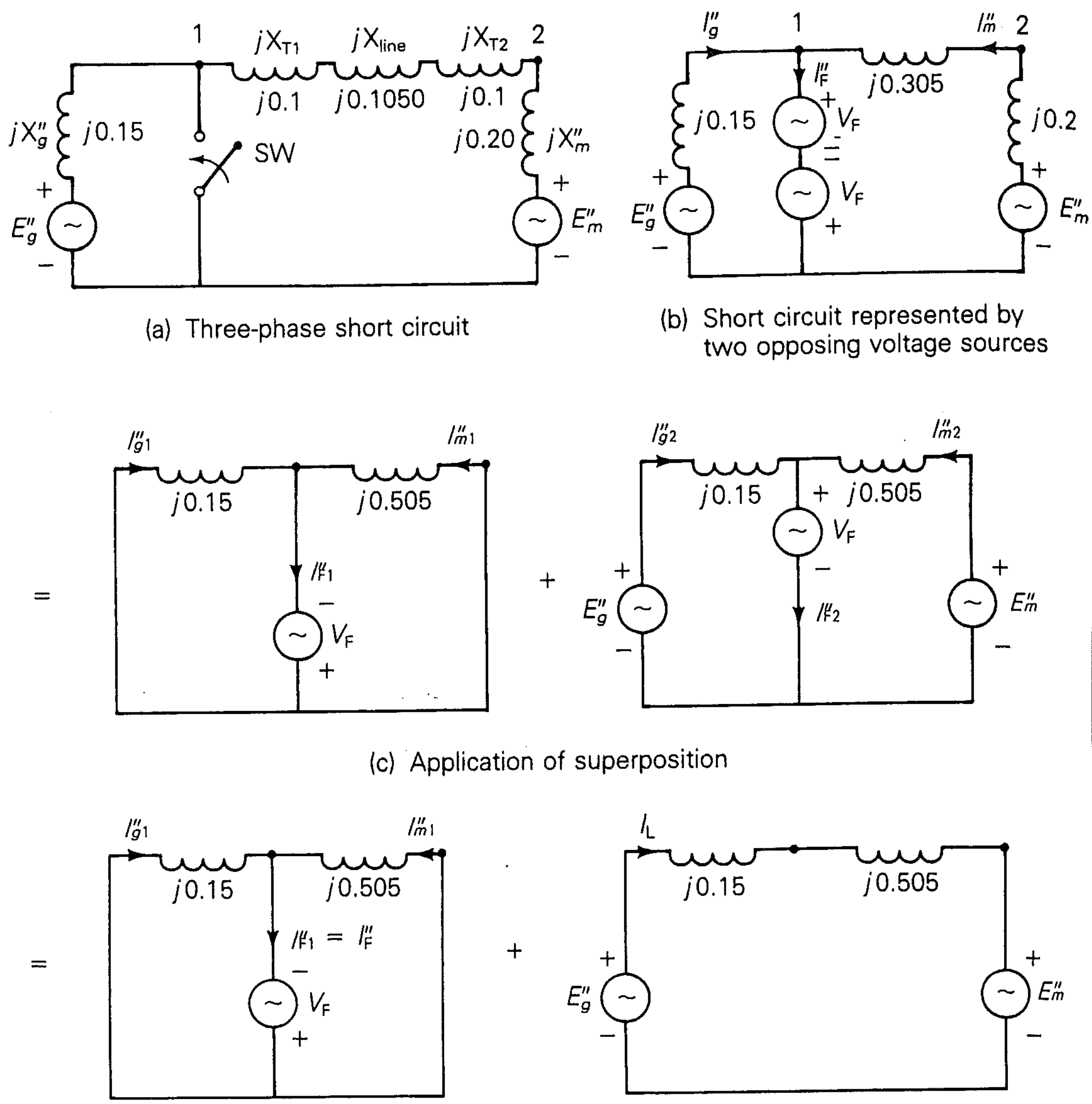


FIGURE 7.3 Single-line diagram of a synchronous generator feeding a synchronous motor



(d) V_F set equal to prefault voltage at fault

FIGURE 7.4 Application of superposition to a power system three-phase short circuit

EXAMPLE 7.3 Three-phase short-circuit currents, power system

The synchronous generator in Figure 7.3 is operating at rated MVA, 0.95 p.f. lagging and at 5% above rated voltage when a bolted three-phase short circuit occurs at bus 1. Calculate the per-unit values of (a) subtransient fault current; (b) subtransient generator and motor currents, neglecting prefault current; and (c) subtransient generator and motor currents including prefault current.

SOLUTION

a. Using a 100-MVA base, the base impedance in the zone of the transmission line is

$$Z_{\text{base, line}} = \frac{(138)^2}{100} = 190.44 \quad \Omega$$

and

$$X_{\text{line}} = \frac{20}{190.44} = 0.1050 \quad \text{per unit}$$

The per-unit reactances are shown in Figure 7.4. From the first circuit in Figure 7.4(d), the Thévenin impedance as viewed from the fault is

$$Z_{\text{Th}} = jX_{\text{Th}} = j\frac{(0.15)(0.505)}{(0.15 + 0.505)} = j0.11565 \quad \text{per unit}$$

and the prefault voltage at the generator terminals is

$$V_{\text{F}} = 1.05\underline{/0^\circ} \quad \text{per unit}$$

The subtransient fault current is then

$$I''_{\text{F}} = \frac{V_{\text{F}}}{Z_{\text{Th}}} = \frac{1.05\underline{/0^\circ}}{j0.11565} = -j9.079 \quad \text{per unit}$$

b. Using current division in the first circuit of Figure 7.4(d),

$$I''_{g1} = \left(\frac{0.505}{0.505 + 0.15}\right) I''_{\text{F}} = (0.7710)(-j9.079) = -j7.000 \quad \text{per unit}$$

$$I''_{m1} = \left(\frac{0.15}{0.505 + 0.15}\right) I''_{\text{F}} = (0.2290)(-j9.079) = -j2.079 \quad \text{per unit}$$

c. The generator base current is

$$I_{\text{base, gen}} = \frac{100}{(\sqrt{3})(13.8)} = 4.1837 \quad \text{kA}$$

and the prefault generator current is

$$I_L = \frac{100}{(\sqrt{3})(1.05 \times 13.8)} \underline{/-\cos^{-1} 0.95} = 3.9845\underline{/-18.19^\circ} \quad \text{kA}$$

$$= \frac{3.9845\underline{/-18.19^\circ}}{4.1837} = 0.9524\underline{/-18.19^\circ}$$

$$= 0.9048 - j0.2974 \quad \text{per unit}$$

The subtransient generator and motor currents, including prefault current, are then

$$I''_g = I''_{g1} + I_L = -j7.000 + 0.9048 - j0.2974$$

$$= 0.9048 - j7.297 = 7.353\underline{/-82.9^\circ} \quad \text{per unit}$$

$$I''_m = I''_{m1} - I_L = -j2.079 - 0.9048 + j0.2974$$

$$= -0.9048 - j1.782 = 1.999\underline{/243.1^\circ} \quad \text{per unit}$$

An alternate method of solving Example 7.3 is to first calculate the internal voltages E''_g and E''_m using the prefault load current I_L. Then, instead of using superposition, the fault currents can be resolved directly from the circuit in Figure 7.4(a) (see Problem 7.11). However, in a system with many synchronous machines, the superposition method has the advantage that all machine voltage sources are shorted, and the prefault voltage is the only source required to calculate the fault current. Also, when calculating the contributions to fault current from each branch, prefault currents are usually small, and hence can be neglected. Otherwise, prefault load currents could be obtained from a power-flow program. ■

BUS IMPEDANCE MATRIX

We now extend the results of the previous section to calculate subtransient fault currents for three-phase faults in an N-bus power system. The system is modeled by its positive-sequence network, where lines and transformers are represented by series reactances and synchronous machines are represented by constant-voltage sources behind subtransient reactances. As before, all resistances, shunt admittances, and nonrotating impedance loads are neglected. For simplicity, we also neglect prefault load currents.

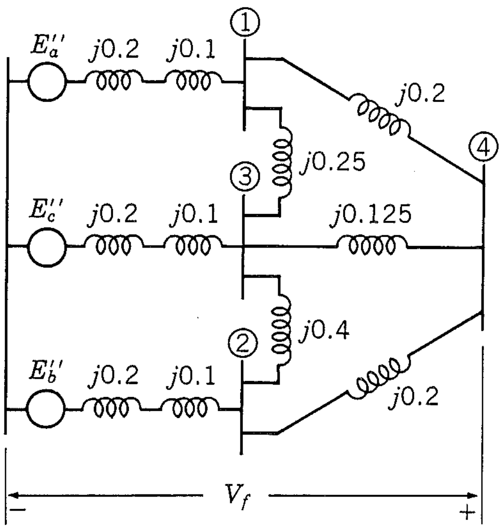


Figure 10.11 Reactance diagram obtained from Fig. 7.3 by substituting subtransient for synchronous reactances of the machines and subtransient internal voltages for no-load generated voltages. Reactance values are marked in per unit.

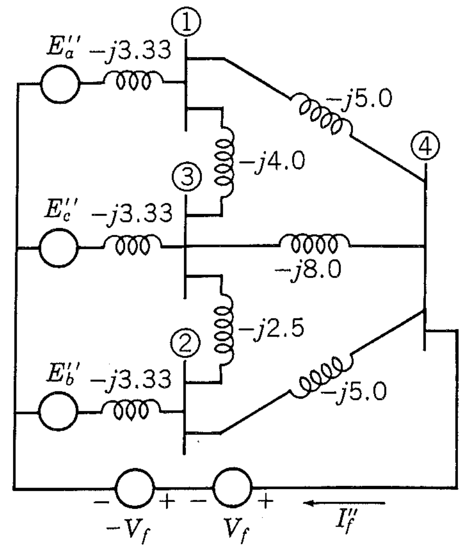


Figure 10.12 Circuit of Fig. 10.11 with admittances marked in per unit and a three-phase fault on bus 4 of the system simulated by V_f and $-V_f$ in series.

A three-phase fault at bus 4 is simulated by the network of Fig. 10.12 where the impedance values of Fig. 10.11 have been changed to admittances. The generated voltages V_f and $-V_f$ in series constitute the short circuit. Generated voltage V_f alone in this branch would cause no current in the branch. With V_f and $-V_f$ in series the branch is a short circuit, and the branch current is I''_f. Admittances rather than impedances have been marked in per unit on this diagram. If E''_a, E''_b, E''_c, and V_f are short-circuited, the voltages and currents are those due only to $-V_f$. Then the only current entering a node from a source is that from $-V_f$ and is $-I''_f$ *into* node 4 (I''_f *from* node 4) since there is no current in this branch until the insertion of $-V_f$. The node equations in matrix form for the network with $-V_f$ the only source are

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ -I''_f \end{bmatrix} = j \begin{bmatrix} -12.33 & 0.0 & 4.0 & 5.0 \\ 0.0 & -10.83 & 2.5 & 5.0 \\ 4.0 & 2.5 & -17.83 & 8.0 \\ 5.0 & 5.0 & 8.0 & -18.0 \end{bmatrix} \begin{bmatrix} V_1^{\Delta} \\ V_2^{\Delta} \\ V_3^{\Delta} \\ -V_f \end{bmatrix} \qquad (10.11)$$

when the superscript Δ indicates that the voltages are due only to $-V_f$. The Δ sign is chosen to indicate the change in voltage due to the fault.

By inverting the bus admittance matrix of the network of Fig. 10.12 we obtain the bus impedance matrix. The bus voltages due to $-V_f$ are given by

$$\begin{bmatrix} V_1^\Delta \\ V_2^\Delta \\ V_3^\Delta \\ -V_f \end{bmatrix} = \mathbf{Z}_{\text{bus}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_f'' \end{bmatrix} \tag{10.12}$$

and so

$$I_f'' = \frac{V_f}{Z_{44}} \tag{10.13}$$

and

$$V_1^\Delta = -I_f'' Z_{14} = -\frac{Z_{14}}{Z_{44}} V_f$$

$$V_2^\Delta = -\frac{Z_{24}}{Z_{44}} V_f \qquad V_3^\Delta = -\frac{Z_{34}}{Z_{44}} V_f \tag{10.14}$$

When the generator voltage $-V_f$ is short-circuited in the network of Fig. 10.12 and E_a'', E_b'', E_c'', and V_f are in the circuit, the currents and voltages everywhere in the network are those existing before the fault. By the principle of superposition these prefault voltages added to those given by Eqs. (10.14) yield the voltages existing after the fault occurs. Usually the faulted network is assumed to have been without loads before the fault. In such a case no current is flowing before the fault, and all voltages throughout the network are the same and equal to V_f. This assumption simplifies our work considerably, and applying the principle of superposition gives

$$\begin{aligned} V_1 &= V_f + V_1^\Delta = V_f - I_f'' Z_{14} \\ V_2 &= V_f + V_2^\Delta = V_f - I_f'' Z_{24} \\ V_3 &= V_f + V_3^\Delta = V_f - I_f'' Z_{34} \\ V_4 &= V_f - V_f = 0 \end{aligned} \tag{10.15}$$

These voltages exist when subtransient current flows and $\mathbf{Z}_{\text{bus}}$ has been formed for a network having subtransient values for generator reactances.

In general terms for a fault on bus k, and neglecting prefault currents,

$$I_f = \frac{V_f}{Z_{kk}} \tag{10.16}$$

and the postfault voltage at bus n is

$$V_n = V_f - \frac{Z_{nk}}{Z_{kk}} V_f \tag{10.17}$$

Using the numerical values of Eq. (10.11), we invert the square matrix $\mathbf{Y}_{\text{bus}}$ of that equation and find

$$\mathbf{Z}_{\text{bus}} = j\begin{bmatrix} 0.1488 & 0.0651 & 0.0864 & 0.0978 \\ 0.0651 & 0.1554 & 0.0799 & 0.0967 \\ 0.0864 & 0.0798 & 0.1341 & 0.1058 \\ 0.0978 & 0.0967 & 0.1058 & 0.1566 \end{bmatrix} \tag{10.18}$$

Usually V_f is assumed to be $1.0\underline{/0^\circ}$ per unit, and with this assumption for our faulted network

$$I''_f = \frac{1}{j0.1566} = -j6.386 \text{ per unit}$$

$$V_1 = 1 - \frac{j0.0978}{j0.1566} = 0.3755 \text{ per unit}$$

$$V_2 = 1 - \frac{j0.0967}{j0.1566} = 0.3825 \text{ per unit}$$

$$V_3 = 1 - \frac{j0.1058}{j0.1566} = 0.3244 \text{ per unit}$$

Currents in any part of the network can be found from the voltages and impedances. For instance, the fault current in the branch connecting nodes 1 and 3 flowing toward node 3 is

$$I''_{13} = \frac{V_1 - V_3}{j0.25} = \frac{0.3755 - 0.3244}{j0.25}$$

$$= -j0.2044 \text{ per unit}$$

From the generator connected to node 1 the current is

$$I''_a = \frac{E''_a - V_1}{j0.3} = \frac{1 - 0.3755}{j0.3}$$

$$= -j2.0817 \text{ per unit}$$

Other currents can be found in a similar manner, and voltages and currents with the fault on any other bus are calculated just as easily from the impedance matrix.

Equation (10.16) is simply an application of Thévenin's theorem, and we recognize that the quantities on the principal diagonal of the bus impedance matrix are the Thévenin impedances of the network for calculating fault current at the various buses. Power companies furnish data to a customer who must determine the fault current to specify circuit breakers for an industrial plant or distribution system connected to the utility system at any point. Usually the data supplied lists the short-circuit megavoltamperes, where

$$\text{Short-circuit MVA} = \sqrt{3} \times (\text{nominal kV}) \times I_{sc} \times 10^{-3} \tag{10.19}$$

With resistance and shunt capacitance neglected, the single-phase Thévenin equivalent circuit which represents the system is an emf equal to the nominal line voltage divided by $\sqrt{3}$ in series with an inductive reactance of

$$X_{th} = \frac{(\text{nominal kV}/\sqrt{3}) \times 1000}{I_{sc}} \ \Omega \tag{10.20}$$

Solving Eq. (10.19) for I_{sc} and substituting in Eq. (10.20) yield

$$X_{th} = \frac{(\text{nominal kV})^2}{\text{short-circuit MVA}} \ \Omega \tag{10.21}$$

If base kilovolts is equal to nominal kilovolts, converting to per unit yields

$$X_{th} = \frac{\text{base MVA}}{\text{short-circuit MVA}} \text{ per unit} \tag{10.22}$$

$$X_{th} = \frac{I_{\text{base}}}{I_{sc}} \text{ per unit} \tag{10.23}$$

10.5 A BUS IMPEDANCE MATRIX EQUIVALENT NETWORK

Although we cannot devise a physically realizable circuit employing the impedances of the bus impedance network, we can draw a circuit with transfer impedances *indicated* between branches. Such a diagram will be helpful in understanding the significance of the equations developed in Sec. 10.4.

In Fig. 10.13 brackets have been drawn between branch 4 and the other three branches of a network which has four nodes in addition to the reference node.† Associated with these brackets are the symbols Z_{14}, Z_{24}, and Z_{34}, which

† This equivalent network is drawn in the manner adopted in J. R. Neuenswander, *Modern Power Systems*, Intext Educational Publishers, New York, 1971, which refers to the bus impedance matrix equivalent network as the *rake equivalent*.

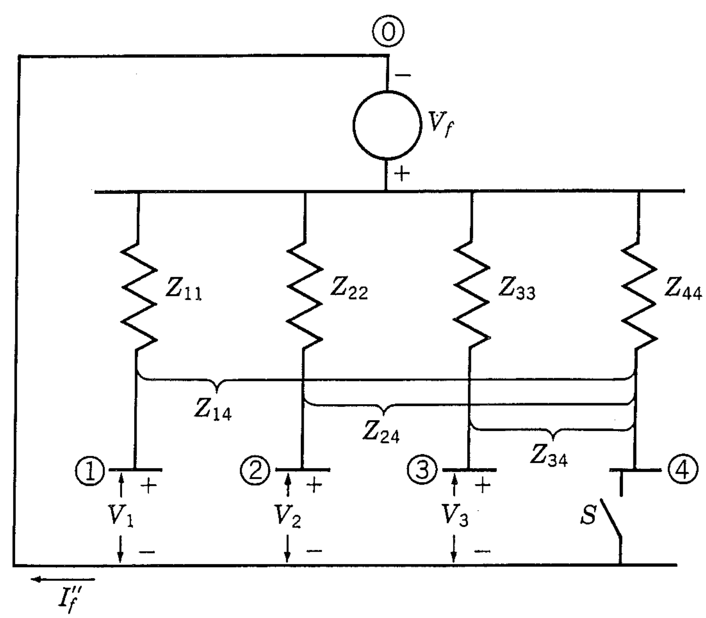


Figure 10.13 Bus impedance matrix equivalent network with four independent nodes. Closing switch S simulates a fault on node 4. Only the transfer admittances for node 4 are shown.

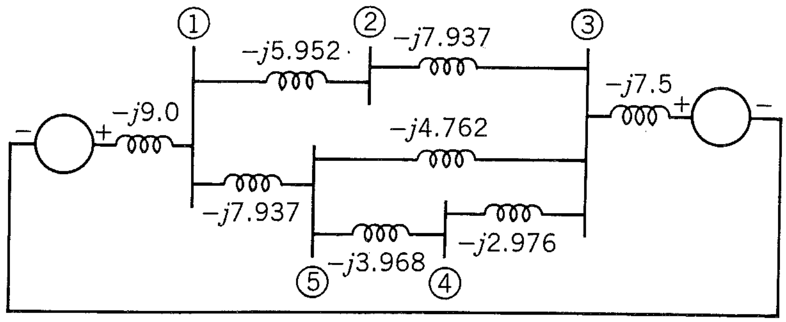


Figure 10.14 Admittance diagram for Example 10.4.

The network with admittances marked in per unit is shown in Fig. 10.14 from which the node admittance matrix is

$$\mathbf{Y}_{\text{bus}} = j\begin{bmatrix} -22.889 & 5.952 & 0.0 & 0.0 & 7.937 \\ 5.952 & -13.889 & 7.937 & 0.0 & 0.0 \\ 0.0 & 7.937 & -23.175 & 2.976 & 4.762 \\ 0.0 & 0.0 & 2.976 & -6.944 & 3.968 \\ 7.937 & 0.0 & 4.762 & 3.968 & -16.667 \end{bmatrix}$$

This 5×5 bus is inverted on a digital computer to yield the short-circuit matrix

$$\mathbf{Z}_{\text{bus}} = j\begin{bmatrix} 0.0793 & 0.0558 & 0.0382 & 0.0511 & 0.0608 \\ 0.0558 & 0.1338 & 0.0664 & 0.0630 & 0.0605 \\ 0.0382 & 0.0664 & 0.0875 & 0.0720 & 0.0603 \\ 0.0511 & 0.0630 & 0.0720 & 0.2321 & 0.1002 \\ 0.0608 & 0.0605 & 0.0603 & 0.1002 & 0.1301 \end{bmatrix}$$

Visualizing a network like that of Fig. 10.13 will help in finding the desired currents and voltages.

The subtransient current in a three-phase fault on bus 4 is

$$I'' = \frac{1.0}{j0.2321} = -j4.308 \text{ per unit}$$

At buses 3 and 5 the voltages are

$$V_3 = 1.0 - (-j4.308)(j0.0720) = 0.6898 \text{ per unit}$$

$$V_5 = 1.0 - (-j4.308)(j0.1002) = 0.5683 \text{ per unit}$$

Currents to the fault are

$$\text{From bus 3:} \quad 0.6898(-j2.976) = -j2.053$$

$$\text{From bus 5:} \quad 0.5683(-j3.968) = \underline{-j2.255}$$

$$-j4.308 \text{ per unit}$$

From the same short-circuit matrix we can find similar information for faults on any of the other buses.