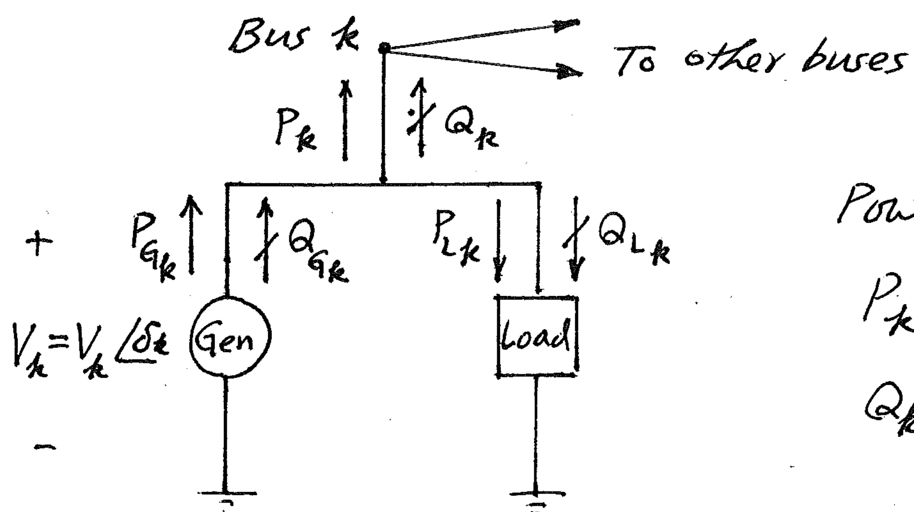
4. Power Flow Problem

(1) Bus Types:

The power-flow problem is to find voltage magnitude and phase angle at each bus in a balanced three-phase power system in steady state.

There are four variables in each bus:

$$V_k,\ \delta_k,\ P_k,\ Q_k$$



Power Injection:

$$P_k = P_{G_k} - P_{L_k}$$

$$Q_k = Q_{G_k} - Q_{L_k}$$

There are three types of buses:

1. Swing bus (slack bus) or (infinite bus): Reference bus.
   with $V_1 \angle \delta_1$, typically $1.0\angle 0^\circ$
   Needs to find P_1 and Q_1

2. Load bus (PQ bus). P and Q are specified
   Needs to find V_k and δ_k.

3. Voltage controlled bus (PV bus) or (Generator bus).
   P_k and V_k are specified
   Needs to find Q_k and δ_k

Examples of PV bus:

Generators, switched shunt capacitors, static var systems.

- Upper and/or lower limits on Q are normally given: $Q_{G_{max}}$, $Q_{G_{min}}$
- If these limits are reached, then the reactive power output of the unit is held at the limit, and the bus is modeled as PQ bus (Load bus).
- Tap-changing transformer: Needs to compute the tap setting.

(2) <u>Network Equations:</u>

$$I = Y_{bus} V$$

or

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}}_{\text{current injection into each bus}} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

current injection into each bus:

$$\Rightarrow \quad I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

(3) <u>Power Equation:</u>

Power injection at each bus is then,

$$S_k = P_k + jQ_k = V_k I_k^*$$
$$= V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^*,$$
$$= |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \, e^{j(\delta_k - \delta_n - \theta_{kn})}$$

where $V_n = |V_n| \angle \delta_n$, $Y_{kn} = |Y_{kn}| \angle \theta_{kn}$

Therefore,

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \end{cases}$$

$$k = 1, 2, \cdots, N$$

We only have $2N$ equations for $4N$ variables.

<u>Alternative Form:</u> Y_{kn} in rectangular form,

$$Y_{kn} = G_{kn} + jB_{kn}$$

Then

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |V_n| \left[G_{kn} \cos(\delta_k - \delta_n) + B_{kn} \sin(\delta_k - \delta_n) \right] \\ Q_k = |V_k| \sum_{n=1}^{N} |V_n| \left[G_{kn} \sin(\delta_k - \delta_n) - B_{kn} \cos(\delta_k - \delta_n) \right] \end{cases}$$

These are "nonlinear" equations to solve.

(4) Power-Flow Solution by Gauss-Seidel:

a) PQ bus: Use the network equation,

(1) $$I_k = \sum_{k=1}^{N} Y_{kn} V_n = Y_{k1} V_1 + \cdots + Y_{kk} V_k + \cdots + Y_{kN} V_N$$

with the current injection at bus k,

(2) $$I_k = \frac{P_k - jQ_k}{V_k^*}$$

which is resulting from the power injection,

$$S_k = P_k + jQ_k = V_k I_k^*$$

Solving (1) for V_k,

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{N} Y_{kn} V_n \right]$$

Applying Gauss-Seidel iteration,

(3) $$\underbrace{V_k^{(i+1)}}_{new} = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{\underbrace{V_k^{*(i)}}_{old}} - \underbrace{\sum_{n=1}^{k-1} Y_{kn} V_n^{(i+1)}}_{Seidel} - \underbrace{\sum_{n=k+1}^{N} Y_{kn} V_n^{(i)}}_{Gauss} \right]$$

1. This can be repeated <u>twice</u> during each iteration, first with $V_k^{*(i)}$, then replacing it with $V_k^{*(i+1)}$.
2. <u>Acceleration</u> is performed to speed up the convergence.

$$V_{k,acc}^{(i+1)} = (1-\alpha) V_k^{(i)} + \alpha V_k^{(i+1)}$$
$$= V_k^{(i)} + \alpha \left(V_k^{(i+1)} - V_k^{(i)} \right), \quad \alpha \sim 1.6$$

b) <u>PV bus:</u> For a voltage controlled bus $|V_k|$ is specified.

1. We first calculate the corresponding reactive power with the power equation:

$$Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \sin(\delta_k - \delta_n - \theta_{kn})$$

or

$$Q_k = -\Im m\{V_k^* I_k\}$$

where $|V_n|$, $n=1,2,\cdots,N$, are the latest values available from the PQ buses.

2. We then check for the limit of the reactive power sources:

$$Q_{G_k} = Q_k + Q_{L_k}, \quad Q_{G_{min}} \le Q_{G_k} \le Q_{G_{max}}$$

where Q_{L_k} is the local load connected at bus k, and Q_{G_k} is the reactive power generation of the var source (Generator, SVC, etc.).

(i) If the limit is not violated, then use (3) to compute

$$V_k = |V_k| \angle \delta_k$$

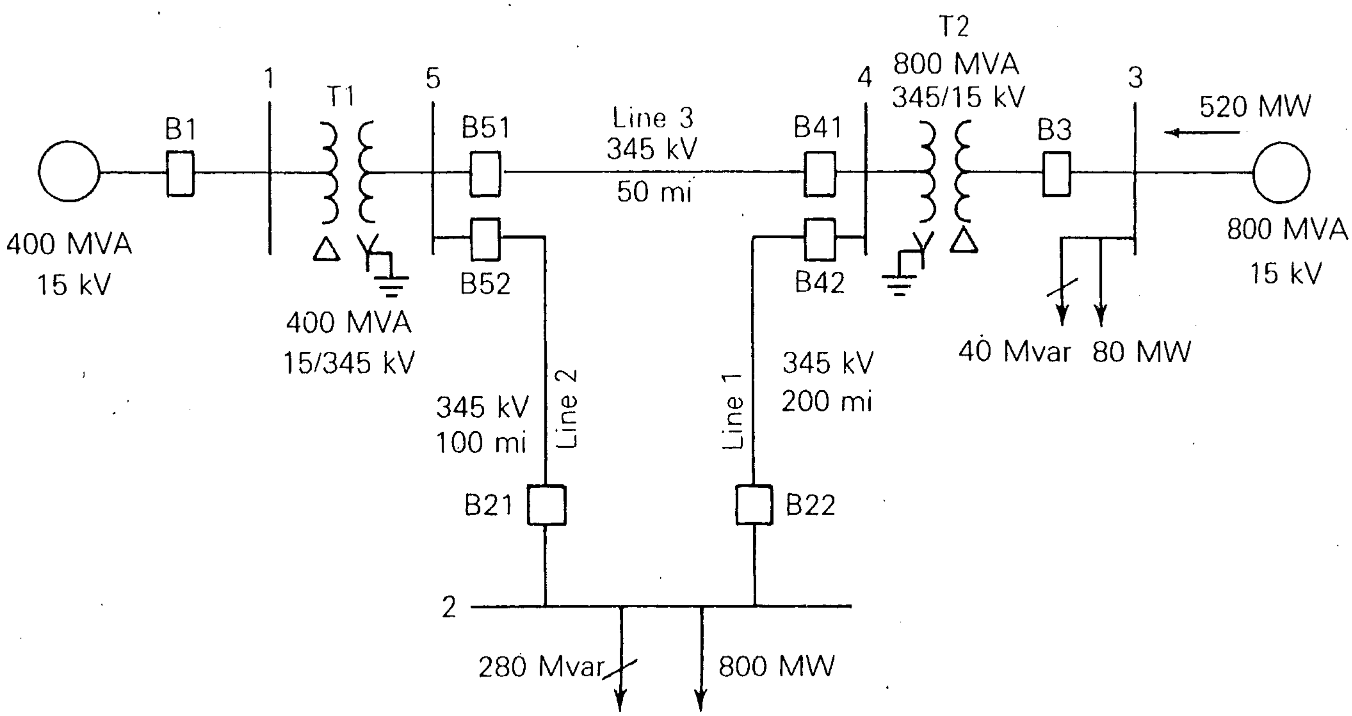
Keep only δ_k, and define $V_k \triangleq |V_k|_{specified} \angle \delta_k$

(ii) If the limit is violated, then treat as PQ bus, with the Q_k set to the limit violated, i.e., $Q_k \triangleq Q_{G\,min\,or\,max} - Q_L$.

c) <u>Swing bus:</u> V_1, δ_1 are specified.

After all PQ and PV bus power-flow calculations are converged, we compute P_1 and Q_1 at the last using the power equation.

<u>Example 6.9:</u>



1. Swing bus: Bus 1
2. PQ bus: Bus 2, 4, 5
3. PV bus: Bus 3

Input Data:

TABLE 6.1

Bus input data for Example 6.9*

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	−2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

* $S_{base} = 100$ MVA, $V_{base} = 15$ kV at buses 1, 3, and 345 kV at buses 2, 4, 5

TABLE 6.2

Line input data for Example 6.9

Bus-to-Bus	R′ per unit	X′ per unit	G′ per unit	B′ per unit	Maximum MVA per unit
2–4	0.0090	0.100	0	1.72	12.0
2–5	0.0045	0.050	0	0.88	12.0
4–5	0.00225	0.025	0	0.44	12.0

TABLE 6.3

Transformer input data for Example 6.9

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1–5	0.00150	0.02	0	0	6.0	—
3–4	0.00075	0.01	0	0	10.0	—

TABLE 6.4

Input data and unknowns for Example 6.9

Bus	Input Data	Unknowns
1	$V_1 = 1.0, \delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	Q_3, δ_3
4	$P_4 = 0, Q_4 = 0$	V_4, δ_4
5	$P_5 = 0, Q_5 = 0$	V_5, δ_5

The elements of $\boldsymbol{Y}_{\text{bus}}$ are computed from (6.4.2). Since buses 1 and 3 are not directly connected to bus 2,

$$Y_{21} = Y_{23} = 0$$

Using (6.4.2),

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \quad \text{per unit}$$

$$= 9.95972\underline{/95.143^\circ} \quad \text{per unit}$$

$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \quad \text{per unit}$$

$$= 19.9195\underline{/95.143^\circ} \quad \text{per unit}$$

$$Y_{22} = \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2}$$

$$= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2}$$

$$= 2.67828 - j28.4590 = 28.5847\underline{/-84.624^\circ} \quad \text{per unit}$$

$$V_2(1) = \frac{1}{Y_{22}}\left\{\frac{P_2 - jQ_2}{V_2^*(0)} - [Y_{21}V_1(1) + Y_{23}V_3(0) + Y_{24}V_4(0) + Y_{25}V_5(0)]\right\}$$

$$= \frac{1}{28.5847\underline{/-84.624^\circ}}\left\{\frac{-8 - j(-2.8)}{1.0\underline{/0^\circ}}\right.$$

$$\left. - [(-1.78552 + j19.83932)(1.0) + (-0.89276 + j9.91964)(1.0)]\right\}$$

$$= \frac{(-8 + j2.8) - (-2.67828 + j29.7589)}{28.5847\underline{/-84.624^\circ}}$$

$$= 0.96132\underline{/-16.543^\circ} \quad \text{per unit}$$

Next, the above value is used in (6.5.2) to recalculate $V_2(1)$:

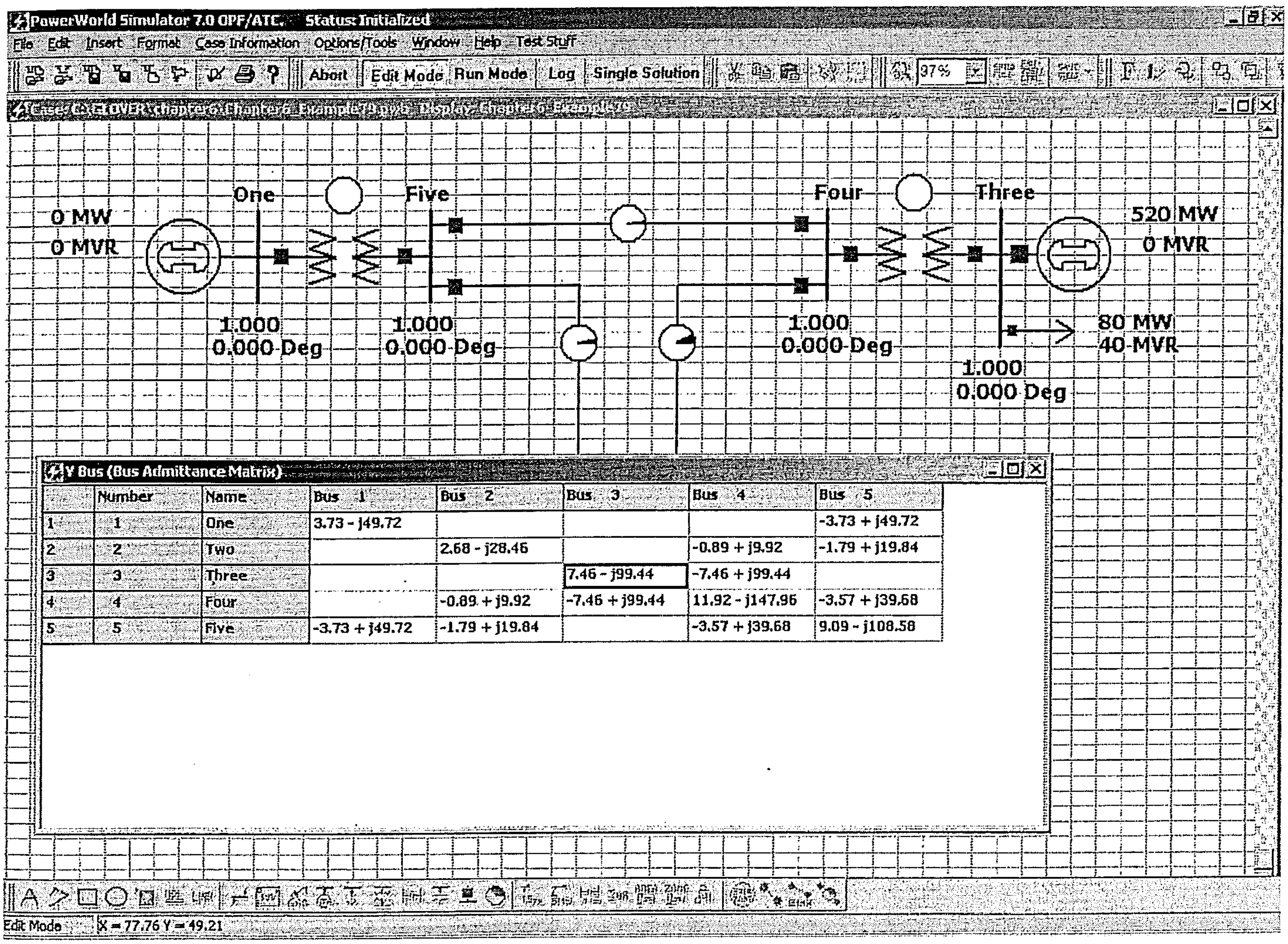
$$V_2(1) = \frac{1}{28.5847\underline{/-84.624^\circ}}\left\{\frac{-8 + j2.8}{0.96132\underline{/16.543^\circ}}\right.$$

$$\left. - [-2.67828 + j29.75829]\right\}$$

$$= \frac{-4.4698 - j24.5973}{28.5847\underline{/-84.624^\circ}} = 0.87460\underline{/-15.675^\circ} \quad \text{per unit}$$

Computations are next performed at buses 3, 4, and 5 to complete the first Gauss–Seidel iteration.

To see the complete convergence of this case, open PowerWorld Simulator case Example 6_10. By default, PowerWorld Simulator uses the Newton–Raphson method described in the next section. However, the case can be solved with the Gauss–Seidel approach by selecting **Simulation, Gauss–Seidel Power Flow**. To avoid getting stuck in an infinite loop if a case does not converge, PowerWorld Simulator places a limit on the maximum number of iterations. Usually for a Gauss–Seidel procedure this number is quite high, perhaps equal to 100 iterations. However, in this example to demonstrate the convergence characteristics of the Gauss–Seidel method it has been set to a single iteration, allowing the voltages to be viewed after each iteration. To step through the solution one iteration at a time, just repeatedly select **Simulation, Gauss–Seidel Power Flow**.



Screen for Example 6.9