5. Lossless lines

For a lossless line, $R = G = 0$, and

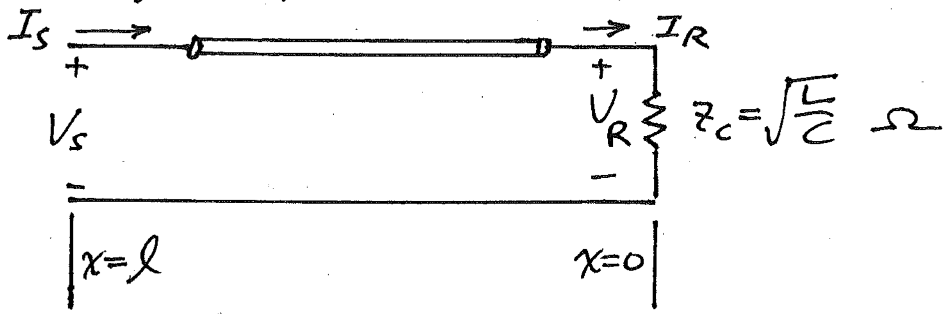
$$z = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \quad [\Omega/m]$$

$$\gamma = \sqrt{zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \quad [m^{-1}]$$

$$\cosh \gamma x = \cosh(j\beta x) = \cos\beta x$$

$$\sinh \gamma x = \sinh(j\beta x) = j\sin\beta x$$

Surge-Impedance Loading (SIL)



$$\begin{aligned}
V(x) &= \cosh\gamma x\, V_R + Z_c \sinh\gamma x\, I_R \\
&= \cos\beta x\, V_R + jZ_c \sin\beta x \frac{V_R}{Z_c} \\
&= (\cos\beta x + j\sin\beta x)\, V_R \\
&= e^{j\beta x}\, V_R
\end{aligned}$$

$\Rightarrow$ $|V(x)| = |V_R|$, flat voltage for all x!

Similary,

$$\begin{aligned}
I(x) &= \frac{1}{Z_c}\sinh\gamma x\, V_R + \cosh\gamma x\, I_R \\
&= \frac{j\sin\beta x}{Z_c} V_R + \cos\beta x \frac{V_R}{Z_c} \\
&= e^{j\beta x}\frac{V_R}{Z_c} = e^{j\beta x} I_R
\end{aligned}$$

$\Rightarrow$ $|I(x)| = |I_R|$, flat current for all x.

Complex power flow at x is

$$S(x) = P(x) + jQ(x) = V(x)\, I^*(x)$$

$$= \left(e^{j\beta x}\, V_R\right)\left(e^{j\beta x}\,\frac{V_R}{Z_c}\right)^*$$

$$= \frac{|V_R|^2}{Z_c}, \quad \text{real \& constant.}$$

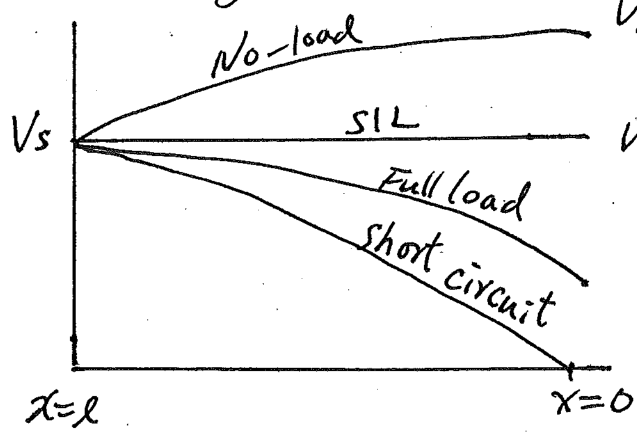
At rated voltage,

$$SIL = \frac{V_{rated}^2}{Z_c}, \quad V_{rated}: \text{line-to-line for } 3\phi$$

Since for 3ϕ, $I_L = \dfrac{|V_L|}{\sqrt{3}\, Z_c}$

$$SIL = \sqrt{3}\,|V_L|\,|I_L| = \frac{|V_L|^2}{Z_c}, \quad \text{same as in single phase.}$$

Voltage Profiles:



$V_{R_{NL}} = \dfrac{V_s}{\cos\beta l}$

$V_{R_{SIL}} = V_s$

$V_{R_{FL}}$

$V_{R_{SC}} = 0$

1. At no load, $I_{R_{NL}} = 0 \Rightarrow V_{NL}(x) = (\cos\beta x)\, V_{R_{NL}}$

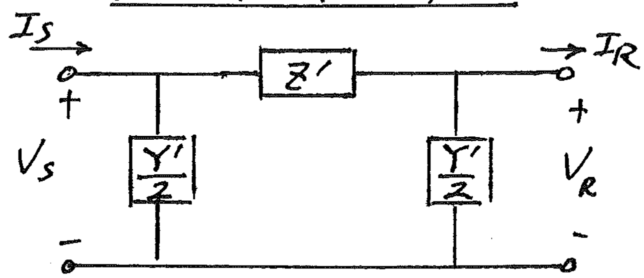
   $V_s = (\cos\beta l)\, V_{R_{NL}}$

2. SIL, flat voltage

3. $V_{R_{SC}} = 0 \Rightarrow V_{SC}(x) = jZ_c \sin\beta x\, I_{R_{SC}}$

   $|V_s| = |\sin\beta l\, (Z_c I_{R_{SC}})|$

6. Power Transfer



For general lossy lines,

$A = \cosh \gamma l = |A| \angle \theta_A$

$B = Z' = |B| \angle \theta_B$

$V_S = |V_S| \angle \delta, \quad V_R = |V_R| \angle 0^\circ$

$$V_S = A\, V_R + B\, I_R$$

Solving for the current,

$$I_R = \frac{V_S - A V_R}{B} = \frac{|V_S|}{|B|} \angle \delta - \theta_B - \frac{|A||V_R|}{|B|} \angle \theta_A - \theta_B$$

Complex power delivered,

$$S_R = V_R I_R^* = P_R + j Q_R$$

$$= \frac{|V_R||V_S|}{|B|} \angle \theta_B - \delta - \frac{|A||V_R|^2}{|B|} \angle \theta_B - \theta_A$$

$\Rightarrow$

$$P_R = \frac{|V_R||V_S|}{|B|} \cos(\theta_B - \delta) - \frac{|A||V_R|^2}{|B|} \cos(\theta_B - \theta_A)$$

$$Q_R = \frac{|V_R||V_S|}{|B|} \sin(\theta_B - \delta) - \frac{|A||V_R|^2}{|B|} \sin(\theta_B - \theta_A)$$

<u>Maximum Real Power</u> (Steady-State Stability Limit) is at $\theta_B = \delta$,

$$P_{R\,max} = \frac{|V_R||V_S|}{|B|} - \frac{|A||V_R|^2}{|B|} \cos(\theta_B - \theta_A)$$

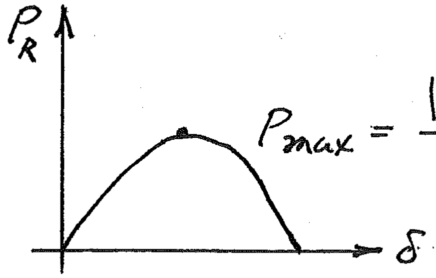
Lossless line: $\theta_A = 0^\circ$, $B = Z' = jX'$, $\theta_B = 90^\circ$,

$$P_R = \frac{|V_R||V_S|}{X'} \cos(90^\circ - \delta) - \frac{|A||V_R|^2}{X'} \cos(90^\circ)$$

$$= \frac{|V_R||V_S|}{X'} \sin\delta = P_S \quad \text{since line is lossless.}$$

$$Q_R = \frac{|V_R||V_S|}{X'} \sin(90^\circ - \delta) - \frac{|A||V_R|^2}{X'} \sin(90^\circ)$$

$$= \frac{|V_R|}{X'} \left[|V_S| \cos\delta - |A||V_R| \right]$$



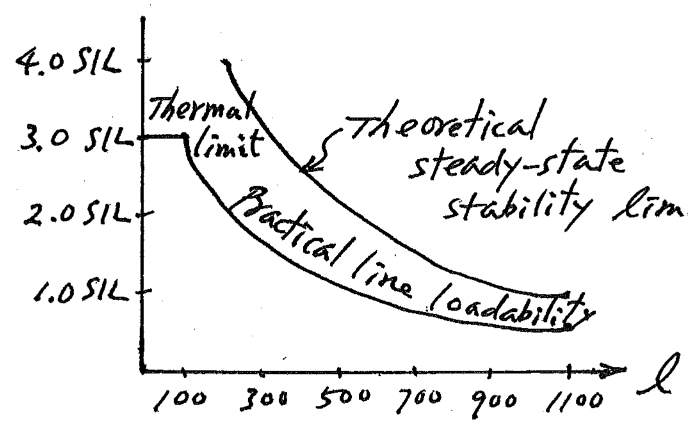
$$P_{max} = \frac{|V_S||V_R|}{X'}$$, theoretical steady-state stability limit

NOTE: $$P_{max} = \frac{V_R V_S}{X'}$$

$$\propto \frac{V^2}{l}$$, increases with square of voltage, decreases with line length.

$$P = \frac{V_S V_R \sin\delta}{X'} = \frac{V_S V_R \sin\delta}{Z_c \sin\beta l} = \frac{V_S}{V_{rated}} \frac{V_R}{V_{rated}} \frac{V_{rated}^2}{Z_c} \frac{\sin\delta}{\sin\frac{2\pi l}{\lambda}}$$

$$= V_{S_{pu}} V_{R_{pu}} (SIL) \frac{\sin\delta}{\sin\frac{2\pi l}{\lambda}} \qquad \lambda = \frac{2\pi}{\beta} \simeq 5000 \text{ km}$$



Practical Line Loadability

Voltage drop limit: $\frac{V_R}{V_S} \geq 0.95$

δ: max. angular displacement of $30 \sim 35^\circ$ across the line or, 45° across line and equiv. system reactances

Thermal limit: short lines less than 80 km.

<u>Example 5.4:</u>

Neglect line losses, find the theoretical steady-state stability limit for a long line in Example 5.2:

765 kV, 60 Hz, 300 km

$z = 0.0165 + j0.3306 = 0.3310 \angle 87.14^\circ \ \Omega/\text{km}$

$y = j\,4.674 \times 10^{-6}$ S/km

$$Z_c = 266.1 \angle -1.43^\circ \ \Omega \simeq 266.1$$

$$\gamma = 0.001244 \angle 88.57^\circ \simeq j\beta$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.001244} = 5050 \simeq 5000 \text{ km}$$

$$SIL = \frac{(765)^2}{266.1} = 2199 \text{ MW}$$

$$P_{max} = \frac{(1)(1)(2199)}{\sin\left(\frac{2\pi \times 300}{5000}\right)} = (2.716)(2199) = 5974 \text{ MW}$$

<u>Note</u>: In Fig. 5.12, the t-s-s stability limit is, for 300 km,

$(2.72)\ SIL = (2.72)(2199) = 5980$ MW

Example 5.5: Theoretical maximum power for the long line in Example 5.2:

$A = \cosh \gamma l = 0.9313 \angle 0.209^\circ$

$B = Z_c \sinh \gamma l = (266.1 \angle -1.43^\circ)(0.3645 \angle 88.63^\circ)$

$= 97.0 \angle 87.2^\circ$

$C = \dfrac{\sinh \gamma l}{Z_c} = \dfrac{0.3645 \angle 88.63^\circ}{266.1 \angle -1.43^\circ} = 1.37 \times 10^{-3} \angle 90.06^\circ$

$|A| = 0.9313, \quad \theta_A = 0.209^\circ$

$|B| = Z' = 97.0, \quad \theta_B = 87.2^\circ$

$Z_c = 266.1$

$$P_{R\,max} = \frac{|V_R|\,|V_S|}{|B|} - \frac{|A|\,|V_R|^2}{|B|} \cos(\theta_B - \theta_A)$$

$$= \frac{(765)^2}{97} - \frac{(0.9313)(765)^2}{97} \cos(87.2^\circ - 0.209^\circ)$$

$= 6033 - 295 = \underline{\underline{5738 \text{ MW}}} < 5974$ by 4%

$= \dfrac{5738}{2199} = 2.61$ pu

or (2.61) SIL

Example 5.6: Practical line loadability & voltage regulation

The 300-km line in Example 5.2, 4×1,272,000-cmil 54/3

Sending-end voltage is held constant at 1.0 pu.

Determine the following:

a. The practical line loadability. Assume $V_R = 0.95$ pu and $\delta = 35^\circ$ maximum across the line.

b. The full-load current at 0.986 p.f. leading

c. The exact receiving-end voltage for the full-load current found in part (b).

d. Percent voltage regulation

e. Thermal limit of the line.

Solution:

a. For practical (lossy) line the power delivered to receiving end: From (5.5.3),

$$P_R = \frac{V_R V_S}{Z'} \cos(\theta_Z - \delta) - \frac{A V_R^2}{Z'} \cos(\theta_z - \theta_A)$$

$V_R = 765$ kV, $V_R = 0.95 \times 765$ kV, $\delta = 35^\circ$;

From Example 5.5:

$|A| = 0.9313, \quad \theta_A = 0.209^\circ$

$|B| = Z' = 97.0, \quad \theta_B = 87.2^\circ = \theta_Z$

$$P_R = \frac{(765)(0.95 \times 765)}{97.0} \cos(87.2^\circ - 35^\circ) - \frac{(0.9313)(0.95 \times 765)^2}{97.0} \cos(87.2^\circ - 0.209^\circ)$$

$= 3513 - 266 = 3247$ MW

$P_R = 3247$ MW is the practical line loadability, provided the thermal and voltage-drop limits are not exceeded. Note, from Figure 5.12 for a 300-km line, the practical line loadability is $(1.49)SIL = (1.49)(2199) = 3277$ MW, about the same.

b. The full-load receiving-end current:

$$I_{R_{FL}} = \frac{P}{\sqrt{3}\, V_R\, (pf)} = \frac{3247}{\sqrt{3}\,(0.95\times 765)(0.986)} = 2.616 \text{ kA}$$

c. To find the receiving-end voltage,

$$V_S = A\, V_{RFL} + B\, I_{RFL}, \quad I_{R_{FL}} = 2.616\angle \cos^{-1} 0.986$$

$$\frac{765}{\sqrt{3}}\angle\delta = (0.9313\angle 0.209^\circ)\, V_{R_{FL}}\angle 0^\circ + (97.0\angle 87.2^\circ)(2.616\angle 9.599^\circ)$$

$$441.7\angle\delta = (0.9313\, V_{R_{FL}} - 30.04) + j\,(0.0034\, V_{R_{FL}} + 251.97)$$

$$(441.7)^2 = (0.9313\, V_{R_{FL}} - 30.04)^2 + (0.0034\, V_{R_{FL}} + 251.97)^2$$

$$= 0.8673\, V_{R_{FL}}^2 - 54.24\, V_{R_{FL}} + 64{,}391$$

$$\Rightarrow \quad V_{R_{FL}} = 420.7 \text{ kV}_{LN}$$

$$= 420.7\sqrt{3} = 728.7 \text{ kV}_{LL} = 0.953 \text{ p.u.}$$

d. $$\% VR = \frac{|V_{R_{NL}}| - |V_{R_{FL}}|}{|V_{R_{FL}}|} \times 100$$

$$V_S = A\, V_R + B\, I_R$$

$$\Rightarrow \quad V_{R_{NL}} = \frac{V_S}{A} = \frac{765}{0.9313} = 821.4 \text{ kV}_{LL}$$

$$\% VR = \frac{821.4 - 728.7}{728.7} \times 100 = 12.72\ \%$$

e. From Table A.4, the approximate current-carrying capacity of four 1,272,000-cmil 54/3 ACSR conductors is $4 \times 1.2 = 4.8$ kA.

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Cc

Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (inches)	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)
				Strand Diameter (inches)		Strand Diameter (inches)						
Joree	2 515 000	76	...	0.1819	19	0.0849	1.880		61 700		0.0621	
Thrasher	2 312 000	76	...	0.1744	19	0.0814	1.802		57 300		0.0595	
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570	
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588	
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534	
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160

<u>NOTES</u>:

1. $V_S = 1.0$, $V_{RFL} = 0.953$ satisfies the voltage-drop limit $V_R / V_S \geq 0.95$.
2. Full-load current 2.616 kA is well below the thermal limit of 4.8 kA.
3. Thus, the steady-state stability is the factor that limits the line loadability.
4. The 12% voltage regulation is too high and compensation technique is needed to reduce V_{RNL}.

Example 5.7: Selection of Line Voltage & Number of Lines

From a plant 9000 MW to transfer over 500 km. Determine number of 3ϕ lines, with one line out of service for reliability.

a) 345 kV $\quad Z_c = 297\ \Omega$

b) 500 kV $\quad Z_c = 277\ \Omega$

c) 765 kV $\quad Z_c = 266\ \Omega$

Assume $\quad V_s = 1.0$ pu, $V_R = 0.95$ pu, $\delta = 35^\circ$

Solution:

a) $$SIL = \frac{(345)^2}{297} = 401 \text{ MW}$$

$$P = \frac{(1.0)(0.95)(401)\sin 35^\circ}{\sin\left(\frac{2\pi \times 500}{5000}\right)} = (401)(0.927) = 372 \text{ MW/line}$$

# of lines with one line out of service:

$$\frac{9000}{372} + 1 = 24.2 + 1 \approx 26 \text{ lines}$$

b) $$SIL = \frac{(500)^2}{277} = 903 \text{ MW}$$

$$P = (903)(0.927) = 837 \text{ MW/line}$$

$$\# \text{ of lines} = \frac{9000}{837} + 1 = 10.8 + 1 \simeq 12 \text{ lines}$$

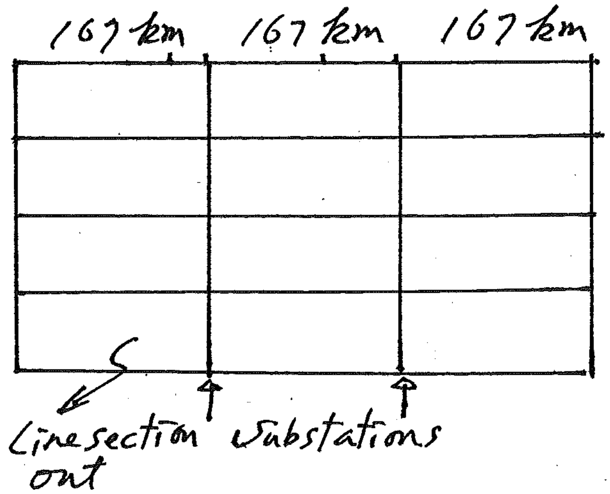
c) $$SIL = \frac{(765)^2}{266} = 2200 \text{ MW}$$

$$P = (2200)(0.927) = 2039 \text{ MW/line}$$

$$\# \text{ of lines} = \frac{9000}{2039} + 1 = 4.4 + 1 \simeq 6 \text{ lines}$$

Example 5.8: Intermediate Substation

Divide the lines into three sections with substations. Can we reduce the number of lines into 5?



500 km 765 kV line has

$$X' = (266)\sin\frac{2\pi \times 500}{5000}$$
$$= 156.35\ \Omega$$

With one section out of serve for one line, the equivalent reactance of the parallel lines:

$$X_{eq} = \frac{1}{5}\left(\frac{2}{3}X'\right) + \frac{1}{4}\left(\frac{X'}{3}\right) = 0.216X'$$
$$= 33.88\ \Omega$$

Then, with $\delta = 35°$,

$$P = \frac{(765)(765 \times 0.95)\sin 35°}{33.88} = \underline{9412\ \text{MW}}$$

Discounting the loss of 3~4%,

$P \simeq 9100$ MW, which is eneough for 9000 MW transfer.

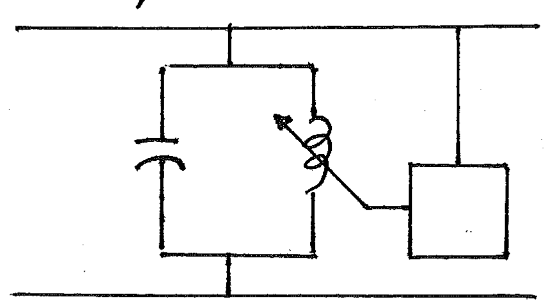
7. Reactive Compensation

Inductors and capacitors are used to increase line loadability and to maintain voltages near rated values.

Shunt reactors (inductors), used in EHV lines
- Reduce overvoltage during light load
- Reduce transient overvoltage due to switching
- Need to be removed during full load

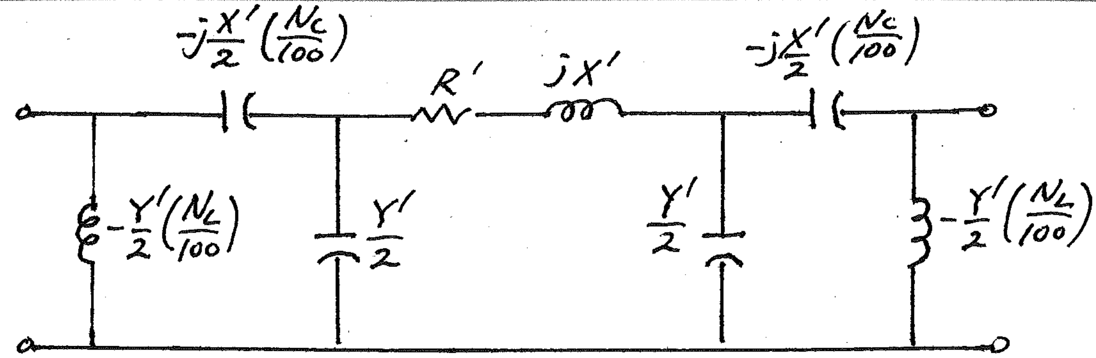
Shunt capacitors
- Increase voltages during heavy load

Static Var Compensators (SVC)
- Thyristor-switched reactors with parallel capacitors



Synchronous condensors
- Synchronous motor with no load

Series capacitors
- Reduce net series impedance - reduce line-voltage drop, increase the steady-state stability limit
- subsynchronous resonance - damage turbine-generator shafts



N_C : % series capacitor compensation
N_L : % shunt reactor compensation

Ex 5.9 300 km line in Ex 5.2

Use shunt reactors to provide 75% compensation during light load conditions.

Reactors are removed during heavy load:

Full load is 1.90 kA at unity pf, 730 kV.

Find: a. % voltage regulation before compensation

b. Equivalent shunt admittance & series impedance of the compensated line

c. % voltage regulation after compensation

(Ex 5.2 lines): 765 kV 300 km

$$z = 0.0165 + j0.3306 = 0.3310 \angle 87.14^\circ \ \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \ \text{S/km}$$

$$Z_c = \sqrt{\frac{z}{y}} = 266.1 \angle -1.43^\circ \ \Omega$$

$$\gamma l = \sqrt{yz}\, l = 0.00931 + j0.3730 \ \text{pu}$$

$$A = D = \cosh(\gamma l) = 0.9313 \angle 0.209^\circ \ \text{pu}$$

$$B = 97.0 \angle 87.2^\circ \ \Omega$$

$$C = 1.37 \times 10^{-3} \angle 90.06^\circ \ \text{S}$$

a) % voltage regulation

$$V_S = A\ V_{R_{FL}} + B\ I_{R_{FL}}$$
$$= A\left(\frac{730}{\sqrt{3}}\angle 0^\circ\right) + B\ (1.9\angle 0^\circ)$$
$$= 442.3\ \angle 24.8^\circ \quad kV_{LN}$$

$$V_S = \sqrt{3}\ (442.3) = 766.0 \quad kV_{LL}$$

$$V_{R_{NL}} = \frac{V_S}{A} = \frac{766}{0.9313} = 822.6 \quad kV_{LL}$$

$$\therefore \%\ VR = \frac{822.6 - 730}{730} \times 100 = \underline{12.68\%}$$

b) Compensation: 75% during light load.

$$\frac{Y'}{2} = \frac{Y}{2}\ \frac{\tanh \frac{\gamma l}{2}}{\frac{\gamma l}{2}} = (7.011\times 10^{-4}\angle 90^\circ)(1.012\angle -0.03^\circ)$$

(from Ex. 5.3)

$$= 3.7\times 10^{-7} + j\,7.094\times 10^{-4} \quad S$$

or $Y' = 7.4\times 10^{-7} + j\,14.188\times 10^{-4} \quad S$

With 75% shunt compensation,

$$Y_{eq} = 7.4\times 10^{-7} + j\,14.188\times 10^{-4}\left(1 - \frac{75}{100}\right)$$
$$= 3.547\times 10^{-4}\ \angle 89.88^\circ \quad S$$

$$Z_{eq} = Z' = Z\ \frac{\sinh \gamma l}{\gamma l} = (99.3\angle 87.14^\circ)(0.9769\angle 0.06^\circ)$$

(from Ex. 5.3)

$$= 97.0\ \angle 87.2^\circ \quad \Omega$$

c) Equivalent circuit after compensation:

$$A_{eq} = 1 + \frac{Y_{eq} Z_{eq}}{2} = 1 + \frac{(3.547\times10^{-4})(97.0\angle 87.2^\circ)}{2}$$

$$= 1 + 0.0172\angle 177.1^\circ$$

$$= 0.9828\angle 0.05^\circ \quad pu$$

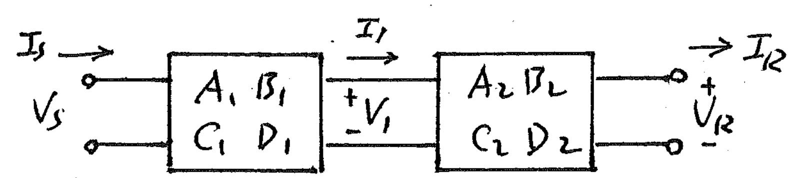
Then,

$$V_{R_{NL}} = \frac{V_S}{A_{eq}} = \frac{766}{0.9828} = 779.4 \ kV_{LL}$$

$V_{R_{FL}} = 730 \ kV_{LL}$, shunt reactors are removed during heavy load

$$\therefore \% VR = \frac{779.4 - 730}{730} \times 100 = \underline{6.77\%}$$

Series (Cascaded) Networks:



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} V_1 \\ I_1 \end{bmatrix}, \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \left[\begin{array}{c|c} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{array}\right]\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

EX. 5.10 Series compensation with 30%

Find theoretical maximum power for the compensated line, $V_S = V_R = 765$ kV

From EX 5.3, $Z' = 97.0 \angle 87.2^\circ \ \Omega$

$\Rightarrow \quad X' = 97 \sin(87.2^\circ) = 96.88 \ \Omega$

30% compensation:

$$Z_{cap} = -jX_{cap} = -j\frac{1}{2}(0.3)(96.88) = -j14.53$$

Series networks:

$$\begin{bmatrix} 1 & -j14.53 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.9313\angle 0.209^\circ & 97.0\angle 87.2^\circ \\ 1.37\times10^{-3}\angle 90.06^\circ & 0.9313\angle 0.209^\circ \end{bmatrix} \begin{bmatrix} 1 & -j14.53 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} 0.9512\angle 0.205^\circ & 69.70\angle 86.02^\circ \\ 1.37\times10^{-3}\angle 90.06^\circ & 0.9512\angle 0.205^\circ \end{bmatrix}$$

$$P_{max} = \frac{V_R V_S}{Z'} - \frac{A V_R^2}{Z'}\cos(\theta_B - \theta_A)$$

$$= \frac{765^2}{69.70} - \frac{(0.9512)(765)^2}{69.70}\cos(86.02^\circ - 0.205^\circ)$$

$$= 8396 - 583 = \underline{7813} \text{ MW}$$

: 36.2% larger than 5738 MW (HW 5.5)