Transmission Lines

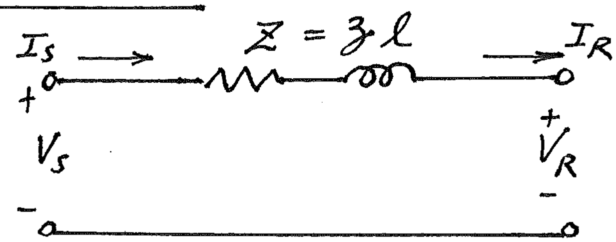
Equivalent Circuits:

The short line approximation: for less than 80 km (50 miles)

The Medium Line approximation: for 80 ~ 250 km (150 miles)

The Long Line approximation: for longer than 250 km

1. Short line: < 80 km (50 miles)



$z = R + j\omega L$ Ω/m

$Z = z l$

l: length in [m]

V_S: line-to-neutral voltage.

Relation between the sending-end and the receiving-end variables:

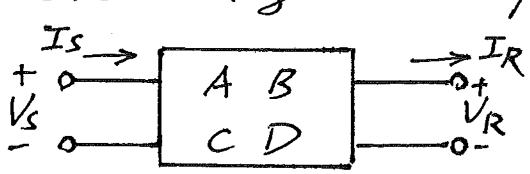
$$V_S = V_R + Z I_R$$

$$I_S = I_R$$

In a matrix form,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Generalized two-port network:



$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

NOTE: For linear, passive, bilateral network,

$$AD - BC = 1$$

For symmetric network,

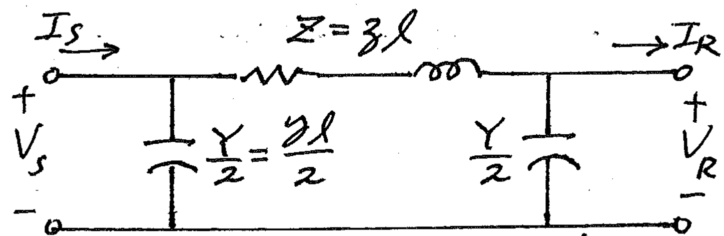
$$A = D$$

Inverse relationship:

Since the determinant of the (A, B, C, D) matrix is $AD - BC = 1$, we have

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

2. <u>Medium-length Line:</u> 80 ~ 250 km (50 ~ 150 miles)



nominal π-circuit

Shunt admittance:

$y = G + j\omega C$ S/m

$Y = y\ell$

$$V_S = V_R + Z\left(I_R + \frac{Y}{2}V_R\right) = \left(1 + \frac{ZY}{2}\right)V_R + ZI_R$$

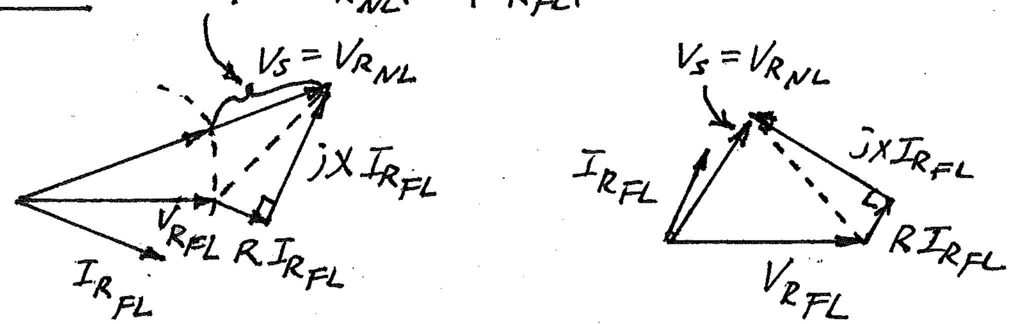
$$\begin{aligned} I_S &= I_R + \frac{Y}{2}V_R + \frac{Y}{2}V_S \\ &= I_R + \frac{Y}{2}V_R + \frac{Y}{2}\left(1 + \frac{ZY}{2}\right)V_R + \frac{Y}{2}ZI_R \\ &= Y\left(1 + \frac{ZY}{4}\right)V_R + \left(1 + \frac{ZY}{2}\right)I_R \end{aligned}$$

$$\therefore \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{ZY}{2} & Z \\ Y\left(1 + \frac{ZY}{4}\right) & 1 + \frac{ZY}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

3. Voltage Regulation:

$$\% VR = \frac{|V_{R_{NL}}| - |V_{R_{FL}}|}{|V_{R_{FL}}|} \times 100$$

Short line: $\Delta V = |V_{R_{NL}}| - |V_{R_{FL}}|$



(a) lagging p.f.
% VR is positive

(b) leading p.f.
% VR is negative

Medium line:

$$V_S = A V_R + B I_R, \quad I_R = 0 \text{ for No Load}$$

$$\Rightarrow \quad V_S = A V_{R_{NL}}$$

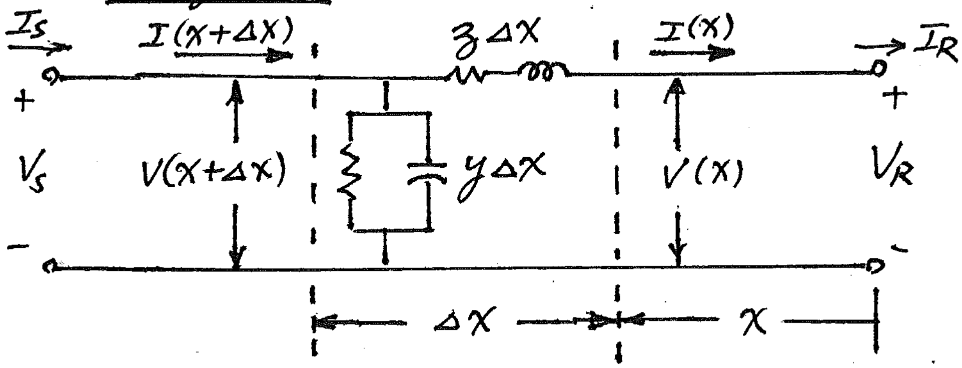
$$\text{or } V_{R_{NL}} = \frac{V_S}{A} \quad (= V_S \text{ for short line})$$

For EHV lines $\pm 5\%$ of rated voltage, i.e., 10 % VR is acceptable.

Line loadability is determined by

- Thermal limit: for short line
- Voltage-drop limit: for longer line upto 300 km
- Steady-state stability limit: over 300 km

4. Long line: > 250 km (150 miles)



z: series impedance Ω/m

y: shunt admittance S/m

For a small section of length Δx,

$$V(x+\Delta x) = V(x) + (z\Delta x)\, I(x)$$
$$I(x+\Delta x) = I(x) + (y\Delta x)\, V(x)$$

$\Rightarrow$

$$\Delta V = V(x+\Delta x) - V(x) = (z\Delta x)\, I(x)$$
$$\Delta I = I(x+\Delta x) - I(x) = (y\Delta x)\, V(x)$$

As $\Delta x \to dx$,

$$dV = I\, z\, dx \qquad\qquad dI = V\, y\, dy$$

or $$\frac{dV}{dx} = z\, I(x) \qquad\qquad \frac{dI}{dx} = y\, V(x) \qquad (1)$$

Differentiating with respect to x,

$$\frac{d^2V}{dx^2} = z\,\frac{dI}{dx} \qquad\qquad \frac{d^2I}{dx^2} = y\,\frac{dV}{dx}$$

or $$\frac{d^2V}{dx^2} = zy\, V(x) \qquad\qquad \frac{d^2I}{dx^2} = zy\, I(x)$$

: These are wave equations

Solution is in the form of

$$V(x) = A\, e^{\sqrt{zy}\, x} + B\, e^{-\sqrt{zy}\, x}$$

Note: $\frac{dV}{dx} = \sqrt{zy}\left(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x}\right)$ (2)

$$\frac{d^2V}{dx^2} = zy\left(A\,e^{\sqrt{zy}\,x} + B\,e^{-\sqrt{zy}\,x}\right)$$

$= zy\,V(x)$, thus satisfying the wave equation.

Since $V(x)$ and $I(x)$ are coupled by (1), with (2),

$$I(x) = \frac{1}{z}\frac{dV}{dx}$$

$$= \frac{1}{\sqrt{\frac{z}{y}}}\left(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x}\right)$$

Define line parameters:

$\gamma = \sqrt{zy}$ $[m^{-1}]$, propagation constant

$= \alpha + j\beta$, α = attenuation constant

β = phase constant

$Z_c = \sqrt{\frac{z}{y}}$ $[\Omega]$, characteristic impedance

Then, the voltage and current at x are:

$$V(x) = A\,e^{\gamma x} + B\,e^{-\gamma x}$$ (3)

$$I(x) = \frac{1}{Z_c}\left(A\,e^{\gamma x} - B\,e^{-\gamma x}\right)$$

To find constants A and B, apply boundary conditions at $x=0$:

$$V_R = V(0) = A + B$$

$$I_R = \frac{1}{Z_c}(A - B)$$

Solving for A & B,

$$A = \frac{V_R + Z_c I_R}{2} \qquad B = \frac{V_R - Z_c I_R}{2}$$

Substituting these in (3),

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2} e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2} e^{-\gamma x}}_{\text{reflected wave}} \qquad (4)$$

<u>NOTE</u>: If $I_R = 0$ (open circuit, no load)

$$V(0)_{\text{incident}} = V(0)_{\text{reflected}} = \frac{V_R}{2}$$

$$I(0)_{\text{incident}} = -I(0)_{\text{reflected}} = -\frac{V_R}{2Z_c}$$

$$\Rightarrow \quad V(0) = \frac{V_R}{2} + \frac{V_R}{2} = V_R$$

$$I(0) = \frac{V_R}{2Z_c} - \frac{V_R}{2Z_c} = 0$$

NOTE: If the line is terminated with characteristic impedance, Z_c, then

$$V_R = Z_c I_R$$

$\Rightarrow$ There is no reflected waves, which is called "Impedance Matching."

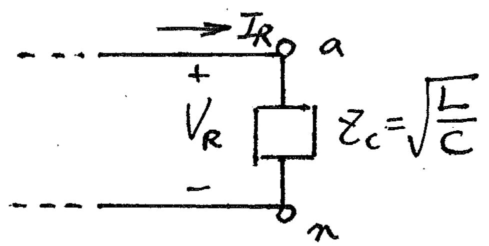
Impedance matching is common in communication systems. In power systems

$$Z_c = \sim 400\,\Omega \angle \sim 15^\circ \quad \text{for single circuit}$$
$$\sim 200\,\Omega \angle \sim 15^\circ \quad \text{for parallel circuit}$$

For loss less line (series resistance and shunt conductance are zero),

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}, \quad \text{surge impedance}$$

Surge Impedance Loading (SIL)



$$V_R = \frac{V_L}{\sqrt{3}}$$

$$I_R = \frac{V_R}{Z_c} = \frac{V_L}{\sqrt{3}\, Z_c}$$

$$SIL = \sqrt{3}\, V_L I_R = \frac{V_L^2}{Z_c}$$

$$= \frac{V_{\text{rated, line-to-line}}^2}{Z_c}$$

Wave Propagation:

$V(x)$ and $I(x)$ propagate (4) withe the propagation constant

$$\gamma = \sqrt{zy} = \alpha + j\beta$$

For loss-less line, $\alpha = 0$ and

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \quad [m^{-1}]$$

β: phase shift in rad/m.

A <u>wave length</u> is the distance required to change the phase of $V(x)$ or $I(x)$ by 2π radians or $360°$, i.e.,

$$\lambda\beta = 2\pi,$$

$\Rightarrow$ Wave length, $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\omega\sqrt{LC}} = \dfrac{1}{f\sqrt{LC}}$ [m]

$$= \frac{3\times10^8}{60} = 5\times10^6\,m = 5000\,km$$

$$= 3100\,mi$$

Velocity of propagation:

$$v = f\cdot\lambda = \frac{1}{\sqrt{LC}} \quad [m/s]$$

$$= \frac{1}{\sqrt{\dfrac{\mu_0}{2\pi}\ln\dfrac{D}{r}\cdot\dfrac{2\pi\varepsilon_0}{\ln\dfrac{D}{r}}}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \simeq 3\times10^8\,m/s$$

: speed of light in free space.

For cables, $\dfrac{\varepsilon}{\varepsilon_0} \simeq 3\sim5$, implying that v is lower than that for overhead lines.

Long Line Model in Hyperbolic Form:

Recall that

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}, \qquad \cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\left[\ \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}, \qquad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\ \right]$$

$\Rightarrow$

$$\sinh\theta + \cosh\theta = e^{\theta}, \quad \cosh\theta - \sinh\theta = e^{-\theta}$$

Thus, from (4),

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$= \frac{V_R + Z_c I_R}{2}(\sinh\gamma x + \cosh\gamma x) + \frac{V_R - Z_c I_R}{2}(\cosh\gamma x - \sinh\gamma x)$$

$$= \cosh(\gamma x)\, V_R + Z_c \sinh(\alpha x)\, I_R$$

$$I(x) = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

$$= \frac{V_R/Z_c + I_R}{2}(\sinh\gamma x + \cosh\gamma x) - \frac{V_R/Z_c - I_R}{2}(\cosh\gamma x - \sinh\gamma x)$$

$$= \frac{\sinh(\gamma x)}{Z_c} V_R + \cosh(\gamma x)\, I_R$$

$$\therefore \quad \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \\ C(x) & D(x) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where

$$A(x) = D(x) = \cosh(\gamma x) \quad [pu]$$

$$B(x) = Z_c \sinh(\gamma x) \quad [\Omega]$$

$$C(x) = \frac{1}{Z_c}\sinh(\gamma x) \quad [S]$$

Finally, at $x = l$, $V(l) = V_s$, $I(l) = I_s$, and

$$A = D = \cosh(\gamma l) \quad [pu]$$

$$B = Z_c \sinh(\gamma l) \quad [\Omega]$$

$$C = \frac{1}{Z_c}\sinh(\gamma l) \quad [S]$$

Computation of Hyperbolic Functions:

Recall that

$$\gamma = \sqrt{zy} = \alpha + j\beta \quad [m^{-1}], \text{ complex}$$

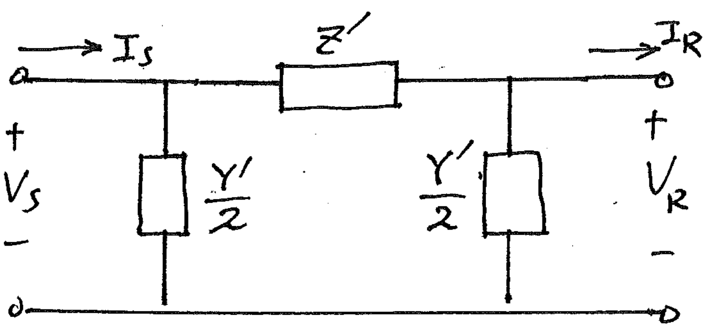
Thus, $$e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l} e^{j\beta l} = e^{\alpha l} \angle \beta l$$

Then,

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\angle\beta l + e^{-\alpha l}\angle{-\beta l}\right)$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\gamma l}\angle\beta l - e^{-\alpha l}\angle{-\beta l}\right)$$

Equivalent π - Circuit:



Recall in the nominal π-equivalent circuit

$$A = D = 1 + \frac{Y'Z'}{2}$$

$$B = Z'$$

$$C = Y'\left(1 + \frac{Y'Z'}{4}\right)$$

Equating these with (A, B, C, D) parameters in the long-line representation,

$$A: \quad 1 + \frac{Y'Z'}{2} = \cosh \gamma l$$

$$B: \quad Z' = Z_c \sinh \gamma l$$

$$C: \quad Y'\left(1 + \frac{Y'Z'}{4}\right) = \frac{\sinh \gamma l}{Z_c}$$

Solving B for Z',

$$Z' = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{\sqrt{zy}\, l} = Z \underbrace{\frac{\sinh \gamma l}{\gamma l}}_{F_1,\ \text{correction factor}} = Z F_1$$

where $Z = zl$, total series impedance,

Solving A for Y',

$$\frac{Y'}{2} = \frac{1}{Z_c} \frac{\cosh \gamma l - 1}{\sinh \gamma l} = \sqrt{\frac{y}{z}} \tanh \frac{\gamma l}{2}$$

$$= \frac{yl}{2} \frac{\tanh \frac{\gamma l}{2}}{\sqrt{zy}\, l/2} = \frac{Y}{2} \underbrace{\frac{\tanh \frac{\gamma l}{2}}{\gamma l/2}}_{F_2}$$

$$= \frac{Y}{2} F_2$$

where $Y = yl$, total shunt admittance.

<u>NOTE</u>: For small γl, $Z' = Z$, $Y' = Y$.