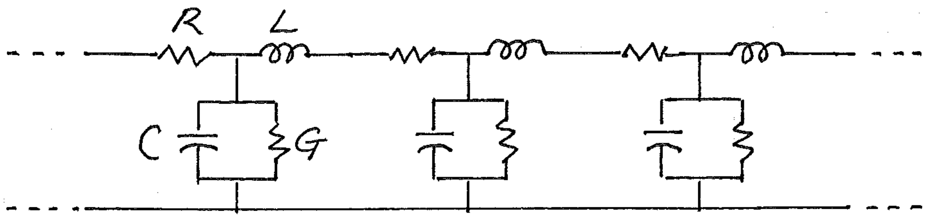
Transmission Lines

Line Parameters



Distributed parameter representation

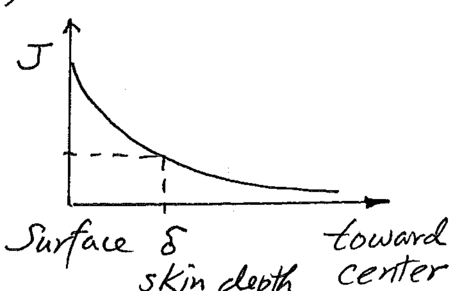
There are four line parameters, which are, in the order of importance,

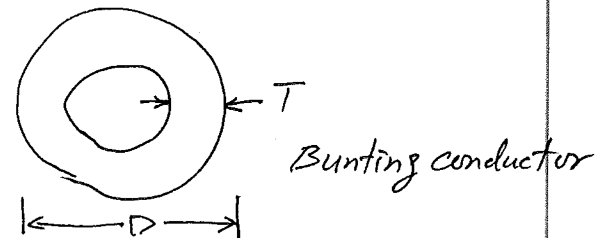
- L: Series inductance H/m
- C: Shunt capacitance F/m
- R: Series resistance Ω/m
- G: Shunt conductance S/m

1. Resistance R: depends on the length of the conductor ℓ, the resistivity of the material ρ (that increases with temperature), and inversely on the effective cross-sectional area A of the conductor:

$$R = \frac{\rho \ell}{A}$$

The effective area A depends on the frequency due to the skin effect, where the current at 60-Hz frequency is not uniformly distributed throughout the cross-section; rather it crowds towards the periphery of the conductor with a higher current-density J for a solid conductor. Therefore, in ACSR conductors, the aluminum thickness T, is kept at the order of the skin depth.





Current-density J decreases exponentially such that at the skin-depth δ, the current density is a factor of e (=2.718) smaller than that at the surface. The skin depth of a material at a frequency f is

$$\delta = \sqrt{\frac{2\rho}{(2\pi f)\mu}}$$

ρ: resistivity
μ: permeability

Ex. Aluminum conductor.

$\rho = 2.65\ \mu\Omega\text{-cm}$

$\mu = 4\pi \times 10^{-7}$ H/m (free space)

$\Rightarrow \delta = 18.75$ mm

In a Bunting conductor, $\frac{T}{D} = 0.3748$, $D = 1.302$ inches

$R_{dc} = 0.0787\ \Omega/\text{mile}$

$R_{bunting} = 0.0811\ \Omega/\text{mile}$

Both at 60 Hz, at the temperature of 25°C

<u>Resistivity</u> increases linearly with temperature:

$$\rho_{T_2} = \rho_{T_1}\left(\frac{T_2 + T}{T_1 + T}\right), \quad T: \text{temperature constant}$$

<u>AC resistance</u> or effective resistance:

$$R_{ac} = \frac{P_{Loss}}{|I|^2}\ \Omega$$

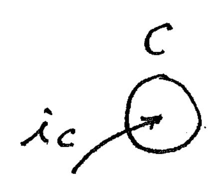
P_{Loss}: real power loss in W; I: rms current in A.

2. Conductance G:

In addition to the power loss due to I^2R in the series resistance, there is a small loss due to leakage current flowing through the insulator. This effect is amplified due to the corona effect where the surrounding air is ionized to conduct and a hissing sound can be heard in misty, foggy weather.

The corona problem can be averted by increasing the conductor size and by the use of conductor bundling. Conductance is usually neglected in power system studies because it is a very small component of the shunt admittance.

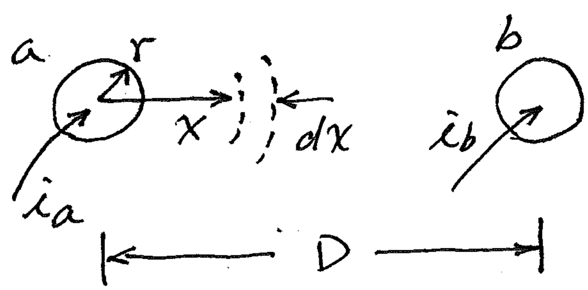
3. Series Inductance L:



Per-phase inductance:

$$L_a \triangleq \frac{\lambda_{a,total}}{i_a} = \frac{\lambda_{a,i_a}+\lambda_{a,i_b}+\lambda_{a,i_c}}{i_a}$$

$\lambda_{a,total}$: superposition of flux due to i_a, i_b, i_c



Considering i_a alone, by Ampere's law at a distance x from conductor-a,

$$H_x = \frac{i_a}{2\pi x}, \quad B_x = \mu_0 H_x = \left(\frac{\mu_0}{2\pi x}\right) i_a$$

Differential flux-linkage in a differential distance dx over a unit length along the conductor ($l = 1$) is

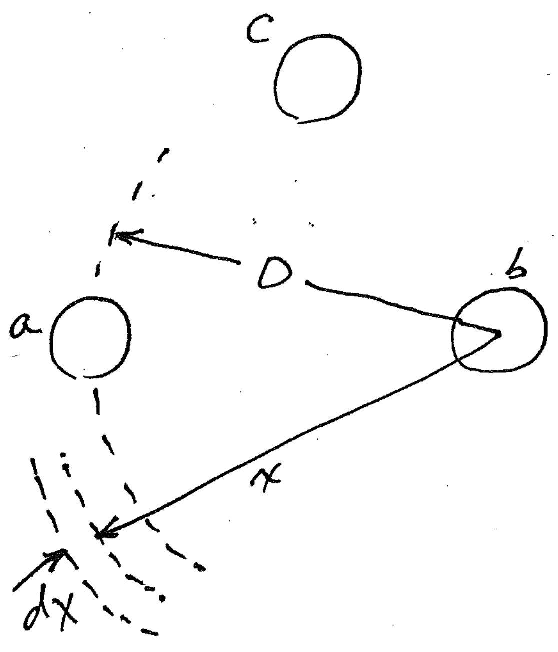
$$d\lambda_{x,i_a} = B_x \cdot dx = \left(\frac{\mu_0}{2\pi x}\right) i_a \cdot dx$$

Assuming the current in each conductor to be at the surface (due to skin effect), integrating x from the conductor radius to infinity,

$$\lambda_{a,i_a} = \int_r^\infty d\lambda_{x,i_a} = \left(\frac{\mu_0}{2\pi}\right) i_a \int_r^\infty \frac{1}{x}\,dx = \left(\frac{\mu_0}{2\pi}\right) i_a \ln\frac{\infty}{r}$$

Next, the mutual flux linking conductor-a due to i_b in conductor-b, noting that $D >> r$,

$$\lambda_{a,i_b} = \int_D^\infty d\lambda_{x,i_b} = \left(\frac{\mu_0}{2\pi}\right) i_b \ln\frac{\infty}{D}$$



Similarly, due to i_c,

$$\lambda_{a,i_c} = \left(\frac{\mu_0}{2\pi}\right) i_c \ln\frac{\infty}{D}$$

By superposition,

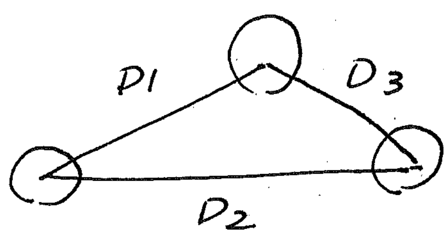
$$\lambda_{a,total} = \lambda_{a,i_a} + \lambda_{a,i_b} + \lambda_{a,i_c}$$
$$= \left(\frac{\mu_0}{2\pi}\right)\left[i_a \ln\frac{\infty}{r} + (i_b + i_c)\ln\frac{\infty}{D}\right]$$

For a balanced three-phase transmission,

$$i_a + i_b + i_c = 0$$

$$\Rightarrow \quad \lambda_{a,total} = \left(\frac{\mu_0}{2\pi}\right) i_a \ln \frac{D}{r}$$

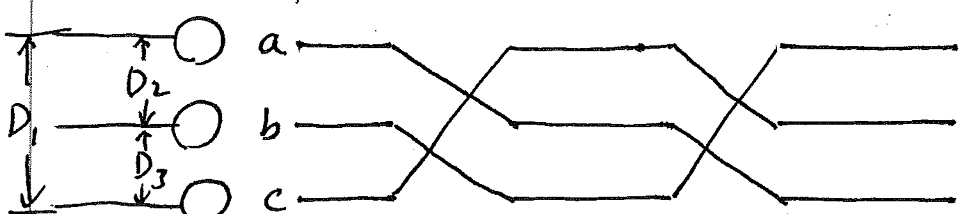
$$\therefore \quad L = \left(\frac{\mu_0}{2\pi}\right) \ln \frac{D}{r} \quad [H/m]$$



For conductors at distances D_1, D_2, D_3 with respect to each other, define the equivalent distance D as the Geometric Mean Distance (GMD):

$$D = \sqrt[3]{D_1 D_2 D_3}$$

for transposed lines.

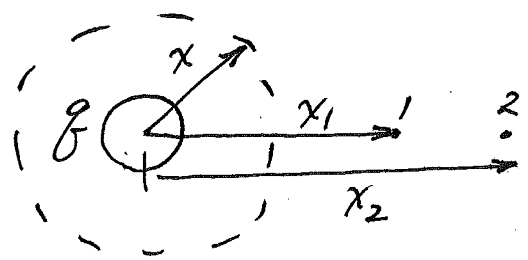


In bundled conductor L is smaller:

by a factor of 0.7 for 3-conductor bundle w/ 18 inches spacing

by a factor of 0.8 for 2-conductor bundle

4. Shunt Capacitance C:



Consider a charge q on a conductor, that results in dielectric flux lines and the electric field intensity E.

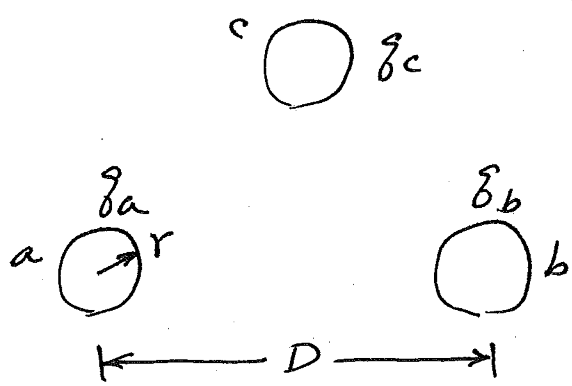
The dielectric flux-density D:

$$D = \frac{q}{(2\pi x)\times 1}, \quad E = \frac{D}{\varepsilon_0} = \frac{q}{(2\pi x)\,\varepsilon_0}$$

where $\varepsilon_0 = 8.85\times 10^{-12}$ F/m, permittivity of air

The voltage of point 1 wrt point 2:

$$v_{12} = -\int_{x_2}^{x_1} E(x)\cdot dx = \left(\frac{q}{2\pi\varepsilon_0}\right)\ln\frac{x_2}{x_1}$$



v_{ab} : voltage of a wrt b due to q_a, q_b, q_c

$$v_{ab,q_a} = \left(\frac{q_a}{2\pi\varepsilon_0}\right)\ln\frac{D}{r}$$

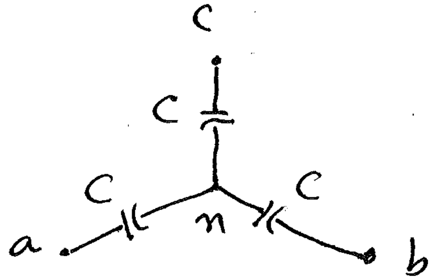
$$v_{ba,q_b} = \left(\frac{q_b}{2\pi\varepsilon_0}\right)\ln\frac{D}{r} = -v_{ab,q_b}$$

where $v_{ab,q_b} = -\left(\frac{q_b}{2\pi\varepsilon_0}\right)\ln\frac{D}{r}$

For equidistance,

q_c does not produce any voltages between a & b.

$$\Rightarrow \quad v_{ab} = v_{ab,q_a} + v_{ab,q_b} = \frac{1}{2\pi\varepsilon_0}(q_a - q_b)\ln\frac{D}{r}$$



Considering a hypothetical neutral point n, the capacitance C from each phase as shown,

$$v_{ab} = v_{an} - v_{bn}$$

$$= \frac{q_a}{C} - \frac{q_b}{C} = \frac{1}{C}(q_a - q_b)$$

Comparing the two eqns,

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{D}{r}} \quad [F/m]$$

For conductors at distances D_1, D_2, D_3, and transposed lines, the equivalent distance (GMD) is

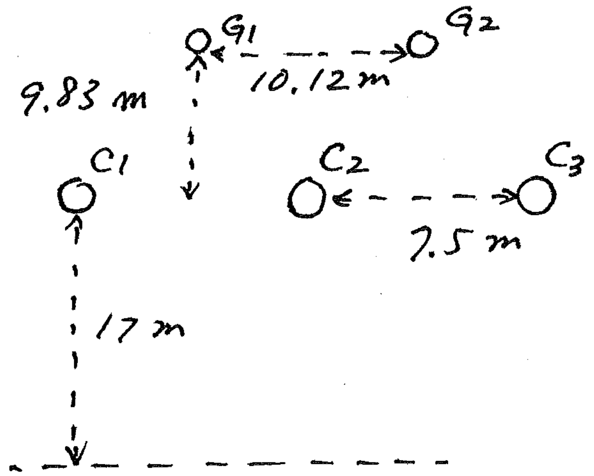
$$D = \sqrt[3]{D_1 D_2 D_3}$$

In bundled conductor C is larger:

by a factor of 1.4 for 3-conductor bundle w/ 18 inches spacing

by a factor of 1.25 for 2-conductor bundle

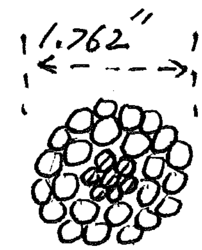
Example.



Tower: $3\angle 3$

Conductors: Bluebird

Ground wires: 7/18" Utility Grade Steel



Aluminum Conductor Steel-Reinforced (ACSR)

Ignoring the effect of ground, ground wires and conductor sags, calculate ωL (Ω/km), ωC ($\mu\mho$/km)

Solution: GMD: $D = \sqrt[3]{7.5 \times 7.5 \times 15} = 9.45$ m

radius: $r = 0.0224$ m

$$\omega L = (2\pi f)\left(\frac{\mu_0}{2\pi}\right) \ln \frac{D}{r}$$

$$= f \mu_0 \ln \frac{D}{r} = (60)(4\pi \times 10^{-7}) \ln \frac{9.45}{0.0224}$$

$$= 0.456 \ \Omega/\text{km}$$

$$\omega C = (2\pi f) \frac{2\pi \varepsilon_0}{\ln \frac{D}{r}} = \frac{(2\pi)^2 (60)(8.85 \times 10^{-12})}{\ln \frac{9.45}{0.0224}}$$

$$= 3.467 \ \mu\mho/\text{km}$$