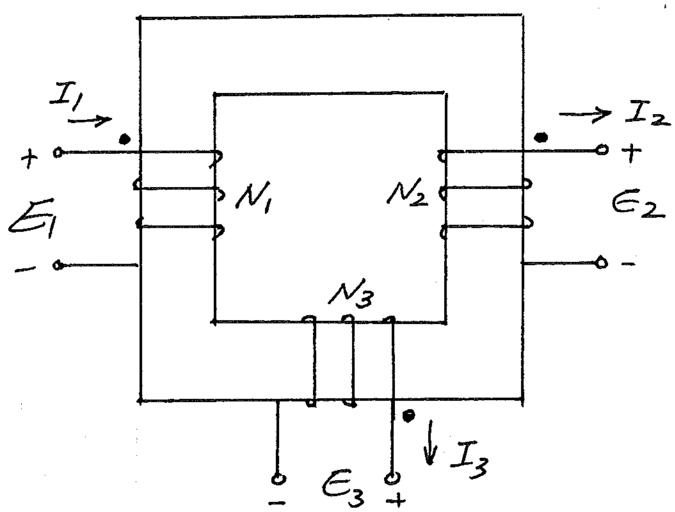
Multi-Winding Transformers



Generalizing the two-winding transformer, additional windings can be used to form multi-winding transformers.

For three-winding transformers the mmf balance in the common flux path is

$$N_1 I_1 = N_2 I_2 + N_3 I_3$$

or $$I_1 = \frac{N_2}{N_1} I_2 + \frac{N_3}{N_1} I_3$$

In per-unit system this becomes

(1) $$I_{1pu} = I_{2pu} + I_{3pu} \qquad (\text{Why?})$$

For voltage, using the Faraday's law,

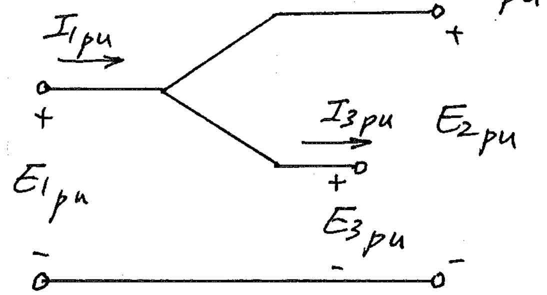
$$E_1 = j\omega N_1 \Phi, \quad E_2 = j\omega N_2 \Phi, \quad E_3 = j\omega N_3 \Phi$$

or $$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

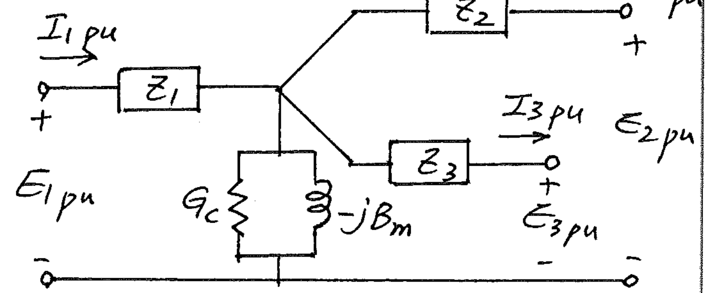
Again, in per-unit system,

(2) $$E_{1pu} = E_{2pu} = E_{3pu} \qquad (\text{Why?})$$

Per-unit equivalent circuit satisfying (1) and (2) is:



Ideal transformer



Practical transformer

The shunt admittance canbe determined by the open-circuit test and the series leakage impedances can be found by the short-circuit test as was done in two-winding transformer.

Let z_{ij} be the per-unit leakage impedance measured from winding i, with winding j shorted and winding k open, where $i, j, k = 1, 2, 3$.

Then

$$z_{12} = z_1 + z_2$$
$$z_{13} = z_1 + z_3$$
$$z_{23} = z_2 + z_3$$

Solving these,

$$z_1 = \frac{1}{2}(z_{12} + z_{13} - z_{23})$$
$$z_2 = \frac{1}{2}(z_{12} + z_{23} - z_{13})$$
$$z_3 = \frac{1}{2}(z_{13} + z_{23} - z_{12})$$

Example 3.9: A 1ϕ tree-winding transformer:

N_1: 300 MVA, 13.8 kV
N_2: 300 MVA, 199.2 kV
N_3: 50 MVA, 19.92 kV

Leakage reactances, from short-circuit tests, are

$X_{12} = 0.1$ pu on 300 MVA, 13.8 kV base
$X_{13} = 0.16$ pu on 50 MVA, 13.8 kV base
$X_{23} = 0.14$ pu on 50 MVA, 199.2 kV base

Find the p.u. equivalent circuit using the base of 300 MVA and 13.8 kV on N_1.

Solution: System base: $S_B = 300$ MVA
Base voltages: $V_{B1} = 13.8$ kV, $V_{B2} = 199.2$ kV, $V_{B3} = 19.92$ kV

New p.u. values:

$$X_{12} = 0.1 \text{ pu}$$
$$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96 \text{ pu}$$
$$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84 \text{ pu}$$

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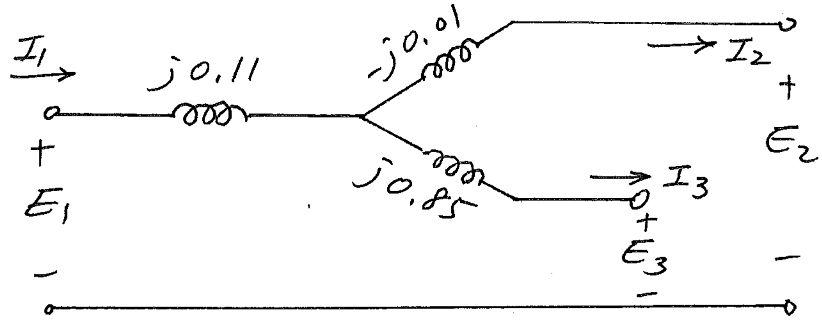
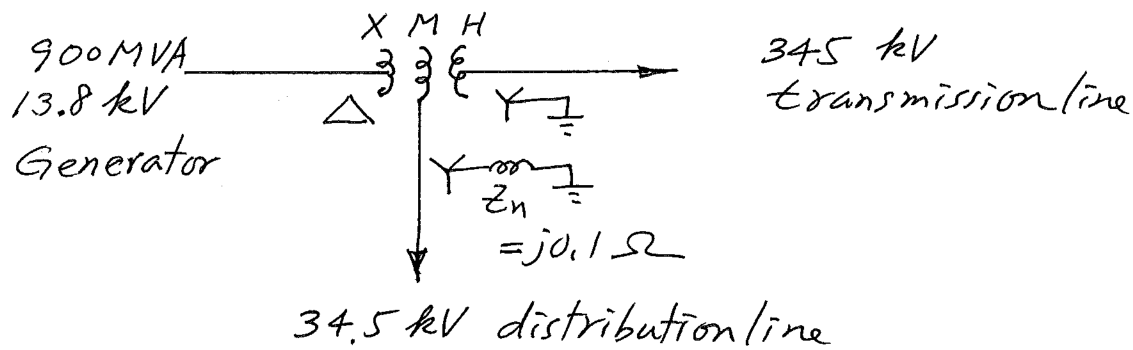
Reactances are

$$X_1 = \tfrac{1}{2}(0.1 + 0.96 - 0.84) = 0.11 \text{ pu}$$
$$X_2 = \tfrac{1}{2}(0.1 + 0.84 - 0.96) = -0.01 \text{ pu}$$
$$X_3 = \tfrac{1}{2}(0.96 + 0.84 - 0.1) = 0.85 \text{ pu}$$

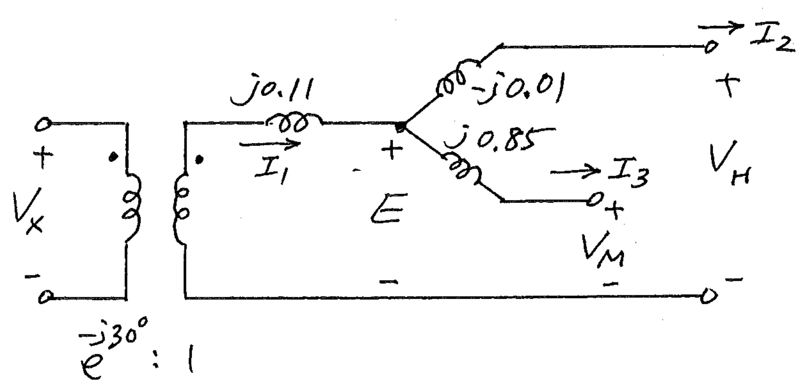


Example 3.10: Three-winding 3ϕ transformer



Draw the per-unit circuit using the 3ϕ base of generator as the system base.

Solution: In American Standard M and H (in Y) should lead X (in Δ) by 30°.



Example: Transmission line delivers 450 MVA at 345 kV with 0.8 pf lagging.

Resistive load of 150 MW at 34.5 kV in the distribution line.

Find the voltage at the generator terminal.

Solution: In per-unit, the transmission line current:

$$I_2 = \frac{450}{900}(0.8 - j0.6) = 0.4 - j0.3 = 0.5\angle{-36.87^\circ}\ \text{pu}$$

Resistance in distribution line is

$$R = (1.0)\left(\frac{900}{150}\right) = 6.0\ \text{pu}$$

Voltage at E due to the load current I_2:

$$E = V_H + Z_2 I_2 = 1.0 + (-j0.01)(0.4 - j0.3)$$
$$= 1.0 - 0.003 - j0.004 = 0.997 - j\,0.004$$
$$\simeq 0.997\angle{-0.23^\circ}$$

Current in the distribution line:

$$I_3 = \frac{E}{6 + j0.85} = \frac{0.997\angle{-0.23^\circ}}{6.06\angle{8.06^\circ}} = 0.165\angle{-8.29^\circ}$$

Current in the low voltage winding:

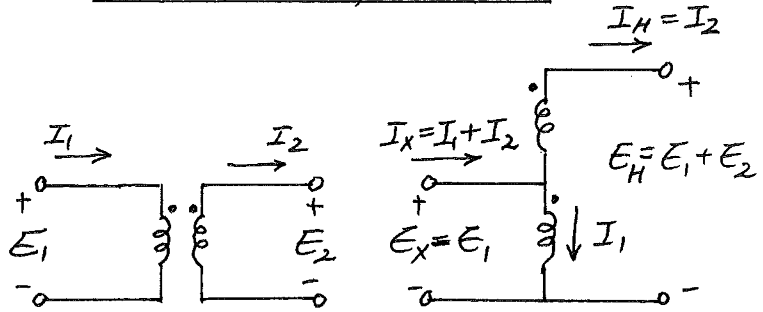
$$I_1 = I_2 + I_3 = 0.4 - j0.3 + (0.163 - j0.024)$$
$$= 0.563 - j0.324 = 0.65\angle{-29.92^\circ}$$

Voltage at the generator terminal:

$$V_x = (e^{-j30^\circ})(E + Z_1 I_1)$$
$$= (e^{-j30^\circ})(0.997 - j0.004 + (j0.11)(0.563 - j0.324))$$
$$= (e^{-j30^\circ})(\quad '' \quad + j0.062 + 0.0356)$$
$$= (e^{-j30^\circ})(1.0326 + j0.058) = 1.0342\angle{3.215^\circ - 30^\circ}$$
$$= 1.0342\angle{-26.79^\circ}$$

$$|V_x| = (1.0342)(13.8\ \text{kV}) = \underline{\underline{14.27}}\ \text{kV}$$

Auto-transformers



By connecting the two windings electrically we have a new transformer with increased voltage ratio.

This will also increase power rating (in kVA).

For two-winding transformer the input & output powers are:

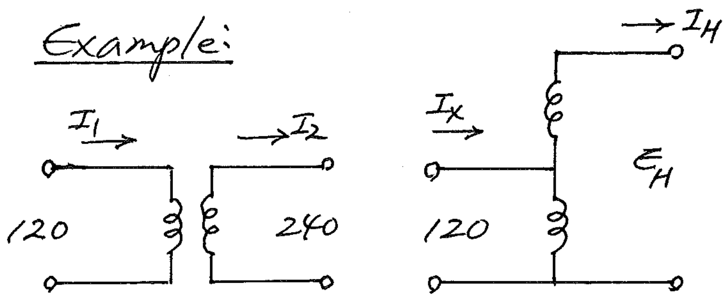
$$S_1 = E_1 I_1^*$$
$$S_2 = E_2 I_2^*$$

<u>NOTE</u>: Current ratings should <u>not</u> exceed the original ratings of the windings.

For auto-transformer the corresponding powers are:

$$S_X = E_1 (I_1 + I_2)^* = E_1 I_1^* + E_1 I_2^*$$
$$S_H = (E_2 + E_1) I_2^* = E_2 I_2^* + E_1 I_2^*$$

<u>Example:</u>



30 kVA
240/120 V

Convert the 30 kVA 240/120 V transformer to an auto-transformer with 120 V as the low voltage.

Find the new ratings of the auto-transformer.

<u>Solution:</u> Current ratings of windings are:

$$I_1 = \frac{30{,}000}{120} = 250\,A, \quad I_2 = \frac{30{,}000}{240} = 125\,A$$

For auto-transformer the ratings are:

$E_X = 120\ V$ $\qquad I_X = I_1 + I_2 = 375\ A$

$E_H = 120 + 240 = 360\ V$ $\qquad I_H = 125\ A$

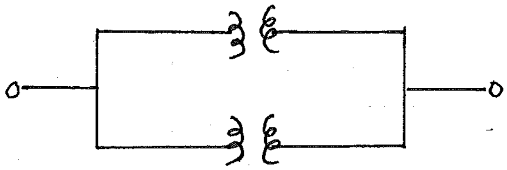
$S_X = 120 \times 375 = 45\ kVA, \quad S_H = 360 \times 125 = 45\ kVA$

<u>NOTE</u>: Per-unit leakage impedance will decrease (Why?)

<u>Hint</u>: $Z_{base\ old} = \dfrac{(240)^2}{30{,}000} = 1.92, \quad Z_{base\ new} = \dfrac{(360)^2}{45{,}000} = 2.88.$

Off-Nominal Turns Ratio

T_1 13.8/345 kV



T_2 13.2/345 kV

When two transformers are connected in parallel transmission lines, base voltages are supposed to be the same on each side of transformers.

However, this is not the case when the two transformers have different voltage ratings, due to:

1. not having identical transformers, or
2. one of the transformers changes taps.

We choose the nominal voltages as base voltages:

$$V_{base_1} = b\, V_{base_2},$$

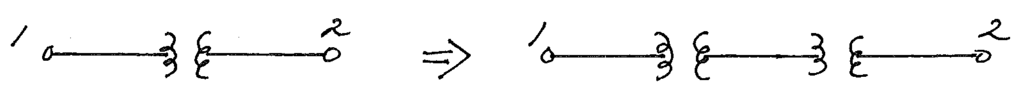
while the rated voltages may be different:

$$V_{1\,rated} = a_t\, V_{2\,rated}, \qquad a_t = \text{turns ratio, complex or real}$$

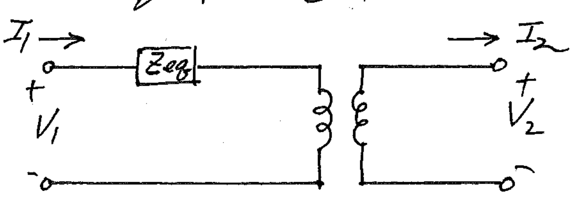
To see how much a_t is different from the nominal voltage ratio b, take the ratio $c = \frac{a_t}{b}$ and rewrite above as

$$V_{1\,rated} = b\left(\frac{a_t}{b}\right) V_{2\,rated} = b\,c\, V_{2\,rated}$$

which can be viewed as two transformers in series:

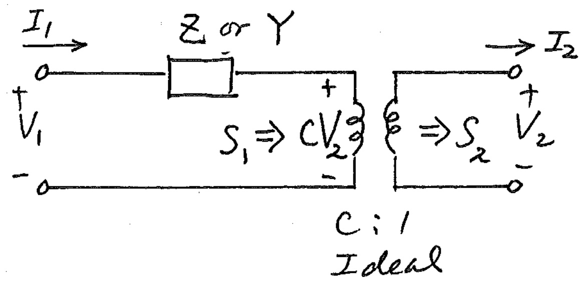


In per-unit only the first transformer disappears, while the second still remains!



$c : 1$
Ideal

Two-port Network Representation



The off-nominal turns ratio (c) can be imbedded in a two-port network model using the admittance formulation (Y-parameters).

Input and output of the ideal transformer are:

$$S_1 = (cV_2) I_1^*, \quad S_2 = V_2 I_2^*$$

Since $S_1 = S_2$, $\quad cV_2 I_1^* = V_2 I_2^* \Rightarrow I_2 = c^* I_1$

Input current is

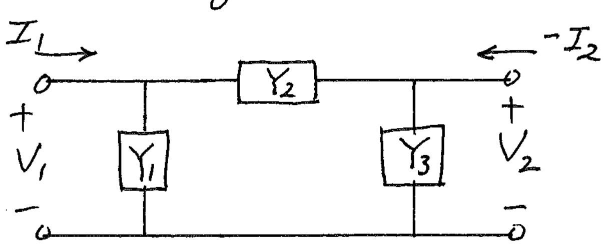
$$I_1 = (V_1 - cV_2) Y$$
$$= YV_1 - cYV_2$$

and output current is $I_2 = c^* I_1 = c^* Y V_1 - |c|^2 Y V_2$

Thus, the node equation in admittance formulation is

$$(1) \quad \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{matrix} Y_{11} = Y & Y_{12} = -cY \\ Y_{21} = -c^* Y & Y_{22} = |c|^2 Y \end{matrix}$$

π-equivalent circuit



If $c = c^*$, real, i.e., magnitude change, then

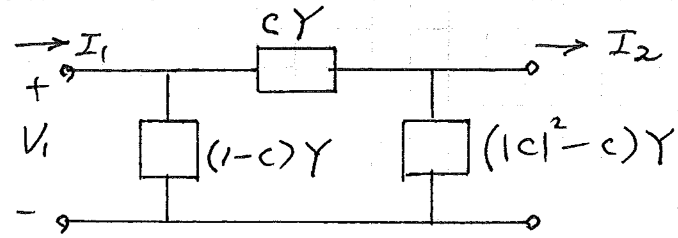
$Y_{12} = Y_{21}$, bi-lateral.

Thus, it can be modeled as a passive, bi-lateral two-port network.

$$I_1 = Y_1 V_1 + Y_2 (V_1 - V_2)$$
$$= (Y_1 + Y_2) V_1 - Y_2 V_2$$
$$-I_2 = Y_2 (V_2 - V_1) + Y_3 V_2$$
$$= -Y_2 V_1 + (Y_2 + Y_3) V_2$$
$$\Rightarrow Y_{11} = Y_1 + Y_2, \quad Y_{12} = Y_{21} = -Y_2$$
$$Y_{22} = Y_2 + Y_3$$

Comparing with (1) for c real,

$$Y_2 = cY$$
$$Y_1 = Y - cY = (1 - c) Y$$
$$Y_3 = (|c|^2 - c) Y$$



π-equivalent circuit for c real

<u>Example:</u> 3ϕ transformer, 13.8 kV Δ/345 kV Y, 1000 MVA $Z_{eq} = j0.1$ pu, has high-voltage winding with $\pm 10\%$ taps.

Use system base: 500 MVA, 13.8 kV/345 kV

Find the per-unit equivalent circuit for:

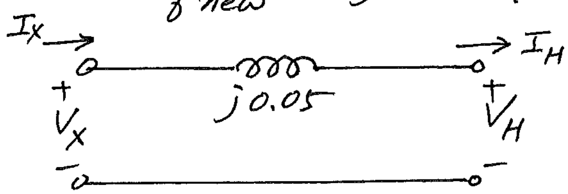
a) Rated tap

b) -10% tap: 10% decrease in high-voltage side

<u>Solution:</u>

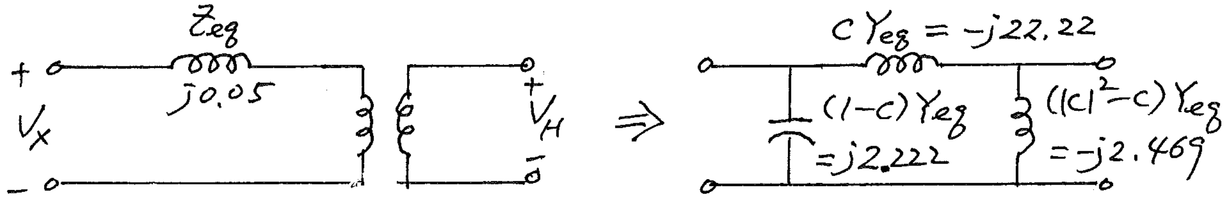
a) For rated tap, $a_t = b = \dfrac{13.8}{345} = 0.04$, $c = 1$

$$Z_{eq\,new} = (j0.1)\left(\frac{500}{1000}\right) = j0.05 \text{ pu}$$



b) For -10% tap, $a_t = \dfrac{13.8}{345(1-0.1)} = \dfrac{b}{1-0.1} = 0.04444$

$$c = \frac{a_t}{b} = \frac{1}{1-0.1} = 1.1111$$



$c = \dfrac{1}{1-0.1} : 1$

or

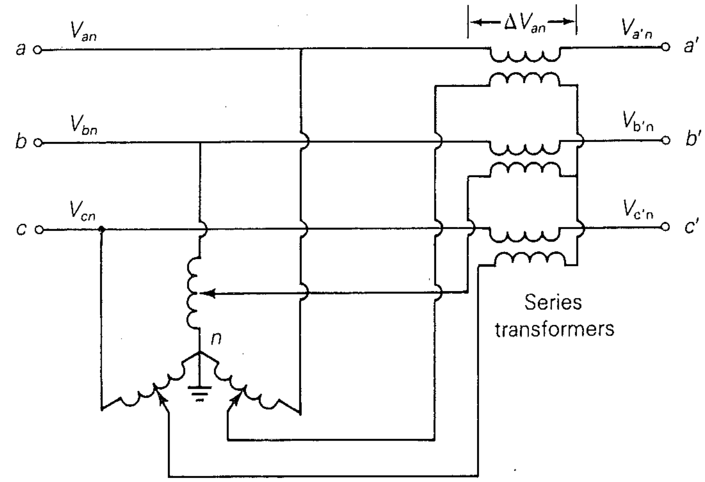
$1 : (1-0.1) = 0.9$

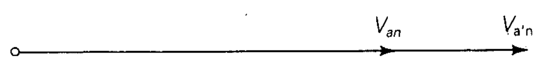
$$Y_{eq} = \frac{1}{Z_{eq}} = \frac{1}{j0.05} = -j20$$

<u>NOTE:</u> In general, the tap is modeled as $\boxed{1 : 1+\Delta V}$

Regulating Transformers

Magnitude-regulating transformer

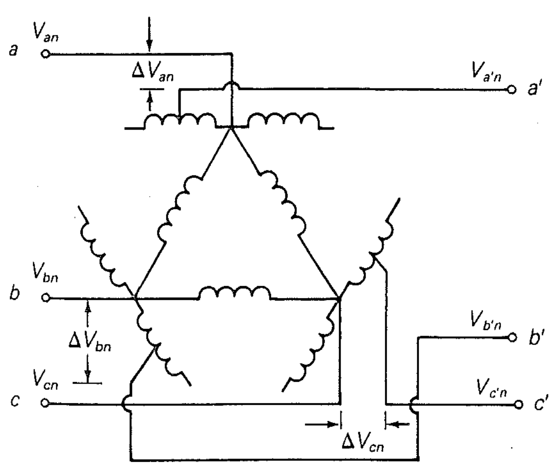


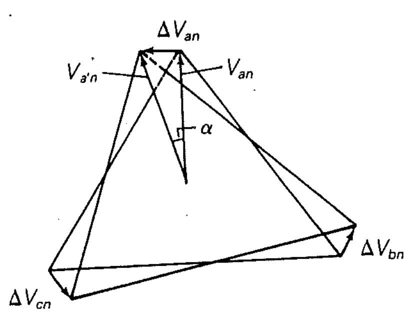


Magnitude regulating transformer:

$$c = (1 + \Delta V)^{-1}$$

Phase-angle regulating transformer



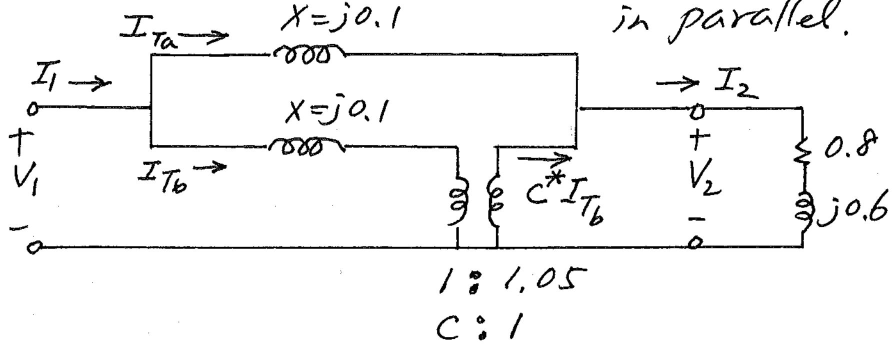


$$
\begin{aligned}
V_{a'b'} &= V_{a'a} + V_{ab} + V_{bb'} \\
&= V_{ab} + \rho V_{cb} + \rho V_{ca} \\
&= V_{ab} + \rho (V_{ca} - V_{bc}) \\
&= V_{ab} + \rho \left(e^{j\frac{2\pi}{3}} - e^{-j\frac{2\pi}{3}} \right) V_{ab} \\
&= V_{ab} (1 + j\rho\sqrt{3}) \\
&\simeq V_{ab} \angle \tan^{-1} \rho\sqrt{3} \\
&= V_{ab} \angle \alpha
\end{aligned}
$$

Phase-angle regulating transformer:

$$c = \angle \alpha$$

Example: (Prob. 3.64) Two identical transformers are in parallel. T_b has tap-setting at the load side.



Load : $0.8 + j0.6$ at $V_2 = 1.0\angle 0°$ pu

a) Magnitude regulation : +5% increase in high-voltage winding

$$c = (1.05)^{-1} = 0.9524$$

Find current in each transformer and complex power delivered to the load through each transformer.

Node equations (Admittance formulation)

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where Y-parameters are for the two transformers in parallel.

Known variables:

$$V_2 = 1.0\angle 0°, \quad I_2 = \frac{1.0\angle 0°}{0.8 + j0.6} = 0.8 - j0.6$$

$\Rightarrow$

From the second equation

(1) $$-I_2 = Y_{21} V_1 + Y_{22} V_2$$

we can find V_1.

The admittance parameters are:

T_a: $Y_{11} = \frac{1}{j0.1} = -j10 = Y_{22}$

$Y_{12} = Y_{21} = -\frac{1}{j0.1} = j10$

T_b: $Y_{11} = \frac{1}{j0.1} = -j10, \quad Y_{22} = |c|^2 Y = (0.9524)^2 \left(\frac{1}{j0.1}\right) = -j9.07$

$Y_{12} = Y_{21} = -cY = -(0.9524)\left(\frac{1}{j0.1}\right) = j9.52$

Parallel: $Y_{11} = -j10 - j10 = -j20, \quad Y_{22} = -j10 - j9.07 = -j19.07$

$Y_{12} = Y_{21} = j10 + j9.52 = j19.52$

From (1),

$$-(0.8-j0.6) = (j19.52)V_1 + (-j19.07)(1\angle 0^\circ)$$

$$V_1 = 1.008 + j0.041$$

$\Rightarrow$ Current through each transformer:

$$I_{Ta} = \left(\frac{1}{j0.1}\right)(V_1 - V_2) = 0.41 - j0.08$$

$$c^* I_{Tb} = I_2 - I_{Ta} = 0.8 - j0.6 - (0.41 - j0.08)$$

$$= 0.39 - j0.52$$

$\Rightarrow$ Complex power through each transformer:

$$S_{Ta} = V_2 I_{Ta}^* = 0.41 + j0.08$$

$$S_{Tb} = V_2 (c^* I_{Tb})^* = 0.39 + j0.52$$

NOTE: Reactive power flow is increased due magnitude increase!

b) Phase-angle regulation: $+3^\circ$ phase shift at the load

$$\Rightarrow \quad c = 1\angle -3^\circ$$

The admittance parameters:

T_b: $Y_{11} = \frac{1}{j0.1} = -j10$, $\quad Y_{22} = |c|^2 Y = (1.0)^2(-j10) = -j10$

$$Y_{12} = -cY = -(1.0\angle -3^\circ)(-j10) = 10\angle 87^\circ$$

$$Y_{21} = -c^* Y = -(1.0\angle +3^\circ)(-j10) = 10\angle 93^\circ$$

T_a & T_b in parallel:

$$Y_{11} = -j10 - j10 = -j20, \qquad Y_{22} = -j10 - j10 = -j20$$

$$Y_{12} = j10 + 10\angle 87^\circ = j10 + (0.5234 + j9.9863)$$

$$= 0.5234 + j19.986$$

$$Y_{21} = j10 + 1.0\angle 93^\circ = -0.5234 + j20$$

From (1),

$$-(0.8-j0.6) = (-0.523 + j20)V_1 + (-j20)(1.0\angle 0^\circ)$$

$$V_1 = 1.03 + j0.013$$

$$I_{Ta} = (V_1 - V_2)\,Y = (0.03 + j0.013)(-j10) = 0.13 - j0.30$$

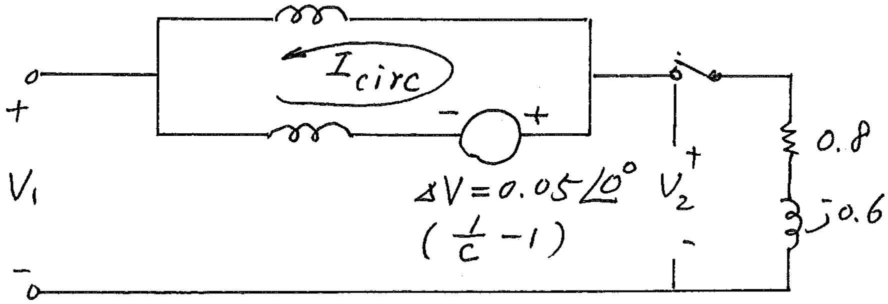
$$c^* I_{Tb} = I_2 - I_{Ta} = 0.8 - j0.6 - (0.13 - j0.30) = 0.67 - j0.30$$

$$S_{Ta} = V_2 I_{Ta}^* = 0.13 + j0.30$$

$$S_{Tb} = V_2 (c^* I_{Tb})^* = 0.67 + j0.30$$

<u>NOTE</u>: Real power is increased due to increase in phase angle!

<u>Example</u>: Approximate by circulating current method.



a) Magnitude regulation:

$$I_{circ} = \frac{0.05}{j0.2} = -j0.25 \text{ pu}$$

$$I_{Ta} = \tfrac{1}{2}(0.8 - j0.6) - (-j0.25) = 0.4 - j0.05$$

$$I_{Tb} = \tfrac{1}{2}(0.8 - j0.6) + (-j0.25) = 0.4 - j0.55$$

$$S_{Ta} = V_2 I_{Ta}^* = 0.4 + j0.05$$

$$S_{Tb} = V_2 I_{Tb}^* = 0.4 + j0.55$$

b) Phase-angle regulation:

$$\Delta V = \tfrac{1}{c} - 1 = 1.0 \angle 3^\circ - 1.0 \angle 0^\circ = 0.0524 \angle 91.5^\circ$$

$$I_{circ} = \frac{0.0524 \angle 91.5^\circ}{j0.2} = 0.262 + j0.0069$$

$$I_{Ta} = 0.4 - j0.3 - (0.262 + j0.007) = 0.138 - j0.307$$

$$I_{Tb} = 0.4 - j0.3 + (0.262 + j0.007) = 0.662 - j0.293$$

$$S_{Ta} = 0.138 + j0.307$$

$$S_{Tb} = 0.662 + j0.293$$