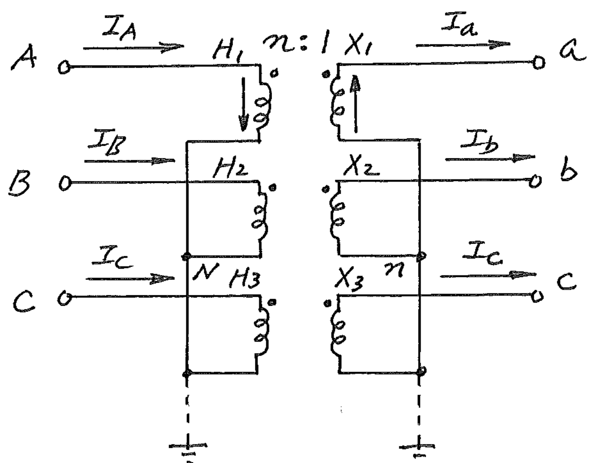
## Three-phase Transformers

Three identical single-phase two-winding transformers may be connected to form a three-phase bank.

Three-phase transformer may also be built with all six windings placed on a common three-phase core.

In both cases, the windings need to be connected in Δ or Y on both sides (primary/secondary, or high-voltage/low-voltage terminals). There are four ways of connecting the windings: Y-Y, Y-Δ, Δ-Y, and Δ-Δ.

### Y-Y connection



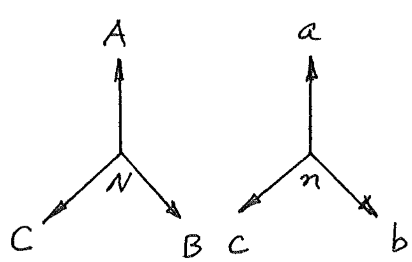

For phase A,

$$V_{AN} = n\, V_{an}$$

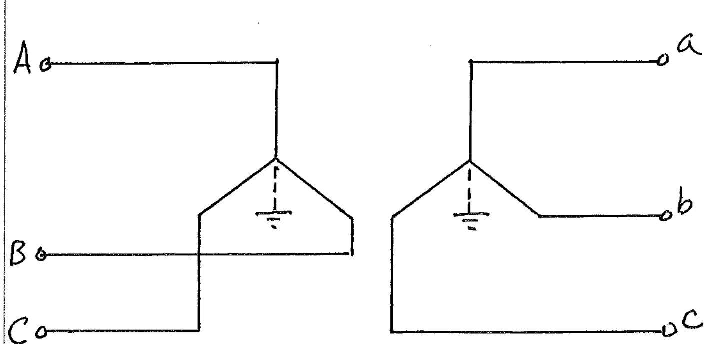
$$I_A = \frac{1}{n} I_a$$

where $n$ is the voltage ratio (turns ratio) of a single-phase transformer.

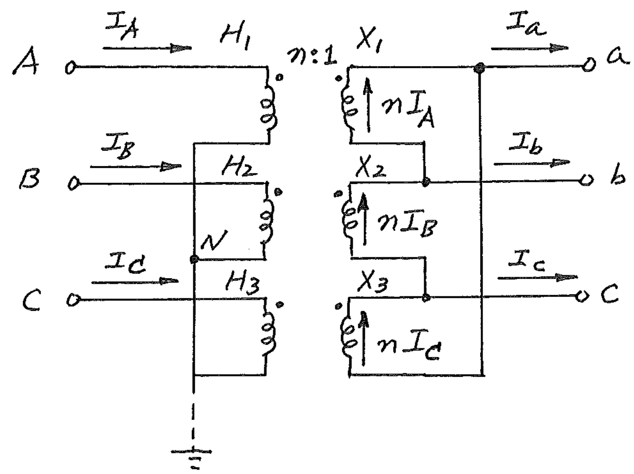
We note that there is no phase shift between corresponding quantities on the low- and high-voltage windings.

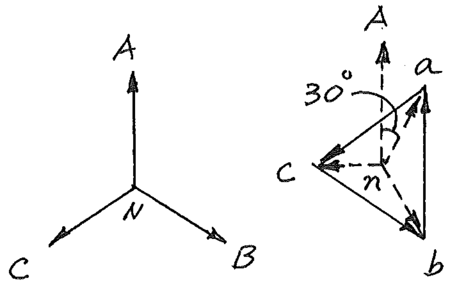


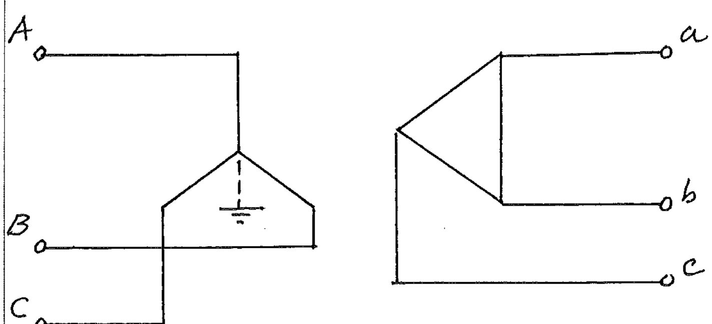

Although it is easy to connect, Y-Y is seldom used because odd harmonics tend to be amplified in the neutral, raising noise level.



## Y-Δ Connection




Between windings $H_1$ and $X_1$

$$V_{AN} = n\,V_{ab}$$
$$= n(V_{an} - V_{bn})$$
$$= n\sqrt{3}\,\underline{/30^\circ}\;V_{an}$$
$$= K\,V_{an}$$

where $K$ is a complex turns ratio

$$K = N\,e^{j30^\circ} = N\,\underline{/30^\circ}$$

and $N = n\sqrt{3}$

is the magnitude ratio.

In Δ, line current is

$$I_a = nI_A - nI_C$$
$$= n(I_A - I_C)$$
$$= n\sqrt{3}\,\underline{/-30^\circ}\;I_A$$
$$= K^*\,I_A$$

or

$$I_A = \frac{1}{K^*}\,I_a = \frac{1}{n\sqrt{3}}\,\underline{/30^\circ}\;I_a$$

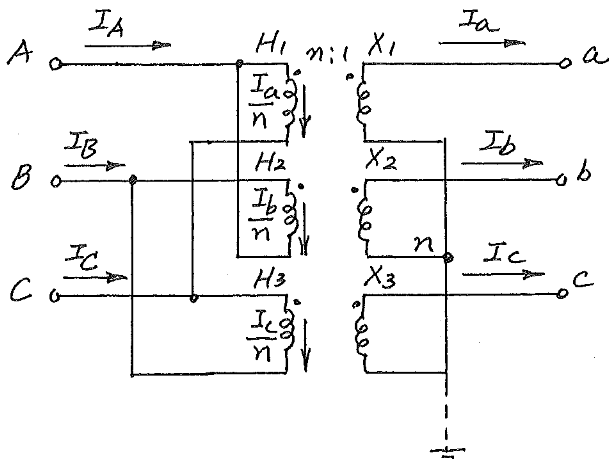
Note: 1. Effective turns ratio is increased to $n\sqrt{3}$

2. There is a phase shift of 30° between corresponding quantities on the low- and high-voltage windings.

### American Standard (ANSI):

In either a Y-Δ or Δ-Y transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by 30°.

## Δ - Y Connection




This time, we connect Δ in high-voltage windings in the order of A - C - B ($H_1 - H_3 - H_2$).

Then, between windings $H_1$ and $X_1$

$$V_{an} = \frac{1}{n} V_{AC}$$

$$= \frac{1}{n}(V_{AN} - V_{CN})$$

$$= \frac{\sqrt{3}}{n} \angle{-30^\circ}\ V_{AN}$$

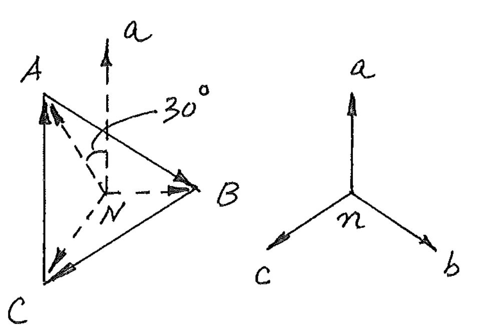
or

$$V_{AN} = \frac{n}{\sqrt{3}} \angle{30^\circ}\ V_{an}$$

$$= K\ V_{an}$$

where

$$K = \frac{n}{\sqrt{3}} \angle{30^\circ} = N \angle{30^\circ}$$

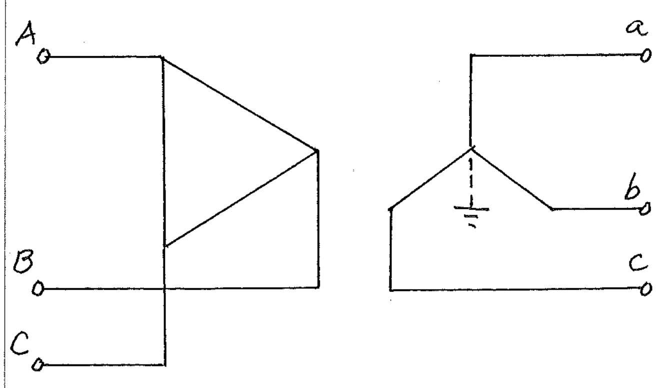



In Δ, the line current is

$$I_A = \frac{I_a}{n} - \frac{I_b}{n} = \frac{1}{n}(I_a - I_b)$$

$$= \frac{\sqrt{3}}{n} \angle{30^\circ}\ I_a$$

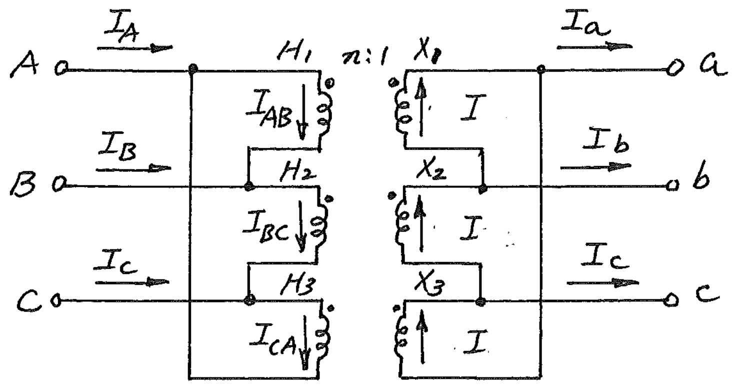
$$= \frac{1}{K^*} I_a = \frac{1}{N} \angle{30^\circ}\ I_a$$

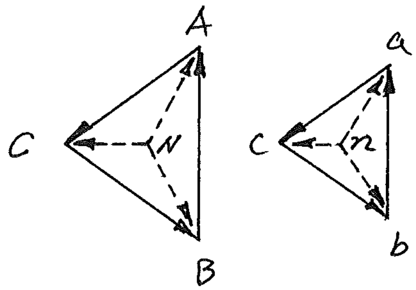


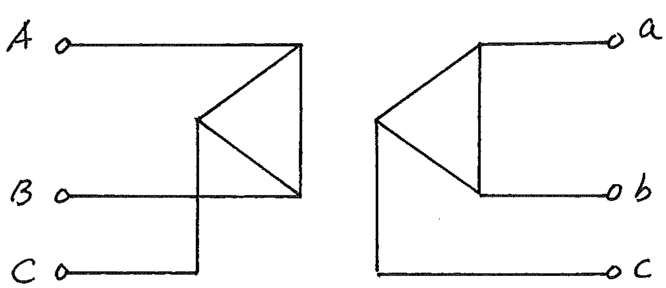

NOTE: 1. Effective turns ratio is decreased to $\frac{n}{\sqrt{3}}$.

2. High-Voltage quantities lead the corresponding low-Voltage quantities by 30°, i.e., ANSI is enforced!

## Δ - Δ Connection




Between windings H & X,

$$V_{AB} = n V_{ab}$$
$$V_{BC} = n V_{bc}$$
$$V_{CA} = n V_{ca}$$

or, equivalently,

$$V_{AN} = n V_{an}$$
$$V_{BN} = n V_{bn}$$
$$V_{CN} = n V_{cn}$$

Similarly, for currents,

$$I_{AB} = \frac{1}{n} I_{ab}$$
$$I_{BC} = \frac{1}{n} I_{bc}$$
$$I_{CA} = \frac{1}{n} I_{ca}$$

or, equivalently,

$$I_A = \frac{1}{n} I_a$$
$$I_B = \frac{1}{n} I_b$$
$$I_C = \frac{1}{n} I_c$$

We note that there is no phase shift between corresponding quantities on the low- and high-voltage windings.

The Δ-Δ connection has the advantage that one phase can be removed for repair or maintenance while the remaining phases continue to operate as a three-phase bank. This open Δ (or V) connection permits balanced three-phase operation with the kVA rating reduced to 58% of the original bank.

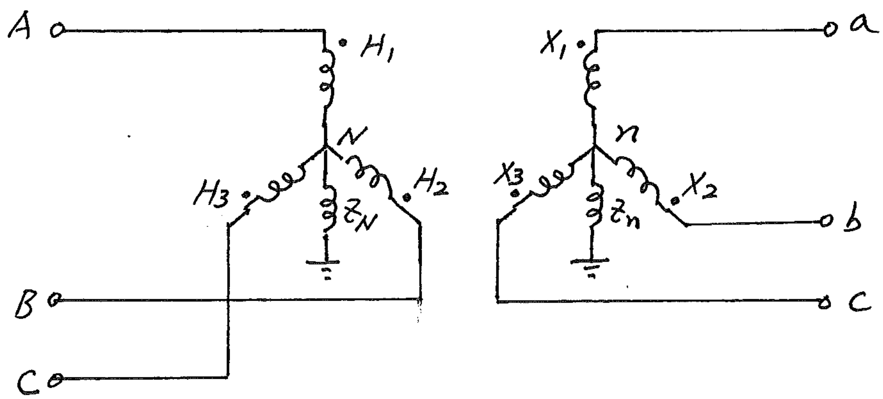
(Why?)

## Per-Unit Equivalent Circuits

By convention, we choose the base quantities as

1. A common $S_{base}$ for both the H and X sides
2. The ratio of the voltage bases $V_{base\,H} / V_{base\,X}$ is selected to be equal to the ratio of the rated line-to-line voltages $V_{rated\,H_{LL}} / V_{rated\,X_{LL}}$

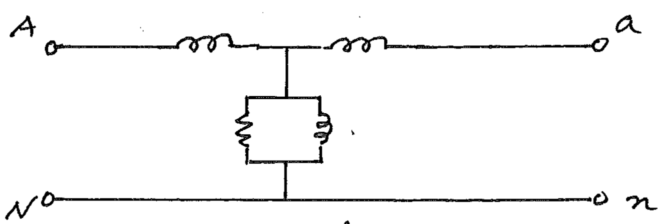
for all connections.



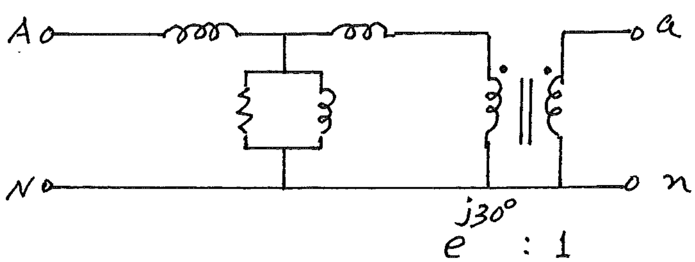

For balanced 3$\phi$ neutral current is zero and there are no voltage drops across the neutral impedances.

Therefore, the per-unit equivalent circuit of the Y-Y transformer is the same as the per-unit single-phase transformer.

- The per-unit equivalent circuit for Δ-Δ transformer is the same as that for Y-Y transformer. (Why?)




Y-Y and Δ-Δ




Y-Δ and Δ-Y

For Δ-Y or Y-Δ transformer, high-voltage side leads the low-voltage side by 30° (ANSI Standard).

Therefore, we add an ideal transformer with a complex turns ratio of $e^{j30°}$.

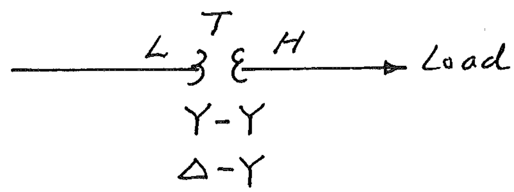
The per-unit impedances do not depend on the winding connections! (Why?)

Furthermore, the per-unit impedance of the 3$\phi$ transformer is the same as the per-unit impedance of the single-phase transformer! (Why?)

Example: Three single-phase transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance $X_{eq} = 0.10$ p.u., are connected to form a three-phase bank. Winding resistances and exciting currents are neglected. The high-voltage windings are connected in Y. A 3φ load connected to the high-voltage side draws 1000 MVA at 0.9 p.f. lagging, with $V_{AN} = 199.2\angle 0°$ kV.

Determine the voltage Van at the low-voltage bus if the low-voltage windings are connected (a) in Y, (b) in Δ.

Solution:




Use the transformer bank ratings as base quantities:

$$S_{base\,3\phi} = 3 \times 400 = 1200 \text{ MVA}$$

$$V_{base\,H_{LL}} = \sqrt{3}\,(199.2) = 345 \text{ kV}$$

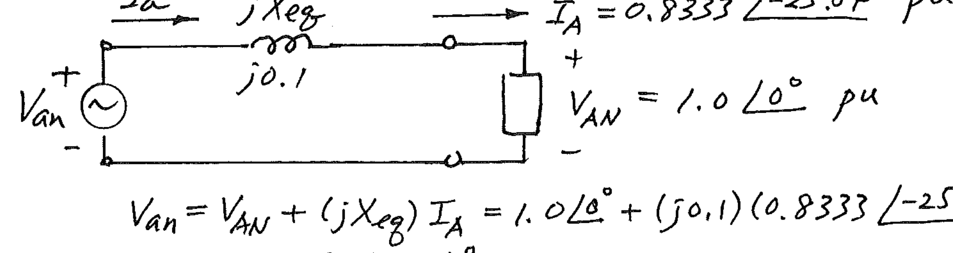
$$I_{base\,H} = \frac{S_{base\,3\phi}}{\sqrt{3}\,V_{base\,H_{LL}}} = \frac{1200}{\sqrt{3} \times 345} = 2.008 \text{ kA}$$

Load current:

$$I_A = \frac{1000}{\sqrt{3}\cdot 345}\angle -\cos^{-1}0.9 = 1.6735\angle -25.84° \text{ kA}$$

$$= 0.8333\angle -25.84° \text{ pu}$$

(a) Y-Y transformer:

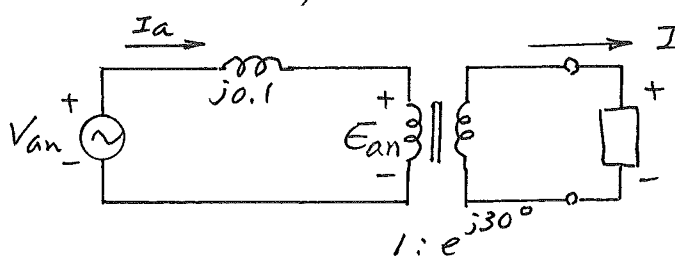



$$V_{an} = V_{AN} + (jX_{eq})\,I_A = 1.0\angle 0° + (j0.1)(0.8333\angle -25.84°)$$

$$= 1.039\angle 4.14°$$

$$\therefore V_{an} = 1.039\,(13.8) = \underline{14.34 \text{ kV}}$$

(b) Δ-Y transformer:




$$E_{an} = e^{-j30°} V_{AN} = 1.0 \angle -30°$$

$$I_a = e^{-j30°} I_A = 0.8333 \angle -25.84° - 30°$$

$$V_{an} = E_{an} + (jX_{eq}) I_a$$

$$= 1.0 \angle -30° + (j1.0)(0.8333 \angle -25.84° - 30°)$$

$$= [\underbrace{1.0 \angle 0° + (j1.0)(0.8333 \angle -25.84°)}_{\text{same as in part (a)}}] \angle -30°$$

$$= 1.039 \angle 4.14° - 30°$$

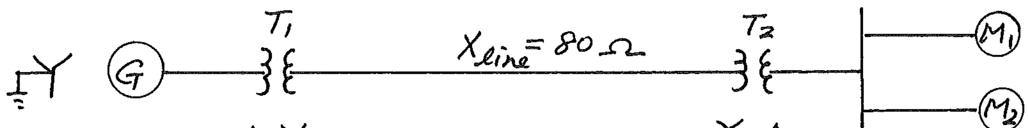
$$= 1.039 \angle -25.86°$$

$$\therefore V_{an} = 1.039 (13.8/\sqrt{3}) = \underline{8.278} \text{ kV}$$

<u>NOTE:</u>

- $|V_{an}|$ is the same in per-unit in Y-Y & Δ-Y.
- We often <u>ignore</u> the phase shift in calculating the magnitude of the voltage, $|V_{an}|$.

Example:




Gen.
30 MVA
13.8 kV
X = 15%

ΔY
35 MVA
13.2/115 kV
X = 10%

YΔ
Three 1φ trans.
each: 10 MVA
12.5/67 kV
X = 10%

$M_1$: 20 MVA
$M_2$: 10 MVA
12.5 kV
X = 20%
for both machines

Use the Generator ratings as the base quantities.

$$S_{B\,3\phi} = 30\text{ MVA}$$

| Zone 1 | Zone 2 | Zone 3 |
|---|---|---|
| $V_{B1} = 13.8\text{ kV}$ | $V_{B2} = 13.8\left(\frac{115}{13.2}\right) = 120\text{ kV}$ | $V_{B3} = 120\left(\frac{12.5}{\sqrt{3}\,(67)}\right) = 12.9\text{ kV}$ |
| | $Z_{B2} = \frac{(120)^2}{30} = 480\,\Omega$ | |

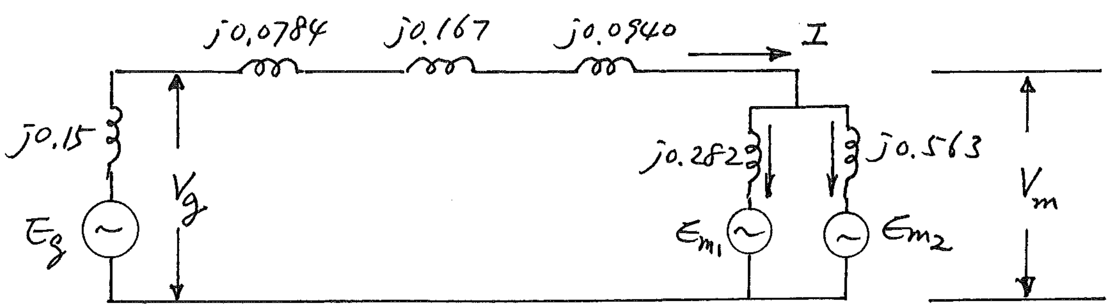
Line : $X_{Line} = \frac{80}{480} = 0.167$ pu

$T_1$ : $X_1 = (0.1)\left(\frac{13.2}{13.8}\right)^2\left(\frac{30}{35}\right) = 0.0784$ pu

$T_2$ : $X_2 = (0.1)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{30}\right) = 0.0940$ pu

$M_1$ : $X_{M_1} = (0.2)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{20}\right) = 0.282$ pu

$M_2$ : $X_{M_2} = (0.2)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{10}\right) = 0.563$ pu




Suppose the motors are operating with:
$M_1 = 16$ MW , $M_2 = 8$ MW @ 12.5 kV & 1.0 pf.

Then find the generator terminal voltage.

Since both machines are operating with the same powerfactor, total power is simply $P = 16 + 8 = 24$ MW at the unity power factor. & at the common voltage of 12.5 kV.

In per-unit, the total power is

$$P_{pu} = \frac{24\ MW}{30\ MVA} = 0.8\ p.u.$$

The voltage at the motor bus:

$$V_{pu} = \frac{12.5}{12.9} = 0.969\ p.u.$$

The current drawn by the combined load:

$$I_{pu} = \frac{0.8}{0.969} = 0.826\ p.u.$$

Therefore, the generator terminal voltage is

$$V_g = V_m + I\,(j0.0784 + j0.167 + j0.0940)$$
$$= 0.969 + (0.826\angle 0^\circ)(j\,0.3394)$$
$$= 1.01\ \underline{\angle 16.1^\circ}\ \ pu$$

$$\therefore\ V_g = 1.01\,(13.8) = \underline{13.92}\ kV$$

<u>Remark</u>: If two machines are not operating at the same power factor, then current drawn by each machine can be computed first.

By adding the currents total load current can be obtained.