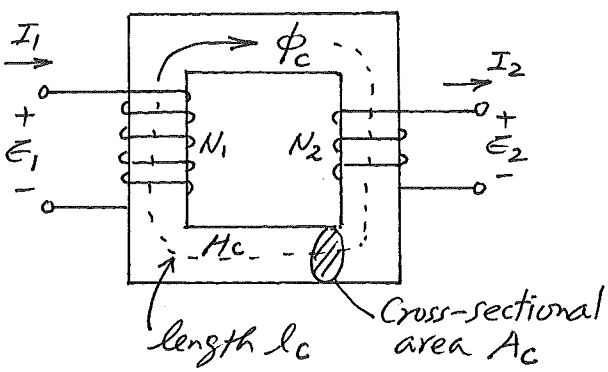
Transformers

Ideal Transformers:



Assume:

1. zero winding resistance (i^2R loss is zero)
2. infinite permeability ($\mu_c = \infty$, zero reluctance, $\mathcal{R}_c = 0$)
3. no leakage flux
4. no core loss

From Ampere's law,

$$\oint H_{tan}\, dl = I_{enclosed}$$

Tangential component of the magnetic field intensity vector integrated along a closed path equals the net current enclosed by that path.

Assuming the magnetic field intensity H_c is uniform along the closed path of length l_c with cross-sectional area A_c,

$$H_c l_c = N_1 I_1 - N_2 I_2$$

Note current I_1 is enclosed N_1 times and I_2 is N_2 times. Also, using the right-hand rule, current I_1 produces flux in clockwise direction while current I_2 produces flux in counter-clockwise direction. Thus, the net current enclosed is $(N_1 I_1 - N_2 I_2)$.

Flux density B_c and the core flux Φ_c are

$$B_c = \mu_c H_c \quad Wb/m^2$$

$$\Phi_c = B_c A_c \quad Wb$$

where μ_c: permeability of core

A_c: cross-sectional area of core

Thus,

$$\underbrace{N_1 I_1 - N_2 I_2}_{\text{mmf}} = \underbrace{\left(\frac{l_c}{\mu_c A_c}\right)}_{\mathcal{R}_c} \Phi_c$$

magneto motive force　　reluctance

$\Rightarrow$

$$\mathcal{F}_c = \mathcal{R}_c \; \Phi_c \quad : \text{Ohm's law in magnetic circuit}$$

For ideal transformer, $\mu_c = \infty$, $\Rightarrow$

$$\mathcal{R}_c = \frac{l_c}{\mu_c A_c} = 0$$

Therefore,

$$N_1 I_1 = N_2 I_2$$

or

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a_t}}, \qquad a_t = \frac{N_1}{N_2} : \text{turns ratio}$$

From Faraday's law, voltage (emf, electro motive force) induced across an N-turn winding by a time-varying flux $\phi(t)$ linking the winding is

$$e(t) = N\frac{d\phi}{dt}, \qquad \phi(t) = \Phi_m \cos(\omega t)$$

$$= -\omega N \Phi_m \sin(\omega t)$$

$$= \underbrace{\omega N \Phi_m}_{E_m} \cos(\omega t + 90^\circ)$$

In phasor,

$$E = \omega N \Phi \angle 90^\circ$$

$$= j\omega N \Phi$$

where Φ and E are phasors for $\phi(t)$ and $e(t)$, respectively.
Note the magnitude of E (in rms) is

$$|E| = \frac{E_m}{\sqrt{2}} = \frac{\omega N \Phi_m}{\sqrt{2}} = \underline{4.44 f N \Phi_m}$$

Thus, the voltage induced is proportional to frequency, number of turns, and the flux in the core.

Since the flux Φ_c links both N_1 and N_2 (no leakage flux), the induced voltages in both windings are

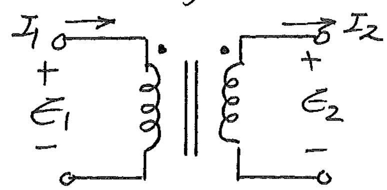
$$E_1 = j\omega N_1 \Phi_c$$

$$E_2 = j\omega N_2 \Phi_c$$

Therefore,

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = a_t}$$

Schematic for ideal transformer:



For ideal transformer, power is not lost:

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2$$

When an impedance Z_2 is connected across winding 2,

$$Z_2 = \frac{E_2}{I_2}$$

The impedance when measured from winding 1, is

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 \frac{E_2}{I_2} = \underline{a_t^2 Z_2}$$

Thus, the impedance Z_2 connected to winding 2 (secondary) is referred to winding 1 (primary) by multiplying Z_2 by a_t^2, the <u>square</u> of the turns ratio.

Practical Transformers: Now, we remove assumptions made for ideal transformers.

1. Include winding resistances R_1 and R_2 in windings N_1 & N_2.
2. Flux leakages in windings N_1 & N_2. When flux leaks it does not couple to other windings and thus, windings corresponding to the leakage is acting as the usual inductances. We call their reactances leakage reactances X_1 and X_2 for windings N_1 & N_2
3. Permeability is finite. Now $\mathcal{R}_c \neq 0$ (since $\mu_c \neq \infty$). Recall

$$N_1 I_1 - N_2 I_2 = \mathcal{R}_c \Phi_c$$

or

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \frac{\mathcal{R}_c}{N_1} \Phi_c \quad , \quad E_1 = j\omega N_1 \Phi_c$$

$$= \frac{\mathcal{R}_c}{j\omega N_1^2} E_1$$

$$= \underbrace{-j\left(\frac{\mathcal{R}_c}{\omega N_1^2}\right)}_{B_m\text{, susceptance [mhos]}} E_1 \triangleq I_m \text{, magnetizing current}$$

Thus, the core draws in magnetizing current I_m, which is lagging by $90°$ to the applied voltage.

4. Core loss is present: There are two types of core loss:

hysteresis loss and eddy current loss

- hysteresis loss can be reduced by using high grade alloy steel.
- eddy current loss can be reduced by using laminated sheets of alloy steel.

Core loss is present even when the secondary is open. Thus, the core loss current I_c is a shunt current and the core loss is $I_c^2 R_c = I_c^2 / G_c = E_1^2 G_c$.

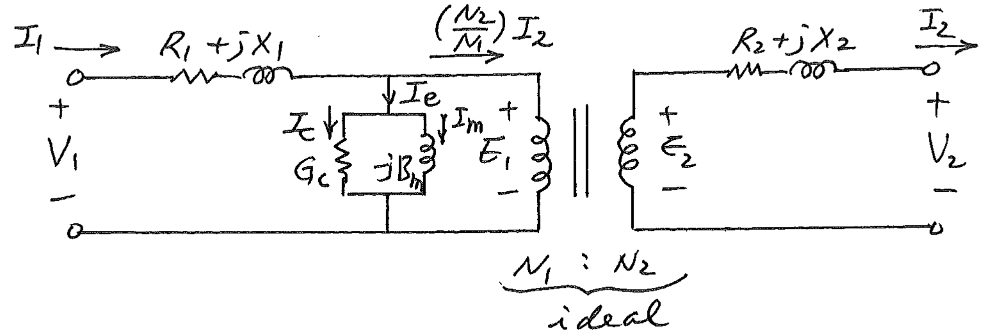
Thus, from (3),

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \underbrace{I_c + I_m}_{I_e\text{: exciting current}} = (G_c - jB_m) E_1$$

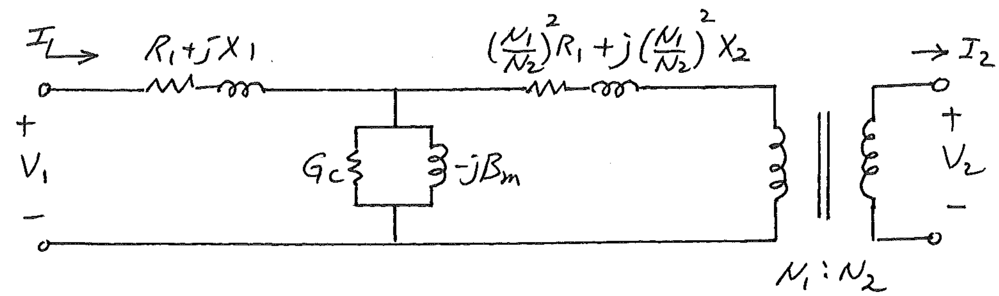
Note: In open circuit, $I_2 = 0$,

$$I_1 = I_c + I_m = I_e$$

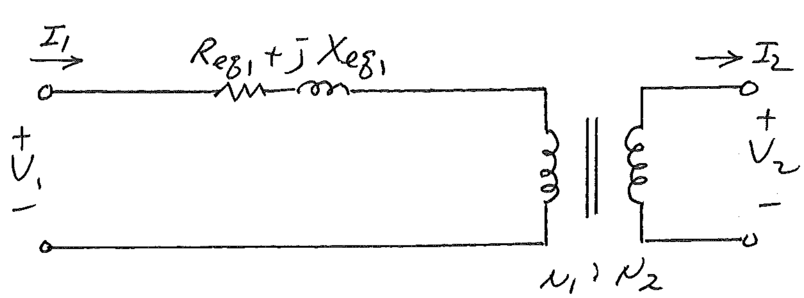
This implies that the exciting current is a shunt current.
The equivalent circuit is, therefore,



Impedances can be moved to either side of the ideal transformer. by multiplying the square of the turns ratio. Thus, an equivalent circuit with impedances referred to primary side is



By neglecting the exciting current, the equivalent circuit can be further simplified as



where
$$R_{eq_1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2$$
$$X_{eq_1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2$$

Voltage Regulation:

When the load at the secondary is disconnected suddenly, the voltage at the secondary terminal changes abruptly. The amount of such change (in %) is defined as voltage regulation. Formally, it can be written as

$$\% \text{ regulation} = \frac{|V_{2,NL}| - |V_{2,FL}|}{|V_{2,FL}|} \times 100 \quad (\%)$$

where

$V_{2,FL}$: full load (rated) voltage at the secondary

$V_{2,NL}$: no load (open-circuit) voltage at the secondary.

When the secondary is open, $I_2 = 0$. Therefore, $I_1 = 0$. Then, V_1 is applied directly to the N_1 winding of the ideal transformer and V_2 is the voltage induced on the N_2 winding:

$$V_{2,NL} = \left(\frac{N_2}{N_1}\right) V_1$$

Per-Unit System

Power system has many components in a wide range of capacities in different sizes. They also have different voltage ratings. Per-unit system normalizes all variables to values in a manageable level. Moreover, transformers will have the same voltages in per-unit on both windings, thus eliminating the ideal transformers in the circuit diagram.

Per-unit value is defined as

$$pu = \frac{\text{actual value}}{\text{base value}}$$

There are four variables to normalize:

$$V \qquad I \qquad Z \qquad S\ (P,\ Q)$$

Normally base values are selected for V and S and then determine base values for I and Z.

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,LN}} = \frac{kVA_{base\,1\phi}}{kV_{base\,LN}}$$

$$Z_{base} = \frac{V_{base\,LN}}{I_{base}} = \frac{V^2_{base\,LN}}{S_{base\,1\phi}} = \frac{(kV_{base\,LN})^2 \times 10^3}{kVA_{base\,1\phi}} = \frac{(kV_{base\,LN})^2}{MVA_{base\,1\phi}}$$

<u>NOTE:</u> $S = P + jQ$ is normalized by $S_{base\,1\phi}$

$$\frac{S}{S_{base\,1\phi}} = \frac{P + jQ}{S_{base\,1\phi}} = \frac{P}{S_{base\,1\phi}} + j\frac{Q}{S_{base\,1\phi}}$$

$$\Rightarrow \quad S_{pu} = P_{pu} + j\,Q_{pu}$$

Thus, we use only <u>one base</u>, $S_{base\,1\phi}$, for S, P, and Q.

Per-unit in 3ϕ system: In 3ϕ system, we are given $V_{base\,LL}$ and $S_{base\,3\phi}$. By converting these to single phase quantities we have:

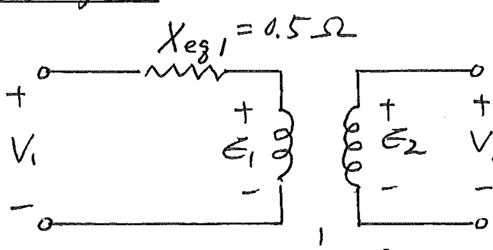
$$S_{base\,1\phi} = \frac{S_{base\,3\phi}}{3}, \quad V_{base\,LN} = \frac{V_{base\,LL}}{\sqrt{3}}$$

Then, the base quantities for I and Z are:

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,LN}} = \frac{S_{base\,3\phi}/3}{V_{base\,LL}/\sqrt{3}} = \frac{S_{base\,3\phi}}{\sqrt{3}\,V_{base\,LL}}$$

$$Z_{base} = \frac{V_{base\,LN}}{I_{base}} = \frac{V_{base\,LL}/\sqrt{3}}{S_{base\,3\phi}/\sqrt{3}\,V_{base\,LL}} = \frac{V_{base\,LL}^2}{S_{base\,3\phi}}$$

Example:



Transformer rating:
5 kVA, 200/100 V

Choose bases: (Transformer ratings)

$$\begin{cases} S_{base} = 5\text{ kVA} \\ V_{1\,base} = 200\text{ V}, \quad V_{2\,base} = 100\text{ V} \end{cases}$$

Zone 1	Zone 2
$V_{1\,base} = 200\text{V}$	$V_{2\,base} = 100\text{V}$
$Z_{1\,base} = \frac{V_{1\,base}^2}{S_{base}} = \frac{(200)^2}{5000} = 8\,\Omega$	$Z_{2\,base} = \frac{V_{2\,base}^2}{S_{base}} = \frac{(100)^2}{5000} = 2\,\Omega$

$X_{eq1} = 0.5\,\Omega \qquad X_{eq2} = \left(\frac{N_2}{N_1}\right)^2 X_{eq1} = \left(\frac{100}{200}\right)^2 (0.5) = 0.125\,\Omega$

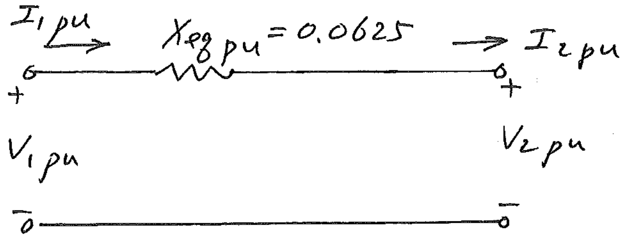
$X_{eq1\,pu} = \frac{0.5}{8}\left(= \frac{X_{eq1}}{Z_{1\,base}}\right) = 0.0625\text{ pu} \qquad X_{eq2\,pu} = \frac{0.125}{2}\left(= \frac{X_{eq2}}{Z_{2\,base}}\right) = 0.0625 = X_{eq1\,pu}$

$$E_{1\,pu} = \frac{E_1}{V_{1\,base}} = \frac{\left(\frac{N_1}{N_2}\right)E_2}{\left(\frac{N_1}{N_2}\right)V_{2\,base}} = \frac{E_2}{V_{2\,base}} = E_{2\,pu}$$

$$I_{1\,pu} = \frac{I_1}{I_{1\,base}} = \frac{\left(\frac{N_2}{N_1}\right)I_2}{\left(\frac{N_2}{N_1}\right)I_{2\,base}} = \frac{I_2}{I_{2\,base}} = I_{2\,pu}$$

Example: Continued

Since $E_{1pu} = E_{2pu}$ and $I_{1pu} = I_{2pu}$, and $X_{1pu} = X_{2pu}$, we have the following equivalent circuit:



Changing Per-Unit Values to Different Bases:

Normally, the per-unit (or percent) impedances of equipment are specified on the equipment base, which generally will be different from the power system base.

Since all impedances in the system must be expressed on the same base for per-unit (or percent) calculations, it is necessary to convert all values to the "common base" selected.

Recall the definition of the per-unit:

$$pu_{old} = \frac{\text{actual}}{\text{base}_{old}} \qquad pu_{new} = \frac{\text{actual}}{\text{base}_{new}}$$

By dividing the two equations:

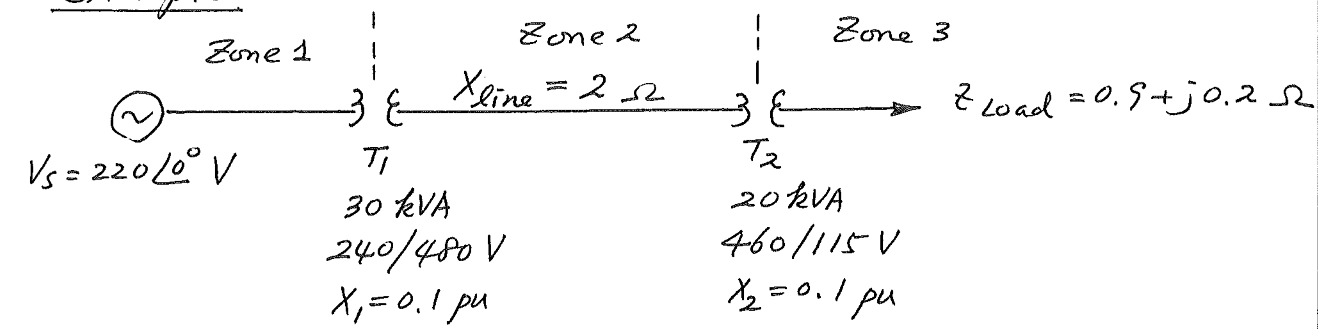
$$\frac{pu_{new}}{pu_{old}} = \frac{\text{base old}}{\text{base new}} \quad \text{or} \quad \boxed{pu_{new} = pu_{old}\left(\frac{\text{base old}}{\text{base new}}\right)}$$

Applying this to per-unit impedance,

$$Z_{pu\,new} = Z_{pu\,old}\frac{Z_{base\ old}}{Z_{base\ new}} = Z_{pu\,old}\frac{\dfrac{V^2_{base\ old}}{S_{base\ old}}}{\dfrac{V^2_{base\ new}}{S_{base\ new}}}$$

$$\therefore \quad \boxed{Z_{pu\,new} = Z_{pu\,old}\left(\frac{V_{base\ old}}{V_{base\ new}}\right)^2\left(\frac{S_{base\ new}}{S_{base\ old}}\right)}$$

Example:



The system is 1ϕ. Use base values of T_1 for the entire system. Draw the per-unit impedance diagram and find the load current.

$S_{base} = 30$ kVA (T_1) for the whole system, System Base.

However, there are three different base voltages, one for each zone: Again using the voltages of T_1,

V_{base}	$V_{base\,1} = 240\,V,$	$V_{base\,2} = 480\,V,$	$V_{base\,3} = 480\left(\frac{115}{460}\right) = 120\ V$
Z_{base}	$Z_{base\,1} = \frac{(240)^2}{30{,}000} = 1.92\ \Omega$	$Z_{base\,2} = \frac{(480)^2}{30{,}000} = 7.68\ \Omega$	$Z_{base\,3} = \frac{(120)^2}{30{,}000} = 0.48\ \Omega$
I_{base}			$I_{base\,3} = \frac{30{,}000}{120} = 250\ A$

Next, convert the per-unit impedance of T_2 using the system base values:

$$X_2 = (0.1)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378\ pu$$

or

$$X_2 = (0.1)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378\ pu$$

The line, located in zone 2, has the per-unit impedance

$$X_{line\,pu} = \frac{X_{line}}{Z_{base\,2}} = \frac{2}{7.68} = 0.2604\ pu$$

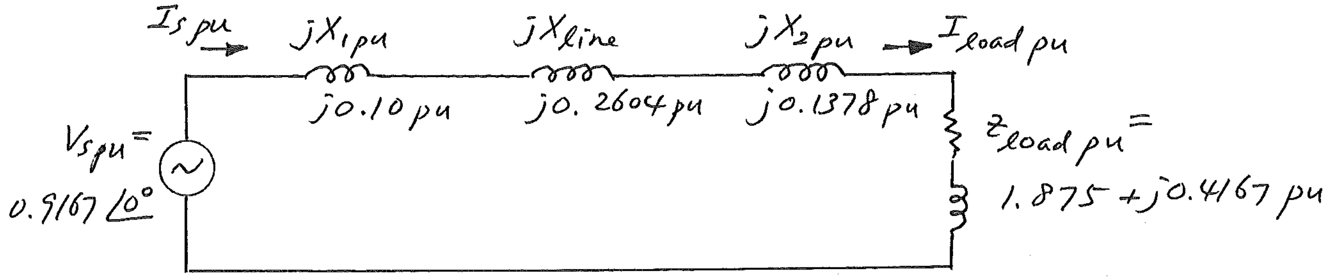
The load, located in zone 3, has the per-unit impedance

$$Z_{load\,pu} = \frac{Z_{load}}{Z_{base\,3}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.416\ pu$$

The source, located in zone 1, has the per-unit voltage

$$V_{S\,pu} = \frac{V_S}{V_{base\,1}} = \frac{220\angle 0°}{240} = 0.9167\angle 0°\ pu$$

The per-unit impedance diagram is now,



The load current is then

$$I_{load\,pu} = I_{s\,pu} = \frac{V_{s\,pu}}{j(X_{1pu} + X_{line\,pu} + X_{2pu}) + Z_{load\,pu}}$$

$$= \frac{0.9167\angle 0^\circ}{j(0.10 + 0.2604 + 0.1378) + (1.875 + 0.4167)}$$

$$= \frac{0.9167\angle 0^\circ}{1.875 + j0.9149} = \frac{0.9167\angle 0^\circ}{2.086\angle 26.01^\circ} = 0.4395\angle -26.01^\circ \text{ pu}$$

The actual load current is

$$I_{load} = (I_{load\,pu})\, I_{base\,3} = (0.4395\angle -26.01^\circ)(250\text{A})$$

$$= \underline{\underline{109.9}}\angle -26.01^\circ \text{ A}$$

<u>Question</u>: How can you find the current from the source?
How about the current in the line?
How can you find the voltage at the load?