ECL 4340

POWER SYSTEMS

LECTURE 5
PER-UNIT SYSTEM

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ANNOUNCEMENT

- Read Chapter 3
- HW 3 is due September 16 in Canvas.

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CALCULATION OF MODEL PARAMETERS

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- The parameters of the model are determined based upon
  - nameplate data: gives the rated voltages and power
  - **open circuit test:** rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
  - **short circuit test:** with secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.

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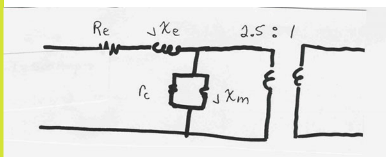
TRANSFORMER EXAMPLE

Example: A single phase, 100 MVA, 200/80 kV transformer has the following test data:

open circuit: 20 amps, with 10 kW losses

short circuit: 30 kV, with 500 kW losses

Determine the model parameters.



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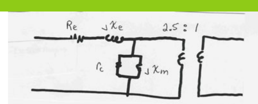
TRANSFORMER EXAMPLE, CONT'D

A single phase, 100 MVA, 200/80 kV

open circuit: 20 amps, with 10 kW losses

short circuit: 30 kV, with 500 kW losses



From the short circuit test

Rated current in primary winding

$$I_{sc} = \frac{100MVA}{200kV} = 500\ A,\ \ |R_e + jX_e| = \frac{30\ kV}{500\ A} = 60\ \Omega$$

$$P_{sc} = R_e I_{sc}^2 = 500\text{ kW} \rightarrow R_e = 2\ \Omega,$$

$$\text{Hence } X_e = \sqrt{60^2 - 2^2} = 60\ \Omega$$

From the open circuit test

$$R_c = \frac{(200\ kV)^2}{10\ kW} = 4M\Omega$$

$$|R_e + jX_e + jX_m| = \frac{200\ kV}{20\ A} = 10{,}000\ \Omega \qquad X_m = 10{,}000\ \Omega$$

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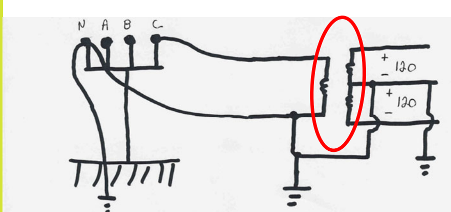
RESIDENTIAL DISTRIBUTION TRANSFORMERS

Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a multi-grounded, common neutral.



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Voltage Regulation

Voltage Regulation:

When the load at the secondary is disconnected suddenly, the voltage at the secondary terminal changes abruptly. The amount of such change (in %) is defined as voltage regulation. Formally, it can be written as

$$\% \text{ regulation} = \frac{|V_{2,NL}| - |V_{2,FL}|}{|V_{2,FL}|} \times 100 \quad (\%)$$

where
$V_{2,FL}$: full load (rated) voltage at the secondary
$V_{2,NL}$: no load (open-circuit) voltage at the secondary.

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Voltage Regulation

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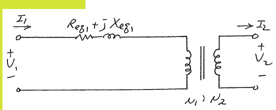

When the secondary is open, $I_2 = 0$. Therefore, $I_1 = 0$. Then, V_1 is applied directly to the N_1 winding of the ideal transformer and V_2 is the voltage induced on the N_2 winding:

$$V_{2,NL} = \left(\frac{N_2}{N_1}\right) V_1$$

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Per-Unit System

Per-Unit System

Power system has many components in a wide range of capacities in different sizes. They also have different voltage ratings. Per-unit system normalizes all variables to values in a manageable level. Moreover, transformers will have the same voltages in per-unit on both windings, thus eliminating the ideal transformers in the circuit diagram.

Per-unit value is defined as

$$pu = \frac{\text{actual value}}{\text{base value}}$$

There are four variables to normalize:

$V \quad I \quad Z \quad S\ (P, Q)$

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Per-Unit System

There are four variables to normalize:

V I Z S (P, Q)

Normally base values are selected for V and S and then determine base values for I and Z.

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,LN}} = \frac{kVA_{base\,1\phi}}{kV_{base\,LN}}$$

$$Z_{base} = \frac{V_{base\,LN}}{I_{base}} = \frac{V_{base\,LN}^2}{S_{base\,1\phi}} = \frac{(kV_{base\,LN})^2 \times 10^3}{kVA_{base\,1\phi}} = \frac{(kV_{base\,LN})^2}{MVA_{base\,1\phi}}$$

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Per-Unit System

NOTE: $S = P + jQ$ is normalized by $S_{base\,1\phi}$

$$\frac{S}{S_{base\,1\phi}} = \frac{P+jQ}{S_{base\,1\phi}} = \frac{P}{S_{base\,1\phi}} + j\frac{Q}{S_{base\,1\phi}}$$

$$\Rightarrow \quad S_{pu} = P_{pu} + j\,Q_{pu}$$

Thus, we use only one base, $S_{base\,1\phi}$, for S, P, and Q.

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Per-Unit System

Per-unit in 3ϕ system: In 3ϕ system, we are given $V_{base\,LL}$ and $S_{base\,3\phi}$. By converting these to single phase quantities we have:

$$S_{base\,1\phi} = \frac{S_{base\,3\phi}}{3}, \quad V_{base\,LN} = \frac{V_{base\,LL}}{\sqrt{3}}$$

Then, the base quantities for I and Z are:

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,LN}} = \frac{S_{base\,3\phi}/3}{V_{base\,LL}/\sqrt{3}} = \frac{S_{base\,3\phi}}{\sqrt{3}\,V_{base\,LL}}$$

$$Z_{base} = \frac{V_{base\,LN}}{I_{base}} = \frac{V_{base\,LL}/\sqrt{3}}{S_{base\,3\phi}/\sqrt{3}\,V_{base\,LL}} = \frac{V_{base\,LL}^2}{S_{base\,3\phi}}$$

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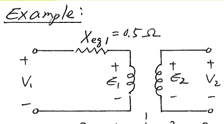
PER-UNIT SYSTEM

Example:



Zone 1

$V_{1\,base} = 200\,V$

$Z_{1\,base} = \dfrac{V_{1\,base}^2}{S_{base}} = \dfrac{(200)^2}{5000} = 8\,\Omega$

Zone 2

$V_{2\,base} = 100\,V$

$Z_{2\,base} = \dfrac{V_{2\,base}^2}{S_{base}} = \dfrac{(100)^2}{5000} = 2\,\Omega$

Transformer rating:

5 kVA, 200/100 V

Choose bases: (Transformer ratings)

$$\begin{cases} S_{base} = 5\,kVA \\ V_{1\,base} = 200\,V, \quad V_{2\,base} = 100\,V \end{cases}$$

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PER-UNIT SYSTEM

$Z_{1\,base} = \dfrac{V_{1\,base}^2}{S_{base}} = \dfrac{(200)^2}{5000} = 8\,\Omega$

$Z_{2\,base} = \dfrac{V_{2\,base}^2}{S_{base}} = \dfrac{(100)^2}{5000} = 2\,\Omega$

$X_{eq_1} = 0.5\,\Omega \qquad X_{eq_2} = \left(\dfrac{N_2}{N_1}\right)^2 X_{eq_1} = \left(\dfrac{100}{200}\right)^2 (0.5) = 0.125\,\Omega$

$X_{eq_1\,pu} = \dfrac{0.5}{8}\left(= \dfrac{X_{eq_1}}{Z_{1\,base}}\right) = 0.0625\ pu \qquad X_{eq_2\,pu} = \dfrac{0.125}{2}\left(= \dfrac{X_{eq_2}}{Z_{2\,base}}\right) = 0.0625 = X_{eq_1\,pu}$

$$E_{1\,pu} = \frac{E_1}{V_{1\,base}} = \frac{\left(\frac{N_1}{N_2}\right)E_2}{\left(\frac{N_1}{N_2}\right)V_{2\,base}} = \frac{E_2}{V_{2\,base}} = E_{2\,pu}$$

$$I_{1\,pu} = \frac{I_1}{I_{1\,base}} = \frac{\left(\frac{N_2}{N_1}\right)I_2}{\left(\frac{N_2}{N_1}\right)I_{2\,base}} = \frac{I_2}{I_{2\,base}} = I_{2\,pu}$$

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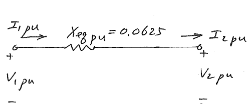
PER-UNIT SYSTEM

Example: Continued

Since $E_{1pu} = E_{2pu}$ and $I_{1pu} = I_{2pu}$, and $X_{1pu} = X_{2pu}$, we have the following equivalent circuit:



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Per Unit Calculations

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

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Per Unit Conversion Procedure, 1Φ

1. Pick a 1ϕ VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

Note, per unit conversion affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., **not** 1 p.u. volts)

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Per Unit Solution Procedure

1. Convert to per unit (p.u.) (many problems are already in per unit)
2. Solve
3. Convert back to actual as necessary

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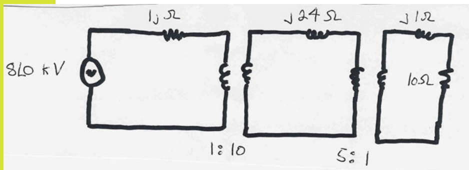
Per Unit Example

Solve for the current, load voltage and load power in the circuit shown below using per unit analysis with an S_B of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV.



Original Circuit

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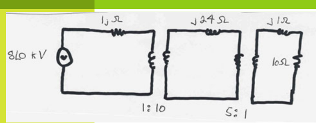
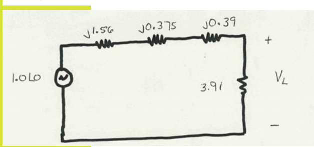
Per Unit Example, cont'd



with an S_B of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV.



$$Z_B^{Left} = \frac{8kV^2}{100MVA} = 0.64\Omega$$

$$Z_B^{Middle} = \frac{80kV^2}{100MVA} = 64\Omega$$

$$Z_B^{Right} = \frac{16kV^2}{100MVA} = 2.56\Omega$$

$$Z^{Left} = \frac{1\Omega}{0.64\Omega} = 1.56\text{p.u.}$$

$$Z^{Middle} = \frac{24\Omega}{64\Omega} = 0.375\,\text{p.u.}$$

$$Z^{Right} = \frac{1\Omega}{2.56\Omega} = 0.39\text{p.u.}$$

Same circuit, with values expressed in per unit.

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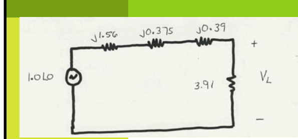
Per Unit Example, cont'd



$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$\begin{aligned} V_L &= 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ \\ &= 0.859\angle -30.8^\circ \text{ p.u.} \end{aligned}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

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Per Unit Example, cont'd

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To convert back to actual values just multiply the per unit values by their per unit base

$$V_{L}^{Actual} = 0.859\angle -30.8° \times 16\text{ kV} = 13.7\angle -30.8°\text{ kV}$$

$$S_{L}^{Actual} = 0.189\angle 0° \times 100\text{ MVA} = 18.9\angle 0°\text{ MVA}$$

$$S_{G}^{Actual} = 0.22\angle 30.8° \times 100\text{ MVA} = 22.0\angle 30.8°\text{ MVA}$$

$$I_{B}^{Middle} = \frac{100\text{ MVA}}{80\text{ kV}} = 1250\text{ Amps}$$

$$I_{Middle}^{Actual} = 0.22\angle -30.8° \times 1250\text{ Amps} = 275\angle -30.8°\text{ A}$$

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Three Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system, $S_B^{3\phi}$
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{\left(\sqrt{3}\,V_{B,LN}\right)^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

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Three Phase Per Unit, cont'd

4. Calculate the current base, I_B

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single phase!

5. Convert actual values to per unit

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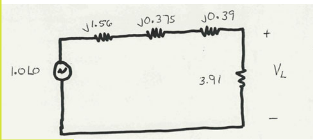
Three Phase Per Unit Example

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of 300 MVA, and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.



Convert to per unit as before. Note the system is exactly the same!

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3Φ Per Unit Example, cont'd

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$\mathrm{V_L} = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Again, analysis is exactly the same!

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3Φ Per Unit Example, cont'd

Differences appear when we convert back to actual values

$$V_{\mathrm{L}}^{\mathrm{Actual}} = 0.859\angle -30.8^\circ \times 27.6 \text{ kV} = 23.8\angle -30.8^\circ \text{ kV}$$

$$S_{\mathrm{L}}^{\mathrm{Actual}} = 0.189\angle 0^\circ \times 300 \text{ MVA} = 56.7\angle 0^\circ \text{ MVA}$$

$$S_{\mathrm{G}}^{\mathrm{Actual}} = 0.22\angle 30.8^\circ \times 300 \text{ MVA} = 66.0\angle 30.8^\circ \text{ MVA}$$

$$\mathrm{I_B^{Middle}} = \frac{300 \text{ MVA}}{\sqrt{3}\,138 \text{ kV}} = 1250 \text{ Amps} \quad \text{(same current!)}$$

$$\mathrm{I_{Middle}^{Actual}} = 0.22\angle -30.8^\circ \times 1250 \text{ Amps} = 275\angle -30.8^\circ \text{ A}$$

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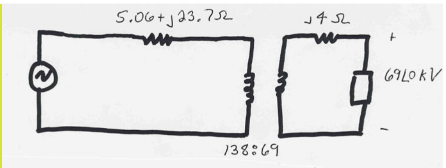
3Φ Per Unit Example 2

Assume a 3ϕ load of 100+j50 MVA with V_{LL} of 69 kV is connected to a source through the below network:



What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA

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Per-Unit Change on New Base

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<u>Changing Per-Unit Values to Different Bases:</u>

Normally, the per-unit (or percent) impedances of equipment are specified on the equipment base, which generally will be different from the <u>power system base</u>.

Since all impedances in the system must be expressed on the same base for per-unit (or percent) calculations, it is necessary to convert all values to the "common base" selected.

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Per-Unit System

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Recall the definition of the per-unit:

$$pu_{old} = \frac{actual}{base_{old}} \qquad pu_{new} = \frac{actual}{base_{new}}$$

By dividing the two equations:

$$\frac{pu_{new}}{pu_{old}} = \frac{base_{old}}{base_{new}} \quad \text{or} \quad \boxed{pu_{new} = pu_{old}\left(\frac{base_{old}}{base_{new}}\right)}$$

Applying this to per-unit impedance,

$$Z_{pu\,new} = Z_{pu\,old}\frac{Z_{base\,old}}{Z_{base\,new}} = Z_{pu\,old}\frac{\dfrac{V^2_{base\,old}}{S_{base\,old}}}{\dfrac{V^2_{base\,new}}{S_{base\,new}}}$$

$$\therefore \boxed{Z_{pu\,new} = Z_{pu\,old}\left(\frac{V_{base\,old}}{V_{base\,new}}\right)^2\boxed{\left(\frac{S_{base\,new}}{S_{base\,old}}\right)}}$$

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Transformer Reactance

- Transformer reactance is often specified as a percentage, say 10%. This is a per unit value (multiplied by 100) on the power base of the transformer.

- Example: A 54 MVA transformer has a leakage reactance of 3.69%. What is the reactance on a 100 MVA base?

$$X_e = 0.0369 \times \boxed{\frac{100}{54}} = 0.0683 \ \text{p.u.}$$

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Transformer Reactance

- Example: A 350 MVA, 230/20 kV transformer has leakage reactance of 10%. What is p.u. value on 100 MVA base? What is value in ohms (230 kV)?

$$X_e = 0.10 \times \boxed{\frac{100}{350}} = 0.0286 \ \text{p.u.}$$

$$0.0286 \times \frac{230^2}{100} = 15.1\,\Omega$$

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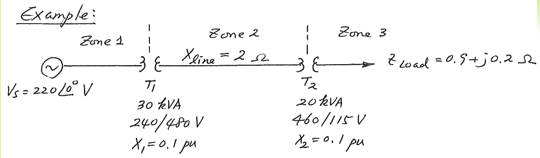
Per-Unit System

Example:



The system is 1ϕ. Use base values of T_1 for the entire system. Draw the per-unit impedance diagram and find the load current.

$S_{base} = 30$ kVA (T_1) for the whole system, System Base.

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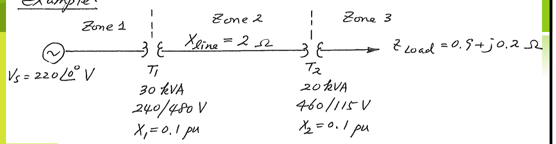
Per-Unit System

Example:



$S_{base} = 30$ kVA (T_1) for the whole system, System Base.

However, there are three different base voltages, one for each zone: Again using the voltages of T_1,

V_{base}	$V_{base\,1} = 240\text{ V}$	$V_{base\,2} = 480\text{ V}$,	$V_{base\,3} = 480\left(\frac{115}{460}\right) = 120\text{ V}$
Z_{base}	$Z_{base\,1} = \frac{(240)^2}{30{,}000} = 1.92\ \Omega$	$Z_{base\,2} = \frac{(480)^2}{30{,}000} = 7.68\ \Omega$	$Z_{base\,3} = \frac{(120)^2}{30{,}000} = 0.48\ \Omega$
I_{base}			$I_{base\,3} = \frac{30{,}000}{120} = 250\text{ A}$

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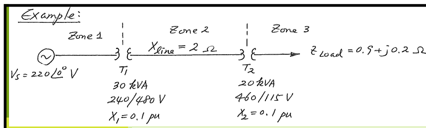
IT System



Next, convert the per-unit impedance of T_2 using the system base values:

$$X_2 = (0.1)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \text{ pu}$$

or

$$X_2 = (0.1)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \text{ pu}$$

The line, located in zone 2, has the per-unit impedance

$$X_{line\,pu} = \frac{X_{line}}{Z_{base\,2}} = \frac{2}{7.68} = 0.2604 \text{ pu}$$

The load, located in zone 3, has the per-unit impedance

$$Z_{load\,pu} = \frac{Z_{load}}{Z_{base\,3}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.416 \text{ pu}$$

The source, located in zone 1, has the per-unit voltage

$$V_{S\,pu} = \frac{V_S}{V_{base\,1}} = \frac{220\angle 0^\circ}{240} = 0.9167\angle 0^\circ \text{ pu}$$

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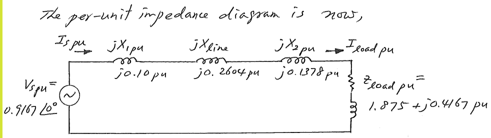
Per-Unit System

The per-unit impedance diagram is now,



The load current is then

$$I_{load\,pu} = I_{S\,pu} = \frac{V_{S\,pu}}{j(X_{1\,pu} + X_{line\,pu} + X_{2\,pu}) + Z_{load\,pu}}$$

$$= \frac{0.9167\angle 0^\circ}{j(0.10 + 0.2604 + 0.1378) + (1.875 + 0.4167)}$$

$$= \frac{0.9167\angle 0^\circ}{1.875 + j0.9149} = \frac{0.9167\angle 0^\circ}{2.086\angle 26.01^\circ} = 0.4395\angle -26.01^\circ \text{ pu}$$

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PER-UNIT SYSTEM

The actual load current is

$$I_{load} = (I_{load\,pu})\, I_{base\,3} = (0.4395\angle{-26.01^\circ})(250\,A)$$
$$= \underline{\underline{109.9}}\,\angle{-26.01^\circ}\ A$$

Question: How can you find the current from the source?
How about the current in the line?
How can you find the voltage at the load?

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