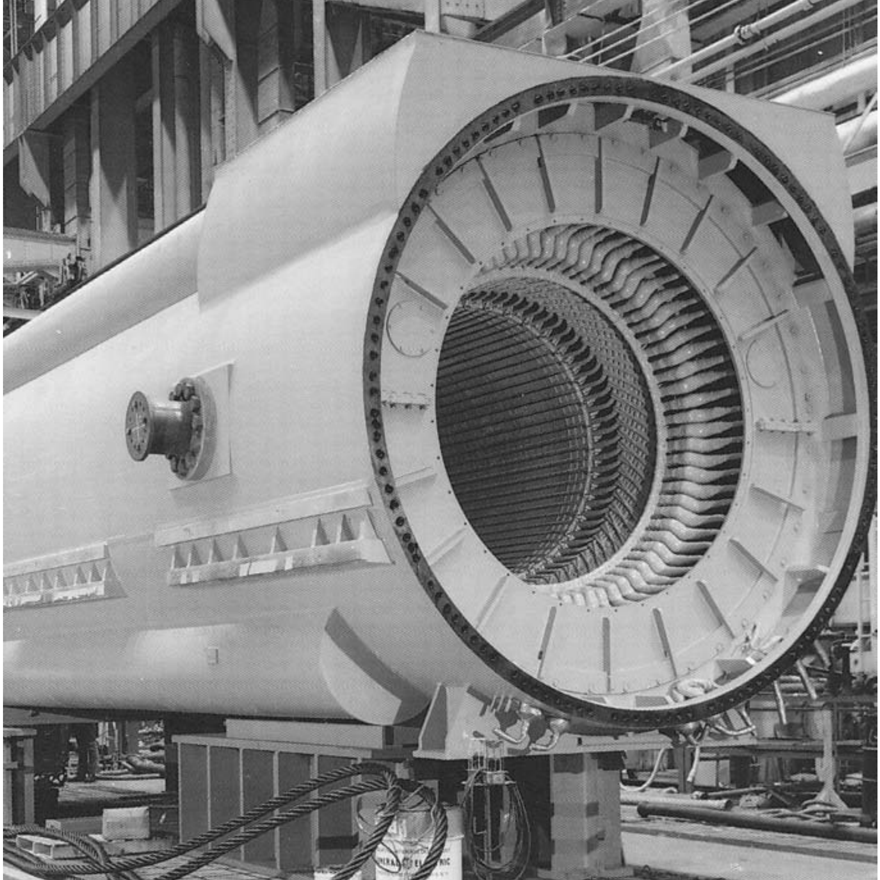

Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric)

8

SYMMETRICAL COMPONENTS

The method of symmetrical components, first developed by C. L. Fortescue in 1918, is a powerful technique for analyzing unbalanced three-phase systems. Fortescue defined a linear transformation from phase components to a new set of components called *symmetrical components*. The advantage of this transformation is that for balanced three-phase networks the equivalent circuits obtained for the symmetrical components, called *sequence networks*, are separated into three uncoupled networks. Furthermore, for unbalanced three-phase systems, the three sequence networks are connected only at points of unbalance. As a result, sequence networks for many cases of unbalanced three-phase systems are relatively easy to analyze.

The symmetrical component method is basically a modeling technique that permits systematic analysis and design of three-phase systems. Decoupling a detailed three-phase network into three simpler sequence networks reveals complicated phenomena in more simplistic terms. Sequence network results can then be superposed to obtain three-phase network results. As an example, the application of symmetrical components to unsymmetrical short-circuit studies (see Chapter 9) is indispensable.

The objective of this chapter is to introduce the concept of symmetrical components in order to lay a foundation and provide a framework for later chapters covering both equipment models as well as power system analysis and design methods. In Section 8.1, we define symmetrical components. In Sections 8.2–8.7, we present sequence networks of loads, series impedances, transmission lines, rotating machines, and transformers. We discuss complex power in sequence networks in Section 8.8. Although Fortescue's original work is valid for polyphase systems with n phases, we will consider only three-phase systems here.

CASE STUDY

The following article provides an overview of circuit breakers with high voltage ratings at or above 72.5 kV [4]. Circuit breakers are broadly classified by the medium used to extinguish the arc: bulk oil, minimum oil, air-blast, vacuum, and sulfur hexafluoride (SF_6). For high voltages, oil circuit breakers dominated in the early 1900s through the 1950s for applications up to 362 kV, with minimum oil circuit breakers developed up to 380 kV. The development of air-blast circuit breakers started in Europe in the 1920s and became prevalent in the 1950s. Air-blast circuit breakers, which use air under high pressure that is blown between the circuit breaker contacts to extinguish the arc, have been used at voltages up to 800 kV and many are still in operation today. Air-blast circuit breakers were manufactured until the 1980s when they were supplanted by lower cost and simpler SF_6 puffer-type circuit breakers. SF_6 gas possesses exceptional arc-interrupting properties that have led to a worldwide change to SF_6 high-voltage circuit breakers, which are more reliable, more efficient and more compact than other types of circuit breakers. Vacuum circuit breakers are commonly used at medium voltages between 1 and 72.5 kV.

Circuit Breakers Go High Voltage: The Low Operating Energy of SF_6 Circuit Breakers Improves Reliability and Reduces Wear and Tear

DENIS DUFOURNET

The first sulfur hexafluoride (SF_6) gas industrial developments were in the medium voltage range. This equipment confirmed the advantages of a technique that uses SF_6 at a low-pressure level concurrently with the auto-pneumatic blast system to interrupt the arc that was called later puffer.

High-voltage SF_6 circuit breakers with self-blast interrupters have found worldwide acceptance because their high current interrupting capability is obtained with a low operating energy that can be provided by low-cost, spring-operated mechanisms. The low-operating energy required reduces the stress and wear of the mechanical components and significantly improves the overall reliability of the circuit breaker. This switching principle was first introduced in the high-voltage area about 20 years ago, starting with the voltage level of 72.5 kV. Today this technique is available up to 800 kV. Furthermore it is used for generator circuit breaker applications with short circuit currents of 63 kA and above.

Service experience shows that when the SF_6 circuit breakers of the self-blast technology were first designed, the expectations of the designers had been fulfilled completely with respect to reliability and day-to-day operation.

A HISTORY OF CIRCUIT BREAKERS

Bulk oil circuit breakers dominated in the early 1900s and remained in use throughout the 1950s, for applications up to 362 kV for which they had eight breaks in series. They were replaced by minimum oil and air-blast circuit breakers for high-voltage applications.

Minimum oil circuit breakers, as shown in Figure 1, have arc control structures that improve the arc cooling process and significantly reduce the volume of oil. They were developed up to 380 kV, in particular for the first 380 kV network in the world (Harsprånget–Halsberg line in Sweden in 1952). There were tentative extensions to 765 kV, 50 kA, but minimum oil circuit breakers were supplanted in the EHV range by air-blast circuit breakers that were the first to be applied in 525, 735, and 765 kV networks, respectively in Russia (1960), Canada (1965), and the United States (1969).

Air-blast circuit breakers, as shown in Figure 2, use air under high pressure that is blown through the arc space between the opening contacts to extinguish the arc. The development of air-blast circuit breakers started in Europe in the 1920s, with further development in 1930s and 1940s, and became prevalent in the 1950s.

Air-blast circuit breakers were very successful in North America and Europe. They had an interrupting capability of 63 kA, later increased to 90 kA in the 1970s. Many circuit breakers of this type are still in operation today, in particular in North America, at 550 and 800 kV.

Air-blast circuit breakers were manufactured until the 1980s when they were supplanted by the lower cost and less complex SF_6 puffer-type circuit breakers.

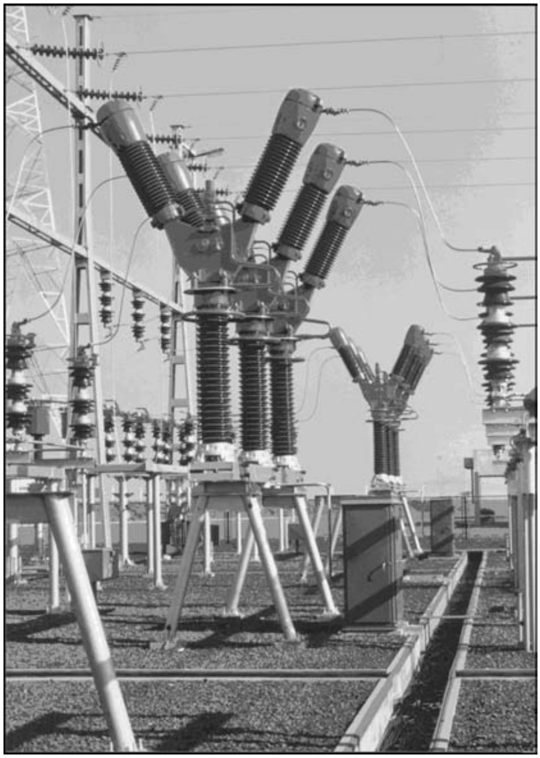

Figure 1
Minimum oil circuit breaker 145 kV type orthojector (Courtesy of Alstom Grid)

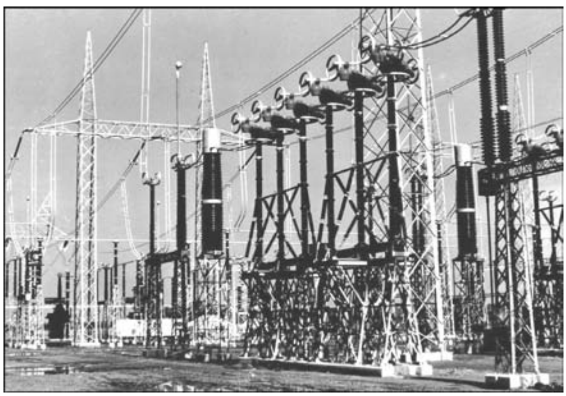

Figure 2
Air-blast circuit breaker type PK12 applied to 765 kV in North America (Courtesy of Alstom Grid)

The first industrial application of SF_6 dates from 1937 when it was used in the United States as an insulating medium for cables (patent by F.S. Cooper of General Electric). With the advent of the nuclear power industry in the 1950s, SF_6 was produced in large quantities and its use extended to circuit breakers as a quenching medium.

The first application of SF_6 for current interruption was done in 1953 when 15–161 kV switches were developed by Westinghouse. The first high-voltage SF_6 circuit breakers were built also by Westinghouse in 1956, the interrupting capability was then limited to 5 kA under 115 kV, with each pole having six interrupting units in series. In 1959, Westinghouse produced the first SF_6 circuit breakers with high current interrupting capabilities: 41.8 kA under 138 kV (10,000 MVA) and 37.8 kA under 230 kV (15,000 MVA). These circuit breakers were of the dual pressure type based on the axial blast principles used in air-blast circuit breakers. They were supplanted by the SF_6 puffer circuit breakers.

In 1967, the puffer-type technique was introduced for high-voltage circuit breakers where the relative movement of a piston and a cylinder linked to the moving contact produced the pressure build-up necessary to blast the arc. The puffer technique, shown in Figure 3, was applied in the first 245 kV metal-enclosed gas insulated circuit breaker installed in France in 1969.

The excellent properties of SF_6 lead to the fast extension of this technique in the 1970s and to its use for the development of circuit breakers with high current interrupting capability, up to 800 kV.

The achievement, around 1983, of the first single-break 245 kV and the corresponding 420 kV, 550 kV, and 800 kV, with, respectively, two, three, and four chambers per pole, lead to the dominance of SF_6 circuit breakers in the complete high-voltage range.

Several characteristics of SF_6 puffer circuit breakers can explain their success:

- simplicity of the interrupting chamber which does not need an auxiliary chamber for breaking
- autonomy provided by the puffer technique
- the possibility to obtain the highest performances, up to 63 kA, with a reduced number of interrupting chambers (Figure 4)

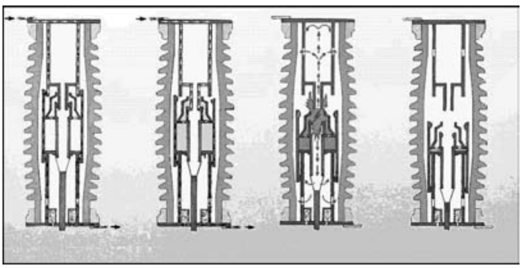

Figure 3
Puffer-type circuit breaker

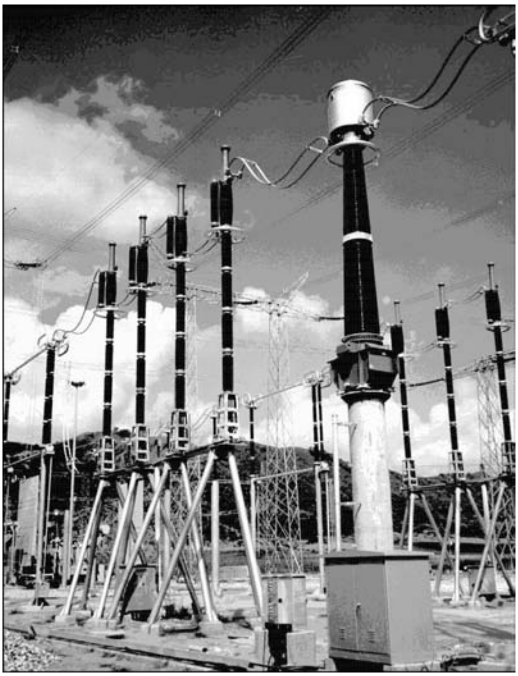

Figure 4
800 kV 50 kA circuit breaker type FX with closing resistors (Courtesy of Alstom Grid)

- short interrupting time of 2-2. 5 cycles at 60 Hz
- high electrical endurance, allowing at least 25 years of operation without reconditioning
- possible compact solutions when used for gas-insulated switchgear (GIS) or hybrid switchgears
- integrated closing resistors or synchronized operations to reduce switching over voltages
- reliability and availability
- low noise level
- no compressor for SF_6 gas.

The reduction in the number of interrupting chambers per pole has led to a considerable simplification of circuit breakers as the number of parts as well as the number of seals was decreased. As a direct consequence, the reliability of circuit breakers was improved, as verified later by CIGRE surveys.

SELF-BLAST TECHNOLOGY

The last 20 years have seen the development of the self-blast technique for SF_6 interrupting chambers. This technique has proven to be very efficient and has been widely applied for high-voltage circuit breakers up to 800 kV. It has allowed the development of new ranges of circuit breakers operated by low energy spring-operated mechanisms.

Another aim of this evolution was to further increase the reliability by reducing dynamic forces in the pole and its mechanism.

These developments have been facilitated by the progress made in digital simulations that were widely used to optimize the geometry of the interrupting chamber and the mechanics between the poles and the mechanism.

The reduction of operating energy was achieved by lowering energy used for gas compression and by making a larger use of arc energy to produce the pressure necessary to quench the arc and obtain current interruption.

Low-current interruption, up to about 30% of rated short-circuit current, is obtained by a puffer blast where the overpressure necessary to quench the arc is produced by gas compression in a volume limited by a fixed piston and a moving cylinder.

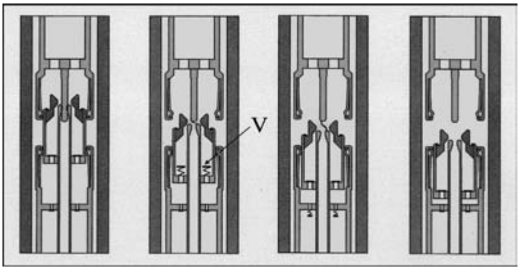

Figure 5
Self blast (or double volume) interrupting chamber

Figure 5 shows the self-blast interruption principle where a valve (V) was introduced between the expansion and the compression volume.

When interrupting low currents, the valve (V) opens under the effect of the overpressure generated in the compression volume. The interruption of the arc is made as in a puffer circuit breaker thanks to the compression of the gas obtained by the piston action.

In the case of high-current interruption, the arc energy produces a high overpressure in the expansion volume, which leads to the closure of the valve (V) and thus isolating the expansion volume from the compression volume. The overpressure necessary for breaking is obtained by the optimal use of the thermal effect and of the nozzle clogging effect produced whenever the cross-section of the arc significantly reduces the exhaust of gas in the nozzle.

This technique, known as self-blast, has been used extensively for more than 15 years for the development of many types of interrupting chambers and circuit breakers (Figure 6).

The better knowledge of arc interruption obtained by digital simulations and validation of performances by interrupting tests has contributed to a higher reliability of these self-blast circuit breakers. In addition, the reduction in

Figure 6
Dead tank circuit breaker 145 kV with spring-operating mechanism and double motion self blast interrupting chambers (Courtesy of Alstom Grid)

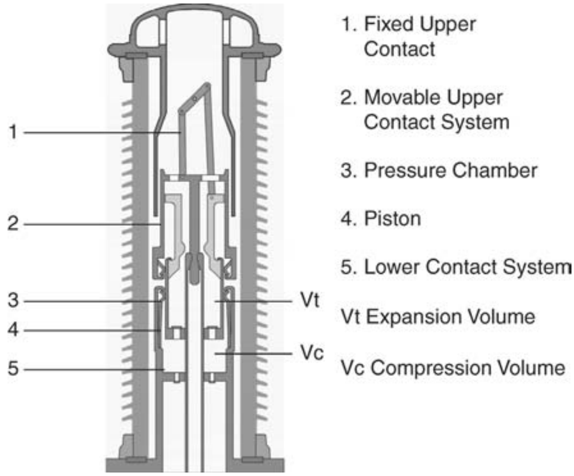

Figure 7
Double motion interrupting chamber

operating energy, allowed by the self-blast technique, leads to a higher mechanical endurance.

DOUBLE MOTION PRINCIPLE

The self-blast technology was further optimized by using the double-motion principle. This leads to further reduction of the operating energy by reducing the kinetic energy consumed during opening. The method consists of displacing the two arcing contacts in opposite directions. With such a system, it was possible to reduce the necessary opening energy for circuit breakers drastically.

Figure 7 shows the arcing chamber of a circuit breaker with the double motion principle. The pole columns are equipped with helical springs mounted in the crankcase.

These springs contain the necessary energy for an opening operation. The energy of the spring is transmitted to the arcing chamber via an insulating rod.

To interrupt an arc, the contact system must have sufficient velocity to avoid reignitions. Furthermore, a pressure rise must be generated to establish a gas flow in the chamber.

The movable upper contact system is connected to the nozzle of the arcing chamber via a linkage system. This allows the movement of both arcing contacts in opposite directions. Therefore the velocity of one contact can be reduced by 50% because the relative velocity of both contacts is still 100%. The necessary kinetic energy scales with the square of the velocity, allowing—theoretically—an energy reduction in the opening spring by a factor of 4. In reality, this value can't be achieved because the moving mass has to be increased. As in the self-blast technique described previously, the arc itself mostly generates the pressure rise.

Because the pressure generation depends on the level of the short-circuit current, an additional small piston is necessary to interrupt small currents (i.e., less than 30% of the rated short-circuit current). Smaller pistons mean less operating energy.

The combination of both double motion of contacts and self-blast technique allows for the significant reduction of opening energy.

GENERATOR CIRCUIT BREAKERS

Generator circuit breakers are connected between a generator and the step-up voltage transformer. They are generally used at the outlet of high-power generators (100–1,800 MVA) to protect them in a sure, quick, and economical manner. Such circuit breakers must be able to allow the passage of high permanent currents under continuous service (6,300–40,000 A), and have a high breaking capacity (63–275 kA).

They belong to the medium voltage range, but the transient recovery voltage (TRV) withstand capability is such that the interrupting principles developed for the high-voltage range has been used. Two particular embodiments of the thermal blast and self-blast techniques have been developed and applied to generator circuit breakers.

Thermal Blast Chamber with Arc-Assisted Opening

In this interruption principle arc energy is used, on the one hand to generate the blast by thermal expansion and, on the other hand, to accelerate the moving part of the circuit breaker when interrupting high currents (Figure 8).

The overpressure produced by the arc energy downstream of the interruption zone is applied on an auxiliary piston linked with the moving part. The resulting force accelerates the moving part, thus increasing the energy available for tripping.

It is possible with this interrupting principle to increase the tripping energy delivered by the operating mechanism by about 30% and to maintain the opening speed irrespective of the short circuit current.

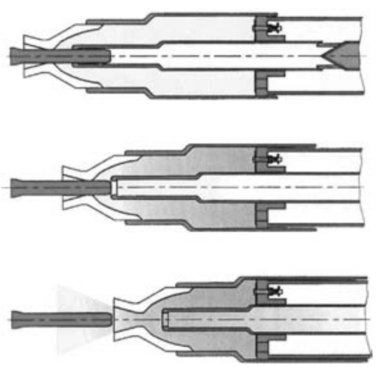

Figure 8
Thermal blast chamber with arc-assisted opening

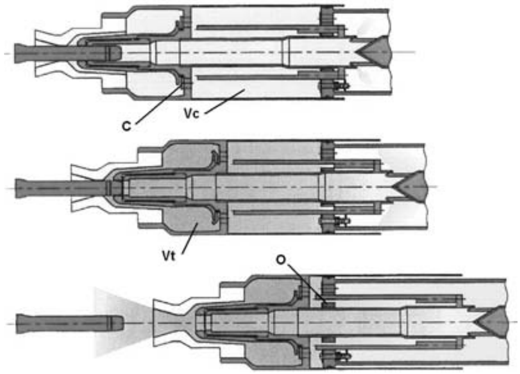

Figure 9
Self-blast chamber with rear exhaust

It is obviously better suited to circuit breakers with high breaking currents such as generator circuit breakers that are required to interrupt currents as high as 120 kA or even 160 kA.

Self-Blast Chamber with Rear Exhaust

This principle works as follows (Figure 9): In the first phase, the relative movement of the piston and the blast cylinder is used to compress the gas in the compression volume Vc. This overpressure opens the valve C and is then transmitted to expansion volume Vt.

In the second phase, gas in volume Vc is exhausted to the rear through openings (O).

The gas compression is sufficient for the interruption of low currents. During high short-circuit current interruption, volume Vt is pressurized by the thermal energy of the arc. This high pressure closes valve C. The pressure in volume Vc on the other hand is limited by an outflow of gas through the openings (O). The high overpressure generated in volume Vt produces the quenching blast necessary to extinguish the arc at current zero.

In this principle the energy that has to be delivered by the operating mechanism is limited and low energy spring operated mechanism can be used.

Figure 10 shows a generator circuit breaker with such type of interrupting chamber.

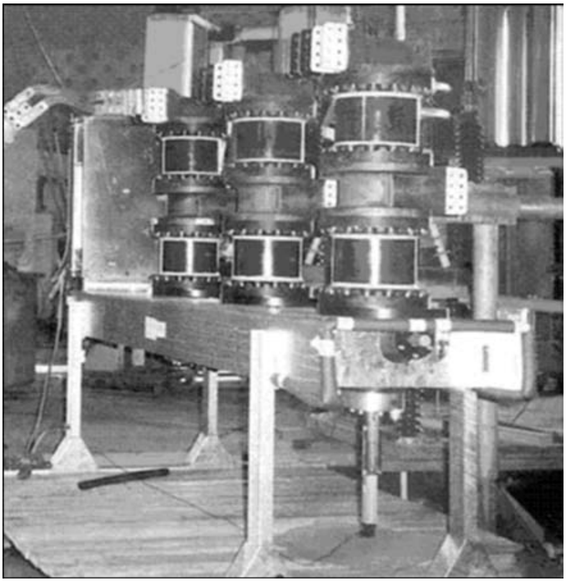

Figure 10
Generator circuit breaker SF_6 17, 5 kV 63 kA 60 Hz

EVOLUTION OF TRIPPING ENERGY

Figure 11 summarizes the evolution of tripping energy for 245 and 420 kV, from 1974 to 2003. It shows that the operating energy has been divided by a factor of five–seven during this period of nearly three decades. This illustrates the great

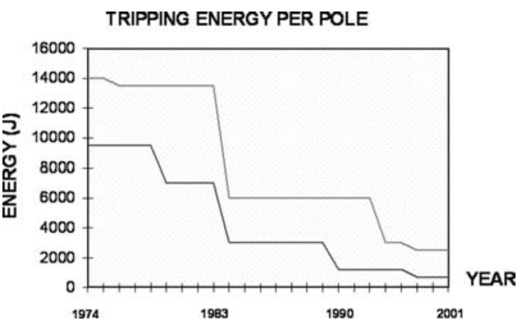

Figure 11
Evolution of tripping energy since 1974 of 245 and 420 kV circuit breakers

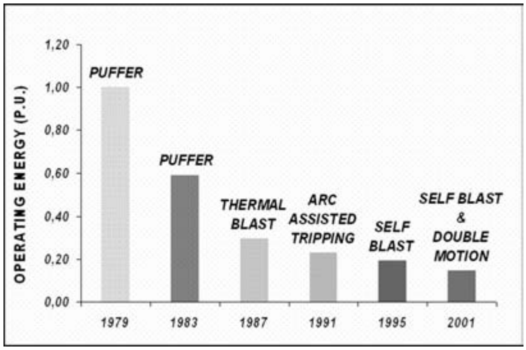

Figure 12
Operating energy as function of interrupting principle

progress that has been made in interrupting techniques for high-voltage circuit breakers during that period.

Figure 12 shows the continuous reduction of the necessary operating energy obtained through the technological progress.

OUTLOOK FOR THE FUTURE

Several interrupting techniques have been presented that all aim to reduce the operating energy of high-voltage circuit breakers. To date they have been widely applied, resulting in the lowering of drive energy, as shown in Figures 11 and 12.

Present interrupting technologies can be applied to circuit breakers with the higher rated interrupting currents (63–80 kA) required in some networks with increasing power generation (Figure 13).

Progress can still be made by the further industrialization of all components and by introducing new drive technologies. Following the remarkable evolution in chamber technology, the operating mechanism represents a not negligible contribution to the moving mass of circuit breakers, especially in the extra high-voltage range ($\geq$ 420 kV). Therefore progress in high-voltage circuit breakers can still be expected with the implementation of the same interrupting principles.

If one looks further in the future, other technology developments could possibly lead to a

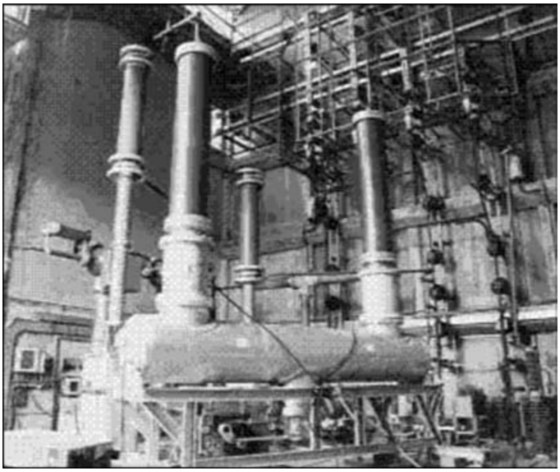

Figure 13
GIS circuit breaker 550 kV 63 kA 50/60 Hz

further reduction in the SF_6 content of circuit breakers.

CONCLUSIONS

Over the last 50 years, high-voltage circuit breakers have become more reliable, more efficient, and more compact because the interrupting capability per break has been increased dramatically. These developments have not only produced major savings, but they have also had a massive impact on the layout of substations with respect to space requirements.

New types of SF_6 interrupting chambers, which implement innovative interrupting principles, have been developed during the last three decades with the objective of reducing the operating energy of the circuit breaker. This has led to reduced stress and wear of the mechanical components and consequently to an increased reliability of circuit breakers.

Service experience shows that the expectations of the designers, with respect to reliability and day-to-day operation, have been fulfilled.

FOR FURTHER READING

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BIOGRAPHY

Denis Dufournet is with AREVA T&D.

8.1

DEFINITION OF SYMMETRICAL COMPONENTS

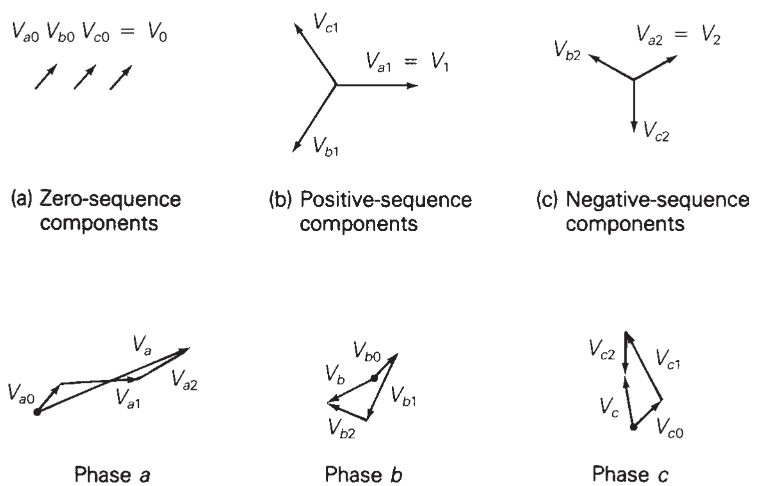
Assume that a set of three-phase voltages designated V_a, V_b, and V_c is given. In accordance with Fortescue, these phase voltages are resolved into the following three sets of sequence components:

1. *Zero-sequence* components, consisting of three phasors with equal magnitudes and with zero phase displacement, as shown in Figure 8.1(a)
2. *Positive-sequence* components, consisting of three phasors with equal magnitudes, $\pm 120°$ phase displacement, and positive sequence, as in Figure 8.1(b)
3. *Negative-sequence* components, consisting of three phasors with equal magnitudes, $\pm 120°$ phase displacement, and negative sequence, as in Figure 8.1(c)

In this text we will work only with the zero-, positive-, and negative-sequence components of phase a, which are V_{a0}, V_{a1}, and V_{a2}, respectively. For simplicity, we drop the subscript a and denote these sequence components as V_0, V_1, and V_2. They are defined by the following transformation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \tag{8.1.1}$$

FIGURE 8.1 Resolving phase voltages into three sets of sequence components



where

$$a = 1\angle 120^\circ = \frac{-1}{2} + j\frac{\sqrt{3}}{2} \tag{8.1.2}$$

Writing (8.1.1) as three separate equations:

$$V_a = V_0 + V_1 + V_2 \tag{8.1.3}$$

$$V_b = V_0 + a^2 V_1 + a V_2 \tag{8.1.4}$$

$$V_c = V_0 + a V_1 + a^2 V_2 \tag{8.1.5}$$

In (8.1.2), a is a complex number with unit magnitude and a 120° phase angle. When any phasor is multiplied by a, that phasor rotates by 120° (counterclockwise). Similarly, when any phasor is multiplied by $a^2 = (1\angle 120^\circ)\cdot(1\angle 120^\circ) = 1\angle 240^\circ$, the phasor rotates by 240°. Table 8.1 lists some common identities involving a.

The complex number a is similar to the well-known complex number $j = \sqrt{-1} = 1\angle 90^\circ$. Thus the only difference between j and a is that the angle of j is 90°, and that of a is 120°.

Equation (8.1.1) can be rewritten more compactly using matrix notation. We define the following vectors $\boldsymbol{V}_p$ and $\boldsymbol{V}_s$, and matrix $\boldsymbol{A}$:

$$\boldsymbol{V}_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{8.1.6}$$

$$\boldsymbol{V}_s = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \tag{8.1.7}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{8.1.8}$$

TABLE 8.1

Common identities involving $a = 1\angle 120^\circ$

$a^4 = a = 1\angle 120^\circ$
$a^2 = 1\angle 240^\circ$
$a^3 = 1\angle 0^\circ$
$1 + a + a^2 = 0$
$1 - a = \sqrt{3}\angle -30^\circ$
$1 - a^2 = \sqrt{3}\angle +30^\circ$
$a^2 - a = \sqrt{3}\angle 270^\circ$
$ja = 1\angle 210^\circ$
$1 + a = -a^2 = 1\angle 60^\circ$
$1 + a^2 = -a = 1\angle -60^\circ$
$a + a^2 = -1 = 1\angle 180^\circ$

$\boldsymbol{V}_p$ is the column vector of phase voltages, $\boldsymbol{V}_s$ is the column vector of sequence voltages, and $\boldsymbol{A}$ is a 3×3 transformation matrix. Using these definitions, (8.1.1) becomes

$$\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s \tag{8.1.9}$$

The inverse of the $\boldsymbol{A}$ matrix is

$$\boldsymbol{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{8.1.10}$$

Equation (8.1.10) can be verified by showing that the product $\boldsymbol{A}\boldsymbol{A}^{-1}$ is the unit matrix. Also, premultiplying (8.1.9) by $\boldsymbol{A}^{-1}$ gives

$$\boldsymbol{V}_s = \boldsymbol{A}^{-1}\boldsymbol{V}_p \tag{8.1.11}$$

Using (8.1.6), (8.1.7), and (8.1.10), then (8.1.11) becomes

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{8.1.12}$$

Writing (8.1.12) as three separate equations,

$$V_0 = \tfrac{1}{3}(V_a + V_b + V_c) \tag{8.1.13}$$

$$V_1 = \tfrac{1}{3}(V_a + aV_b + a^2V_c) \tag{8.1.14}$$

$$V_2 = \tfrac{1}{3}(V_a + a^2V_b + aV_c) \tag{8.1.15}$$

Equation (8.1.13) shows that there is no zero-sequence voltage in a *balanced* three-phase system because the sum of three balanced phasors is zero. In an unbalanced three-phase system, line-to-neutral voltages may have a zero-sequence component. However, line-to-line voltages never have a zero-sequence component, since by KVL their sum is always zero.

The symmetrical component transformation can also be applied to currents, as follows. Let

$$\boldsymbol{I}_p = \boldsymbol{A}\boldsymbol{I}_s \tag{8.1.16}$$

where $\boldsymbol{I}_p$ is a vector of phase currents,

$$\boldsymbol{I}_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.1.17}$$

and $\boldsymbol{I}_s$ is a vector of sequence currents,

$$\boldsymbol{I}_s = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{8.1.18}$$

Also,

$$\boldsymbol{I}_s = \boldsymbol{A}^{-1}\boldsymbol{I}_p \tag{8.1.19}$$

Equations (8.1.16) and (8.1.19) can be written as separate equations as follows. The phase currents are

$$I_a = I_0 + I_1 + I_2 \tag{8.1.20}$$

$$I_b = I_0 + a^2I_1 + aI_2 \tag{8.1.21}$$

$$I_c = I_0 + aI_1 + a^2I_2 \tag{8.1.22}$$

and the sequence currents are

$$I_0 = \tfrac{1}{3}(I_a + I_b + I_c) \tag{8.1.23}$$

$$I_1 = \tfrac{1}{3}(I_a + aI_b + a^2I_c) \tag{8.1.24}$$

$$I_2 = \tfrac{1}{3}(I_a + a^2I_b + aI_c) \tag{8.1.25}$$

In a three-phase Y-connected system, the neutral current I_n is the sum of the line currents:

$$I_n = I_a + I_b + I_c \tag{8.1.26}$$

Comparing (8.1.26) and (8.1.23),

$$I_n = 3I_0 \tag{8.1.27}$$

The neutral current equals three times the zero-sequence current. In a balanced Y-connected system, line currents have no zero-sequence component, since the neutral current is zero. Also, in any three-phase system with no neutral path, such as a Δ-connected system or a three-wire Y-connected system with an ungrounded neutral, line currents have no zero-sequence component.

The following three examples further illustrate symmetrical components.

EXAMPLE 8.1 Sequence components: balanced line-to-neutral voltages

Calculate the sequence components of the following balanced line-to-neutral voltages with *abc* sequence:

$$\boldsymbol{V}_p = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 277\underline{/-120^\circ} \\ 277\underline{/+120^\circ} \end{bmatrix} \quad \text{volts}$$

SOLUTION Using (8.1.13)–(8.1.15):

$$V_0 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/-120^\circ} + 277\underline{/+120^\circ}] = 0$$

$$V_1 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/(-120^\circ + 120^\circ)} + 277\underline{/(120^\circ + 240^\circ)}]$$

$$= 277\underline{/0^\circ} \quad \text{volts} = V_{an}$$

$$V_2 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/(-120^\circ + 240^\circ)} + 277\underline{/(120^\circ + 120^\circ)}]$$

$$= \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/120^\circ} + 277\underline{/240^\circ}] = 0$$

This example illustrates the fact that balanced three-phase systems with *abc* sequence (or positive sequence) have no zero-sequence or negative-sequence components. For this example, the positive-sequence voltage V_1 equals V_{an}, and the zero-sequence and negative-sequence voltages are both zero. ■

EXAMPLE 8.2 Sequence components: balanced *acb* currents

A Y-connected load has balanced currents with *acb* sequence given by

$$\boldsymbol{I}_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 10\underline{/+120^\circ} \\ 10\underline{/-120^\circ} \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents.

SOLUTION Using (8.1.23)–(8.1.25):

$$I_0 = \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/120^\circ} + 10\underline{/-120^\circ}] = 0$$

$$I_1 = \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/(120^\circ + 120^\circ)} + 10\underline{/(-120^\circ + 240^\circ)}]$$

$$= \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/240^\circ} + 10\underline{/120^\circ}] = 0$$

$$I_2 = \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/(120^\circ + 240^\circ)} + 10\underline{/(-120^\circ + 120^\circ)}]$$

$$= 10\underline{/0^\circ} \text{ A} = I_a$$

This example illustrates the fact that balanced three-phase systems with *acb* sequence (or negative sequence) have no zero-sequence or positive-sequence components. For this example the negative-sequence current I_2 equals I_a, and the zero-sequence and positive-sequence currents are both zero. ■

EXAMPLE 8.3 Sequence components: unbalanced currents

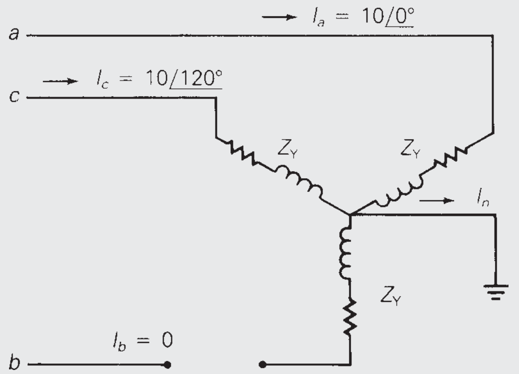
A three-phase line feeding a balanced-Y load has one of its phases (phase *b*) open. The load neutral is grounded, and the unbalanced line currents are

$$\boldsymbol{I}_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 0 \\ 10\underline{/120^\circ} \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents and the neutral current.

FIGURE 8.2

Circuit for Example 8.3



SOLUTION The circuit is shown in Figure 8.2. Using (8.1.23)–(8.1.25):

$$I_0 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/120^\circ}]$$
$$= 3.333\underline{/60^\circ} \quad \text{A}$$
$$I_1 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 240^\circ)}] = 6.667\underline{/0^\circ} \quad \text{A}$$
$$I_2 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 120^\circ)}]$$
$$= 3.333\underline{/-60^\circ} \quad \text{A}$$

Using (8.1.26) the neutral current is

$$I_n = (10\underline{/0^\circ} + 0 + 10\underline{/120^\circ})$$
$$= 10\underline{/60^\circ} \text{ A} = 3I_0$$

This example illustrates the fact that *unbalanced* three-phase systems may have nonzero values for all sequence components. Also, the neutral current equals three times the zero-sequence current, as given by (8.1.27). ■

8.2 SEQUENCE NETWORKS OF IMPEDANCE LOADS

Figure 8.3 shows a balanced-Y impedance load. The impedance of each phase is designated Z_Y, and a neutral impedance Z_n is connected between the load neutral and ground. Note from Figure 8.3 that the line-to-ground voltage V_{ag} is

$$V_{ag} = Z_Y I_a + Z_n I_n$$
$$= Z_Y I_a + Z_n(I_a + I_b + I_c)$$
$$= (Z_Y + Z_n)I_a + Z_n I_b + Z_n I_c \tag{8.2.1}$$

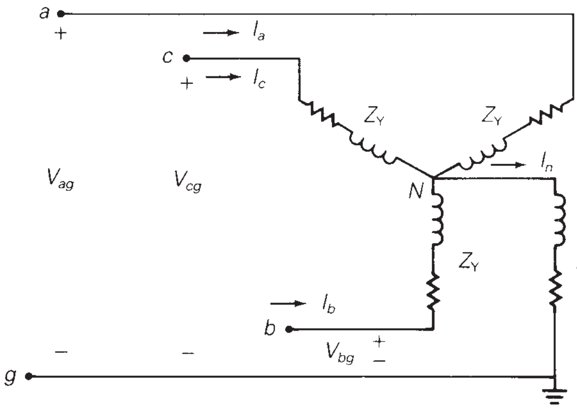


FIGURE 8.3 Balanced-Y impedance load

Similar equations can be written for V_{bg} and V_{cg}:

$$V_{bg} = Z_n I_a + (Z_Y + Z_n) I_b + Z_n I_c \tag{8.2.2}$$

$$V_{cg} = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c \tag{8.2.3}$$

Equations (8.2.1)–(8.2.3) can be rewritten in matrix format:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.2.4}$$

Equation (8.2.4) is written more compactly as

$$\boldsymbol{V_p} = \boldsymbol{Z_p I_p} \tag{8.2.5}$$

where $\boldsymbol{V_p}$ is the vector of line-to-ground voltages (or phase voltages), $\boldsymbol{I_p}$ is the vector of line currents (or phase currents), and $\boldsymbol{Z_p}$ is the 3×3 phase impedance matrix shown in (8.2.4). Equations (8.1.9) and (8.1.16) can now be used in (8.2.5) to determine the relationship between the sequence voltages and currents, as follows:

$$\boldsymbol{AV_s} = \boldsymbol{Z_p A I_s} \tag{8.2.6}$$

Premultiplying both sides of (8.2.6) of $\boldsymbol{A}^{-1}$ gives

$$\boldsymbol{V_s} = (\boldsymbol{A}^{-1} \boldsymbol{Z_p A}) \boldsymbol{I_s} \tag{8.2.7}$$

or

$$\boldsymbol{V_s} = \boldsymbol{Z_s I_s} \tag{8.2.8}$$

where

$$\boldsymbol{Z_s} = \boldsymbol{A}^{-1} \boldsymbol{Z_p A} \tag{8.2.9}$$

The impedance matrix $\boldsymbol{Z_s}$ defined by (8.2.9) is called the *sequence impedance matrix*. Using the definition of $\boldsymbol{A}$, its inverse $\boldsymbol{A}^{-1}$, and $\boldsymbol{Z_p}$ given by (8.1.8), (8.1.10), and (8.2.4), the sequence impedance matrix $\boldsymbol{Z}_s$ for the balanced-Y load is

$$\boldsymbol{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{8.2.10}$$

Performing the indicated matrix multiplications in (8.2.10), and using the identity $(1 + a + a^2) = 0$,

$$\boldsymbol{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} (Z_Y + 3Z_n) & Z_Y & Z_Y \\ (Z_Y + 3Z_n) & a^2 Z_Y & aZ_Y \\ (Z_Y + 3Z_n) & aZ_Y & a^2 Z_Y \end{bmatrix}$$

$$= \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \tag{8.2.11}$$

As shown in (8.2.11), the sequence impedance matrix $\boldsymbol{Z}_s$ for the balanced-Y load of Figure 8.3 is a diagonal matrix. Since $\boldsymbol{Z}_s$ is diagonal, (8.2.8) can be written as three *uncoupled* equations. Using (8.1.7), (8.1.18), and (8.2.11) in (8.2.8),

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{8.2.12}$$

Rewriting (8.2.12) as three separate equations,

$$V_0 = (Z_Y + 3Z_n)I_0 = Z_0 I_0 \tag{8.2.13}$$

$$V_1 = Z_Y I_1 = Z_1 I_1 \tag{8.2.14}$$

$$V_2 = Z_Y I_2 = Z_2 I_2 \tag{8.2.15}$$

As shown in (8.2.13), the zero-sequence voltage V_0 depends only on the zero-sequence current I_0 and the impedance $(Z_Y + 3Z_n)$. This impedance is called the *zero-sequence impedance* and is designated Z_0. Also, the positive-sequence voltage V_1 depends only on the positive-sequence current I_1 and an impedance $Z_1 = Z_Y$ called the *positive-sequence impedance*. Similarly, V_2 depends only on I_2 and the *negative-sequence impedance* $Z_2 = Z_Y$.

Equations (8.2.13)–(8.2.15) can be represented by the three networks shown in Figure 8.4. These networks are called the *zero-sequence*, *positive-sequence*, and *negative-sequence networks*. As shown, each sequence network

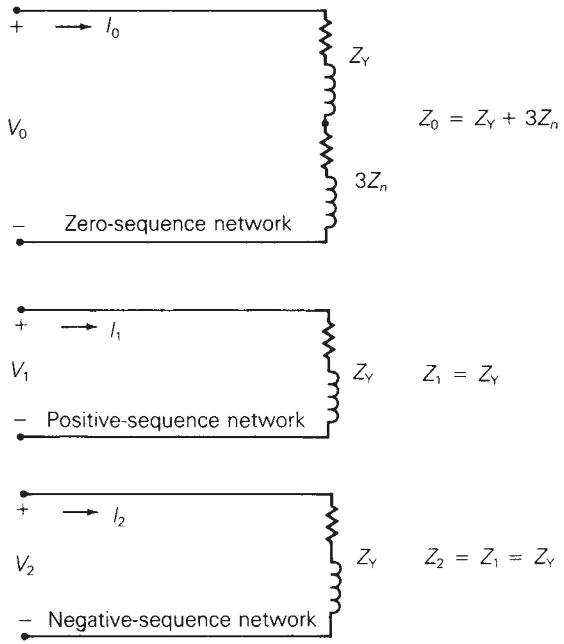


FIGURE 8.4

Sequence networks of a balanced-Y load

is separate, uncoupled from the other two. The separation of these sequence networks is a consequence of the fact that $\boldsymbol{Z}_s$ is a diagonal matrix for a balanced-Y load. This separation underlies the advantage of symmetrical components.

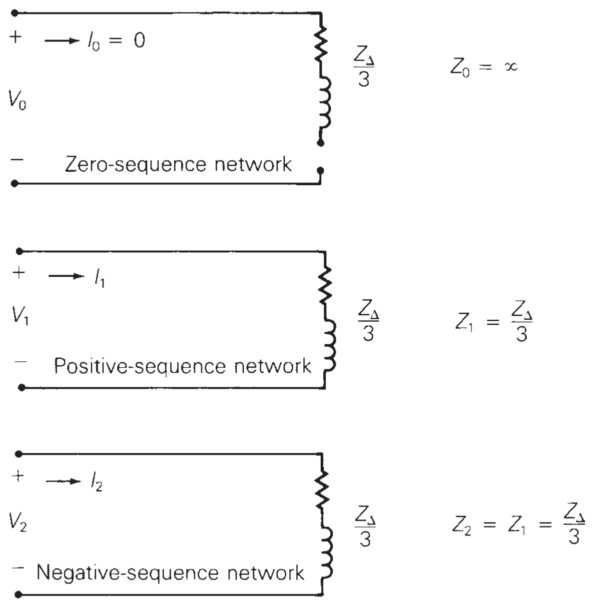
Note that the neutral impedance does not appear in the positive- and negative-sequence networks of Figure 8.4. This illustrates the fact that positive- and negative-sequence currents do not flow in neutral impedances. However, the neutral impedance is multiplied by 3 and placed in the zero-sequence network of the figure. The voltage $I_0(3Z_n)$ across the impedance $3Z_n$ is the voltage drop (I_nZ_n) across the neutral impedance Z_n in Figure 8.3, since $I_n = 3I_0$.

When the neutral of the Y load in Figure 8.3 has no return path, then the neutral impedance Z_n is infinite and the term $3Z_n$ in the zero-sequence network of Figure 8.4 becomes an open circuit. Under this condition of an open neutral, no zero-sequence current exists. However, when the neutral of the Y load is solidly grounded with a zero-ohm conductor, then the neutral impedance is zero and the term $3Z_n$ in the zero-sequence network becomes a short circuit. Under this condition of a solidly grounded neutral, zero-sequence current I_0 can exist when there is a zero-sequence voltage caused by unbalanced voltages applied to the load.

Figure 2.15 shows a balanced-Δ load and its equivalent balanced-Y load. Since the Δ load has no neutral connection, the equivalent Y load in Figure 2.15 has an open neutral. The sequence networks of the equivalent Y load corresponding to a balanced-Δ load are shown in Figure 8.5. As shown,

FIGURE 8.5

Sequence networks for an equivalent Y representation of a balanced-Δ load



the equivalent Y impedance $Z_Y = Z_\Delta/3$ appears in each of the sequence networks. Also, the zero-sequence network has an open circuit, since $Z_n = \infty$ corresponds to an open neutral. No zero-sequence current occurs in the equivalent Y load.

The sequence networks of Figure 8.5 represent the balanced-Δ load as viewed from its terminals, but they do not represent the internal load characteristics. The currents I_0, I_1, and I_2 in Figure 8.5 are the sequence components of the line currents feeding the Δ load, not the load currents within the Δ. The Δ load currents, which are related to the line currents by (2.5.14), are not shown in Figure 8.5.

EXAMPLE 8.4 Sequence networks: balanced-Y and balanced-Δ loads

A balanced-Y load is in parallel with a balanced-Δ-connected capacitor bank. The Y load has an impedance $Z_Y = (3 + j4)\ \Omega$ per phase, and its neutral is grounded through an inductive reactance $X_n = 2\ \Omega$. The capacitor bank has a reactance $X_c = 30\ \Omega$ per phase. Draw the sequence networks for this load and calculate the load-sequence impedances.

SOLUTION The sequence networks are shown in Figure 8.6. As shown, the Y-load impedance in the zero-sequence network is in series with three times the neutral impedance. Also, the Δ-load branch in the zero-sequence network is open, since no zero-sequence current flows into the Δ load. In the positive- and negative-sequence circuits, the Δ-load impedance is divided by 3 and placed in parallel with the Y-load impedance. The equivalent sequence impedances are

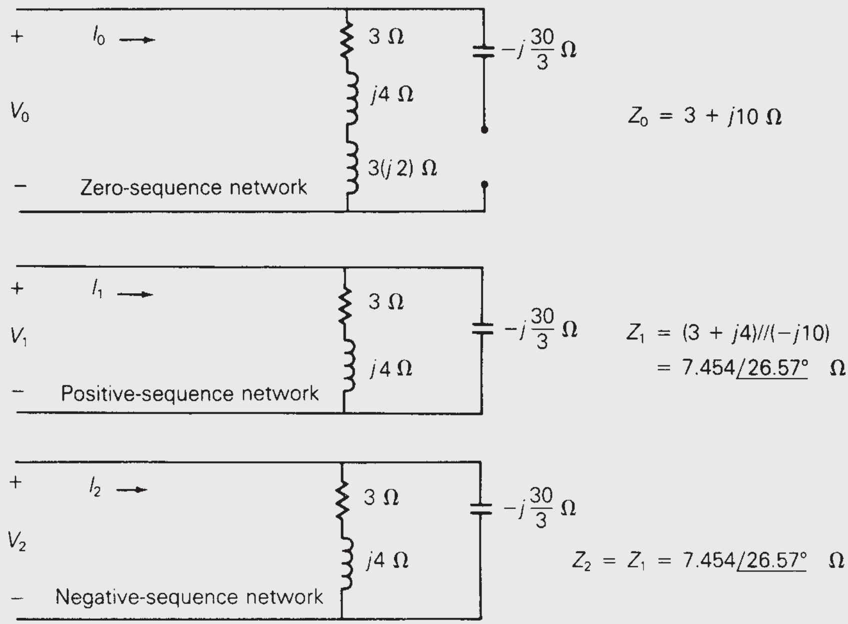


FIGURE 8.6
Sequence networks for Example 8.4

$$Z_0 = Z_Y + 3Z_n = 3 + j4 + 3(j2) = 3 + j10 \quad \Omega$$

$$Z_1 = Z_Y /\!/ (Z_\Delta/3) = \frac{(3 + j4)(-j30/3)}{3 + j4 - j(30/3)}$$

$$= \frac{(5\angle 53.13^\circ)(10\angle -90^\circ)}{6.708\angle -63.43^\circ} = 7.454\angle 26.57^\circ \quad \Omega$$

$$Z_2 = Z_1 = 7.454\angle 26.57^\circ \quad \Omega$$

■

Figure 8.7 shows a general three-phase linear impedance load. The load could represent a balanced load such as the balanced-Y or balanced-Δ load, or an unbalanced impedance load. The general relationship between the line-to-ground voltages and line currents for this load can be written as

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.2.16}$$

or

$$\boldsymbol{V_p} = \boldsymbol{Z_p I_p} \tag{8.2.17}$$

where $\boldsymbol{V_p}$ is the vector of line-to-neutral (or phase) voltages, $\boldsymbol{I_p}$ is the vector of line (or phase) currents, and $\boldsymbol{Z_p}$ is a 3×3 phase impedance matrix. It is assumed here that the load is nonrotating, and that $\boldsymbol{Z_p}$ is a symmetric matrix, which corresponds to a bilateral network.

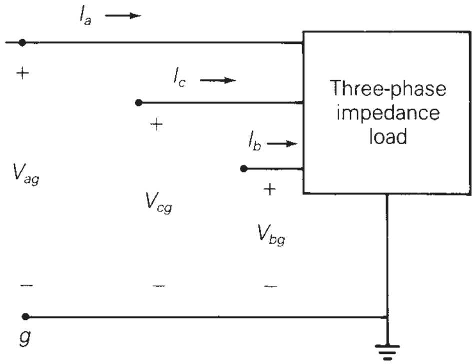


FIGURE 8.7 General three-phase impedance load (linear, bilateral network, nonrotating equipment)

Since (8.2.17) has the same form as (8.2.5), the relationship between the sequence voltages and currents for the general three-phase load of Figure 8.6 is the same as that of (8.2.8) and (8.2.9), which are rewritten here:

$$\boldsymbol{V}_s = \boldsymbol{Z}_s \boldsymbol{I}_s \tag{8.2.18}$$

$$\boldsymbol{Z}_s = \boldsymbol{A}^{-1} \boldsymbol{Z}_p \boldsymbol{A} \tag{8.2.19}$$

The sequence impedance matrix $\boldsymbol{Z}_s$ given by (8.2.19) is a 3×3 matrix with nine sequence impedances, defined as follows:

$$\boldsymbol{Z}_s = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} \tag{8.2.20}$$

The diagonal impedances Z_0, Z_1, and Z_2 in this matrix are the self-impedances of the zero-, positive-, and negative-sequence networks. The off-diagonal impedances are the mutual impedances between sequence networks. Using the definitions of $\boldsymbol{A}, \boldsymbol{A}^{-1}, \boldsymbol{Z}_p$, and $\boldsymbol{Z}_s$, (8.2.19) is

$$\begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{8.2.21}$$

Performing the indicated multiplications in (8.2.21), and using the identity $(1 + a + a^2) = 0$, the following separate equations can be obtained (see Problem 8.18):

Diagonal sequence impedances

$$Z_0 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc}) \tag{8.2.22}$$

$$Z_1 = Z_2 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc}) \tag{8.2.23}$$

Off-diagonal sequence impedances

$$Z_{01} = Z_{20} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc} - aZ_{ab} - a^2 Z_{ac} - Z_{bc}) \tag{8.2.24}$$

$$Z_{02} = Z_{10} = \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc} - a^2 Z_{ab} - aZ_{ac} - Z_{bc}) \tag{8.2.25}$$

$$Z_{12} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc} + 2aZ_{ab} + 2a^2 Z_{ac} + 2Z_{bc}) \tag{8.2.26}$$

$$Z_{21} = \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc} + 2a^2 Z_{ab} + 2aZ_{ac} + 2Z_{bc}) \tag{8.2.27}$$

A *symmetrical load* is defined as a load whose sequence impedance matrix is diagonal; that is, all the mutual impedances in (8.2.24)–(8.2.27) are zero. Equating these mutual impedances to zero and solving, the following conditions for a symmetrical load are determined. When both

$$Z_{aa} = Z_{bb} = Z_{cc} \tag{8.2.28}$$

and

$$Z_{ab} = Z_{ac} = Z_{bc} \tag{8.2.29}$$

$\Bigg\}$ conditions for a symmetrical load

then

$$Z_{01} = Z_{10} = Z_{02} = Z_{20} = Z_{12} = Z_{21} = 0 \tag{8.2.30}$$

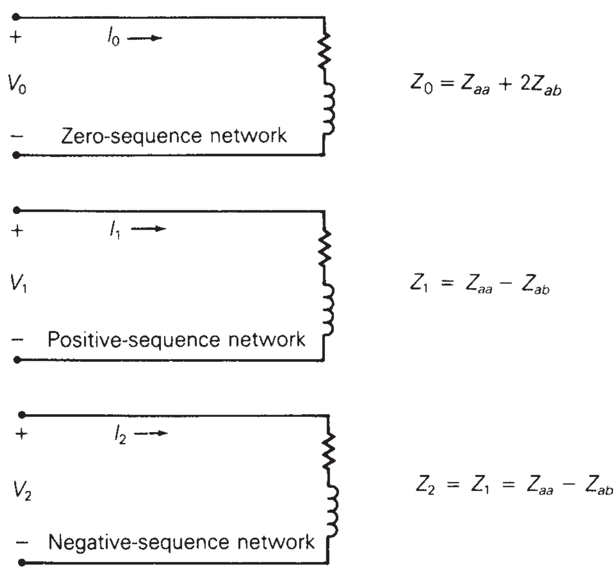
$$Z_0 = Z_{aa} + 2Z_{ab} \tag{8.2.31}$$

$$Z_1 = Z_2 = Z_{aa} - Z_{ab} \tag{8.2.32}$$

The conditions for a symmetrical load are that the diagonal phase impedances be equal and that the off-diagonal phase impedances be equal. These conditions can be verified by using (8.2.28) and (8.2.29) with the

FIGURE 8.8

Sequence networks of a three-phase symmetrical impedance load (linear, bilateral network, nonrotating equipment)



identity $(1 + a + a^2) = 0$ in (8.2.24)–(8.2.27) to show that all the mutual sequence impedances are zero. Note that the positive- and negative-sequence impedances are equal for a symmetrical load, as shown by (8.2.32), and for a nonsymmetrical load, as shown by (8.2.23). This is always true for linear, symmetric impedances that represent nonrotating equipment such as transformers and transmission lines. However, the positive- and negative-sequence impedances of rotating equipment such as generators and motors are generally not equal. Note also that the zero-sequence impedance Z_0 is not equal to the positive- and negative-sequence impedances of a symmetrical load unless the mutual phase impedances $Z_{ab} = Z_{ac} = Z_{bc}$ are zero.

The sequence networks of a symmetrical impedance load are shown in Figure 8.8. Since the sequence impedance matrix $\boldsymbol{Z_s}$ is diagonal for a symmetrical load, the sequence networks are separate or uncoupled.

8.3 SEQUENCE NETWORKS OF SERIES IMPEDANCES

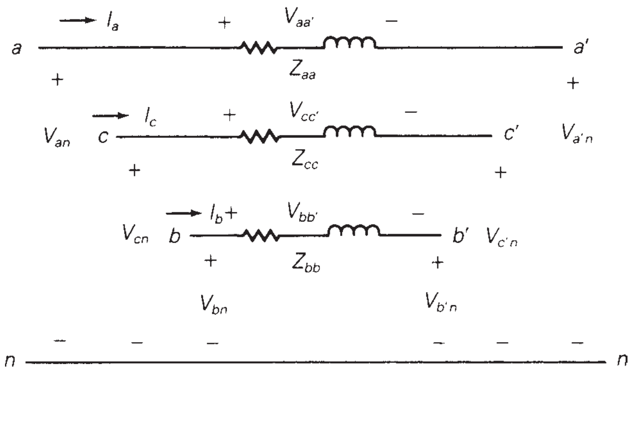
Figure 8.9 shows series impedances connected between two three-phase buses denoted abc and $a'b'c'$. Self-impedances of each phase are denoted Z_{aa}, Z_{bb}, and Z_{cc}. In general, the series network may also have mutual impedances between phases. The voltage drops across the series-phase impedances are given by

$$\begin{bmatrix} V_{an} - V_{a'n} \\ V_{bn} - V_{b'n} \\ V_{cn} - V_{c'n} \end{bmatrix} = \begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.3.1}$$

Both self-impedances and mutual impedances are included in (8.3.1). It is assumed that the impedance matrix is symmetric, which corresponds to a bilateral network. It is also assumed that these impedances represent

FIGURE 8.9

Three-phase series impedances (linear, bilateral network, nonrotating equipment)



nonrotating equipment. Typical examples are series impedances of transmission lines and of transformers. Equation (8.3.1) has the following form:

$$\boldsymbol{V}_p - \boldsymbol{V}_{p'} = \boldsymbol{Z}_p \boldsymbol{I}_p \tag{8.3.2}$$

where $\boldsymbol{V}_p$ is the vector of line-to-neutral voltages at bus abc, $\boldsymbol{V}_{p'}$ is the vector of line-to-neutral voltages at bus $a'b'c'$, $\boldsymbol{I}_p$ is the vector of line currents, and $\boldsymbol{Z}_p$ is the 3×3 phase impedance matrix for the series network. Equation (8.3.2) is now transformed to the sequence domain in the same manner that the load-phase impedances were transformed in Section 8.2. Thus,

$$\boldsymbol{V}_s - \boldsymbol{V}_{s'} = \boldsymbol{Z}_s \boldsymbol{I}_s \tag{8.3.3}$$

where

$$\boldsymbol{Z}_s = \boldsymbol{A}^{-1} \boldsymbol{Z}_p \boldsymbol{A} \tag{8.3.4}$$

From the results of Section 8.2, this sequence impedance $\boldsymbol{Z}_s$ matrix is diagonal under the following conditions:

$$\left.\begin{array}{l} Z_{aa} = Z_{bb} = Z_{cc} \\ \text{and} \\ Z_{ab} = Z_{ac} = Z_{bc} \end{array}\right\} \begin{array}{l}\text{conditions for} \\ \text{symmetrical} \\ \text{series impedances}\end{array} \tag{8.3.5}$$

When the phase impedance matrix $\boldsymbol{Z}_p$ of (8.3.1) has both equal self-impedances and equal mutual impedances, then (8.3.4) becomes

$$\boldsymbol{Z}_s = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \tag{8.3.6}$$

where

$$Z_0 = Z_{aa} + 2Z_{ab} \tag{8.3.7}$$

and

$$Z_1 = Z_2 = Z_{aa} - Z_{ab} \tag{8.3.8}$$

and (8.3.3) becomes three uncoupled equations, written as follows:

$$V_0 - V_{0'} = Z_0 I_0 \tag{8.3.9}$$

$$V_1 - V_{1'} = Z_1 I_1 \tag{8.3.10}$$

$$V_2 - V_{2'} = Z_2 I_2 \tag{8.3.11}$$

Equations (8.3.9)–(8.3.11) are represented by the three uncoupled sequence networks shown in Figure 8.10. From the figure it is apparent that for symmetrical series impedances, positive-sequence currents produce only positive-sequence voltage drops. Similarly, negative-sequence currents produce only negative-sequence voltage drops, and zero-sequence currents produce only zero-sequence voltage drops. However, if the series impedances

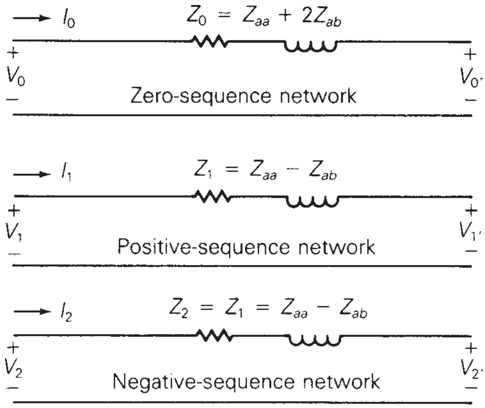


FIGURE 8.10 Sequence networks of three-phase symmetrical series impedances (linear, bilateral network, nonrotating equipment)

are not symmetrical, then $\boldsymbol{Z}_s$ is not diagonal, the sequence networks are coupled, and the voltage drop across any one sequence network depends on all three sequence currents.

8.4 SEQUENCE NETWORKS OF THREE-PHASE LINES

Section 4.7 develops equations suitable for computer calculation of the series phase impedances, including resistances and inductive reactances, of three-phase overhead transmission lines. The series phase impedance matrix $\boldsymbol{Z}_{\mathrm{P}}$ for an untransposed line is given by Equation (4.7.19), and $\hat{\boldsymbol{Z}}_{\mathrm{P}}$ for a completely transposed line is given by (4.7.21)–(4.7.23). Equation (4.7.19) can be transformed to the sequence domain to obtain

$$\boldsymbol{Z}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Z}_{\mathrm{P}}\boldsymbol{A} \tag{8.4.1}$$

$\boldsymbol{Z}_{\mathrm{S}}$ is the 3×3 series sequence impedance matrix whose elements are

$$\boldsymbol{Z}_{\mathrm{S}} = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} \quad \Omega/\mathrm{m} \tag{8.4.2}$$

In general $\boldsymbol{Z}_{\mathrm{S}}$ is not diagonal. However, if the line is completely transposed,

$$\hat{\boldsymbol{Z}}_{\mathrm{S}} = \boldsymbol{A}^{-1}\hat{\boldsymbol{Z}}_{\mathrm{P}}\boldsymbol{A} = \begin{bmatrix} \hat{Z}_0 & 0 & 0 \\ 0 & \hat{Z}_1 & 0 \\ 0 & 0 & \hat{Z}_2 \end{bmatrix} \tag{8.4.3}$$

where, from (8.3.7) and (8.3.8),

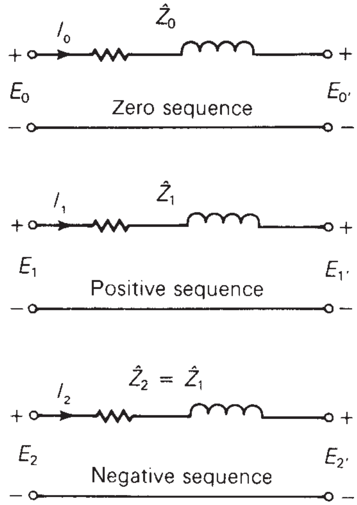


FIGURE 8.11 Circuit representation of the series sequence impedances of a completely transposed three-phase line

$$\hat{Z}_0 = \hat{Z}_{aaeq} + 2\hat{Z}_{abeq} \tag{8.4.4}$$

$$\hat{Z}_1 = \hat{Z}_2 = \hat{Z}_{aaeq} - \hat{Z}_{abeq} \tag{8.4.5}$$

A circuit representation of the series sequence impedances of a completely transposed three-phase line is shown in Figure 8.11.

Section 4.11 develops equations suitable for computer calculation of the shunt phase admittances of three-phase overhead transmission lines. The shunt admittance matrix $\boldsymbol{Y}_{\mathrm{P}}$ for an untransposed line is given by Equation (4.11.16), and $\hat{\boldsymbol{Y}}_{\mathrm{P}}$ for a completely transposed three-phase line is given by (4.11.17).

Equation (4.11.16) can be transformed to the sequence domain to obtain

$$\boldsymbol{Y}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Y}_{\mathrm{P}}\boldsymbol{A} \tag{8.4.6}$$

where

$$\boldsymbol{Y}_{\mathrm{S}} = \mathbf{G}_{\mathrm{S}} + j(2\pi f)\mathbf{C}_{\mathrm{S}} \tag{8.4.7}$$

$$\mathbf{C}_{\mathrm{S}} = \begin{bmatrix} \mathrm{C}_0 & \mathrm{C}_{01} & \mathrm{C}_{02} \\ \mathrm{C}_{10} & \mathrm{C}_1 & \mathrm{C}_{12} \\ \mathrm{C}_{20} & \mathrm{C}_{21} & \mathrm{C}_2 \end{bmatrix} \quad \mathrm{F/m} \tag{8.4.8}$$

In general, $\mathbf{C}_{\mathrm{S}}$ is not diagonal. However, for the completely transposed line,

$$\hat{\boldsymbol{Y}}_{\mathrm{S}} = \boldsymbol{A}^{-1}\hat{\boldsymbol{Y}}_{\mathrm{P}}\boldsymbol{A} = \begin{bmatrix} \hat{y}_0 & 0 & 0 \\ 0 & \hat{y}_1 & 0 \\ 0 & 0 & \hat{y}_2 \end{bmatrix} = j(2\pi f)\begin{bmatrix} \hat{\mathrm{C}}_0 & 0 & 0 \\ 0 & \hat{\mathrm{C}}_1 & 0 \\ 0 & 0 & \hat{\mathrm{C}}_2 \end{bmatrix} \tag{8.4.9}$$

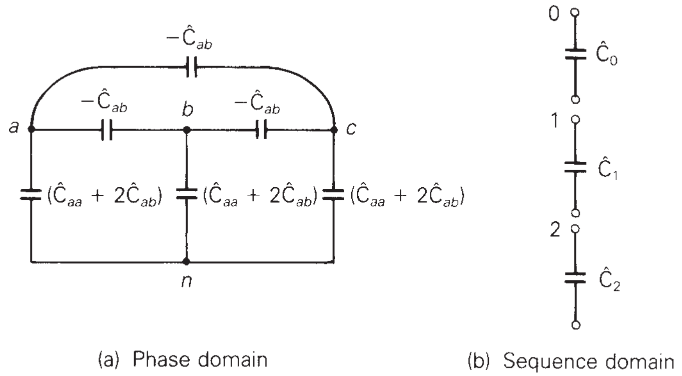


FIGURE 8.12 Circuit representations of the capacitances of a completely transposed three-phase line

where

$$\hat{C}_0 = \hat{C}_{aa} + 2\hat{C}_{ab} \quad \text{F/m} \tag{8.4.10}$$

$$\hat{C}_1 = \hat{C}_2 = \hat{C}_{aa} - \hat{C}_{ab} \quad \text{F/m} \tag{8.4.11}$$

Since $\hat{C}_{ab}$ is negative, the zero-sequence capacitance $\hat{C}_0$ is usually much less than the positive- or negative-sequence capacitance.

Circuit representations of the phase and sequence capacitances of a completely transposed three-phase line are shown in Figure 8.12.

8.5 SEQUENCE NETWORKS OF ROTATING MACHINES

A Y-connected synchronous generator grounded through a neutral impedance Z_n is shown in Figure 8.13. The internal generator voltages are designated E_a, E_b, and E_c, and the generator line currents are designated I_a, I_b, and I_c.

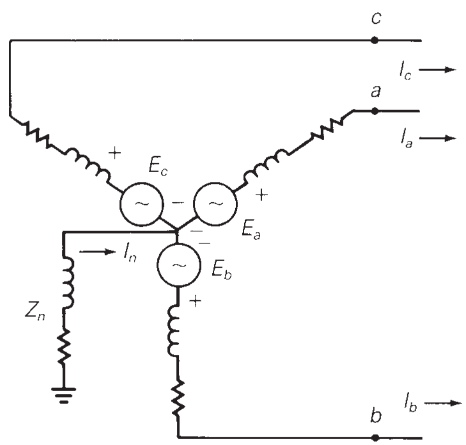


FIGURE 8.13 Y-connected synchronous generator

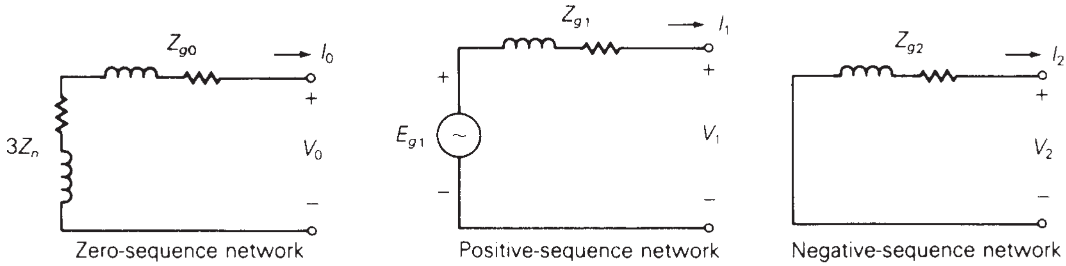


FIGURE 8.14 Sequence networks of a Y-connected synchronous generator

The sequence networks of the generator are shown in Figure 8.14. Since a three-phase synchronous generator is designed to produce balanced internal phase voltages E_a, E_b, E_c with only a positive-sequence component, a source voltage E_{g1} is included only in the positive-sequence network. The sequence components of the line-to-ground voltages at the generator terminals are denoted V_0, V_1, and V_2 in Figure 8.14.

The voltage drop in the generator neutral impedance is $Z_n I_n$, which can be written as $(3Z_n)I_0$, since, from (8.1.27), the neutral current is three times the zero-sequence current. Since this voltage drop is due only to zero-sequence current, an impedance $(3Z_n)$ is placed in the zero-sequence network of Figure 8.14 in series with the generator zero-sequence impedance Z_{g0}.

The sequence impedances of rotating machines are generally not equal. A detailed analysis of machine-sequence impedances is given in machine theory texts. We give only a brief explanation here.

When a synchronous generator stator has balanced three-phase positive-sequence currents under steady-state conditions, the net mmf produced by these positive-sequence currents rotates at the synchronous rotor speed in the same direction as that of the rotor. Under this condition, a high value of magnetic flux penetrates the rotor, and the positive-sequence impedance Z_{g1} has a high value. Under steady-state conditions, the positive-sequence generator impedance is called the *synchronous impedance*.

When a synchronous generator stator has balanced three-phase negative-sequence currents, the net mmf produced by these currents rotates at synchronous speed in the direction opposite to that of the rotor. With respect to the rotor, the net mmf is not stationary but rotates at twice synchronous speed. Under this condition, currents are induced in the rotor windings that prevent the magnetic flux from penetrating the rotor. As such, the negative-sequence impedance Z_{g2} is less than the positive-sequence synchronous impedance.

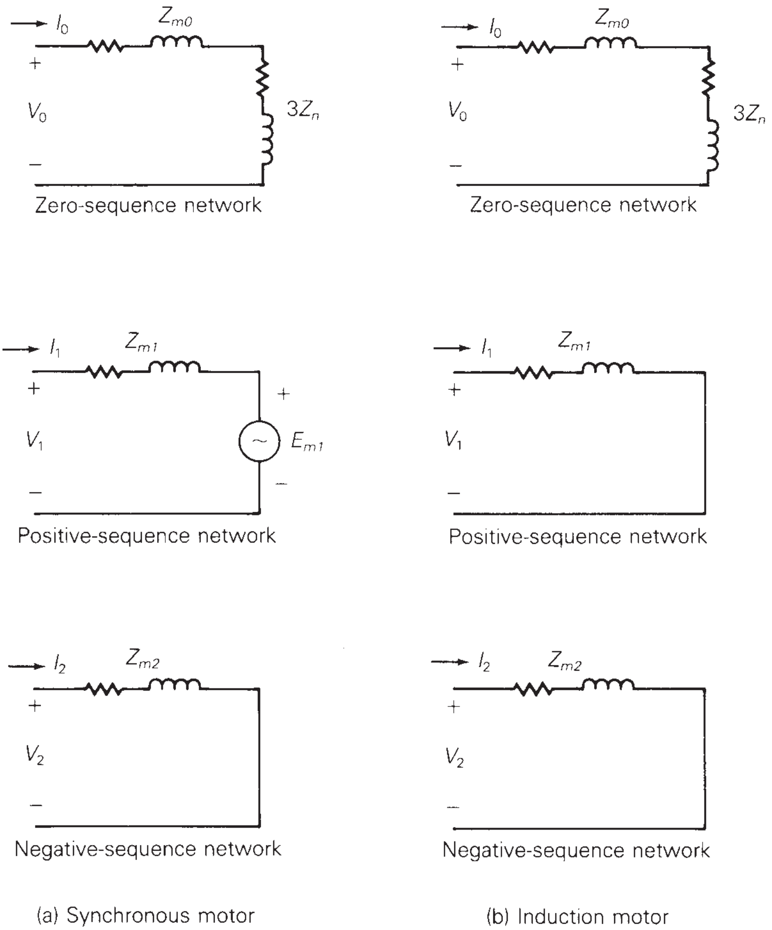
When a synchronous generator has only zero-sequence currents, which are line (or phase) currents with equal magnitude and phase, then the net mmf produced by these currents is theoretically zero. The generator zero-sequence impedance Z_{g0} is the smallest sequence impedance and is due to leakage flux, end turns, and harmonic flux from windings that do not produce a perfectly sinusoidal mmf.

Typical values of machine-sequence impedances are listed in Table A.1 in the Appendix. The positive-sequence machine impedance is synchronous, transient, or subtransient. *Synchronous* impedances are used for steady-state conditions, such as in power-flow studies, which are described in Chapter 6. *Transient* impedances are used for stability studies, which are described in Chapter 13, and *subtransient* impedances are used for short-circuit studies, which are described in Chapters 7 and 9. Unlike the positive-sequence impedances, a machine has only one negative-sequence impedance and only one zero-sequence impedance.

The sequence networks for three-phase synchronous motors and for three-phase induction motors are shown in Figure 8.15. Synchronous motors have the same sequence networks as synchronous generators, except that the sequence currents for synchronous motors are referenced *into* rather than out of the sequence networks. Also, induction motors have the same sequence networks as synchronous motors, except that the positive-sequence voltage

FIGURE 8.15

Sequence networks of three-phase motors



source E_{m1} is removed. Induction motors do not have a dc source of magnetic flux in their rotor circuits, and therefore E_{m1} is zero (or a short circuit).

The sequence networks shown in Figures 8.14 and 8.15 are simplified networks for rotating machines. The networks do not take into account such phenomena as machine saliency, saturation effects, and more complicated transient effects. These simplified networks, however, are in many cases accurate enough for power system studies.

EXAMPLE 8.5 Currents in sequence networks

Draw the sequence networks for the circuit of Example 2.5 and calculate the sequence components of the line current. Assume that the generator neutral is grounded through an impedance $Z_n = j10\ \Omega$, and that the generator sequence impedances are $Z_{g0} = j1\ \Omega$, $Z_{g1} = j15\ \Omega$, and $Z_{g2} = j3\ \Omega$.

SOLUTION The sequence networks are shown in Figure 8.16. They are obtained by interconnecting the sequence networks for a balanced-Δ load, for

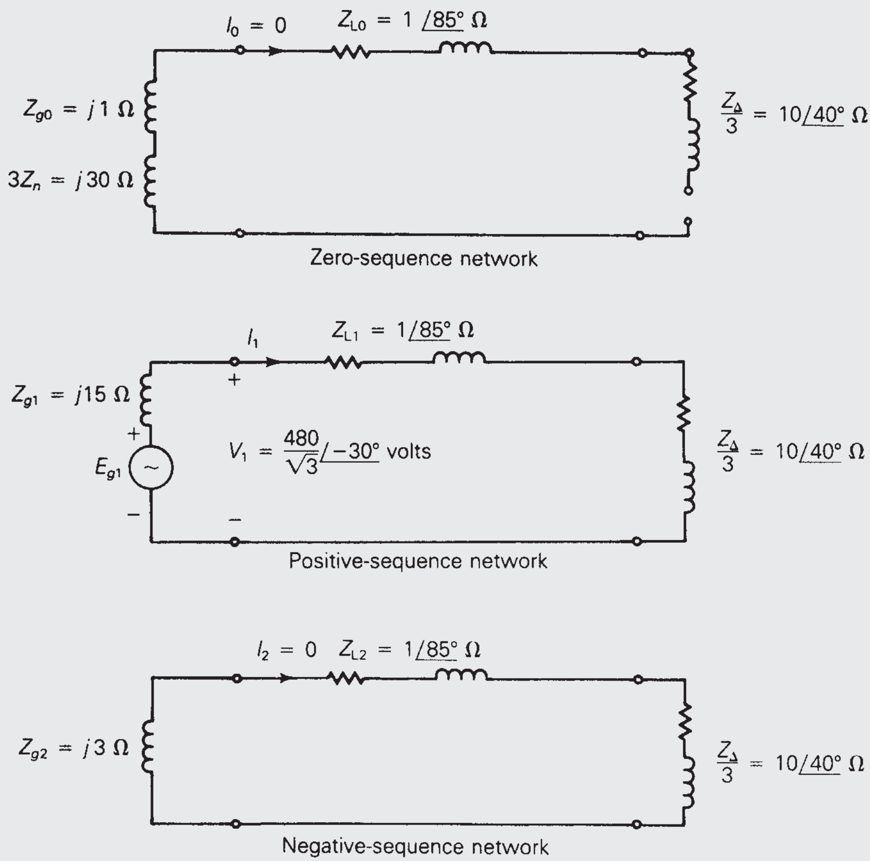


FIGURE 8.16 Sequence networks for Example 8.5

series-line impedances, and for a synchronous generator, which are given in Figures 8.5, 8.10, and 8.14.

It is clear from Figure 8.16 that $I_0 = I_2 = 0$ since there are no sources in the zero- and negative-sequence networks. Also, the positive-sequence generator terminal voltage V_1 equals the generator line-to-neutral terminal voltage. Therefore, from the positive-sequence network shown in the figure and from the results of Example 2.5,

$$I_1 = \frac{V_1}{\left(Z_{L1} + \frac{1}{3}Z_\Delta\right)} = 25.83\underline{/-73.78^\circ}\ \mathrm{A} = I_a$$

Note that from (8.1.20), I_1 equals the line current I_a, since $I_0 = I_2 = 0$. ■

The following example illustrates the superiority of using symmetrical components for analyzing unbalanced systems.

EXAMPLE 8.6 Solving unbalanced three-phase networks using sequence components

A Y-connected voltage source with the following unbalanced voltage is applied to the balanced line and load of Example 2.5.

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 260\underline{/-120^\circ} \\ 295\underline{/+115^\circ} \end{bmatrix} \quad \text{volts}$$

The source neutral is solidly grounded. Using the method of symmetrical components, calculate the source currents I_a, I_b, and I_c.

SOLUTION Using (8.1.13)–(8.1.15), the sequence components of the source voltages are:

$$\begin{aligned} V_0 &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ} + 295\underline{/115^\circ}) \\ &= 7.4425 + j14.065 = 15.912\underline{/62.11^\circ} \quad \text{volts} \end{aligned}$$

$$\begin{aligned} V_1 &= \tfrac{1}{3}(227\underline{/0^\circ} + 260\underline{/-120^\circ + 120^\circ} + 295\underline{/115^\circ + 240^\circ}) \\ &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/0^\circ} + 295\underline{/-5^\circ}) \\ &= 276.96 - j8.5703 = 277.1\underline{/-1.772^\circ} \quad \text{volts} \end{aligned}$$

$$\begin{aligned} V_2 &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ + 240^\circ} + 295\underline{/115^\circ + 120^\circ}) \\ &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/120^\circ} + 295\underline{/235^\circ}) \\ &= -7.4017 - j5.4944 = 9.218\underline{/216.59^\circ} \quad \text{volts} \end{aligned}$$

These sequence voltages are applied to the sequence networks of the line and load, as shown in Figure 8.17. The sequence networks of this figure

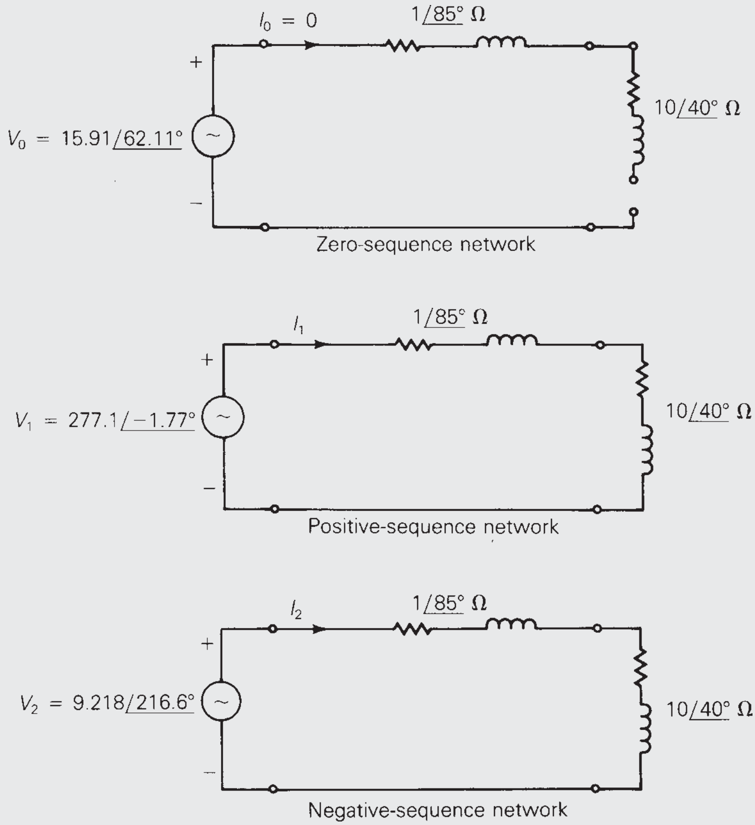


FIGURE 8.17

Sequence networks for Example 8.6

are uncoupled, and the sequence components of the source currents are easily calculated as follows:

$$I_0 = 0$$

$$I_1 = \frac{V_1}{Z_{L1} + \dfrac{Z_\Delta}{3}} = \frac{277.1\underline{/-1.772°}}{10.73\underline{/43.78°}} = 25.82\underline{/-45.55°} \quad \text{A}$$

$$I_2 = \frac{V_2}{Z_{L2} + \dfrac{Z_\Delta}{3}} = \frac{9.218\underline{/216.59°}}{10.73\underline{/43.78°}} = 0.8591\underline{/172.81°} \quad \text{A}$$

Using (8.1.20)–(8.1.22), the source currents are:

$$I_a = (0 + 25.82\underline{/-45.55°} + 0.8591\underline{/172.81°})$$
$$= 17.23 - j18.32 = 25.15\underline{/-46.76°} \quad \text{A}$$

$$I_b = (0 + 25.82\underline{/-45.55° + 240°} + 0.8591\underline{/172.81° + 120°})$$
$$= (25.82\underline{/194.45°} + 0.8591\underline{/292.81°})$$
$$= -24.67 - j7.235 = 25.71\underline{/196.34°} \quad \text{A}$$

$$I_c = (0 + 25.82\underline{/-45.55^\circ + 120^\circ} + 0.8591\underline{/172.81^\circ + 240^\circ})$$
$$= (25.82\underline{/74.45^\circ} + 0.8591\underline{/52.81^\circ})$$
$$= 7.441 + j25.56 = 26.62\underline{/73.77^\circ} \quad \text{A}$$

You should calculate the line currents for this example without using symmetrical components, in order to verify this result and to compare the two solution methods (see Problem 8.33). Without symmetrical components, coupled KVL equations must be solved. With symmetrical components, the conversion from phase to sequence components decouples the networks as well as the resulting KVL equations, as shown above. ■

8.6

PER-UNIT SEQUENCE MODELS OF THREE-PHASE TWO-WINDING TRANSFORMERS

Figure 8.18(a) is a schematic representation of an ideal Y–Y transformer grounded through neutral impedances Z_N and Z_n. Figures 8.18(b–d) show the per-unit sequence networks of this ideal transformer.

When balanced positive-sequence currents or balanced negative-sequence currents are applied to the transformer, the neutral currents are zero and there are no voltage drops across the neutral impedances. Therefore, the per-unit positive- and negative-sequence networks of the ideal Y–Y transformer, Figures 8.18(b) and (c), are the same as the per-unit single-phase ideal transformer, Figure 3.9(a).

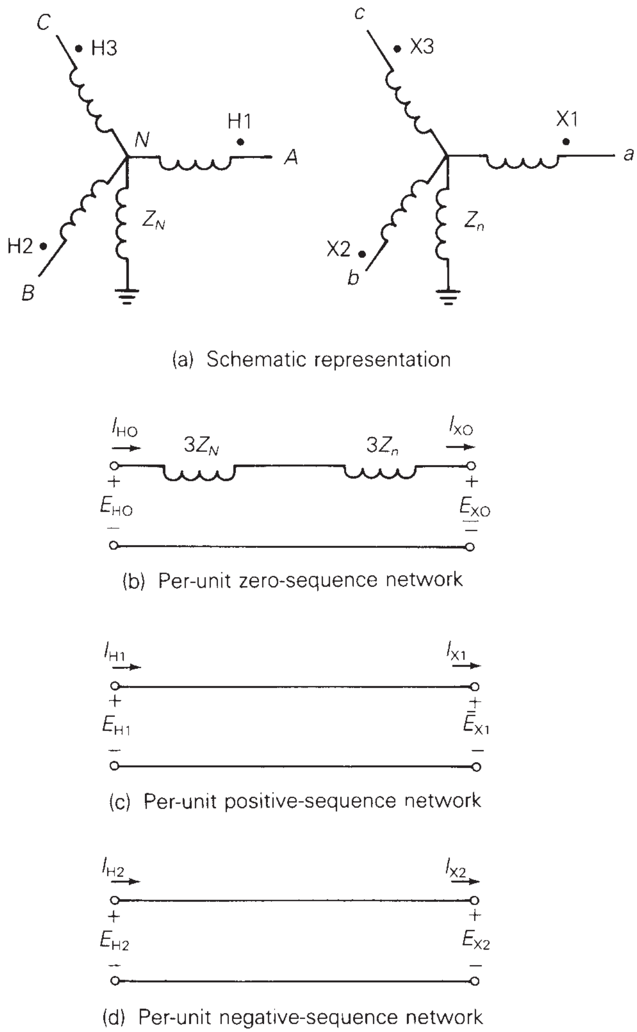
Zero-sequence currents have equal magnitudes and equal phase angles. When per-unit sequence currents $I_{A0} = I_{B0} = I_{C0} = I_0$ are applied to the high-voltage windings of an ideal Y–Y transformer, the neutral current $I_N = 3I_0$ flows through the neutral impedance Z_N, with a voltage drop $(3Z_N)I_0$. Also, per-unit zero-sequence current I_0 flows in each low-voltage winding [from (3.3.9)], and therefore $3I_0$ flows through neutral impedance Z_n, with a voltage drop $(3I_0)Z_n$. The per-unit zero-sequence network, which includes the impedances $(3Z_N)$ and $(3Z_n)$, is shown in Figure 8.18(b).

Note that if either one of the neutrals of an ideal transformer is ungrounded, then no zero sequence can flow in either the high- or low-voltage windings. For example, if the high-voltage winding has an open neutral, then $I_N = 3I_0 = 0$, which in turn forces $I_0 = 0$ on the low-voltage side. This can be shown in the zero-sequence network of Figure 8.18(b) by making $Z_N = \infty$, which corresponds to an open circuit.

The per-unit sequence networks of a practical Y–Y transformer are shown in Figure 8.19(a). These networks are obtained by adding external impedances to the sequence networks of the ideal transformer, as follows. The leakage impedances of the high-voltage windings are series impedances like the series impedances shown in Figure 8.9, with no coupling between phases

FIGURE 8.18

Ideal Y–Y transformer



$(Z_{ab} = 0)$. If the phase a, b, and c windings have equal leakage impedances $Z_H = R_H + jX_H$, then the series impedances are *symmetrical* with sequence networks, as shown in Figure 8.10, where $Z_{H0} = Z_{H1} = Z_{H2} = Z_H$. Similarly, the leakage impedances of the low-voltage windings are symmetrical series impedances with $Z_{X0} = Z_{X1} = Z_{X2} = Z_X$. These series leakage impedances are shown in per-unit in the sequence networks of Figure 8.19(a).

The shunt branches of the practical Y–Y transformer, which represent exciting current, are equivalent to the Y load of Figure 8.3. Each phase in Figure 8.3 represents a core loss resistor in parallel with a magnetizing inductance. Assuming these are the same for each phase, then the Y load is *symmetrical*, and the sequence networks are shown in Figure 8.4. These shunt

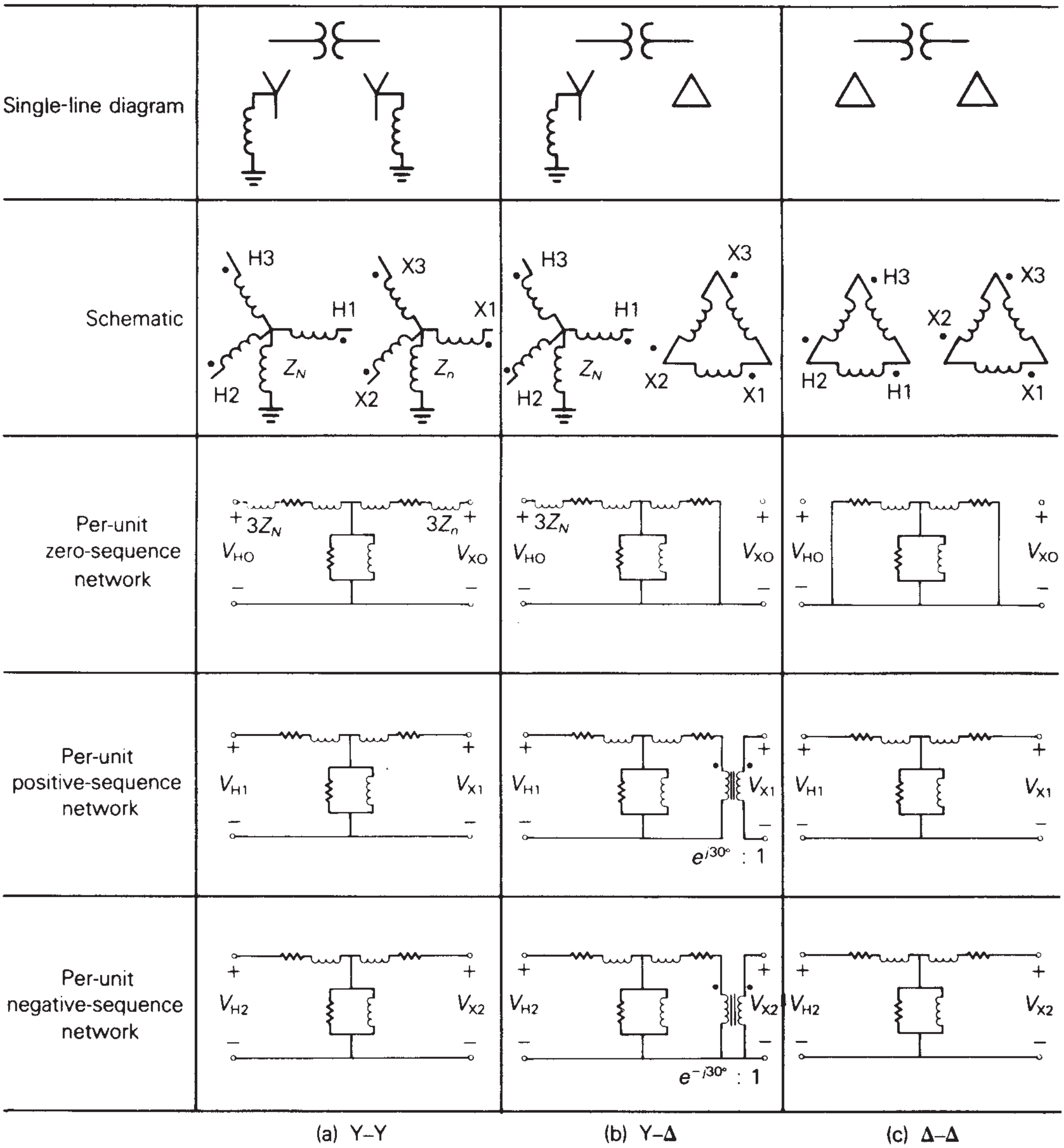


FIGURE 8.19 Per-unit sequence networks of practical Y–Y, Y–Δ, and Δ–Δ transformers

branches are also shown in Figure 8.19(a). Note that $(3Z_N)$ and $(3Z_n)$ have already been included in the zero-sequence network.

The per-unit positive- and negative-sequence transformer impedances of the practical Y–Y transformer in Figure 8.19(a) are identical, which is always true for nonrotating equipment. The per-unit zero-sequence network, however, depends on the neutral impedances Z_N and Z_n.

The per-unit sequence networks of the Y–Δ transformer, shown in Figure 8.19(b), have the following features:

1. The per-unit impedances do not depend on the winding connections. That is, the per-unit impedances of a transformer that is connected Y–Y, Y–Δ, Δ–Y, or Δ–Δ are the same. However, the base voltages do depend on the winding connections.

2. A phase shift is included in the per-unit positive- and negative-sequence networks. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30°. For negative sequence, the high-voltage quantities lag by 30°.

3. Zero-sequence currents can flow in the Y winding if there is a neutral connection, and corresponding zero-sequence currents flow within the Δ winding. However, no zero-sequence current enters or leaves the Δ winding.

The phase shifts in the positive- and negative-sequence networks of Figure 8.19(b) are represented by the phase-shifting transformer of Figure 3.4. Also, the zero-sequence network of Figure 8.19(b) provides a path on the Y side for zero-sequence current to flow, but no zero-sequence current can enter or leave the Δ side.

The per-unit sequence networks of the Δ–Δ transformer, shown in Figure 8.19(c), have the following features:

1. The positive- and negative-sequence networks, which are identical, are the same as those for the Y–Y transformer. It is assumed that the windings are labeled so there is no phase shift. Also, the per-unit impedances do not depend on the winding connections, but the base voltages do.

2. Zero-sequence currents *cannot* enter or leave either Δ winding, although they can circulate within the Δ windings.

EXAMPLE 8.7 Solving unbalanced three-phase networks with transformers using per-unit sequence components

A 75-kVA, 480-volt Δ/208-volt Y transformer with a solidly grounded neutral is connected between the source and line of Example 8.6. The transformer leakage reactance is $X_{eq} = 0.10$ per unit; winding resistances and exciting current are neglected. Using the transformer ratings as base quantities, draw the per-unit sequence networks and calculate the phase a source current I_a.

SOLUTION The base quantities are $S_{base1\phi} = 75/3 = 25$ kVA, $V_{baseHLN} = 480/\sqrt{3} = 277.1$ volts, $V_{baseXLN} = 208/\sqrt{3} = 120.1$ volts, and $Z_{baseX} = (120.1)^2/25{,}000 = 0.5770\ \Omega$. The sequence components of the actual source voltages are given in Figure 8.17. In per-unit, these voltages are

$$V_0 = \frac{15.91\underline{/62.11^\circ}}{277.1} = 0.05742\underline{/62.11^\circ} \quad \text{per unit}$$

$$V_1 = \frac{277.1\underline{/-1.772^\circ}}{277.1} = 1.0\underline{/-1.772^\circ} \quad \text{per unit}$$

$$V_2 = \frac{9.218\underline{/216.59^\circ}}{277.1} = 0.03327\underline{/216.59^\circ} \quad \text{per unit}$$

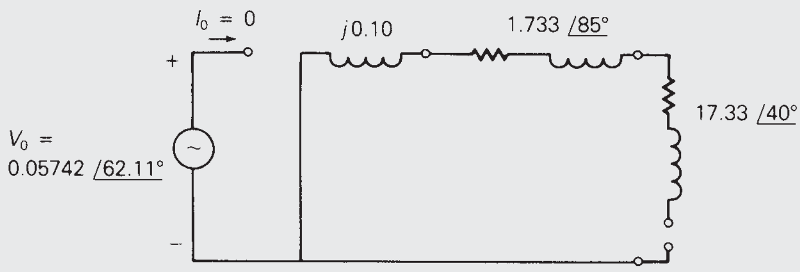
The per-unit line and load impedances, which are located on the low-voltage side of the transformer, are

$$Z_{L0} = Z_{L1} = Z_{L2} = \frac{1\underline{/85^\circ}}{0.577} = 1.733\underline{/85^\circ} \quad \text{per unit}$$

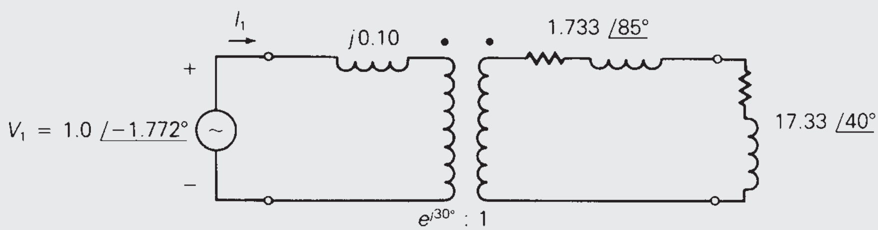
$$Z_{\text{load1}} = Z_{\text{load2}} = \frac{Z_\Delta}{3(0.577)} = \frac{10\underline{/40^\circ}}{0.577} = 17.33\underline{/40^\circ} \quad \text{per unit}$$

FIGURE 8.20

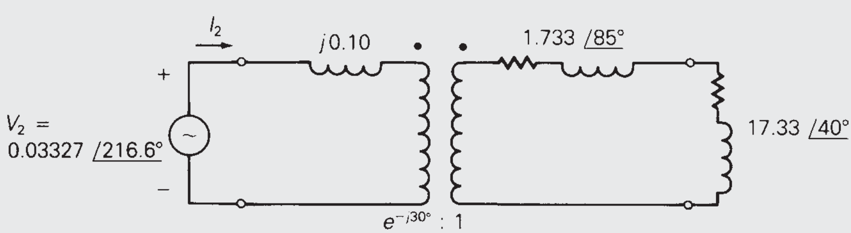
Per-unit sequence networks for Example 8.7



(a) Per-unit zero-sequence network



(b) Per-unit positive-sequence network



(c) Per-unit negative-sequence network

The per-unit sequence networks are shown in Figure 8.20. Note that the per-unit line and load impedances, when referred to the high-voltage side of the phase-shifting transformer, do not change [(see (3.1.26)]. Therefore, from Figure 8.20, the sequence components of the source currents are

$$I_0 = 0$$

$$I_1 = \frac{V_1}{jX_{eq} + Z_{L1} + Z_{load1}} = \frac{1.0\underline{/-1.772^\circ}}{j0.10 + 1.733\underline{/85^\circ} + 17.33\underline{/40^\circ}}$$

$$= \frac{1.0\underline{/-1.772^\circ}}{13.43 + j12.97} = \frac{1.0\underline{/-1.772^\circ}}{18.67\underline{/44.0^\circ}} = 0.05356\underline{/-45.77^\circ} \quad \text{per unit}$$

$$I_2 = \frac{V_2}{jX_{eq} + Z_{L2} + Z_{load2}} = \frac{0.03327\underline{/216.59^\circ}}{18.67\underline{/44.0^\circ}}$$

$$= 0.001782\underline{/172.59^\circ} \quad \text{per unit}$$

The phase a source current is then, using (8.1.20),

$$I_a = I_0 + I_1 + I_2$$

$$= 0 + 0.05356\underline{/-45.77^\circ} + 0.001782\underline{/172.59^\circ}$$

$$= 0.03511 - j0.03764 = 0.05216\underline{/-46.19^\circ} \quad \text{per unit}$$

Using $I_{baseH} = \dfrac{75{,}000}{480\sqrt{3}} = 90.21$ A,

$$I_a = (0.05216)(90.21)\underline{/-46.19^\circ} = 4.705\underline{/-46.19^\circ} \quad \text{A}$$

■

8.7 PER-UNIT SEQUENCE MODELS OF THREE-PHASE THREE-WINDING TRANSFORMERS

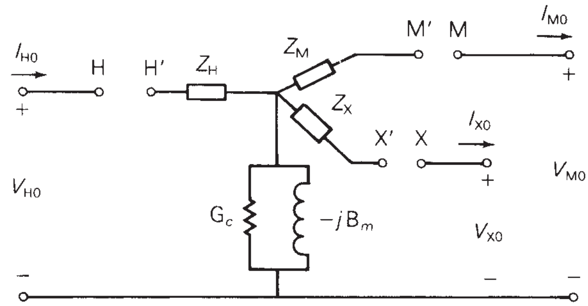
Three identical single-phase three-winding transformers can be connected to form a three-phase bank. Figure 8.21 shows the general per-unit sequence networks of a three-phase three-winding transformer. Instead of labeling the windings 1, 2, and 3, as was done for the single-phase transformer, the letters H, M, and X are used to denote the high-, medium-, and low-voltage windings, respectively. By convention, a common S_{base} is selected for the H, M, and X terminals, and voltage bases V_{baseH}, V_{baseM}, and V_{baseX} are selected in proportion to the rated line-to-line voltages of the transformer.

For the general zero-sequence network, Figure 8.21(a), the connection between terminals H and H′ depends on how the high-voltage windings are connected, as follows:

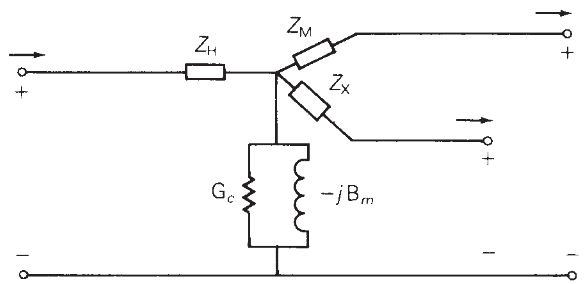
1. Solidly grounded Y—Short H to H′.
2. Grounded Y through Z_N—Connect $(3Z_N)$ from H to H′.

FIGURE 8.21

Per-unit sequence networks of a three-phase three-winding transformer



(a) Per-unit zero-sequence network



(b) Per-unit positive- or negative-sequence network (phase shift not shown)

3. Ungrounded Y—Leave H–H′ open as shown.
4. Δ—Short H′ to the reference bus.

Terminals X–X′ and M–M′ are connected in a similar manner.

The impedances of the per-unit negative-sequence network are the same as those of the per-unit positive-sequence network, which is always true for nonrotating equipment. Phase-shifting transformers, not shown in Figure 8.21(b), can be included to model phase shift between Δ and Y windings.

EXAMPLE 8.8 Three-winding three-phase transformer: per-unit sequence networks

Three transformers, each identical to that described in Example 3.9, are connected as a three-phase bank in order to feed power from a 900-MVA, 13.8-kV generator to a 345-kV transmission line and to a 34.5-kV distribution line. The transformer windings are connected as follows:

13.8-kV windings (X):	Δ, to generator
199.2-kV windings (H):	solidly grounded Y, to 345-kV line
19.92-kV windings (M):	grounded Y through $Z_n = j0.10\ \Omega$, to 34.5-kV line

The positive-sequence voltages and currents of the high- and medium-voltage Y windings lead the corresponding quantities of the low-voltage Δ winding by 30°. Draw the per-unit sequence networks, using a three-phase base of 900 MVA and 13.8 kV for terminal X.

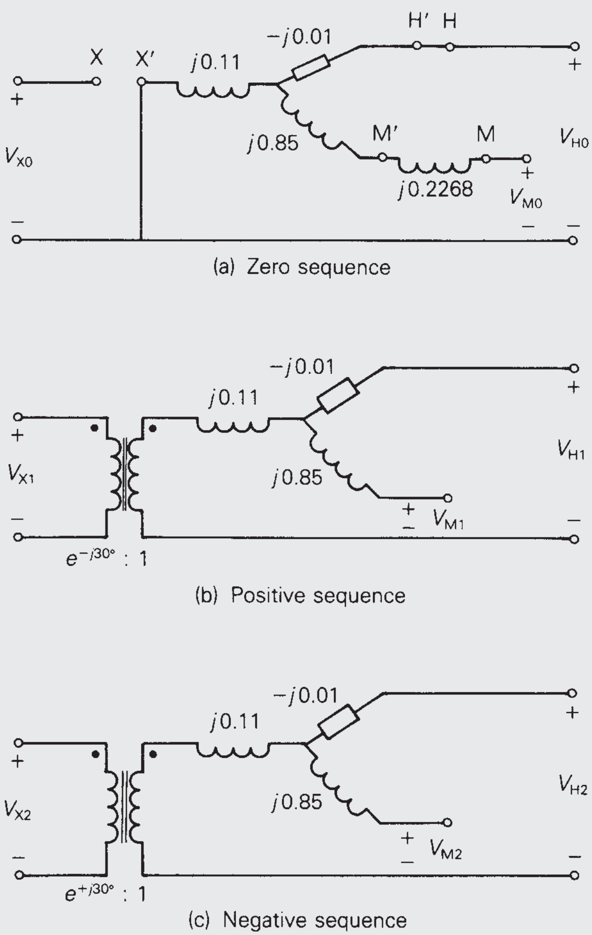
SOLUTION The per-unit sequence networks are shown in Figure 8.22. Since $V_{baseX} = 13.8$ kV is the rated line-to-line voltage of terminal X, $V_{baseM} = \sqrt{3}(19.92) = 34.5$ kV, which is the rated line-to-line voltage of terminal M. The base impedance of the medium-voltage terminal is then

$$Z_{baseM} = \frac{(34.5)^2}{900} = 1.3225 \quad \Omega$$

Therefore, the per-unit neutral impedance is

$$Z_n = \frac{j0.10}{1.3225} = j0.07561 \quad \text{per unit}$$

FIGURE 8.22 Per-unit sequence networks for Example 8.8



and $(3Z_n) = j0.2268$ is connected from terminal M to M′ in the per-unit zero-sequence network. Since the high-voltage windings have a solidly grounded neutral, H to H′ is shorted in the zero-sequence network. Also, phase-shifting transformers are included in the positive- and negative-sequence networks. ■

8.8

POWER IN SEQUENCE NETWORKS

The power delivered to a three-phase network can be determined from the power delivered to the sequence networks. Let S_p denote the total complex power delivered to the three-phase load of Figure 8.7, which can be calculated from

$$S_p = V_{ag}I_a^* + V_{bg}I_b^* + V_{cg}I_c^* \tag{8.8.1}$$

Equation (8.8.1) is also valid for the total complex power delivered by the three-phase generator of Figure 8.13, or for the complex power delivered to any three-phase bus. Rewriting (8.8.1) in matrix format,

$$\begin{aligned} S_p &= [V_{ag}\, V_{bg}\, V_{cg}] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} \\ &= \boldsymbol{V}_p^{\mathrm{T}} \boldsymbol{I}_p^* \end{aligned} \tag{8.8.2}$$

where T denotes transpose and * denotes complex conjugate. Now, using (8.1.9) and (8.1.16),

$$\begin{aligned} S_p &= (\boldsymbol{A}\boldsymbol{V}_s)^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{I}_s)^* \\ &= \boldsymbol{V}_s^{\mathrm{T}}[\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}^*]\boldsymbol{I}_s^* \end{aligned} \tag{8.8.3}$$

Using the definition of $\boldsymbol{A}$, which is (8.1.8), to calculate the term within the brackets of (8.8.3), and noting that a and a^2 are conjugates,

$$\begin{aligned} \boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}^* &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^* \\ &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\boldsymbol{U} \end{aligned} \tag{8.8.4}$$

Equation (8.8.4) can now be used in (8.8.3) to obtain

$$S_p = 3\boldsymbol{V}_s^{\mathrm{T}}\boldsymbol{I}_s^*$$

$$= 3[V_0 + V_1 + V_2]\begin{bmatrix} I_0^* \\ I_1^* \\ I_2^* \end{bmatrix} \tag{8.8.5}$$

$$S_p = 3(V_0 I_0^* + V_1 I_1^* + V_2 I_2^*)$$

$$= 3S_s \tag{8.8.6}$$

Thus, the total complex power S_p delivered to a three-phase network equals *three* times the total complex power S_s delivered to the sequence networks.

The factor of 3 occurs in (8.8.6) because $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}^* = 3\boldsymbol{U}$, as shown by (8.8.4). It is possible to eliminate this factor of 3 by defining a new transformation matrix $\boldsymbol{A}_1 = (1/\sqrt{3})\boldsymbol{A}$ such that $\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{A}_1^* = \boldsymbol{U}$, which means that $\boldsymbol{A}_1$ is a *unitary* matrix. Using $\boldsymbol{A}_1$ instead of $\boldsymbol{A}$, the total complex power delivered to three-phase networks would equal the total complex power delivered to the sequence networks. However, standard industry practice for symmetrical components is to use $\boldsymbol{A}$, defined by (8.1.8).

EXAMPLE 8.9 Power in sequence networks

Calculate S_p and S_s delivered by the three-phase source in Example 8.6. Verify that $S_p = 3S_s$.

SOLUTION Using (8.5.1),

$$S_p = (277\underline{/0^\circ})(25.15\underline{/+46.76^\circ}) + (260\underline{/-120^\circ})(25.71\underline{/-196.34^\circ})$$

$$+ (295\underline{/115^\circ})(26.62\underline{/-73.77^\circ})$$

$$= 6967\underline{/46.76^\circ} + 6685\underline{/43.66^\circ} + 7853\underline{/41.23^\circ}$$

$$= 15{,}520 + j14{,}870 = 21{,}490\underline{/43.78^\circ} \quad \text{VA}$$

In the sequence domain,

$$S_s = V_0 I_0^* + V_1 I_1^* + V_2 I_2^*$$

$$= 0 + (277.1\underline{/-1.77^\circ})(25.82\underline{/45.55^\circ})$$

$$+ (9.218\underline{/216.59^\circ})(0.8591\underline{/-172.81^\circ})$$

$$= 7155\underline{/43.78^\circ} + 7.919\underline{/43.78^\circ}$$

$$= 5172 + j4958 = 7163\underline{/43.78^\circ} \quad \text{VA}$$

Also,

$$3S_s = 3(7163\underline{/43.78^\circ}) = 21{,}490\underline{/43.78^\circ} = S_p$$

■

MULTIPLE CHOICE QUESTIONS

SECTION 8.1

8.1 Positive-sequence components consist of three phasors with __________ magnitudes, and __________ phase displacement in positive sequence; negative-sequence components consist of three phasors with __________ magnitudes, and __________ phase displacement in negative sequence; and zero-sequence components consist of three phasors with __________ magnitudes, and __________ phase displacement. Fill in the Blanks.

8.2 In symmetrical-component theory, express the complex-number operator $a = 1\underline{/120^\circ}$ in exponential and rectangular forms.

8.3 In terms of sequence components of phase a given by $V_{a0} = V_0, V_{a1} = V_1$ and $V_{a2} = V_2$, give expressions for the phase voltages V_a, V_b, and V_c.
$V_a =$ __________________; $V_b =$ __________________; $V_c =$ __________________

8.4 The sequence components V_0, V_1, and V_2 can be expressed in terms of phase components V_a, V_b, and V_c.
$V_0 =$ __________________; $V_1 =$ __________________; $V_2 =$ __________________

8.5 In a balanced three-phase system, what is the zero-sequence voltage?
$V_0 =$ __________________

8.6 In an unblanced three-phase system, line-to-neutral voltage ___________ have a zero-sequence component, whereas line-to-line voltages ___________ have a zero-sequence component. Fill in the Blanks.

8.7 Can the symmetrical component transformation be applied to currents, just as applied to voltages?
(a) Yes (b) No

8.8 In a three-phase Wye-connected system with a neutral, express the neutral current in terms of phase currents and sequence-component terms.
$I_n =$ __________________ $=$ __________________

8.9 In a balanced Wye-connected system, what is the zero-sequence component of the line currents?

8.10 In a delta-connected three-phase system, line currents have no zero-sequence component.
(a) True (b) False

8.11 Balanced three-phase systems with positive sequence do not have zero-sequence and negative-sequence components.
(a) True (b) False

8.12 Unbalanced three-phase systems may have nonzero values for all sequence components.
(a) True (b) False

SECTION 8.2

8.13 For a balanced-Y impedance load with per-phase impedance of Z_Y and A neutral impedance Z_n connected between the load neutral and the ground, the 3×3 phase-impedance matrix will consist of equal diagonal elements given by __________, and equal nondiagonal elements given by __________. Fill in the Blanks.

8.14 Express the sequence impedance matrix $\boldsymbol{Z}_s$ in terms of the phase-impedance matrix $\boldsymbol{Z}_p$, and the transformation matrix $\boldsymbol{A}$ which relates $\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s$ and $\boldsymbol{I}_p = \boldsymbol{A}\boldsymbol{I}_s$.
$\boldsymbol{Z}_s =$ __________. Fill in the Blank.

8.15 The sequence impedance matrix Z_s for a balanced-Y load is a diagonal matrix and the sequence networks are uncoupled.
(a) True (b) False

8.16 For a balanced-Y impedance load with per-phase impedance of Z_Y and a neutral impedance Z_n, the zero-sequence voltage $V_0 = Z_0 I_0$, where $Z_0 =$ _________. Fill in the Blank.

8.17 For a balanced-Δ load with per-phase impedance of Z_Δ the equivalent Y-load will have an open neutral; for the corresponding uncoupled sequence networks, $Z_0 =$ __________, $Z_1 =$ __________, and $Z_2 =$ __________. Fill in the Blanks.

8.18 For a three-phase symmetrical impedance load, the sequence impedance matrix is __________ and hence the sequence networks are coupled/uncoupled.

SECTION 8.3

8.19 Sequence networks for three-phase symmetrical series impedances are coupled/uncoupled; positive-sequence currents produce only _________ voltage drops.

SECTION 8.4

8.20 The series sequence impedance matrix of a completely transposed three-phase line is _________, with its nondiagonal elements equal to _________. Fill in the Blanks.

SECTION 8.5

8.21 A Y-connected synchronous generator grounded through a neutral impedance Z_n, with a zero-sequence impedance Z_{g0}, will have zero-sequence impedance $Z_0 =$ _________ in its zero-sequence network. Fill in the Blank.

8.22 In sequence networks, a Y-connected synchronous generator is represented by its source per-unit voltage only in _________ network, while synchronous/transient/subtransient impedance is used in positive-sequence network for short-circuit studies.

8.23 In the positive-sequence network of a synchronous motor, a source voltage is represented, whereas in that of an induction motor, the source voltage does/does not come into picture.

8.24 With symmetrical components, the conversion from phase to sequence components decouples the networks and the resulting kVL equations.
(a) True (b) False

SECTION 8.6

8.25 Consider the per-unit sequence networks of Y-Y, Y-Δ, and $\Delta - \Delta$ transformers, with neutral impedances of Z_N on the high-voltage Y-side, and Z_n on the low-voltage Y-side. Answer the following:
(i) Zero-sequence currents can/cannot flow in the Y winding with a neutral connection; corresponding zero-sequence currents do/do not flow within the delta winding;

however zero-sequence current does/does not enter or leave the Δ winding. In zero-sequence network, 1/2/3 times the neutral impedance comes into play in series.

(ii) In Y(HV)- Δ(LV) transformers, if a phase shift is included as per the American-standard notation, the ratio _________ is used in positive-sequence network, and the ratio _________ is used in the negative-sequence network.

(iii) The base voltages depend on the winding connections; the per-unit impedances do/do not depend on the winding connections.

SECTION 8.7

8.26 In per-unit sequence models of three-phase three-winding transformers, for the general zero-sequence network, the connection between terminals H and H′ depends on how the high-voltage windings are connected:

(i) For solidly grounded Y, ________ H to H′.
(ii) For grounded Y through Z_n, connect _________ from H to H′.
(iii) For ungrounded Y, leave H–H′ __________.
(iv) For Δ, ________ H′ to the reference bus.

SECTION 8.8

8.27 The total complex power delivered to a three-phase network equals 1/2/3 times the total complex power delivered to the sequence networks.

8.28 Express the complex power S_s Delivered to the sequence networks in terms of sequence voltages and sequence currents.
$S_s =$ ___________

PROBLEMS

SECTION 8.1

8.1 Using the operator $a = 1\underline{/120^\circ}$, evaluate the following in polar form: (a) $(a-1)/(1+a-a^2)$, (b) $(a^2+a+j)/(ja+a^2)$, (c) $(1+a)(1+a^2)$, (d) $(a-a^2)(a^2-1)$.

8.2 Using $a = 1\underline{/120^\circ}$, evaluate the following in rectangular form:

a. a^{10}

b. $(ja)^{10}$

c. $(1-a)^3$

d. e^a

Hint for (d): $e^{(x+jy)} = e^x e^{jy} = e^x\underline{/y}$, where y is in radians.

8.3 Determine the symmetrical components of the following line currents: (a) $I_a = 5\underline{/90^\circ}$, $I_b = 5\underline{/320^\circ}$, $I_c = 5\underline{/220^\circ}$ A; (b) $I_a = j50$, $I_b = 50$, $I_c = 0$ A.

8.4 Find the phase voltages V_{an}, V_{bn}, and V_{cn} whose sequence components are: $V_0 = 50\underline{/80^\circ}$, $V_1 = 100\underline{/0^\circ}$, $V_2 = 50\underline{/90^\circ}$ V.

8.5 For the unbalanced three-phase system described by

$$I_a = 12\underline{/0^\circ}\text{A}, \quad I_b = 6\underline{/-90^\circ}\text{A}, \quad I_C = 8\underline{/150^\circ}\text{A}$$

compute the symmetrical components I_0, I_1, I_2.

8.6 (a) Given the symmetrical components to be

$$V_0 = 10\underline{/0^\circ}\,V, \quad V_1 = 80\underline{/30^\circ}\,V, \quad V_2 = 40\underline{/-30^\circ}\,V$$

determine the unbalanced phase voltages V_a, V_b, and V_c.
(b) Using the results of part (a), calculate the line-to-line voltages V_{ab}, V_{bc}, and V_{ca}. Then determine the symmetrical components of these ling-to-line voltages, the symmetrical components of the corresponding phase voltages, and the phase voltages. Compare them with the result of part (a). Comment on why they are different, even though either set will result in the same line-to-line voltages.

8.7 One line of a three-phase generator is open circuited, while the other two are short-circuited to ground. The line currents are $I_a = 0$, $I_b = 1000\underline{/150^\circ}$, and $I_c = 1000\underline{/+30^\circ}$ A. Find the symmetrical components of these currents. Also find the current into the ground.

8.8 Let an unbalanced, three-phase, Wye-connected load (with phase impedances of Z_a, Z_b, and Z_c) be connected to a balanced three-phase supply, resulting in phase voltages of V_a, V_b, and V_c across the corresponding phase impedances.
Choosing V_{ab} as the reference, show that

$$V_{ab,0} = 0; \quad V_{ab,1} = \sqrt{3}V_{a,1}e^{j30^\circ}; \quad V_{ab,2} = \sqrt{3}V_{a,2}e^{-j30^\circ}.$$

8.9 Reconsider Problem 8.8 and choosing V_{bc} as the reference, show that

$$V_{bc,0} = 0; \quad V_{bc,1} = -j\sqrt{3}V_{a,1}; \quad V_{bc,2} = j\sqrt{3}V_{a,2}.$$

8.10 Given the line-to-ground voltages $V_{ag} = 280\underline{/0^\circ}$, $V_{bg} = 250\underline{/-110^\circ}$, and $V_{cg} = 290\underline{/130^\circ}$ volts, calculate (a) the sequence components of the line-to-ground voltages, denoted V_{Lg0}, V_{Lg1}, and V_{Lg2}; (b) line-to-line voltages V_{ab}, V_{bc}, and V_{ca}; and (c) sequence components of the line-to-line voltages V_{LL0}, V_{LL1}, and V_{LL2}. Also, verify the following general relation: $V_{LL0} = 0$, $V_{LL1} = \sqrt{3}V_{Lg1}\underline{/+30^\circ}$, and $V_{LL2} = \sqrt{3}V_{Lg2}\underline{/-30^\circ}$ volts.

8.11 A balanced Δ-connected load is fed by a three-phase supply for which phase C is open and phase A is carrying a current of $10\underline{/0^\circ}$ A. Find the symmetrical components of the line currents. (Note that zero-sequence currents are not present for any three-wire system.)

8.12 A Y-connected load bank with a three-phase rating of 500 kVA and 2300 V consists of three identical resistors of 10.58 Ω. The load bank has the following applied voltages: $V_{ab} = 1840\underline{/82.8^\circ}$, $V_{bc} = 2760\underline{/-41.4^\circ}$, and $V_{ca} = 2300\underline{/180^\circ}$ V. Determine the symmetrical components of (a) the line-to-line voltages V_{ab0}, V_{ab1}, and V_{ab2}; (b) the line-to-neutral voltages V_{an0}, V_{an1}, and V_{an2}; (c) and the line currents I_{a0}, I_{a1}, and I_{a2}. (Note that the absence of a neutral connection means that zero-sequence currents are not present.)

SECTION 8.2

8.13 The currents in a Δ load are $I_{ab} = 10\underline{/0^\circ}$, $I_{bc} = 15\underline{/-90^\circ}$, and $I_{ca} = 20\underline{/90^\circ}$ A. Calculate (a) the sequence components of the Δ-load currents, denoted $I_{\Delta 0}$, $I_{\Delta 1}$, $I_{\Delta 2}$; (b) the line currents I_a, I_b, and I_c, which feed the Δ load; and (c) sequence components of the line currents I_{L0}, I_{L1}, and I_{L2}. Also, verify the following general relation: $I_{L0} = 0$, $I_{L1} = \sqrt{3}I_{\Delta 1}\underline{/-30^\circ}$, and $I_{L2} = \sqrt{3}I_{\Delta 2}\underline{/+30^\circ}$ A.

8.14 The voltages given in Problem 8.10 are applied to a balanced-Y load consisting of $(12 + j16)$ ohms per phase. The load neutral is solidly grounded. Draw the sequence networks and calculate I_0, I_1, and I_2, the sequence components of the line currents. Then calculate the line currents I_a, I_b, and I_c.

8.15 Repeat Problem 8.14 with the load neutral open.

8.16 Repeat Problem 8.14 for a balanced-Δ load consisting of $(12 + j16)$ ohms per phase.

8.17 Repeat Problem 8.14 for the load shown in Example 8.4 (Figure 8.6).

8.18 Perform the indicated matrix multiplications in (8.2.21) and verify the sequence impedances given by (8.2.22)–(8.2.27).

8.19 The following unbalanced line-to-ground voltages are applied to the balanced-Y load shown in Figure 3.3: $V_{ag} = 100\underline{/0^\circ}$, $V_{bg} = 75\underline{/180^\circ}$, and $V_{cg} = 50\underline{/90^\circ}$ volts. The Y load has $Z_Y = 3 + j4\ \Omega$ per phase with neutral impedance $Z_n = j1\ \Omega$. (a) Calculate the line currents I_a, I_b, and I_c without using symmetrical components. (b) Calculate the line currents I_a, I_b, and I_c using symmetrical components. Which method is easier?

8.20 (a) Consider three equal impedances of $(j27)\ \Omega$ connected in Δ. Obtain the sequence networks.
(b) Now, with a mutual impedance of $(j6)\ \Omega$ between each pair of adjacent branches in the Δ-connected load of part (a), how would the sequence networks change?

8.21 The three-phase impedance load shown in Figure 8.7 has the following phase impedance matrix:

$$Z_p = \begin{bmatrix} (6 + j10) & 0 & 0 \\ 0 & (6 + j10) & 0 \\ 0 & 0 & (6 + j10) \end{bmatrix} \quad \Omega$$

Determine the sequence impedance matrix Z_s for this load. Is the load symmetrical?

8.22 The three-phase impedance load shown in Figure 8.7 has the following sequence impedance matrix:

$$Z_S = \begin{bmatrix} (8 + j12) & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \Omega$$

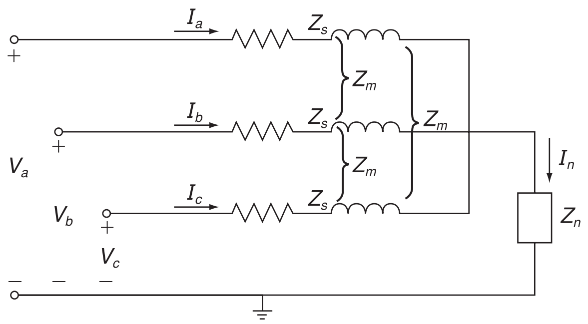
Determine the phase impedance matrix Z_p for this load. Is the load symmetrical?

8.23 Consider a three-phase balanced Y-connected load with self and mutual impedances as shown in Figure 8.23. Let the load neutral be grounded through an impedance Z_n. Using Kirchhoff's laws, develop the equations for line-to-neutral voltages, and then determine the elements of the phase impedance matrix. Also find the elements of the corresponding sequence impedance matrix.

8.24 A three-phase balanced voltage source is applied to a balanced Y-connected load with ungrounded neutral. The Y-connected load consists of three mutually coupled reactances, where the reactance of each phase is $j12\ \Omega$ and the mutual coupling between any two phases is $j4\ \Omega$. The line-to-line source voltage is $100\sqrt{3}$ V. Determine the line currents (a) by mesh analysis without using symmetrical components, and (b) using symmetrical components.

FIGURE 8.23

Problem 8.23



8.25 A three-phase balanced Y-connected load with series impedances of $(8 + j24)\ \Omega$ per phase and mutual impedance between any two phases of $j4\ \Omega$ is supplied by a three-phase unbalanced source with line-to-neutral voltages of $V_{an} = 200\underline{/25^\circ}$, $V_{bn} = 100\underline{/-155^\circ}$, $V_{cn} = 80\underline{/100^\circ}$ V. The load and source neutrals are both solidly grounded. Determine: (a) the load sequence impedance matrix, (b) the symmetrical components of the line-to-neutral voltages, (c) the symmetrical components of the load currents, and (d) the load currents.

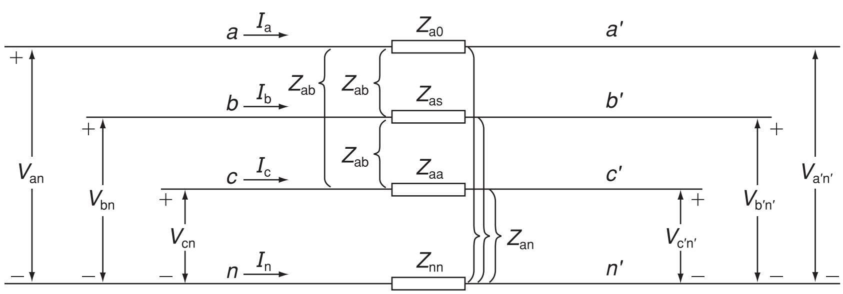
SECTION 8.3

8.26 Repeat Problem 8.14 but include balanced three-phase line impedances of $(3 + j4)$ ohms per phase between the source and load.

8.27 Consider the flow of unbalanced currents in the symmetrical three-phase line section with neutral conductor as shown in Figure 8.24. (a) Express the voltage drops across the line conductors given by $V_{aa'}$, $V_{bb'}$, and $V_{cc'}$ in terms of line currents, self-impedances defined by $Z_s = Z_{aa} + Z_{nn} - 2Z_{an}$, and mutual impedances defined by $Z_m = Z_{ab} + Z_{nn} - 2Z_{an}$. (b) Show that the sequence components of the voltage drops between the ends of the line section can be written as $V_{aa'0} = Z_0 I_{a0}$, $V_{aa'1} = Z_1 I_{a1}$, and $V_{aa'2} = Z_2 I_{a2}$, where $Z_0 = Z_s + 2Z_m = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an}$ and $Z_1 = Z_2 = Z_s - Z_m = Z_{aa} - Z_{ab}$.

FIGURE 8.24

Problem 8.27



8.28 Let the terminal voltages at the two ends of the line section shown in Figure 8.24 be given by:

$$V_{an} = (182 + j70)\ \text{kV} \qquad V_{an'} = (154 + j28)\ \text{kV}$$
$$V_{bn} = (72.24 - j32.62)\ \text{kV} \qquad V_{bn'} = (44.24 + j74.62)\ \text{kV}$$
$$V_{cn} = (-170.24 + j88.62)\ \text{kV} \qquad V_{cn'} = (-198.24 + j46.62)\ \text{kV}$$

The line impedances are given by:

$$Z_{aa} = j60\ \Omega \qquad Z_{ab} = j20\ \Omega \qquad Z_{nn} = j80\ \Omega \qquad Z_{an} = 0$$

(a) Compute the line currents using symmetrical components. (*Hint:* See Problem 8.27.) (b) Compute the line currents without using symmetrical components.

8.29 A completely transposed three-phase transmission line of 200 km in length has the following symmetrical sequence impedances and sequence admittances:

$$Z_1 = Z_2 = j0.5\ \Omega/\text{km}; \quad Z_0 = j2\ \Omega/\text{km}$$
$$Y_1 = Y_2 = j3 \times 10^{-9}\ \text{s/m}; \quad Y_0 = j1 \times 10^{-9}\ \text{s/m}$$

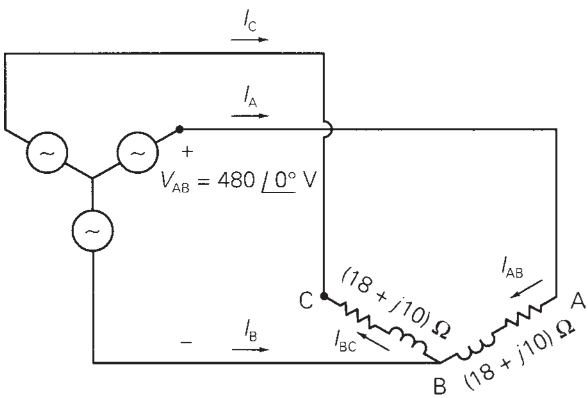
Set up the nominal Π sequence circuits of this medium-length line.

SECTION 8.5

8.30 As shown in Figure 8.25, a balanced three-phase, positive-sequence source with $V_{AB} = 480\underline{/0^\circ}$ volts is applied to an unbalanced Δ load. Note that one leg of the Δ is open. Determine: (a) the load currents I_{AB} and I_{BC}; (b) the line currents I_A, I_B, and I_C, which feed the Δ load; and (c) the zero-, positive-, and negative-sequence components of the line currents.

FIGURE 8.25

Problem 8.30



8.31 A balanced Y-connected generator with terminal voltage $V_{bc} = 200\underline{/0^\circ}$ volts is connected to a balanced-Δ load whose impedance is $10\underline{/40^\circ}$ ohms per phase. The line impedance between the source and load is $0.5\underline{/80^\circ}$ ohm for each phase. The generator neutral is grounded through an impedance of $j5$ ohms. The generator sequence impedances are given by $Z_{g0} = j7$, $Z_{g1} = j15$, and $Z_{g2} = j10$ ohms. Draw the sequence networks for this system and determine the sequence components of the line currents.

8.32 In a three-phase system, a synchronous generator supplies power to a 200-volt synchronous motor through a line having an impedance of $0.5\underline{/80^\circ}$ ohm per phase. The motor draws 5 kW at 0.8 p.f. leading and at rated voltage. The neutrals of both the generator and motor are grounded through impedances of $j5$ ohms. The sequence impedances of both machines are $Z_0 = j5$, $Z_1 = j15$, and $Z_2 = j10$ ohms. Draw the sequence networks for this system and find the line-to-line voltage at the generator terminals. Assume balanced three-phase operation.

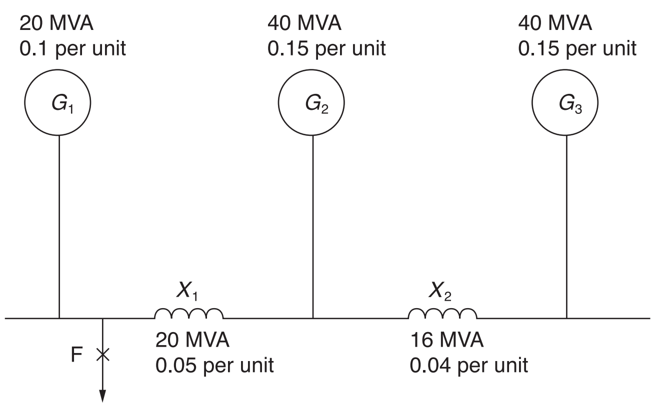
8.33 Calculate the source currents in Example 8.6 without using symmetrical components. Compare your solution method with that of Example 8.6. Which method is easier?

8.34 A Y-connected synchronous generator rated 20 MVA at 13.8 kV has a positive-sequence reactance of $j2.38\ \Omega$, negative-sequence reactance of $j3.33\ \Omega$, and zero-sequence reactance of $j0.95\ \Omega$. The generator neutral is solidly grounded. With the generator operating unloaded at rated voltage, a so-called single line-to-ground fault occurs at the machine terminals. During this fault, the line-to-ground voltages at the generator terminals are $V_{ag} = 0$, $V_{bg} = 8.071\underline{/-102.25^\circ}$, and $V_{cg} = 8.071\underline{/102.25^\circ}$ kV. Determine the sequence components of the generator fault currents and the generator fault currents. Draw a phasor diagram of the pre-fault and post-fault generator terminal voltages. (*Note:* For this fault, the sequence components of the generator fault currents are all equal to each other.)

8.35 Figure 8.26 shows a single-line diagram of a three-phase, interconnected generator-reactor system, in which the given per-unit reactances are based on the ratings of the individual pieces of equipment. If a three-phase short-circuit occurs at fault point F, obtain the fault MVA and fault current in kA, if the pre-fault busbar line-to-line voltage is 13.2 kV. Choose 100 MVA as the base MVA for the system.

FIGURE 8.26

One-line diagram for Problem 8.35



8.36 Consider Figures 8.13 and 8.14 of the text with reference to a Y-connected synchronous generator (grounded through a neutral impedance Z_n) operating at no load. For a line-to-ground fault occurring on phase a of the generator, list the constraints on the currents and voltages in the phase domain, transform those into the sequence domain, and then obtain a sequence-network representation. Also, find the expression for the fault current in phase a.

8.37 Reconsider the synchronous generator of Problem 8.36. Obtain sequence-network representations for the following fault conditions.
(a) A short-circuit between phases b and c.
(b) A double line-to-ground fault with phases b and c grounded.

SECTION 8.6

8.38 Three single-phase, two-winding transformers, each rated 450 MVA, 20 kV/288.7 kV, with leakage reactance $X_{eq} = 0.12$ per unit, are connected to form a three-phase bank. The high-voltage windings are connected in Y with a solidly grounded neutral. Draw the per-unit zero-, positive-, and negative-sequence networks if the low-voltage windings are connected: (a) in Δ with American standard phase shift, (b) in Y with an open neutral. Use the transformer ratings as base quantities. Winding resistances and exciting current are neglected.

8.39 The leakage reactance of a three-phase, 500-MVA, 345 Y/23 Δ-kV transformer is 0.09 per unit based on its own ratings. The Y winding has a solidly grounded neutral. Draw the sequence networks. Neglect the exciting admittance and assume American standard phase shift.

8.40 Choosing system bases to be 360/24 kV and 100 MVA, redraw the sequence networks for Problem 8.39.

8.41 Draw the zero-sequence reactance diagram for the power system shown in Figure 3.33. The zero-sequence reactance of each generator and of the synchronous motor is 0.05 per unit based on equipment ratings. Generator 2 is grounded through a neutral reactor of 0.06 per unit on a 100-MVA, 18-kV base. The zero-sequence reactance of each transmission line is assumed to be three times its positive-sequence reactance. Use the same base as in Problem 3.29.

8.42 Three identical Y-connected resistors of $1.0\underline{/0^\circ}$ per unit form a load bank, which is supplied from the low-voltage Y-side of a $Y - \Delta$ transformer. The neutral of the load is not connected to the neutral of the system. The positive- and negative-sequence currents flowing toward the resistive load are given by

$$I_{a,1} = 1\underline{/4.5^\circ} \text{ per unit}; \quad I_{a,2} = 0.25\underline{/250^\circ} \text{ per unit}$$

and the corresponding voltages on the low-voltage Y-side of the transformer are

$$V_{an,1} = 1\underline{/45^\circ} \text{ per unit (Line-to-neutral voltage base)}$$

$$V_{an,2} = 0.25\underline{/250^\circ} \text{ per unit (Line-to-neutral voltage base)}$$

Determine the line-to-line voltages and the line currents in per unit on the high-voltage side of the transformer. Account for the phase shift.

SECTION 8.7

8.43 Draw the positive-, negative-, and zero-sequence circuits for the transformers shown in Figure 3.34. Include ideal phase-shifting transformers showing phase shifts determined in Problem 3.32. Assume that all windings have the same kVA rating and that the equivalent leakage reactance of any two windings with the third winding open is 0.10 per unit. Neglect the exciting admittance.

8.44 A single-phase three-winding transformer has the following parameters: $Z_1 = Z_2 = Z_3 = 0 + j0.05$, $G_c = 0$, and $B_m = 0.2$ per unit. Three identical transformers, as described, are connected with their primaries in Y (solidly grounded neutral) and with their secondaries and tertiaries in Δ. Draw the per-unit sequence networks of this transformer bank.

SECTION 8.8

8.45 For Problem 8.14, calculate the real and reactive power delivered to the three-phase load.

8.46 A three-phase impedance load consists of a balanced-Δ load in parallel with a balanced-Y load. The impedance of each leg of the Δ load is $Z_\Delta = 6 + j6\ \Omega$, and the impedance of each leg of the Y load is $Z_Y = 2 + j2\ \Omega$. The Y load is grounded through a neutral impedance $Z_n = j1\ \Omega$. Unbalanced line-to-ground source voltages V_{ag}, V_{bg}, and V_{cg} with sequence components $V_0 = 10\underline{/60^\circ}$, $V_1 = 100\underline{/0^\circ}$, and $V_2 = 15\underline{/200^\circ}$ volts are applied to the load. (a) Draw the zero-, positive-, and negative-sequence networks. (b) Determine the complex power delivered to each sequence network. (c) Determine the total complex power delivered to the three-phase load.

8.47 For Problem 8.12, compute the power absorbed by the load using symmetrical components. Then verify the answer by computing directly without using symmetrical components.

8.48 For Problem 8.25, determine the complex power delivered to the load in terms of symmetrical components. Verify the answer by adding up the complex power of each of the three phases.

8.49 Using the voltages of Problem 8.6(a) and the currents of Problem 8.5, compute the complex power dissipated based on (a) phase components, and (b) symmetrical components.

CASE STUDY QUESTIONS

A. What are the advantages of SF_6 circuit breakers for applications at or above 72.5 kV?

B. What are the properties of SF_6 that make it make it advantageous as a medium for interrupting an electric arc?

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