Fossil-fuel (oil/gas) power plant with two 850-MVA generating units (Courtesy of PacifiCorp)



2

FUNDAMENTALS

The objective of this chapter is to review basic concepts and establish terminology and notation. In particular, we review phasors, instantaneous power, complex power, network equations, and elementary aspects of balanced threephase circuits. Students who have already had courses in electric network theory and basic electric machines should find this chapter to be primarily refresher material.

CASE STUDY

Throughout most of the 20th-century, electric utility companies built increasingly larger generation plants, primarily hydro or thermal (using coal, gas, oil, or nuclear fuel). At the end of the twentieth century, following the ongoing deregulation of the electric utility industry with increased competition in the United States and in other countries, smaller generation sources that connect directly to distribution systems have emerged. Distributed energy resources are sources of energy including generation and storage devices that are located near local loads. Distributed generation sources include renewable technologies (including geothermal, ocean tides, solar and wind) and nonrenewable technologies (including internal combustion engines, combustion turbines, combined cycle, microturbines, and fuel cells). Microgrids are systems that have distribution systems. The following article describes the benefits of microgrids and several microgrid technologies under development in the United States and other countries. [5].

Making Microgrids Work

Distributed energy resources (DER), including distributed generation (DG) and distributed storage (DS), are sources of energy located near local loads and can provide a variety of benefits including improved reliability if they are properly operated in the electrical distribution system. Microgrids are systems that have at least one distributed energy resource and associated loads and can form intentional islands in the electrical distribution systems. Within microgrids, loads and energy sources can be disconnected from and reconnected to the area or local electric power system with minimal disruption to the local loads. Any time a microgrid is implemented in an electrical distribution system, it needs to be well planned to avoid causing problems.

For microgrids to work properly, an upstream switch must open (typically during an unacceptable power quality condition), and the DER must be able to carry the load on the islanded section. This includes maintaining suitable voltage and frequency levels for all islanded loads. Depending on switch technology, momentary interruptions may occur during transfer from grid-connected to islanded mode. In this case, the DER assigned to carry the island loads should be able to restart and pick up the island load after the switch has opened. Power flow analysis of island scenarios should be performed to insure that proper voltage regulation is maintained and to establish that the DER can handle inrush during "starting" of the island. The DER must be able to supply the real and

("Making Microgrids Work" by Benjamin Kroposki et al. © 2008 IEEE. Reprinted, with permission, from IEEE Power & Energy (May/June 2008), pg. 40–53) reactive power requirements during islanded operation and to sense if a fault current has occurred downstream of the switch location. When power is restored on the utility side, the switch must not close unless the utility and "island" are synchronized. This requires measuring the voltage on both sides of the switch to allow synchronizing the island and the utility.

Microgrids' largest impact will be in providing higher reliability electric service and better power quality to the end customers. Microgrids can also provide additional benefits to the local utility by providing dispatchable power for use during peak power conditions and alleviating or postponing distribution system upgrades.

MICROGRID TECHNOLOGIES

Microgrids consist of several basic technologies for operation. These include DG, DS, interconnection switches, and control systems. One of the technical challenges is the design, acceptance, and availability of low-cost technologies for installing and using microgrids. Several technologies are under development to allow the safe interconnection and use of microgrids (see Figure 1).

DISTRIBUTED GENERATION

DG units are small sources of energy located at or near the point of use. DG technologies (Figures 2–5) typically include photovoltaic (PV), wind, fuel cells, microturbines, and reciprocating internal combustion engines with generators. These systems may be powered by either fossil or renewable fuels.





Some types of DG can also provide combined heat and power by recovering some of the waste heat generated by the source such as the microturbine in Figure 2. This can significantly increase the efficiency of the DG unit. Most of the DG technologies require a power electronics interface in order to convert the energy into gridcompatible ac power. The power electronics interface contains the necessary circuitry to convert power from one form to another. These converters may include both a rectifier and an inverter or just an inverter. The converter is compatible in voltage and frequency with the electric power system to which it will be connected and



Figure 2 Microturbines with heat recovery

contains the necessary output filters. The power electronics interface can also contain protective functions for both the distributed energy system and the local electric power system that allow paralleling and disconnection from the electric power system. These power electronic interfaces provide a unique capability to the DG units and can enhance the operations of a microgrid.

DISTRIBUTED STORAGE

DS technologies are used in microgrid applications where the generation and loads of the microgrid cannot be exactly matched. Distributed storage provides a bridge in meeting the power and energy requirements of the microgrid. Storage capacity is

defined in terms of the time that the nominal energy capacity can cover the load at rated power. Storage capacity can be then categorized in terms of energy density requirements (for medium- and long-term needs) or in terms of power density requirements (for short- and very short-term needs). Distributed storage enhances the overall performance of microgrid systems in three ways. First, it stabilizes and permits

DG units to run at a constant and stable output, despite load fluctuations. Second, it provides the ride-through capability when there are dynamic variations of primary energy (such as those of sun, wind, and hydropower sources). Third, it permits DG to seamlessly operate as a dispatchable unit. Moreover, energy storage can benefit power by damping systems peak surges in electricity demand, countering momentary power disturbances, providing outage ride-through while backup generators respond, and reserving energy for future demand.



Figure 3 Wind turbine



Figure 4 Fuel cell



Figure 5 PV array



Figure 6 Large lead-acid battery bank

There are several forms of energy storage available that can be used in microgrids; these include batteries, supercapacitors, and flywheels. Battery systems store electrical energy in the form of chemical energy (Figure 6). Batteries are dc power systems that require power electronics to convert the energy to and from ac power. Many utility connections for batteries have bidirectional converters, which allow energy to be stored and taken from the batteries. Supercapacitors, also known as ultracapacitors, are electrical energy storage devices that offer high power density and extremely high cycling capability. Flywheel systems have recently regained consideration as a viable means of supporting critical load during grid power interruption because of their fast response compared to electrochemical energy storage. Advances in power electronics and digitally controlled fields have led to better flywheel designs that deliver a cost-effective alternative in the power quality market. Typically, an electric motor supplies mechanical energy to the flywheel and a generator is coupled on the same shaft that outputs the energy, when needed, through a converter. It is also possible to design a bidirectional system with one machine that is capable of motoring and regenerating operations.

INTERCONNECTION SWITCH

The interconnection switch (Figure 7) ties the point of connection between the microgrid and the rest of the distribution system. New technology in this area consolidates the various power and switching functions (e.g., power switching, protective relaying, metering, and communications) traditionally provided by relays, hardware, and other components at the utility interface into a single system with a digital signal processor (DSP). Grid conditions are measured both on the utility and microgrid sides of the switch through current transformers (CTs) and potential transformers (PTs) to determine operational conditions (Figure 8). The interconnection switches are designed to meet grid interconnection standards (IEEE 1547 and UL 1741 for North America) to minimize custom engineering and site-specific approval processes and lower cost. To maximize applicability and functionality, the controls are also designed to be technology neutral and can be used with a circuit breaker as well as faster semiconductor-based static switches like thyristors and integrated gate bipolar transistor technologies and are applicable to a variety of DG assets with conventional generators or power converters.



Figure 7 Interconnection switch and control board

actual reactive power consumed by the load; and protect the internal microgrid.

In stand-alone mode, frequency control is a challenging problem. The frequency response of larger systems is based on rotating masses and these are regarded as



Figure 8

Schematic diagram of a circuit breaker-based interconnection switch

CONTROL SYSTEMS

The control system of a microgrid is designed to safely operate the system in gridconnected and standalone modes. This system may be based on a central controller or imbedded as autonomous parts of each distributed generator. When the utility is disconnected the control system must control the local voltage and frequency, provide (or absorb) the instantaneous real power difference between genand eration loads. provide the difference between generated reactive power and the essential for the inherent stability of these systems. In contrast, microgrids are inherently converter-dominated grids without or with very little directly connected rotating masses, like flywheel energy storage coupled through a converter. Since microturbines and fuel cells have slow response to control signals and are inertia-less, isolated operation is technically demanding and raises load-tracking problems. The converter control systems must be adapted to provide the response previously obtained from directly connected rotating masses. The frequency control strategy should exploit, in a cooperative way, the capabilities of the micro sources to change their active power, through frequency control droops, the response of the storage devices, and load shedding.

Appropriate voltage regulation is necessary for local reliability and stability. Without effective local voltage control, systems with high penetration of distributed energy resources are likely to experience voltage and/or reactive power excursions and oscillations. Voltage control requires that there are no large circulating reactive currents between sources. Since the voltage control is inherently a local problem, voltage regulation faces the same problems in both modes of operation; i.e., isolated or interconnected. In the grid-interconnected mode, it is conceivable to consider that DG units can provide ancillary services in the form of local voltage support. The capability of modern power electronic interfaces offers solutions to the provision of reactive power locally by the adoption of a voltage versus reactive current droop controller, similar to the droop controller for frequency control.

MICROGRID TESTING EXPERIENCE

Around the world, there are several active experiments in the microgrid area covering an array of technologies. As part of this research, microgrid topologies and operational configurations are being defined and design criteria established for all possibilities of microgrid applications.

TESTING EXPERIENCE IN THE UNITED STATES

Consortium for Electric Reliability Solutions (CERTS) Testbed

The objective of the CERTS microgrid testbed is to demonstrate a mature system approach that allows for high penetration of DER equipment by providing a resilient platform for plug-andplay operation, use of waste heat and intermittent sources, and enhancement of the robustness and reliability of the customers' electrical supply. The CERTS microgrid has two main components: a static switch and autonomous sources. The static switch has the ability to autonomously island the microgrid from disturbances such as faults, IEEE 1547 events, or power quality events. After islanding, the reconnection of the microgrid is achieved autonomously after the tripping event is no longer present. This synchronization is achieved by using the frequency difference between the islanded microgrid and the utility grid. Each source can seamlessly balance the power on the islanded microgrid using real power versus frequency droop and maintain voltage using the reactive power versus voltage droop. The coordination between sources is through frequency, and the voltage controller provides local stability. Without local voltage control, systems with high penetrations of DG could experience voltage and/or reactive power oscillations. Voltage control must also insure that there are no large circulating reactive currents between sources. This requires a voltage versus reactive power droop controller so that, as the reactive power generated by the source becomes more capacitive, the local voltage set point is reduced. Conversely, as reactive power becomes more inductive, the voltage set point is increased.

The CERTS microgrid has no "master" controller or source. Each source is connected in a peer-to-peer fashion with a localized control scheme implemented with each component. This arrangement increases the reliability of the system in comparison to a master–slave or centralized control scheme. In the case of a master–slave architecture, the failure of the master controller could compromise the operation of the whole system. The CERTS testbed uses a central communication system to dispatch DG set points as needed to improve overall system operation. However, this communication network is not used for the dynamic operation of the microgrid. This plug-and-play approach allows expansion of the microgrid to meet the requirements of the site without extensive re-engineering.

The CERTS testbed (Figure 9) is located at American Electric Power's Walnut test site in Columbus, Ohio. It consists of three 60-kW converter based sources and a thyristor based static switch. The prime mover in this case is an automobile internal combustion engine converted to run on natural gas. It drives a synchronous generator at variable speeds to achieve maximum efficiencies over a wide range of loads. The output is rectified and inverted to insure a constant ac frequency



Figure 9 CERTS/AEP microgrid testbed

at the microgrid. To insure that the converter can provide the necessary energy demanded by the CERTS controls there is storage on the dc bus. This also insures that the dynamics of the permanent magnet and generator are decoupled from the dynamics of the converter. This insures that a variety of energy sources can have the same dynamic response as the sources used at the testbed.

The testbed has three feeders, two of which have DG units connected and can be islanded. One of these feeders has two sources separated by 170 m of cable. The other feeder has a single source, which allows for testing parallel operation of sources. The third feeder stays connected to the utility but can receive power from the micro sources when the static switch is closed without injecting power into the utility. The objective of the testing is to demonstrate the system dynamics of each component of the CERTS microgrid. This includes smooth transitions from grid-connected to islanded operation and back, high power quality, system protection, speed of response of the sources, operation under difficult loads, and autonomous load tracking.

Figure 10 is an example of islanding dynamics between two sources on a single feeder at the CERTS testbed. Initially, the microgrid is utility connected with unit A and unit B output at 6 kW and 54 kW, respectively. The load is such that the grid provides 42 kW. Upon islanding, unit B exceeds 60 kW and quickly settles at its maximum steady-state operating point of 60 kW with a reduced frequency of 59.8 Hz due to the power versus frequency droop. Unit A increases to 42 kW and converges to the same islanded frequency. The smoothness and speed of the transition is seen in the invert currents and the microgrid voltages. The loads do not see the islanding event.

Figure 11 shows voltage across the switch and the phase currents through the static switch during autonomous synchronization. This synchronization is achieved by using the frequency difference between the islanded







Figure 11

Synchronization of the microgrid to the utility

microgrid and the utility grid. This results in a lowfrequency beat voltage across the switch. When the two voltages come in phase due to this frequency difference the switch will close. The phase currents display a smooth transition due to closing at zero voltage phase difference. The unbalanced currents are driven by a utility voltage unbalance of around 1% and a balanced voltage created by the DG source. All loads see balanced voltages provided by the DG sources. The neutral third harmonic current and phase current distortion are due to transformer magnetization currents.

The fundamental and third-harmonic frequency component from the transformer magnetization is apparent. As the loading of the transformer increases, the distortion becomes a smaller component of the total current.

Interconnection Switch Testing

The National Renewable Energy Laboratory has worked with a variety of U.S. interconnection switch manufacturers on the development of advanced interconnection technologies that allow paralleling of distributed generators with the utility for uninterrupted electrical service and the ability to parallel and sell electricity back to the utility. This research promotes the development of new products and technologies that enable faster switching, greater reliability, and lower fault currents on the electrical grids, thereby providing fewer disruptions for customers while expanding capabilities as an energyintensive world becomes more energy efficient in the future.

Testing of the various switch technologies includes typical protective relay function tests such as detection and tripping for over- and undervoltage, over- and underfrequency, phase sequence, reverse power, instantaneous over-current, and discrete event trip tests. To evaluate the switches' interconnection requirements, conformance tests to the IEEE 1547.1 standard are conducted. These tests evaluate if the unit detects and trips for over- and undervoltage, over- and underfrequency, synchronization, unintentional islanding, reconnection, and open-phase tests. To evaluate the power quality functions of the switch, tests are performed to verify that the switch responded as expected, which was to disconnect the grid and DG terminals when a power quality event occurred. Figure 12 shows results from the power quality testing done on a circuit-breaker-based switch. This testing showed that there is a minimum trip time for the breaker (0.005 s) and that the control logic for the breaker needs



Figure 12 Testing of a circuit breaker-based microgrid switch versus the ITI curve

to be more accurately tuned to stay within the Information Technology Industry (ITI) Council curve.

TESTING EXPERIENCE IN JAPAN

The New Energy and Industrial Technology Development Organization (NEDO) is currently supporting a variety of microgrid demonstration projects applying renewable and distributed generation. The first group of projects, called Regional Power Grids with Various New Energies, was implemented at three locations in Japan: Expo 2005 Aichi, recently moved to the Central Japan Airport City (Aichi project), Kyoto Eco-Energy project (Kyotango project), and Regional Power Grid with Renewable Energy Resources in Hachinohe City (Hachinohe project). In these three projects, control systems capable of matching energy demand and supply for microgrid operation were established. An important target in all of the projects is achieving a matched supply and demand of electricity. In each project, a standard for the margin of error between supplied energy and consumed energy over a certain period was set as a control target.

In the Aichi project, a power supply system utilizing fuel cells, PV, and a battery storage system, all equipped with converters, was constructed. A block diagram of the supply system for the project is shown in Figure 13. The fuel cells adopted for the system include two molten carbonate fuel cells (MCFCs) with capacities of 270 kW and 300 kW, one 25-kW solid oxide fuel cell (SOFC), and four 200-kW phosphoric acid fuel cells (PAFCs). The total capacity of the installed PV systems is 330 kW, and the adopted cell types include multicrystalline silicon, amorphous silicon, and a single crystalline silicon bifacial type. A sodium-sulfur (NaS) battery is used to store energy within the supply system and it plays an important role in matching supply and demand. In the Aichi project, the load-generation balancing has been maintained at 3% for as short as ten-minute intervals. The Aichi project experienced a second grid-independent operation mode in September 2007. In this operational mode, the NaS battery converter controls voltage and balancing of the load.

In the Kyotango project, the energy supply facilities and demand sites are connected to a utility grid and are integrated by a master control system. The energy supply system functions as a "virtual microgrid." A management system for matching the demand and supply of electricity is being demonstrated and a reduction in imbalances to within 3% of expected demand for five-minute intervals was achieved. Several criteria related to power quality (outages, voltage fluctuations, and frequency fluctuations) are being





monitored during the demonstration period to determine if the system can achieve and maintain the same power quality level as a utility network. In the plant, gas engines with a total capacity of 400 kW were installed together with a 250-kW MCFC and a 100-kW lead-acid battery. In remote locations, two PV systems and one 50-kW small wind turbine were also installed. The power generation equipment and end-user demand are managed by remote monitoring and control. One of the interesting features of the system is that it is managed not by a state-of-the-art information network system but by conventional information networks, which are the only network systems available in rural areas.

The Hachinohe project (Figure 14) features a microgrid system constructed using a private distribution line measuring more than 5 km. The private distribution line was constructed to transmit electricity primarily generated by the gas engine system. Several PV systems and small wind turbines are also connected to the microgrid. At the sewage plant, three 170-kW gas engines and a 50-kW PV system have been installed. To support the creation of digestion gas by the sewage plant, a wood-waste steam boiler was also installed due to a shortage of thermal heat to safeguard the bacteria. Between the sewage plant and city office, four schools and a water supply authority office are connected to the private distribution line. At the school sites, renewable energy resources are used to create a power supply that fluctuates according to weather conditions in order to prove the microgrid's control system's capabilities to match demand and supply. The control system used to balance supply and demand consists of three facets: weekly supply and demand planning, economic dispatch control once every three minutes, and second-by-second power flow control at interconnection points. The control target is a margin of error between supply and demand of less than 3% for every six-minute interval. During testing, a margin of error rate of less than 3% was achieved during 99.99% of the system's operational time. The Hachinohe project experienced one week of grid-independent operation in November 2007. In this operational mode, imbalance among the three phases was compensated by the PV converter.

The New Power Network Systems project is evaluating new test equipment installed on a test distribution network (Figure 15) constructed at the Akagi Test Center of the Central Research Institute of the Electric Power Industry (CRIEPI). This equipment includes a static var compensator (SVC), a step voltage regulator (SVR), and loop balance controllers (LBCs). The SVC and SVR are



Figure 14 Overview of the Hachinohe project



Figure 15 Structure of test network at CRIEPI

used for controlling the voltage on a distribution line, and they are sometimes applied on an actual utility network. In this project, the effects of integrated control of this equipment are being examined. LBCs are a new type of distribution network equipment that can control the power flow between two distribution feeders by means of a backto-back (BTB) type converter. The LBCs allow connections of two sources with different voltages, frequencies, and phase angles by providing a dc link.

A final microgrid project is evaluating the possibility that grid technology can create value for consumers and various energy service levels. In Sendai City a microgrid consisting of two 350-kW gas engine generators, one 250-kW MCFC, and various types of compensating equipment is being evaluated to demonstrate four levels of customer power. Two of the service levels will have compensating equipment that includes an integrated power quality backup system that supplies high-quality power that reduces interruptions and voltage drops. In one of these cases, the wave pattern is guaranteed. Two additional lower service levels have only short-term voltage drops compensated by a series compensator. This work will evaluate the possibility of providing various service levels to customers located in the same area. Since summer of 2007, the Sendai system has been in operation and has improved the power quality at the site. Before starting actual operation, the compensation equipment was tested by using a BTB power supply system to create artificial voltage sag.

In addition to the NEDO-sponsored projects, there are several private microgrid projects. Tokyo Gas has been evaluating a 100-kW microgrid test facility since September 2006 at the Yokohama Research Institute, consisting of gas-engine combined heat and power (CHP), PV, wind power, and battery-incorporated power electronics. Shimizu Corp. has developed a microgrid control system with a small microgrid that consists of gas engines, gas turbines, PV, and batteries. The system is designed for load following and includes load forecasting and integrated control for heat and power.

TESTING EXPERIENCE IN CANADA

Planned microgrid islanding application, also known as intentional islanding, is an early utility adaptation of the microgrid concept that has been implemented by BC Hydro and Hydro Quebec, two of the major utility companies in Canada. The main objective of planned islanding projects is to enhance customer-based power supply reliability on rural feeders by utilizing an appropriately located independent power producer (IPP), which is, for instance, located on the same or adjacent feeder of a distribution substation. In one case, the customers in Boston Bar town, part of the BC Hydro rural areas, which is supplied by three 25-kV medium-voltage distribution feeders, had been exposed to power outages of 12 to 20 hrs two or three times per year. This area, as shown in Figure 16, is supplied by a 69/25-kV distribution substation and is connected to the BC Hydro high-voltage system through 60 km of 69-kV line. Most of the line is built off a highway in a canyon that is difficult to access with high potential of rock/mud/snow slides. The implemented option to reduce sustained power-outage durations is based on utilizing a local IPP to operate in an intentional island mode and supply the town load on one or more feeders of the substation. The Boston Bar IPP has two 3.45-MW hydro power generators and is connected to one of the three feeders with a peak load of 3.0 MW. Depending on the water level, the Boston Bar IPP can supply the community load on one or more of the feeders during the islanding operation. If the water level is not sufficient, the load on one feeder can be sectioned to adequate portions.

Based on the BC Hydro islanding guideline, to perform planned islanding, an IPP should be equipped with additional equipment and control systems for voltage regulation, frequency stabilization, and fault protection. In addition, the island-load serving capability of an IPP needs to be tested prior to and during the project commissioning to ensure



Figure 16

System configuration for the Boston Bar IPP and BC Hydro planned islanding site

that the IPP can properly respond to load transients such as a step change in load and still sustain the island.

The functional requirements added to the Boston Bar IPP to support planned islanding are as follows:

- governor speed control with fixed-frequency (isochronous) mode for single-unit operation and speeddroop settings for two-unit operation in parallel
- engineering mass of generators and hydro turbines to increase inertia and improve transient response
- excitation system control with positive voltage field forcing for output current boost during the feeder fault to supply high fault current for proper coordination of protection relays
- automatic voltage regulation control to regulate voltages at the point of common coupling
- two sets of overcurrent protection set-points for the grid-connected and the islanding operating modes
- real-time data telemetry via a leased telephone line between the IPP remote control site and the utility area control center
- black start capability via an onsite 55-kW diesel generator.

In addition to the above upgrades, the auto-recloser on the connecting IPP feeder is equipped with a secondary voltage supervision function for voltage supervisory close and blocking of the auto-reclosing action. Remote auto-synchronization capability was also added at the substation level to synchronize and connect the island area to the 69-kV feeder without causing load interruption. When a sustain power outage event, such as a permanent fault or line breakdown, occurs on the utility side of the substation, the main circuit breaker and feeder reclosers are opened (Figure 16). Then, the substation breaker open position is telemetered to the IPP operator. Subsequently, the IPP changes the control and protection settings to the island mode and attempts to hold the island downstream of the feeder 2 recloser. If the IPP fails to sustain the island, the IPP activates a black-start procedure and picks up the dead feeder load under the utility supervision. The island load may be supplied by one generator or both generators in parallel.

Two sets of tests were performed during the generator commissioning as follows:

 grid parallel operation tests including a) the automatic and manual synchronization, and b) output load, voltage and frequency controls, and load rejection tests

2) island operation tests comprising a) load pick-up and drop-off tests in 350-kW increments, b) dead load pick-up of 1.2 MW when only one of the two generators is in operation, and c) islanded operation and load following capability when one unit is generating and/or both units are operating in parallel.

The planned islanding operation of the Boston Bar IPP has been successfully demonstrated and performed several times during power outages caused by adverse environmental effects. Building on the knowledge and experience gained from this project, BC Hydro has recently completed a second case of planned islanding and is presently assessing a third project.

TESTING IN EUROPE

At the international level, the European Union has supported two major research efforts devoted exclusively to microgrids: the Microgrids and More Microgrids projects. The Microgrids project focused on the operation of a single microgrid, has successfully investigated appropriate control techniques, and demonstrated the feasibility of microgrid operation through laboratory experiments. The Microgrids project investigated a microgrid central controller (MCC) that promotes technical and economical operation, interfaces with loads and micro sources and demand-side management, and provides set points or supervises local control to interruptible loads and micro sources. A pilot installation was installed in Kythnos Island, Greece, that evaluated a variety of DER to create a microgrid.

Continuing microgrid projects in Greece include a laboratory facility (Figure 17) that has been set up at the National Technical University of Athens (NTUA), with the objective to test small-scale equipment and control strategies for micro-grid operation. The system comprises two poles, each equipped with local (PV and wind) generation and battery storage, connected to each other via a low-voltage line as well as to the main grid. Each pole may operate as a micro-grid via its own connection to the grid, or both poles may be connected via the low-voltage line to form a two-bus micro-grid connected to the main grid at one end. The battery converters are the main regulating units in island mode, regulated via active power-frequency and reactive power-voltage droops. Multi-agent technology has been implemented for the control of the sources and the loads.





Figure 18 shows indicative test results demonstrating the seamless transition of the microgrid from gridconnected to island mode and vice-versa (one-pole microgrid operation). The first diagram illustrates the variation of the frequency and the second of the voltage. The change of the component power flows is shown in the third illustration. While the load and the PV continue operating at the same power, the output of the battery converter and the power flow from the grid change to maintain the power equilibrium in the microgrid.

Testing on microgrid components has also been extensively conducted by ISET in Germany. Figure 19 shows



Figure 18

Changes of the microgrid operating mode from island to interconnected mode and vice-versa: (a) frequency, (b) voltage, and (c) component powers

testing conducted to examine voltage and current transient when microgrids transfer from grid-connected to islanded mode. This figure shows that with proper design, there can be minimal load disruption during the transfer.



Figure 19

Voltage and current changes as the microgrid switches to islanded mode

The More Microgrids project aims at the increase of penetration of microgeneration in electrical networks through the exploitation and extension of the Microgrids concept, involving the investigation of alternative microgenerator control strategies and alternative network designs, development of new tools for multimicrogrid management operation and standardization of technical and commercial protocols, and field trials on actual microgrids and evaluation of the system performance on power system operation.

One of the More Microgrids projects is located at Bronsbergen Holiday Park, located near Zutphen in the Netherlands. It comprises 210 cottages, 108 of which are equipped with grid-connected PV systems. The park is electrified by a traditional three-phase 400-V network, which is connected to a 10-kV medium-voltage network via a distribution transformer located on the premises (Figure 20). The distribution transformer does not feed any low-voltage loads outside of the holiday park. Internally in the park, the 400-V supply from the distribution transformer is distributed over four cables, each protected by 200-A fuses on the three phases. The peak load is approximately 90 kW. The installed power of all the PV systems together is 315 kW. The objective of this project is experimental validation of islanded microgrids by means of smart storage (coupled by a flexible ac distribution system) including evaluation of islanded operation, automatic isolation and reconnection, fault level of the microgrid, harmonic voltage distortion, energy management and lifetime optimization of the storage system, and parallel operation of converters.

Another More Microgrids project involves field test on the transfer between interconnected and islanding mode with German utility MVV Energie. MVV Energie is planning to develop an efficient solution to cope with the expected future high penetration of renewable energy sources and distributed generation in the low-voltage distribution grid. If integrated in an intelligent way, these new players in the distribution grid will improve independence from energy imports, reliability, and power quality at lower cost than the "business as usual" regarding replacement or reinforcement of the regional energy infrastructure. A successful transfer between interconnected and islanding



Figure 20

Schematic for the Bronsbergen Holiday Park microgrid

mode would provide a substantial benefit for the grid operator.

This project will evaluate decentralized control in a residential site in the ecological settlement in Mannheim-Wallstadt. The new control structures for the decentralized control with agents will be tested and allow the transition from grid connection to islanding operation without interruptions. This would improve reliability of the grid and support for black start after failure of the grid.

The CESI RICERCA test facility in Italy will also be used to experiment, demonstrate, and validate the operation of an actual microgrid field test of different microgrid topologies at steady and transient state and power quality analysis. During a transient state, the behavior during short-duration voltage variation for single/threephase ac faults, or dynamic response to sudden load changes and to conditions of phase imbalance or loss of phase, the islanding conditions following interruption of the supply will be analyzed.

CONCLUSIONS

Microgrids will provide improved electric service reliability and better power quality to end customers and can also benefit local utilities by providing dispatchable load for use during peak power conditions and alleviating or postponing distribution system upgrades. There are a number of active microgrid projects around the world involved with testing and evaluation of these advanced operating concepts for electrical distribution systems.

FOR FURTHER READING

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2.I

PHASORS

A sinusoidal voltage or current at constant frequency is characterized by two parameters: a maximum value and a phase angle. A voltage

$$v(t) = V_{\max} \cos(\omega t + \delta) \tag{2.1.1}$$

has a maximum value V_{max} and a phase angle δ when referenced to $\cos(\omega t)$. The root-mean-square (rms) value, also called *effective value*, of the sinusoidal voltage is

$$V = \frac{V_{max}}{\sqrt{2}}$$
(2.1.2)

Euler's identity, $e^{j\phi} = \cos \phi + j \sin \phi$, can be used to express a sinusoid in terms of a phasor. For the above voltage,

$$v(t) = \operatorname{Re}[\operatorname{V}_{\max} e^{j(\omega t + \delta)}]$$
$$= \operatorname{Re}[\sqrt{2}(\operatorname{V} e^{j\delta})e^{j\omega t}]$$
(2.1.3)

where $j = \sqrt{-1}$ and Re denotes "real part of." The rms phasor representation of the voltage is given in three forms—exponential, polar, and rectangular:

$$V = \underbrace{Ve^{j\delta}}_{\text{exponential}} = \underbrace{V/\delta}_{\text{polar}} = \underbrace{V\cos\delta + jV\sin\delta}_{\text{rectangular}}$$
(2.1.4)

A phasor can be easily converted from one form to another. Conversion from polar to rectangular is shown in the phasor diagram of Figure 2.1. Euler's identity can be used to convert from exponential to rectangular form. As an example, the voltage

$$v(t) = 169.7 \cos(\omega t + 60^{\circ})$$
 volts (2.1.5)

has a maximum value $V_{max} = 169.7$ volts, a phase angle $\delta = 60^{\circ}$ when referenced to $\cos(\omega t)$, and an rms phasor representation in polar form of

$$V = 120/60^{\circ}$$
 volts (2.1.6)

Also, the current

$$i(t) = 100\cos(\omega t + 45^{\circ})$$
 A (2.1.7)

FIGURE 2.1 Imaginary axis

Phasor diagram for converting from polar to rectangular form



FIGURE 2.2

Summary of relationships between phasors V and I for constant R, L, and C elements with sinusoidalsteady-state excitation



has a maximum value $I_{max} = 100$ A, an rms value $I = 100/\sqrt{2} = 70.7$ A, a phase angle of 45°, and a phasor representation

$$I = 70.7/45^{\circ} = 70.7e^{j45} = 50 + j50 \quad A \tag{2.1.8}$$

The relationships between the voltage and current phasors for the three passive elements—resistor, inductor, and capacitor—are summarized in Figure 2.2, where sinusoidal-steady-state excitation and constant values of R, L, and C are assumed.

When voltages and currents are discussed in this text, lowercase letters such as v(t) and i(t) indicate instantaneous values, uppercase letters such as V and I indicate rms values, and uppercase letters in italics such as V and I indicate rms phasors. When voltage or current values are specified, they shall be rms values unless otherwise indicated.

2.2

INSTANTANEOUS POWER IN SINGLE-PHASE AC CIRCUITS

Power is the rate of change of energy with respect to time. The unit of power is a watt, which is a joule per second. Instead of saying that a load absorbs energy at a rate given by the power, it is common practice to say that a load absorbs power. The instantaneous power in watts absorbed by an electrical load is the product of the instantaneous voltage across the load in volts and the instantaneous current into the load in amperes. Assume that the load voltage is

$$v(t) = V_{\text{max}} \cos(\omega t + \delta)$$
 volts (2.2.1)

We now investigate the instantaneous power absorbed by purely resistive, purely inductive, purely capacitive, and general RLC loads. We also introduce the concepts of real power, power factor, and reactive power. The physical significance of real and reactive power is also discussed.

PURELY RESISTIVE LOAD

For a purely resistive load, the current into the load is in phase with the load voltage, I = V/R, and the current into the resistive load is

$$i_{\rm R}(t) = I_{\rm Rmax} \cos(\omega t + \delta)$$
 A (2.2.2)

where $I_{Rmax} = V_{max}/R$. The instantaneous power absorbed by the resistor is

$$p_{R}(t) = v(t)i_{R}(t) = V_{\max}I_{R\max}\cos^{2}(\omega t + \delta)$$
$$= \frac{1}{2}V_{\max}I_{R\max}\{1 + \cos[2(\omega t + \delta)]\}$$
$$= VI_{R}\{1 + \cos[2(\omega t + \delta)]\} \quad W$$
(2.2.3)

As indicated by (2.2.3), the instantaneous power absorbed by the resistor has an average value

$$P_R = VI_R = \frac{V^2}{R} = I_R^2 R \quad W \tag{2.2.4}$$

plus a double-frequency term VI_R $\cos[2(\omega t + \delta)]$.

PURELY INDUCTIVE LOAD

For a purely inductive load, the current lags the voltage by 90°, $I_{\rm L} = V/(jX_{\rm L})$, and

$$i_{\rm L}(t) = I_{\rm Lmax} \cos(\omega t + \delta - 90^{\circ}) \quad A \tag{2.2.5}$$

where $I_{Lmax} = V_{max}/X_L$, and $X_L = \omega L$ is the inductive reactance. The instantaneous power absorbed by the inductor is*

$$p_{\rm L}(t) = v(t)i_{\rm L}(t) = V_{\rm max}I_{\rm Lmax}\cos(\omega t + \delta)\cos(\omega t + \delta - 90^{\circ})$$
$$= \frac{1}{2}V_{\rm max}I_{\rm Lmax}\cos[2(\omega t + \delta) - 90^{\circ}]$$
$$= VI_{\rm L}\sin[2(\omega t + \delta)] \quad W$$
(2.2.6)

As indicated by (2.2.6), the instantaneous power absorbed by the inductor is a double-frequency sinusoid with *zero* average value.

PURELY CAPACITIVE LOAD

For a purely capacitive load, the current leads the voltage by 90°, $I_{\rm C} = V/(-jX_{\rm C})$, and

$$i_{\rm C}(t) = I_{\rm Cmax} \cos(\omega t + \delta + 90^\circ) \quad A \tag{2.2.7}$$

where $I_{Cmax}=V_{max}/X_C,$ and $X_C=1/(\omega C)$ is the capacitive reactance. The instantaneous power absorbed by the capacitor is

$$p_{\rm C}(t) = v(t)i_{\rm C}(t) = V_{\rm max}I_{\rm Cmax}\cos(\omega t + \delta)\cos(\omega t + \delta + 90^{\circ})$$
$$= \frac{1}{2}V_{\rm max}I_{\rm Cmax}\cos[2(\omega t + \delta) + 90^{\circ})]$$
$$= -VI_{\rm C}\sin[2(\omega t + \delta)] \quad W$$
(2.2.8)

The instantaneous power absorbed by a capacitor is also a double-frequency sinusoid with *zero* average value.

GENERAL RLC LOAD

For a general load composed of RLC elements under sinusoidal-steady-state excitation, the load current is of the form

$$i(t) = I_{\text{max}} \cos(\omega t + \beta) \quad A$$
 (2.2.9)

The instantaneous power absorbed by the load is then*

$$p(t) = v(t)i(t) = V_{\max}I_{\max}\cos(\omega t + \delta)\cos(\omega t + \beta)$$

= $\frac{1}{2}V_{\max}I_{\max}\{\cos(\delta - \beta) + \cos[2(\omega t + \delta) - (\delta - \beta)]\}$
= VI $\cos(\delta - \beta) + VI \cos(\delta - \beta) \cos[2(\omega t + \delta)]$
+ VI $\sin(\delta - \beta) \sin[2(\omega t + \delta)]$
 $p(t) = VI \cos(\delta - \beta)\{1 + \cos[2(\omega t + \delta)]\} + VI \sin(\delta - \beta) \sin[2(\omega t + \delta)]$

Letting I $\cos(\delta - \beta) = I_R$ and I $\sin(\delta - \beta) = I_X$ gives

$$p(t) = \underbrace{\text{VI}_{R}\{1 + \cos[2(\omega t + \delta)]\}}_{p_{R}(t)} + \underbrace{\text{VI}_{X} \sin[2(\omega t + \delta)]}_{p_{X}(t)}$$
(2.2.10)

As indicated by (2.2.10), the instantaneous power absorbed by the load has two components: One can be associated with the power $p_{\rm R}(t)$ absorbed by the resistive component of the load, and the other can be associated with the power $p_{\rm X}(t)$ absorbed by the reactive (inductive or capacitive) component of the load. The first component $p_{\rm R}(t)$ in (2.2.10) is identical to (2.2.3), where $I_{\rm R} = I \cos(\delta - \beta)$ is the component of the load current in phase with the load voltage. The phase angle $(\delta - \beta)$ represents the angle between the voltage and current. The second component $p_{\rm X}(t)$ in (2.2.10) is identical to (2.2.6) or (2.2.8), where $I_{\rm X} = I \sin(\delta - \beta)$ is the component of load current 90° out of phase with the voltage.

^{*} Use the identity: $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)].$

REAL POWER

Equation (2.2.10) shows that the instantaneous power $p_R(t)$ absorbed by the resistive component of the load is a double-frequency sinusoid with average value P given by

$$P = VI_R = VI\cos(\delta - \beta) \quad W \tag{2.2.11}$$

The *average power* P is also called *real power* or *active power*. All three terms indicate the same quantity P given by (2.2.11).

POWER FACTOR

The term $\cos(\delta - \beta)$ in (2.2.11) is called the *power factor*. The phase angle $(\delta - \beta)$, which is the angle between the voltage and current, is called the *power factor angle*. For dc circuits, the power absorbed by a load is the product of the dc load voltage and the dc load current; for ac circuits, the average power absorbed by a load is the product of the rms load voltage V, rms load current I, and the power factor $\cos(\delta - \beta)$, as shown by (2.2.11). For inductive loads, the current lags the voltage, which means β is less than δ , and the power factor is said to be *lagging*. For capacitive loads, the current leads the voltage, which means β is greater than δ , and the power factor is said to be *leading*. By convention, the power factor $\cos(\delta - \beta)$ is positive. If $|\delta - \beta|$ is greater than 90°, then the reference direction for current may be reversed, resulting in a positive value of $\cos(\delta - \beta)$.

REACTIVE POWER

The instantaneous power absorbed by the reactive part of the load, given by the component $p_X(t)$ in (2.2.10), is a double-frequency sinusoid with zero average value and with amplitude Q given by

$$Q = VI_X = VI \sin(\delta - \beta) \quad \text{var} \tag{2.2.12}$$

The term Q is given the name *reactive power*. Although it has the same units as real power, the usual practice is to define units of reactive power as volt-amperes reactive, or var.

EXAMPLE 2.1 Instantaneous, real, and reactive power; power factor

The voltage $v(t) = 141.4 \cos(\omega t)$ is applied to a load consisting of a 10- Ω resistor in parallel with an inductive reactance $X_L = \omega L = 3.77 \Omega$. Calculate the instantaneous power absorbed by the resistor and by the inductor. Also calculate the real and reactive power absorbed by the load, and the power factor.



SOLUTION The circuit and phasor diagram are shown in Figure 2.3(a). The load voltage is

$$V = \frac{141.4}{\sqrt{2}} / \frac{0^{\circ}}{\sqrt{2}} = 100 / \frac{0^{\circ}}{\sqrt{2}}$$
 volts

The resistor current is

$$I_{\rm R} = \frac{V}{\rm R} = \frac{100}{10} \underline{/0^{\circ}} = 10 \underline{/0^{\circ}} \, \text{A}$$

The inductor current is

$$I_{\rm L} = \frac{V}{jX_{\rm L}} = \frac{100}{(j3.77)} \underline{/0^{\circ}} = 26.53 \underline{/-90^{\circ}} \quad A$$

The total load current is

$$I = I_{\rm R} + I_{\rm L} = 10 - j26.53 = 28.35/-69.34^{\circ}$$
 A

The instantaneous power absorbed by the resistor is, from (2.2.3),

$$p_{\rm R}(t) = (100)(10)[1 + \cos(2\omega t)]$$

= 1000[1 + cos(2\omega t)] W

The instantaneous power absorbed by the inductor is, from (2.2.6),

$$p_{\rm L}(t) = (100)(26.53) \sin(2\omega t)$$

= 2653 sin(2\omega t) W

The real power absorbed by the load is, from (2.2.11),

$$P = VI \cos(\delta - \beta) = (100)(28.53) \cos(0^{\circ} + 69.34^{\circ})$$
$$= 1000 \text{ W}$$

(*Note*: P is also equal to $VI_R = V^2/R$.) The reactive power absorbed by the load is, from (2.2.12),

Q = VI sin(
$$\delta - \beta$$
) = (100)(28.53) sin(0° + 69.34°)
= 2653 var

(*Note*: Q is also equal to $VI_L = V^2/X_L$.) The power factor is

p.f. =
$$\cos(\delta - \beta) = \cos(69.34^{\circ}) = 0.3528$$
 lagging

Voltage, current, and power waveforms are shown in Figure 2.3(b).

As shown for this parallel RL load, the resistor absorbs real power (1000 W) and the inductor absorbs reactive power (2653 var). The resistor current $i_R(t)$ is in phase with the load voltage, and the inductor current $i_L(t)$ lags the load voltage by 90°. The power factor is lagging for an RL load.

Note that $p_R(t)$ and $p_X(t)$, given by (2.2.10), are strictly valid only for a parallel R-X load. For a general RLC load, the voltages across the resistive and reactive components may not be in phase with the source voltage v(t), resulting in additional phase shifts in $p_R(t)$ and $p_X(t)$ (see Problem 2.13). However, (2.2.11) and (2.2.12) for P and Q are valid for a general RLC load.

PHYSICAL SIGNIFICANCE OF REAL AND REACTIVE POWER

The physical significance of real power P is easily understood. The total energy absorbed by a load during a time interval T, consisting of one cycle of the sinusoidal voltage, is PT watt-seconds (Ws). During a time interval of *n* cycles, the energy absorbed is P(nT) watt-seconds, all of which is absorbed by the resistive component of the load. A kilowatt-hour meter is designed to measure the energy absorbed by a load during a time interval $(t_2 - t_1)$, consisting of an integral number of cycles, by integrating the real power P over the time interval $(t_2 - t_1)$.

The physical significance of reactive power Q is not as easily understood. Q refers to the maximum value of the instantaneous power absorbed by the reactive component of the load. The instantaneous reactive power, given by the second term $p_X(t)$ in (2.2.10), is alternately positive and negative, and it expresses the reversible flow of energy to and from the reactive component of the load. Q may be positive or negative, depending on the sign of $(\delta - \beta)$ in (2.2.12). Reactive power Q is a useful quantity when describing the operation of power systems (this will become evident in later chapters). As one example, shunt capacitors can be used in transmission systems to deliver reactive power and thereby increase voltage magnitudes during heavy load periods (see Chapter 5).

2.3

COMPLEX POWER

For circuits operating in sinusoidal-steady-state, real and reactive power are conveniently calculated from complex power, defined below. Let the voltage across a circuit element be $V = V/\delta$, and the current into the element be $I = I/\beta$. Then the complex power S is the product of the voltage and the conjugate of the current:

$$S = VI^* = [V\underline{\delta}][I\underline{\beta}]^* = VI\underline{\delta} - \beta$$

= VI cos(\delta - \beta) + \beta VI sin(\delta - \beta) (2.3.1)

where $(\delta - \beta)$ is the angle between the voltage and current. Comparing (2.3.1) with (2.2.11) and (2.2.12), S is recognized as

$$\mathbf{S} = \mathbf{P} + j\mathbf{Q} \tag{2.3.2}$$

The magnitude S = VI of the complex power *S* is called the *apparent power*. Although it has the same units as P and Q, it is common practice to define the units of apparent power S as voltamperes or VA. The real power P is obtained by multiplying the apparent power S = VI by the power factor $p.f. = cos(\delta - \beta)$.

The procedure for determining whether a circuit element absorbs or delivers power is summarized in Figure 2.4. Figure 2.4(a) shows the *load*

FIGURE 2.4

Load and generator conventions



(a) Load convention. Current enters positive terminal of circuit element. If P is positive, then positive real power is absorbed. If Q is positive, then positive reactive power is absorbed. If P (Q) is negative, then positive real (reactive) power is *delivered*.

(b) Generator convention. Current leaves positive terminal of the circuit element. If P is positive, then positive real power is *delivered*. If Q is positive, then positive reactive power is *delivered*. If P (Q) is negative, then positive real (reactive) power is *absorbed*. convention, where the current enters the positive terminal of the circuit element, and the complex power absorbed by the circuit element is calculated from (2.3.1). This equation shows that, depending on the value of $(\delta - \beta)$, P may have either a positive or negative value. If P is positive, then the circuit element absorbs positive real power. However, if P is negative, the circuit element absorbs negative real power, or alternatively, it delivers positive real power. Similarly, if Q is positive, the circuit element in Figure 2.4(a) absorbs positive reactive power. However, if Q is negative, the circuit element absorbs negative reactive power, or it delivers positive reactive power.

Figure 2.4(b) shows the *generator convention*, where the current *leaves* the positive terminal of the circuit element, and the complex power *delivered* is calculated from (2.3.1). When P is positive (negative) the circuit element *delivers* positive (negative) real power. Similarly, when Q is positive (negative), the circuit element *delivers* positive (negative) reactive power.

EXAMPLE 2.2 Real and reactive power, delivered or absorbed

A single-phase voltage source with $V = 100/130^{\circ}$ volts delivers a current $I = 10/10^{\circ}$ A, which leaves the positive terminal of the source. Calculate the source real and reactive power, and state whether the source delivers or absorbs each of these.

SOLUTION Since *I* leaves the positive terminal of the source, the generator convention is assumed, and the complex power delivered is, from (2.3.1),

$$S = VI^* = [100/130^\circ][10/10^\circ]^*$$

$$S = 1000/120^\circ = -500 + j866$$

$$P = \text{Re}[S] = -500 \text{ W}$$

$$Q = \text{Im}[S] = +866 \text{ var}$$

where Im denotes "imaginary part of." The source absorbs 500 W and delivers 866 var. Readers familiar with electric machines will recognize that one example of this source is a synchronous motor. When a synchronous motor operates at a leading power factor, it absorbs real power and delivers reactive power.

The *load convention* is used for the RLC elements shown in Figure 2.2. Therefore, the complex power *absorbed* by any of these three elements can be calculated as follows. Assume a load voltage $V = V/\delta$. Then, from (2.3.1),

resistor:
$$S_{\rm R} = VI_{\rm R}^* = [V/\underline{\delta}] \left[\frac{V}{R} / \underline{-\delta} \right] = \frac{V^2}{R}$$
 (2.3.3)

inductor:
$$S_{\rm L} = VI_{\rm L}^* = [V\underline{\delta}] \left[\frac{V}{-jX_{\rm L}} \underline{\delta} \right] = +j \frac{V^2}{X_{\rm L}}$$
 (2.3.4)

capacitor:
$$S_{\rm C} = VI_{\rm C}^* = [V/\underline{\delta}] \left[\frac{V}{jX_{\rm C}} / \underline{-\delta} \right] = -j \frac{V^2}{X_{\rm C}}$$
 (2.3.5)



From these complex power expressions, the following can be stated:

A (positive-valued) resistor absorbs (positive) real power, $P_R = V^2/R$ W, and zero reactive power, $Q_R = 0$ var.

An inductor absorbs zero real power, $P_L = 0$ W, and positive reactive power, $Q_L = V^2/X_L$ var.

A capacitor *absorbs* zero real power, $P_C = 0$ W, and *negative* reactive power, $Q_C = -V^2/X_C$ var. Alternatively, a capacitor *delivers positive* reactive power, $+V^2/X_C$.

For a general load composed of RLC elements, complex power S is also calculated from (2.3.1). The real power P = Re(S) absorbed by a passive load is always positive. The reactive power Q = Im(S) absorbed by a load may be either positive or negative. When the load is inductive, the current lags the voltage, which means β is less than δ in (2.3.1), and the reactive power absorbed is positive. When the load is capacitive, the current leads the voltage, which means β is greater than δ , and the reactive power absorbed is negative; or, alternatively, the capacitive load delivers positive reactive power.

Complex power can be summarized graphically by use of the power triangle shown in Figure 2.5. As shown, the apparent power S, real power P, and reactive power Q form the three sides of the power triangle. The power factor angle $(\delta - \beta)$ is also shown, and the following expressions can be obtained:

$$\mathbf{S} = \sqrt{\mathbf{P}^2 + \mathbf{Q}^2} \tag{2.3.6}$$

$$(\delta - \beta) = \tan^{-1}(\mathbf{Q}/\mathbf{P}) \tag{2.3.7}$$

$$\mathbf{Q} = \mathbf{P} \tan(\delta - \beta) \tag{2.3.8}$$

p.f. =
$$\cos(\delta - \beta) = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$
 (2.3.9)

EXAMPLE 2.3 Power triangle and power factor correction

A single-phase source delivers 100 kW to a load operating at a power factor of 0.8 lagging. Calculate the reactive power to be delivered by a capacitor connected in parallel with the load in order to raise the source power factor



to 0.95 lagging. Also draw the power triangle for the source and load. Assume that the source voltage is constant, and neglect the line impedance between the source and load.

SOLUTION The circuit and power triangle are shown in Figure 2.6. The real power $P = P_S = P_R$ delivered by the source and absorbed by the load is not changed when the capacitor is connected in parallel with the load, since the capacitor delivers only reactive power Q_C . For the load, the power factor angle, reactive power absorbed, and apparent power are

$$\begin{aligned} \theta_{\rm L} &= (\delta - \beta_{\rm L}) = \cos^{-1}(0.8) = 36.87^{\circ} \\ Q_{\rm L} &= P \tan \theta_{\rm L} = 100 \tan(36.87^{\circ}) = 75 \quad \text{kvan} \\ S_{\rm L} &= \frac{P}{\cos \theta_{\rm L}} = 125 \quad \text{kVA} \end{aligned}$$

After the capacitor is connected, the power factor angle, reactive power delivered, and apparent power of the source are

$$\theta_{\rm S} = (\delta - \beta_{\rm S}) = \cos^{-1}(0.95) = 18.19^{\circ}$$
$$Q_{\rm S} = P \tan \theta_{\rm S} = 100 \tan(18.19^{\circ}) = 32.87 \text{ kvas}$$
$$S_{\rm S} = \frac{P}{\cos \theta_{\rm S}} = \frac{100}{0.95} = 105.3 \text{ kVA}$$

The capacitor delivers

 $Q_C = Q_L - Q_S = 75 - 32.87 = 42.13$ kvar



The method of connecting a capacitor in parallel with an inductive load is known as *power factor correction*. The effect of the capacitor is to increase the power factor of the source that delivers power to the load. Also, the source apparent power S_S decreases. As shown in Figure 2.6, the source apparent power for this example decreases from 125 kVA without the capacitor to 105.3 kVA with the capacitor. The source current $I_S = S_S/V$ also decreases. When line impedance between the source and load is included, the decrease in source current results in lower line losses and lower line-voltage drops. The end result of power factor correction is improved efficiency and improved voltage regulation.

To see an animated view of this example, open PowerWorld Simulator case Example 2.3 (see Figure 2.7). From the Ribbon select the green and black "Play" button to begin the simulation. The speed and size of the green arrows are proportional to the real power supplied to the load bus, and the blue arrows are proportional to the reactive power. Here reactive compensation can be supplied in discrete 20-kVar steps by clicking on the arrows in the capacitor's kvar field, and the load can be varied by clicking on the arrows in the load field. Notice that increasing the reactive compensation decreases both the reactive power flow on the supply line and the kVA power supplied by the generator; the real power flow is unchanged.

2.4

NETWORK EQUATIONS

For circuits operating in sinusoidal-steady-state, Kirchhoff's current law (KCL) and voltage law (KVL) apply to phasor currents and voltages. Thus the sum of all phasor currents entering any node is zero and the sum of the phasor-voltage drops around any closed path is zero. Network analysis techniques based on Kirchhoff's laws, including nodal analysis, mesh or loop analysis, superposition, source transformations, and Thévenin's theorem or Norton's theorem, are useful for analyzing such circuits.

Various computer solutions of power system problems are formulated from nodal equations, which can be systematically applied to circuits. The circuit shown in Figure 2.8, which is used here to review nodal analysis, is assumed to be operating in sinusoidal-steady-state; source voltages are represented by phasors E_{S1} , E_{S2} , and E_{S3} ; circuit impedances are specified in ohms. Nodal equations are written in the following three steps:

STEP I For a circuit with (N + 1) nodes (also called buses), select one bus as the reference bus and define the voltages at the remaining buses with respect to the reference bus.

The circuit in Figure 2.8 has four buses—that is, N + 1 = 4 or N = 3. Bus 0 is selected as the reference bus, and bus voltages V_{10} , V_{20} , and V_{30} are then defined with respect to bus 0.

STEP 2 Transform each voltage source in series with an impedance to an equivalent current source in parallel with that impedance. Also, show admittance values instead of impedance values on the circuit diagram. Each current source is equal to the voltage source divided by the source impedance.







FIGURE 2.9

Circuit of Figure 2.8 with equivalent current sources replacing voltage sources. Admittance values are also shown



In Figure 2.9 equivalent current sources I_1 , I_2 , and I_3 are shown, and all impedances are converted to corresponding admittances.

STEP 3 Write nodal equations in matrix format as follows:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2N} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3N} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ \vdots \\ V_{N0} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix}$$
(2.4.1)

Using matrix notation, (2.4.1) becomes

$$YV = I \tag{2.4.2}$$

where Y is the $N \times N$ bus admittance matrix, V is the column vector of N bus voltages, and I is the column vector of N current sources. The elements Y_{kn} of the bus admittance matrix Y are formed as follows:

| diagonal elements: | $Y_{kk} = \text{sum of admittances}$ connected to bus k (k = 1, 2,, N) (2.4.3) |) |
|------------------------|--|---------|
| off-diagonal elements: | $Y_{kn} = -(\text{sum of admittances} \text{ connected between buses } k$ and n) $(k \neq n)$ (2.4.4 | / .) |

The diagonal element Y_{kk} is called the *self-admittance* or the *driving-point admittance* of bus k, and the off-diagonal element Y_{kn} for $k \neq n$ is called the *mutual admittance* or the *transfer admittance* between buses k and n. Since $Y_{kn} = Y_{nk}$, the matrix Y is symmetric.

For the circuit of Figure 2.9, (2.4.1) becomes

$$\begin{bmatrix} (j3-j10) & -(j3) & 0 \\ -(j3) & (j3-j1+j1-j2) & -(j1-j2) \\ 0 & -(j1-j2) & (j1-j2-j4) \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix}$$
$$= \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$
$$j \begin{bmatrix} -7 & -3 & 0 \\ -3 & 1 & 1 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$
(2.4.5)

The advantage of this method of writing nodal equations is that a digital computer can be used both to generate the admittance matrix Y and to solve (2.4.2) for the unknown bus voltage vector V. Once a circuit is specified with the reference bus and other buses identified, the circuit admittances and their bus connections become computer input data for calculating the elements Y_{kn} via (2.4.3) and (2.4.4). After Y is calculated and the current source vector I is given as input, standard computer programs for solving simultaneous linear equations can then be used to determine the bus voltage vector V.

When double subscripts are used to denote a voltage in this text, the voltage shall be that at the node identified by the first subscript with respect to the node identified by the second subscript. For example, the voltage V_{10} in Figure 2.9 is the voltage at node 1 with respect to node 0. Also, a current I_{ab} shall indicate the current from node a to node b. Voltage polarity marks (+/-) and current reference arrows (\rightarrow) are not required when double subscript notation is employed. The polarity marks in Figure 2.9 for V_{10}, V_{20} , and V_{30} , although not required, are shown for clarity. The reference arrows for sources I_1, I_2 , and I_3 in Figure 2.9 are required, however, since single subscripts are used for these currents. Matrices and vectors shall be indicated in this text by boldface type (for example, Y or V).

2.5

BALANCED THREE-PHASE CIRCUITS

In this section we introduce the following topics for balanced three-phase circuits: Y connections, line-to-neutral voltages, line-to-line voltages, line currents, Δ loads, Δ -Y conversions, and equivalent line-to-neutral diagrams.

FIGURE 2.10

Circuit diagram of a three-phase Y-connected source feeding a balanced-Y load



BALANCED-Y CONNECTIONS

Figure 2.10 shows a three-phase Y-connected (or "wye-connected") voltage source feeding a balanced-Y-connected load. For a Y connection, the neutrals of each phase are connected. In Figure 2.10 the source neutral connection is labeled bus n and the load neutral connection is labeled bus N. The three-phase source is assumed to be ideal since source impedances are neglected. Also neglected are the line impedances between the source and load terminals, and the neutral impedance between buses n and N. The three-phase load is *balanced*, which means the load impedances in all three phases are identical.

BALANCED LINE-TO-NEUTRAL VOLTAGES

In Figure 2.10, the terminal buses of the three-phase source are labeled a, b, and c, and the source line-to-neutral voltages are labeled E_{an} , E_{bn} , and E_{cn} . The source is *balanced* when these voltages have equal magnitudes and an equal 120°-phase difference between any two phases. An example of balanced three-phase line-to-neutral voltages is

FIGURE 2.11

Phasor diagram of balanced positive-sequence line-to-neutral voltages with *E*_{an} as the reference



$$E_{an} = 10/0^{\circ}$$

$$E_{bn} = 10/-120^{\circ} = 10/+240^{\circ}$$

$$E_{cn} = 10/+120^{\circ} = 10/-240^{\circ}$$
 volts (2.5.1)

where the line-to-neutral voltage magnitude is 10 volts and E_{an} is the reference phasor. The phase sequence is called *positive sequence* or *abc* sequence when E_{an} leads E_{bn} by 120° and E_{bn} leads E_{cn} by 120°. The phase sequence is called *negative sequence* or *acb* sequence when E_{an} leads E_{cn} by 120° and E_{cn} leads E_{bn} by 120°. The voltages in (2.5.1) are positive-sequence voltages, since E_{an} leads E_{bn} by 120°. The corresponding phasor diagram is shown in Figure 2.11.

BALANCED LINE-TO-LINE VOLTAGES

The voltages E_{ab} , E_{bc} , and E_{ca} between phases are called line-to-line voltages. Writing a KVL equation for a closed path around buses a, b, and n in Figure 2.10,

$$E_{ab} = E_{an} - E_{bn} \tag{2.5.2}$$

For the line-to-neutral voltages of (2.5.1),

$$E_{ab} = 10/0^{\circ} - 10/-120^{\circ} = 10 - 10 \left[\frac{-1 - j\sqrt{3}}{2}\right]$$

$$E_{ab} = \sqrt{3}(10) \left(\frac{\sqrt{3} + j1}{2}\right) = \sqrt{3}(10/30^{\circ}) \quad \text{volts}$$
(2.5.3)

Similarly, the line-to-line voltages E_{bc} and E_{ca} are

$$E_{bc} = E_{bn} - E_{cn} = 10 / -120^{\circ} - 10 / +120^{\circ}$$

= $\sqrt{3}(10 / -90^{\circ})$ volts (2.5.4)
$$E_{ca} = E_{cn} - E_{an} = 10 / +120^{\circ} - 10 / 0^{\circ}$$

= $\sqrt{3}(10 / 150^{\circ})$ volts (2.5.5)

The line-to-line voltages of (2.5.3)–(2.5.5) are also balanced, since they have equal magnitudes of $\sqrt{3}(10)$ volts and 120° displacement between any two phases. Comparison of these line-to-line voltages with the line-to-neutral voltages of (2.5.1) leads to the following conclusion:

In a balanced three-phase Y-connected system with positive-sequence sources, the line-to-line voltages are $\sqrt{3}$ times the line-to-neutral voltages and lead by 30°. That is,

$$E_{ab} = \sqrt{3}E_{an}/+30^{\circ}$$

$$E_{bc} = \sqrt{3}E_{bn}/+30^{\circ}$$

$$E_{ca} = \sqrt{3}E_{cn}/+30^{\circ}$$
(2.5.6)

This very important result is summarized in Figure 2.12. In Figure 2.12(a) each phasor begins at the origin of the phasor diagram. In Figure 2.12(b) the line-to-line voltages form an equilateral triangle with vertices labeled a, b, c corresponding to buses a, b, and c of the system; the line-to-neutral voltages begin at the vertices and end at the center of the triangle, which is labeled n for neutral bus n. Also, the clockwise sequence of the vertices abc in Figure 2.12(b) indicates positive-sequence voltages. In both diagrams, E_{an} is the reference. However, the diagrams could be rotated to align with any other reference.

Since the balanced line-to-line voltages form a closed triangle in Figure 2.12, their sum is zero. In fact, the sum of line-to-line voltages $(E_{ab} + E_{bc} + E_{ca})$



is *always* zero, even if the system is unbalanced, since these voltages form a closed path around buses *a*, *b*, and *c*. Also, in a balanced system the sum of the line-to-neutral voltages $(E_{an} + E_{bn} + E_{cn})$ equals zero.

BALANCED LINE CURRENTS

Since the impedance between the source and load neutrals in Figure 2.10 is neglected, buses *n* and *N* are at the same potential, $E_{nN} = 0$. Accordingly, a separate KVL equation can be written for each phase, and the line currents can be written by inspection:

$$I_{a} = E_{an}/Z_{Y}$$

$$I_{b} = E_{bn}/Z_{Y}$$

$$I_{c} = E_{cn}/Z_{Y}$$
(2.5.7)

For example, if each phase of the Y-connected load has an impedance $Z_{\rm Y} = 2/30^{\circ} \Omega$, then

$$I_{a} = \frac{10/0^{\circ}}{2/30^{\circ}} = 5/-30^{\circ} \quad A$$

$$I_{b} = \frac{10/-120^{\circ}}{2/30^{\circ}} = 5/-150^{\circ} \quad A$$

$$I_{c} = \frac{10/+120^{\circ}}{2/30^{\circ}} = 5/90^{\circ} \quad A$$
(2.5.8)

The line currents are also balanced, since they have equal magnitudes of 5 A and 120° displacement between any two phases. The neutral current I_n is determined by writing a KCL equation at bus N in Figure 2.10.

$$I_n = I_a + I_b + I_c (2.5.9)$$

FIGURE 2.13

Using the line currents of (2.5.8),

Phasor diagram of line currents in a balanced three-phase system



$$I_n = 5\underline{/-30^\circ} + 5\underline{/-150^\circ} + 5\underline{/90^\circ}$$
$$I_n = 5\left(\frac{\sqrt{3} - j1}{2}\right) + 5\left(\frac{-\sqrt{3} - j1}{2}\right) + j5 = 0$$
(2.5.10)

The phasor diagram of the line currents is shown in Figure 2.13. Since these line currents form a closed triangle, their sum, which is the neutral current I_n , is zero. In general, the sum of any balanced three-phase set of phasors is zero, since balanced phasors form a closed triangle. Thus, although the impedance between neutrals n and N in Figure 2.10 is assumed to be zero, the neutral current will be zero for *any* neutral impedance ranging from short circuit (0 Ω) to open circuit ($\infty \Omega$), as long as the system is balanced. If the system is not balanced—which could occur if the source voltages, load impedances, or line impedances were unbalanced—then the line currents will not be balanced and a neutral current I_n may flow between buses n and N.

BALANCED Δ LOADS

Figure 2.14 shows a three-phase Y-connected source feeding a balanced- Δ connected (or "delta-connected") load. For a balanced- Δ connection, equal load impedances Z_{Δ} are connected in a triangle whose vertices form the buses, labeled A, B, and C in Figure 2.14. The Δ connection does not have a neutral bus.

Since the line impedances are neglected in Figure 2.14, the source line-to-line voltages are equal to the load line-to-line voltages, and the Δ -load currents I_{AB} , I_{BC} , and I_{CA} are

$$I_{AB} = E_{ab}/Z_{\Delta}$$

$$I_{BC} = E_{bc}/Z_{\Delta}$$

$$I_{CA} = E_{ca}/Z_{\Delta}$$
(2.5.11)



FIGURE 2.14

Circuit diagram of a Yconnected source feeding a balanced- Δ load For example, if the line-to-line voltages are given by (2.5.3)–(2.5.5) and if $Z_{\Delta} = 5/30^{\circ} \Omega$, then the Δ -load currents are

$$I_{AB} = \sqrt{3} \left(\frac{10/30^{\circ}}{5/30^{\circ}} \right) = 3.464/0^{\circ} \quad A$$

$$I_{BC} = \sqrt{3} \left(\frac{10/-90^{\circ}}{5/30^{\circ}} \right) = 3.464/-120^{\circ} \quad A$$

$$I_{CA} = \sqrt{3} \left(\frac{10/150^{\circ}}{5/30^{\circ}} \right) = 3.464/+120^{\circ} \quad A$$
(2.5.12)

Also, the line currents can be determined by writing a KCL equation at each bus of the Δ load, as follows:

$$I_{a} = I_{AB} - I_{CA} = 3.464 \underline{/0^{\circ}} - 3.464 \underline{/120^{\circ}} = \sqrt{3}(3.464 \underline{/-30^{\circ}})$$

$$I_{b} = I_{BC} - I_{AB} = 3.464 \underline{/-120^{\circ}} - 3.464 \underline{/0^{\circ}} = \sqrt{3}(3.464 \underline{/-150^{\circ}}) \qquad (2.5.13)$$

$$I_{c} = I_{CA} - I_{BC} = 3.464 \underline{/120^{\circ}} - 3.464 \underline{/-120^{\circ}} = \sqrt{3}(3.464 \underline{/+90^{\circ}})$$

Both the Δ -load currents given by (2.5.12) and the line currents given by (2.5.13) are balanced. Thus the sum of balanced Δ -load currents $(I_{AB} + I_{BC} + I_{CA})$ equals zero. The sum of line currents $(I_a + I_b + I_c)$ is always zero for a Δ -connected load even if the system is unbalanced, since there is no neutral wire. Comparison of (2.5.12) and (2.5.13) leads to the following conclusion:

For a balanced- Δ load supplied by a balanced positive-sequence source, the line currents into the load are $\sqrt{3}$ times the Δ -load currents and lag by 30°. That is,

$$I_{a} = \sqrt{3}I_{AB}/-30^{\circ}$$

$$I_{b} = \sqrt{3}I_{BC}/-30^{\circ}$$

$$I_{c} = \sqrt{3}I_{CA}/-30^{\circ}$$
(2.5.14)

This result is summarized in Figure 2.15.

FIGURE 2.15

Phasor diagram of line currents and load currents for a balanced-∆ load





 Δ -Y conversion for balanced loads



$\Delta\text{-}Y$ conversion for balanced loads

Figure 2.16 shows the conversion of a balanced- Δ load to a balanced-Y load. If balanced voltages are applied, then these loads will be equivalent as viewed from their terminal buses A, B, and C when the line currents into the Δ load are the same as the line currents into the Y load. For the Δ load,

$$I_{\rm A} = \sqrt{3} I_{\rm AB} / \underline{-30^{\circ}} = \frac{\sqrt{3} E_{\rm AB} / \underline{-30^{\circ}}}{Z_{\Delta}}$$
(2.5.15)

and for the Y load,

$$I_{\rm A} = \frac{E_{\rm AN}}{Z_{\rm Y}} = \frac{E_{\rm AB}/-30^{\circ}}{\sqrt{3}Z_{\rm Y}}$$
(2.5.16)

Comparison of (2.5.15) and (2.5.16) indicates that I_A will be the same for both the Δ and Y loads when

$$Z_{\rm Y} = \frac{Z_{\Delta}}{3} \tag{2.5.17}$$

Also, the other line currents $I_{\rm B}$ and $I_{\rm C}$ into the Y load will equal those into the Δ load when $Z_{\rm Y} = Z_{\Delta}/3$, since these loads are balanced. Thus a balanced- Δ load can be converted to an equivalent balanced-Y load by dividing the Δ -load impedance by 3. The angles of these Δ - and equivalent Y-load impedances are the same. Similarly, a balanced-Y load can be converted to an equivalent balanced- Δ load using $Z_{\Delta} = 3Z_{\rm Y}$.

EXAMPLE 2.4 Balanced Δ and Y loads

A balanced, positive-sequence, Y-connected voltage source with $E_{ab} = 480/0^{\circ}$ volts is applied to a balanced- Δ load with $Z_{\Delta} = 30/40^{\circ} \Omega$. The line impedance between the source and load is $Z_{\rm L} = 1/85^{\circ} \Omega$ for each phase. Calculate the line currents, the Δ -load currents, and the voltages at the load terminals.

FIGURE 2.17

Circuit diagram for Example 2.4



SOLUTION The solution is most easily obtained as follows. First, convert the Δ load to an equivalent Y. Then connect the source and Y-load neutrals with a zero-ohm neutral wire. The connection of the neutral wire has no effect on the circuit, since the neutral current $I_n = 0$ in a balanced system. The resulting circuit is shown in Figure 2.17. The line currents are

$$I_{A} = \frac{E_{an}}{Z_{L} + Z_{Y}} = \frac{\frac{480}{\sqrt{3}} / -30^{\circ}}{1 / (85^{\circ}) + \frac{30}{3} / 40^{\circ}}$$

= $\frac{277.1 / -30^{\circ}}{(0.0872 + j0.9962) + (7.660 + j6.428)}$ (2.5.18)
= $\frac{277.1 / -30^{\circ}}{(7.748 + j7.424)} = \frac{277.1 / -30^{\circ}}{10.73 / 43.78^{\circ}} = 25.83 / -73.78^{\circ}$ A
 $I_{B} = 25.83 / 166.22^{\circ}$ A
 $I_{C} = 25.83 / 46.22^{\circ}$ A

The Δ -load currents are, from (2.5.14),

$$I_{AB} = \frac{I_a}{\sqrt{3}} / +30^{\circ} = \frac{25.83}{\sqrt{3}} / -73.78^{\circ} + 30^{\circ} = 14.91 / -43.78^{\circ} \quad A$$

$$I_{BC} = 14.91 / -163.78^{\circ} \quad A$$

$$I_{CA} = 14.91 / +76.22^{\circ} \quad A$$

(2.5.19)

The voltages at the load terminals are

$$E_{AB} = Z_{\Delta}I_{AB} = (30/40^{\circ})(14.91/-43.78^{\circ}) = 447.3/-3.78^{\circ}$$

$$E_{BC} = 447.3/-123.78^{\circ}$$

$$E_{CA} = 447.3/116.22^{\circ} \quad \text{volts}$$

(2.5.20)

FIGURE 2.18

Equivalent line-toneutral diagram for the circuit of Example 2.4



EQUIVALENT LINE-TO-NEUTRAL DIAGRAMS

When working with balanced three-phase circuits, only one phase need be analyzed. Δ loads can be converted to Y loads, and all source and load neutrals can be connected with a zero-ohm neutral wire without changing the solution. Then one phase of the circuit can be solved. The voltages and currents in the other two phases are equal in magnitude to and $\pm 120^{\circ}$ out of phase with those of the solved phase. Figure 2.18 shows an equivalent line-to-neutral diagram for one phase of the circuit in Example 2.4.

When discussing three-phase systems in this text, voltages shall be rms line-to-line voltages unless otherwise indicated. This is standard industry practice.

2.6

POWER IN BALANCED THREE-PHASE CIRCUITS

In this section, we discuss instantaneous power and complex power for balanced three-phase generators and motors and for balanced-Y and Δ -impedance loads.

INSTANTANEOUS POWER: BALANCED THREE-PHASE GENERATORS

Figure 2.19 shows a Y-connected generator represented by three voltage sources with their neutrals connected at bus n and by three identical generator impedances Z_g . Assume that the generator is operating under balanced steady-state conditions with the instantaneous generator terminal voltage given by

$$v_{an}(t) = \sqrt{2} V_{LN} \cos(\omega t + \delta)$$
 volts (2.6.1)

and with the instantaneous current leaving the positive terminal of phase a given by

$$i_a(t) = \sqrt{2} I_L \cos(\omega t + \beta) \quad A \tag{2.6.2}$$



where V_{LN} is the rms line-to-neutral voltage and I_L is the rms line current. The instantaneous power $p_a(t)$ delivered by phase *a* of the generator is

$$p_{a}(t) = v_{an}(t)i_{a}(t)$$

= $2V_{LN}I_{L}\cos(\omega t + \delta)\cos(\omega t + \beta)$
= $V_{LN}I_{L}\cos(\delta - \beta) + V_{LN}I_{L}\cos(2\omega t + \delta + \beta)$ W (2.6.3)

Assuming balanced operating conditions, the voltages and currents of phases b and c have the same magnitudes as those of phase a and are $\pm 120^{\circ}$ out of phase with phase a. Therefore the instantaneous power delivered by phase b is

$$p_b(t) = 2V_{LN}I_L \cos(\omega t + \delta - 120^\circ) \cos(\omega t + \beta - 120^\circ)$$

= $V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta - 240^\circ)$ W (2.6.4)

and by phase c,

$$p_c(t) = 2V_{LN}I_L \cos(\omega t + \delta + 120^\circ) \cos(\omega t + \beta + 120^\circ)$$

= $V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta + 240^\circ)$ W (2.6.5)

The total instantaneous power $p_{3\phi}(t)$ delivered by the three-phase generator is the sum of the instantaneous powers delivered by each phase. Using (2.6.3)–(2.6.5):

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$

= $3V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L [\cos(2\omega t + \delta + \beta) + \cos(2\omega t + \delta + \beta - 240^\circ)] + \cos(2\omega t + \delta + \beta + 240^\circ)] W$ (2.6.6)

The three cosine terms within the brackets of (2.6.6) can be represented by a *balanced* set of three phasors. Therefore, the sum of these three terms is zero

for any value of δ , for any value of β , and for all values of *t*. Equation (2.6.6) then reduces to

$$p_{3\phi}(t) = \mathbf{P}_{3\phi} = 3\mathbf{V}_{\rm LN}\mathbf{I}_{\rm L}\,\cos(\delta - \beta) \quad \mathbf{W} \tag{2.6.7}$$

Equation (2.6.7) can be written in terms of the line-to-line voltage V_{LL} instead of the line-to-neutral voltage V_{LN} . Under balanced operating conditions,

$$V_{LN} = V_{LL}/\sqrt{3}$$
 and $P_{3\phi} = \sqrt{3}V_{LL}I_L\cos(\delta - \beta)$ W (2.6.8)

Inspection of (2.6.8) leads to the following conclusion:

The total instantaneous power delivered by a three-phase generator under balanced operating conditions is not a function of time, but a constant, $p_{3\phi}(t) = P_{3\phi}$.

INSTANTANEOUS POWER: BALANCED THREE-PHASE MOTORS AND IMPEDANCE LOADS

The total instantaneous power absorbed by a three-phase motor under balanced steady-state conditions is also a constant. Figure 2.19 can be used to represent a three-phase motor by reversing the line currents to enter rather than leave the positive terminals. Then (2.6.1)–(2.6.8), valid for power *delivered* by a generator, are also valid for power *absorbed* by a motor. These equations are also valid for the instantaneous power absorbed by a balanced three-phase impedance load.

COMPLEX POWER: BALANCED THREE-PHASE GENERATORS

The phasor representations of the voltage and current in (2.6.1) and (2.6.2) are

$$V_{an} = V_{LN} / \delta$$
 volts (2.6.9)

$$I_a = I_L / \beta \quad A \tag{2.6.10}$$

where I_a leaves positive terminal "a" of the generator. The complex power S_a delivered by phase *a* of the generator is

$$S_a = V_{an} I_a^* = V_{LN} I_L / (\delta - \beta)$$

= $V_{LN} I_L \cos(\delta - \beta) + j V_{LN} I_L \sin(\delta - \beta)$ (2.6.11)

Under balanced operating conditions, the complex powers delivered by phases *b* and *c* are identical to S_a , and the total complex power $S_{3\phi}$ delivered by the generator is

$$S_{3\phi} = S_a + S_b + S_c = 3S_a$$

= $3V_{LN}I_L/(\delta - \beta)$
= $3V_{LN}I_L \cos(\delta - \beta) + j3V_{LN}I_L \sin(\delta - \beta)$ (2.6.12)

In terms of the total real and reactive powers,

$$S_{3\phi} = \mathbf{P}_{3\phi} + j\mathbf{Q}_{3\phi} \tag{2.6.13}$$

where

$$P_{3\phi} = \operatorname{Re}(S_{3\phi}) = 3V_{LN}I_L \cos(\delta - \beta)$$
$$= \sqrt{3}V_{LL}I_L \cos(\delta - \beta) \quad W$$
(2.6.14)

and

$$Q_{3\phi} = \operatorname{Im}(S_{3\phi}) = 3V_{LN}I_L \sin(\delta - \beta)$$

= $\sqrt{3}V_{LL}I_L \sin(\delta - \beta)$ var (2.6.15)

Also, the total apparent power is

$$S_{3\phi} = |S_{3\phi}| = 3V_{LN}I_L = \sqrt{3}V_{LL}I_L$$
 VA (2.6.16)

COMPLEX POWER: BALANCED THREE-PHASE MOTORS

The preceding expressions for complex, real, reactive, and apparent power *delivered* by a three-phase generator are also valid for the complex, real, reactive, and apparent power *absorbed* by a three-phase motor.

COMPLEX POWER: BALANCED-Y AND BALANCED- Δ IMPEDANCE LOADS

Equations (2.6.13)–(2.6.16) are also valid for balanced-Y and - Δ impedance loads. For a balanced-Y load, the line-to-neutral voltage across the phase *a* load impedance and the current entering the positive terminal of that load impedance can be represented by (2.6.9) and (2.6.10). Then (2.6.11)–(2.6.16) are valid for the power absorbed by the balanced-Y load.

For a balanced- Δ load, the line-to-line voltage across the phase a-b load impedance and the current into the positive terminal of that load impedance can be represented by

$$V_{ab} = V_{LL} / \delta \quad \text{volts} \tag{2.6.17}$$

$$I_{ab} = I_{\Delta} \underline{/\beta} \quad A \tag{2.6.18}$$

where V_{LL} is the rms line-to-line voltage and I_{Δ} is the rms Δ -load current. The complex power S_{ab} absorbed by the phase a-b load impedance is then

$$S_{ab} = V_{ab}I_{ab}^* = \mathcal{V}_{LL}\mathcal{I}_{\Delta}/(\delta - \beta)$$
(2.6.19)

The total complex power absorbed by the Δ load is

$$S_{3\phi} = S_{ab} + S_{bc} + S_{ca} = 3S_{ab}$$

= $3V_{LL}I_{\Delta}/(\delta - \beta)$
= $3V_{LL}I_{\Delta}\cos(\delta - \beta) + j3V_{LL}I_{\Delta}\sin(\delta - \beta)$ (2.6.20)

Rewriting (2.6.19) in terms of the total real and reactive power,

$$S_{3\phi} = \mathbf{P}_{3\phi} + j\mathbf{Q}_{3\phi} \tag{2.6.21}$$

$$P_{3\phi} = \operatorname{Re}(S_{3\phi}) = 3V_{\mathrm{LL}}I_{\Delta}\cos(\delta - \beta)$$

$$= \sqrt{3} \mathbf{V}_{\mathrm{LL}} \mathbf{I}_{\mathrm{L}} \cos(\delta - \beta) \quad \mathbf{W}$$
 (2.6.22)

 $Q_{3\phi} = Im(S_{3\phi}) = 3V_{LL}I_{\Delta}\sin(\delta - \beta)$

$$= \sqrt{3} V_{LL} I_L \sin(\delta - \beta) \quad \text{var} \tag{2.6.23}$$

where the Δ -load current I_{Δ} is expressed in terms of the line current $I_{L} = \sqrt{3}I_{\Delta}$ in (2.6.22) and (2.6.23). Also, the total apparent power is

$$S_{3\phi} = |S_{3\phi}| = 3V_{LL}I_{\Delta} = \sqrt{3}V_{LL}I_L$$
 VA (2.6.24)

Equations (2.6.21)–(2.6.24) developed for the balanced- Δ load are identical to (2.6.13)–(2.6.16).

EXAMPLE 2.5 Power in a balanced three-phase system

Two balanced three-phase motors in parallel, an induction motor drawing 400 kW at 0.8 power factor lagging and a synchronous motor drawing 150 kVA at 0.9 power factor leading, are supplied by a balanced, three-phase 4160-volt source. Cable impedances between the source and load are neglected, (a) Draw the power triangle for each motor and for the combined-motor load. (b) Determine the power factor of the combined-motor load. (c) Determine the magnitude of the line current delivered by the source. (d) A delta-connected capacitor bank is now installed in parallel with the combined-motor load. What value of capacitive reactance is required in each leg of the capacitor bank to make the source power factor unity? (e) Determine the magnitude of the line current delivered by the capacitor bank installed.

SOLUTION (a) For the induction motor, P = 400 kW and:

S = P/p.f. = 400/0.8 = 500 kVA
Q =
$$\sqrt{S^2 - P^2} = \sqrt{(500)^2 - (400)^2} = 300$$
 kvar absorbed



For the synchronous motor, S = 150 kVA and

P = S(p.f.) = 150(0.9) = 135 kW
Q =
$$\sqrt{S^2 - P^2} = \sqrt{(150)^2 - (135)^2} = 65.4$$
 kvar delivered

For the combined-motor load:

$$P = 400 + 135 = 535 \text{ kW}$$
 $Q = 300 - 65.4 = 234.6 \text{ kvar absorbed}$

$$S = \sqrt{P^2 + Q^2} = \sqrt{(535)^2 + (234.6)^2} = 584.2 \text{ kVA}$$

- (a) The power triangles for each motor and the combined-motor load are shown in Figure 2.20.
- (b) The power factor of the combined-motor load is p.f. = P/S = 535/ 584.2 = 0.916 lagging.
- (c) The line current delivered by the source is $I = S/(\sqrt{3} V)$, where S is the three-phase apparent power of the combined-motor load and V is the magnitude of the line-to-line load voltage, which is the same as the source voltage for this example. I = 584.2/($\sqrt{3}$ 4160 V) = 0.0811 kA = 81.1 per phase.
- (d) For unity power factor, the three-phase reactive power supplied by the capacitor bank should equal the three-phase reactive power absorbed by the combined-motor load. That is, $Q_c = 234.6$ kvar. For a delta-connected capacitor bank, $Q_c = 3V^2/X_{\Delta}$ where V is the lineto-line voltage across the bank and X_{Δ} the capacitive reactance of each leg of the bank. The capacitive reactance of each leg is

$$X_{\Delta} = 3V^2/Q_c = 3(4160^2)/234.6 \times 10^3 = 221.3 \ \Omega$$

(e) With the capacitor bank installed, the source power factor is unity and the apparent power S delivered by the source is the same as the real power P delivered by the source. The line current magnitude is

$$I = S/(\sqrt{3} V) = P/(\sqrt{3} V) = 535/(\sqrt{3} 4160) = 0.0743 kA = 74.3 A$$
 per phase

In this example, the source voltage of 4160 V is not specified as a line-toline voltage or line-to-neutral voltage, rms or peak. Therefore, it is assumed to be an rms line-to-line voltage, which is the convention throughout this text and a standard practice in the electric power industry. The combinedmotor load absorbs 535 kW of real power. The induction motor, which operates at lagging power factor, absorbs reactive power (300 kvar) and the synchronous motor, which operates at leading power factor, delivers reactive power (65.4 kvar). The capacitor bank also delivers reactive power (234.6 kvar). Note that the line current delivered by the source is reduced from 81.1 A without the capacitor bank to 74.3 A with the capacitor bank. Any I²R losses due to cable resistances and voltage drops due to cable reactances between the source and loads (not included in this example) would also be reduced.

2.7

ADVANTAGES OF BALANCED THREE-PHASE VERSUS SINGLE-PHASE SYSTEMS

Figure 2.21 shows three separate single-phase systems. Each single-phase system consists of the following identical components: (1) a generator represented by a voltage source and a generator impedance Z_g ; (2) a forward and return conductor represented by two series line impedances Z_L ; (3) a load represented by an impedance Z_Y . The three single-phase systems, although completely separated, are drawn in a Y configuration in the figure to illustrate two advantages of three-phase systems.

Each separate single-phase system requires that *both* the forward and return conductors have a current capacity (or *ampacity*) equal to or greater than the load current. However, if the source and load neutrals in Figure 2.21 are connected to form a three-phase system, and if the source voltages are



balanced with equal magnitudes and with 120° displacement between phases, then the neutral current will be zero [see (2.5.10)] and the three neutral conductors can be removed. Thus, the balanced three-phase system, while delivering the same power to the three load impedances Z_Y , requires only half the number of conductors needed for the three separate single-phase systems. Also, the total I²R line losses in the three-phase system are only half those of the three separate single-phase systems, and the line-voltage drop between the source and load in the three-phase system is half that of each single-phase system. Therefore, one advantage of balanced three-phase systems over separate single-phase systems is reduced capital and operating costs of transmission and distribution, as well as better voltage regulation.

Some three-phase systems such as Δ -connected systems and three-wire Y-connected systems do not have any neutral conductor. However, the majority of three-phase systems are four-wire Y-connected systems, where a grounded neutral conductor is used. Neutral conductors are used to reduce transient overvoltages, which can be caused by lightning strikes and by line-switching operations, and to carry unbalanced currents, which can occur during unsymmetrical short-circuit conditions. Neutral conductors for transmission lines are typically smaller in size and ampacity than the phase conductors because the neutral current is nearly zero under normal operating conditions. Thus, the cost of a neutral conductor is substantially less than that of a phase conductor. The capital and operating costs of three-phase transmission and distribution systems with or without neutral conductors are substantially less than those of separate single-phase systems.

A second advantage of three-phase systems is that the total instantaneous electric power delivered by a three-phase generator under balanced steady-state conditions is (nearly) constant, as shown in Section 2.6. A threephase generator (constructed with its field winding on one shaft and with its three-phase windings equally displaced by 120° on the stator core) will also have a nearly constant mechanical input power under balanced steady-state conditions, since the mechanical input power equals the electrical output power plus the small generator losses. Furthermore, the mechanical shaft torque, which equals mechanical input power divided by mechanical radian frequency ($T_{mech} = P_{mech}/\omega_m$) is nearly constant.

On the other hand, the equation for the instantaneous electric power delivered by a single-phase generator under balanced steady-state conditions is the same as the instantaneous power delivered by one phase of a three-phase generator, given by $p_a(t)$ in (2.6.3). As shown in that equation, $p_a(t)$ has two components: a constant and a double-frequency sinusoid. Both the mechanical input power and the mechanical shaft torque of the single-phase generator will have corresponding double-frequency components that create shaft vibration and noise, which could cause shaft failure in large machines. Accordingly, most electric generators and motors rated 5 kVA and higher are constructed as three-phase machines in order to produce nearly constant torque and thereby minimize shaft vibration and noise.

MULTIPLE CHOICE QUESTIONS

SECTION 2.1

- 2.1 The rms value of $v(t) = V_{max} \cos(\omega t + \delta)$ is given by (a) V_{max} (b) $V_{max}/\sqrt{2}$ (c) 2 V_{max} (d) $\sqrt{2} V_{max}$
- **2.2** If the rms phasor of a voltage is given by $V = 120/60^{\circ}$ volts, then the corresponding v(t) is given by
 - (a) $120\sqrt{2}\cos(\omega t + 60^{\circ})$ (b) $120\cos(\omega t + 60^{\circ})$
 - (c) $120\sqrt{2} \sin(\omega t + 60^{\circ})$
- **2.3** If a phasor representation of a current is given by $I = 70.7/45^{\circ}$ A, it is equivalent to (a) 100 $e^{j45^{\circ}}$ (b) 100 + j100
 - (c) 50 + j50
- **2.4** With sinusoidal steady-state excitation, for a purely resistive circuit, the voltage and current phasors are
 - (a) in phase
 - (b) perpendicular with each other with V leading I
 - (c) perpendicular with each other with I leading V.
- **2.5** For a purely inductive circuit, with sinusoidal steady-state excitation, the voltage and current phasors are
 - (a) in phase
 - (b) perpendicular to each other with V leading I
 - (c) perpendicular to each other with I leading V.
- **2.6** For a purely capacitive circuit, with sinusoidal steady-state excitation, the voltage and current phasors are
 - (a) in phase
 - (b) perpendicular to each other with V leading I
 - (c) perpendicular to each other with I leading V.

SECTION 2.2

2.7 With sinusoidal steady-state excitation, the average power in a single-phase ac circuit with a purely resistive load is given by $(1) V^2 = V^2$

(a) $I_{\rm rms}^2 R$ (b) $V_{\rm max}^2/R$ (c) Zero

2.8 The average power in a single-phase ac circuit with a purely inductive load, for sinusoidal steady-state excitation, is

(a)
$$I_{\rm rms}^2 X_L$$
 (b) $V_{\rm max}^2/X_L$ (c) Zero

[Note: $X_L - \omega L$ is the inductive reactance]

- **2.9** The average power in a single-phase ac circuit with a purely capacitive load, for sinusoidal steady-state excitation, is
 - (a) zero (b) V_{max}^2/X_C (c) $I_{rms}^2X_C$

[Note: $X_C = 1/(\omega L_c)$ is the capacitive reactance]

2.10 The average value of a double-frequency sinusoid, $\sin 2(\omega t + \delta)$, is given by (a) 1 (b) δ (c) zero

- 2.11 The power factor for an inductive circuit (R-L load), in which the current lags the voltage, is said to be (a) Lagging (b) Leading (c) Zero
- The power factor for a capacitive circuit (R-C load), in which the current leads the 2.12 voltage, is said to be (a) Lagging (b) Leading (c) One

SECTION 2.3

- In a single-phase ac circuit, for a general load composed of RLC elements under 2.13 sinusoidal-steady-state excitation, the average reactive power is given by
 - (a) $V_{rms} I_{rms} \cos \phi$ (b) $V_{rms} I_{rms} \sin \phi$
 - (c) zero

[Note: ϕ is the power-factor angle]

- 2.14 The instantaneous power absorbed by the load in a single-phase ac circuit, for a general RLC load under sinusoidal-steady-state excitation, is
 - (a) Nonzero constant (b) zero
 - (c) containing double-frequency components
- 2.15 With load convention, where the current enters the positive terminal of the circuit element, if Q is positive then positive reactive power is absorbed. (a) True (b) False
- 2.16 With generator convention, where the current leaves the positive terminal of the circuit element, if P is positive then positive real power is delivered. (a) False (b) True
- 2.17 Consider the load convention that is used for the *RLC* elements shown in Figure 2.2 of the text.

A. If one says that an inductor absorbs zero real power and positive reactive power, is it

(a) True (b) False

B. If one says that a capacitor absorbs zero real power and negative reactive power (or delivers positive reactive power), is it

- (a) False
- (b) True

C. If one says that a (positive-valued) resistor absorbs (positive) real power and zero reactive power, is it

(a) True

- (b) False
- 2.18 In an ac circuit, power factor connection or improvement is achieved by
 - (a) connecting a resistor in parallel with the inductive load.
 - (b) connecting an inductor in parallel with the inductive load.
 - (c) connecting a capacitor in parallel with the inductive load.

SECTION 2.4

The admittance of the impedance $-j\frac{1}{2}\Omega$ is given by (a) -j2 S (b) i2 S 2.19 (c) -i4 S

Consider Figure 2.9 of the text. Let the nodal equations in matrix form be given by 2.20 Eq. (2.4) of the text.

A. The element Y_{11} is given by

(c) -j 7 (a) 0 (b) *j* 13

B. The element Y_{31} is given by

(a) 0 (b) -j 5 (c) j 1

C. The admittance matrix is always symmetric square.

(a) False

SECTION 2.5 AND 2.6

2.21 The three-phase source line-to-neutral voltages are given by $E_{an} = 10/0^{\circ}$, $E_{bn} = 10/+240^{\circ}$, and $E_{cn} = 10/-240^{\circ}$ volts. Is the source balanced? (b) No (a) Yes 2.22 In a balanced 3-phase wye-connected system with positive-sequence source, the lineto-line voltages are $\sqrt{3}$ times the line-to-neutral voltages and lend by 30°. (a) True (b) False 2.23 In a balanced system, the phasor sum of line-to-line voltages and the phasor sum of line-to-neutral voltages are always equal to zero. (a) False (b) True 2.24 Consider a three-phase Y-connected source feeding a balanced-Y load. The phasor sum of the line currents as well as the neutral current are always zero. (a) True (b) False 2.25 For a balanced- Δ load supplied by a balanced positive-sequence source, the line currents into the load are $\sqrt{3}$ times the Δ -load currents and lag by 30°. (a) True (b) False 2.26 A balanced Δ -load can be converted to an equivalent balanced-Y load by dividing the Δ -load impedance by (a) $\sqrt{3}$ (b) 3 (c) 1/32.27 When working with balanced three-phase circuits, per-phase analysis is commonly done after converting Δ loads to Y loads, thereby solving only one phase of the circuit. (a) True (b) False 2.28 The total instantaneous power delivered by a three-phase generator under balanced operating conditions is (a) a function of time (b) a constant 2.29 The total instantaneous power absorbed by a three-phase motor (under balanced steady-state conditions) as well as a balanced three-phase impedance load is (a) a constant (b) a function of time 2.30 Under balanced operating conditions, consider the 3-phase complex power delivered by the 3-phase source to the 3-phase load. Match the following expressions, those on the left to those on the right. (a) $(\sqrt{3} V_{LL} I_L) VA$ (i) Real power, $P_{3\phi}$ (b) $(\sqrt{3} V_{LL} I_L \sin \phi)$ var (ii) Reactive power, $Q_{3\phi}$ (ii) Reactive power, $Q_{3\phi}$ (b) $(\sqrt{3} V_{LL} I_L \sin \phi)$ van (iii) Total apparent power $S_{3\phi}$ (c) $(\sqrt{3} V_{LL} I_L \cos \phi) W$

(b) True

(iv) Complex power, $S_{3\phi}$ (d) $P_{3\phi} + jQ_{3\phi}$

Note that VLL is the rms line-to-line voltage, I_L is the rms line current, and ϕ is the power-factor angle.

- 2.31 One advantage of balanced three-phase systems over separate single-phase systems is reduced capital and operating costs of transmission and distribution.(a) True(b) False
- 2.32 While the instantaneous electric power delivered by a single-phase generator under balanced steady-state conditions is a function of time having two components of a constant and a double-frequency sinusoid, the total instantaneous electric power delivered by a three-phase generator under balanced steady-state conditions is a constant.
 (a) True
 (b) False

PROBLEMS

SECTION 2.1

- **2.1** Given the complex numbers $A_1 = 5/30^\circ$ and $A_2 = -3 + j4$, (a) convert A_1 to rectangular form; (b) convert A_2 to polar and exponential form; (c) calculate $A_3 = (A_1 + A_2)$, giving your answer in polar form; (d) calculate $A_4 = A_1A_2$, giving your answer in rectangular form; (e) calculate $A_5 = A_1/(A_2^*)$, giving your answer in exponential form.
- 2.2 Convert the following instantaneous currents to phasors, using cos(ωt) as the reference. Give your answers in both rectangular and polar form.
 (a) i(t) = 400√2 cos(ωt 30°);
 (b) i(t) = 5 sin(ωt + 15°);
 - (c) $i(t) = 4\cos(\omega t 30^\circ) + 5\sqrt{2}\sin(\omega t + 15^\circ)$.
- **2.3** The instantaneous voltage across a circuit element is $v(t) = 359.3 \sin(\omega t + 15^{\circ})$ volts, and the instantaneous current entering the positive terminal of the circuit element is $i(t) = 100 \cos(\omega t + 5^{\circ})$ A. For both the current and voltage, determine (a) the maximum value, (b) the rms value, (c) the phasor expression, using $\cos(\omega t)$ as the reference.
- **2.4** For the single-phase circuit shown in Figure 2.22, $I = 10/0^{\circ}$ A. (a) Compute the phasors I_1 , I_2 , and V. (b) Draw a phasor diagram showing I, I_1 , I_2 , and V.

FIGURE 2.22

Circuit for Problem 2.4



- **2.5** A 60-Hz, single-phase source with $V = 277/30^{\circ}$ volts is applied to a circuit element. (a) Determine the instantaneous source voltage. Also determine the phasor and instantaneous currents entering the positive terminal if the circuit element is (b) a 20- Ω resistor, (c) a 10-mH inductor, (d) a capacitor with 25- Ω reactance.
- **2.6** (a) Transform $v(t) = 100 \cos(377t 30^\circ)$ to phasor form. Comment on whether $\omega = 377$ appears in your answer. (b) Transform $V = 100/20^\circ$ to instantaneous form. Assume that $\omega = 377$. (c) Add the two sinusoidal functions a(t) and b(t) of the same frequency given as follows: $a(t) = A\sqrt{2}\cos(\omega t + \alpha)$ and $b(t) = B\sqrt{2}\cos(\omega t + \beta)$. Use phasor methods and obtain the resultant c(t). Does the resultant have the same frequency?
- 2.7 Let a 100-V sinusoidal source be connected to a series combination of a 3-Ω resistor, an 8-Ω inductor, and a 4-Ω capacitor. (a) Draw the circuit diagram. (b) Compute the series impedance. (c) Determine the current *I* delivered by the source. Is the current lagging or leading the source voltage? What is the power factor of this circuit?
- **2.8** Consider the circuit shown in Figure 2.23 in time domain. Convert the entire circuit into phasor domain.



2.9 For the circuit shown in Figure 2.24, compute the voltage across the load terminals.



SECTION 2.2

2.10 For the circuit element of Problem 2.3, calculate (a) the instantaneous power absorbed, (b) the real power (state whether it is delivered or absorbed), (c) the reactive power (state whether delivered or absorbed), (d) the power factor (state whether lagging or leading).

[*Note*: By convention the power factor $\cos(\delta - \beta)$ is positive. If $|\delta - \beta|$ is greater than 90°, then the reference direction for current may be reversed, resulting in a positive value of $\cos(\delta - \beta)$].

- **2.11** Referring to Problem 2.5, determine the instantaneous power, real power, and reactive power absorbed by: (a) the 20- Ω resistor, (b) the 10-mH inductor, (c) the capacitor with 25- Ω reactance. Also determine the source power factor and state whether lagging or leading.
- **2.12** The voltage $v(t) = 359.3 \cos(\omega t)$ volts is applied to a load consisting of a 10- Ω resistor in parallel with a capacitive reactance $X_C = 25 \Omega$. Calculate (a) the instantaneous power absorbed by the resistor, (b) the instantaneous power absorbed by the capacitor, (c) the real power absorbed by the resistor, (d) the reactive power delivered by the capacitor, (e) the load power factor.
- **2.13** Repeat Problem 2.12 if the resistor and capacitor are connected in series.
- **2.14** A single-phase source is applied to a two-terminal, passive circuit with equivalent impedance $Z = 2.0/-45^{\circ} \Omega$ measured from the terminals. The source current is $i(t) = 4\sqrt{2} \cos(\omega t)$ kA. Determine the (a) instantaneous power, (b) real power, and (c) reactive power delivered by the source. (d) Also determine the source power factor.
- 2.15 Let a voltage source v(t) = 4 cos(ωt + 60°) be connected to an impedance Z = 2/30° Ω. (a) Given the operating frequency to be 60 Hz, determine the expressions for the current and instantaneous power delivered by the source as functions of time. (b) Plot these functions along with v(t) on a single graph for comparison. (c) Find the frequency and average value of the instantaneous power.
- **2.16** A single-phase, 120-V (rms), 60-Hz source supplies power to a series R-L circuit consisting of R = 10 Ω and L = 40 mH. (a) Determine the power factor of the circuit and state whether it is lagging or leading. (b) Determine the real and reactive power absorbed by the load. (c) Calculate the peak magnetic energy W_{int} stored in the inductor by using the expression $W_{int} = L(I_{rms})^2$ and check whether the reactive power $Q = \omega W$ is satisfied. (*Note*: The instantaneous magnetic energy storage fluctuates between zero and the peak energy. This energy must be sent twice each cycle to the load from the source by means of reactive power flows.)

SECTION 2.3

2.17 Consider a load impedance of $Z = j\omega L$ connected to a voltage V let the current drawn be I.

(a) Develop an expression for the reactive power Q in terms of ω , L, and I, from complex power considerations.

(b) Let the instantaneous current be $i(t) = \sqrt{2I} \cos(\omega t + \theta)$. Obtain an expression for the instantaneous power $\rho(t)$ into L, and then express it in terms of Q.

(c) Comment on the average real power P supplied to the inductor and the instantaneous power supplied.

2.18 Let a series R-L-C network be connected to a source voltage V, drawing a current I.(a) In terms of the load impedance Z = Z < Z, find expressions for P and Q, from complex power considerations.

(b) Express $\rho(t)$ in terms of P and Q, by choosing $i(t) = \sqrt{2}I \cos \omega t$.

(c) For the case of $Z = R + j\omega L + 1/j\omega c$, interpret the result of part (b) in terms of P, Q_L , and Q_C . In particular, if $\omega^2 LC = 1$, when the inductive and capacitive reactances cancel, comment on what happens.

- **2.19** Consider a single-phase load with an applied voltage $v(t) = 150 \cos(\omega t + 10^{\circ})$ volts and load current $i(t) = 5 \cos(\omega t 50^{\circ})$ A. (a) Determine the power triangle. (b) Find the power factor and specify whether it is lagging or leading. (c) Calculate the reactive power supplied by capacitors in parallel with the load that correct the power factor to 0.9 lagging.
- **2.20** A circuit consists of two impedances, $Z_1 = 20/30^{\circ} \Omega$ and $Z_2 = 25/60^{\circ} \Omega$, in parallel, supplied by a source voltage $V = 100/60^{\circ}$ volts. Determine the power triangle for each of the impedances and for the source.
- **2.21** An industrial plant consisting primarily of induction motor loads absorbs 500 kW at 0.6 power factor lagging. (a) Compute the required kVA rating of a shunt capacitor to improve the power factor to 0.9 lagging. (b) Calculate the resulting power factor if a synchronous motor rated 500 hp with 90% efficiency operating at rated load and at unity power factor is added to the plant instead of the capacitor. Assume constant voltage. (1 hp = 0.746 kW)
- **2.22** The real power delivered by a source to two impedances, $Z_1 = 3 + j4 \Omega$ and $Z_2 = 10 \Omega$, connected in parallel, is 1100 W. Determine (a) the real power absorbed by each of the impedances and (b) the source current.
- **2.23** A single-phase source has a terminal voltage $V = 120/0^{\circ}$ volts and a current $I = 10/30^{\circ}$ A, which leaves the positive terminal of the source. Determine the real and reactive power, and state whether the source is delivering or absorbing each.
- **2.24** A source supplies power to the following three loads connected in parallel: (1) a lighting load drawing 10 kW, (2) an induction motor drawing 10 kVA at 0.90 power factor lagging, and (3) a synchronous motor operating at 10 hp, 85% efficiency and 0.95 power factor leading (1 hp = 0.746 kW). Determine the real, reactive, and apparent power delivered by the source. Also, draw the source power triangle.
- **2.25** Consider the series R-L-C circuit of Problem 2.7 and calculate the complex power absorbed by each of the elements R, L, and C, as well as the complex power absorbed by the total load. Draw the resultant power triangle. Check whether the complex power delivered by the source equals the total complex power absorbed by the load.
- **2.26** A small manufacturing plant is located 2 km down a transmission line, which has a series reactance of $0.5 \Omega/\text{km}$. The line resistance is negligible. The line voltage at the plant is $480/0^{\circ}$ V (rms), and the plant consumes 120 kW at 0.85 power factor lagging. Determine the voltage and power factor at the sending end of the transmission line by using (a) a complex power approach and (b) a circuit analysis approach.
- **2.27** An industrial load consisting of a bank of induction motors consumes 50 kW at a power factor of 0.8 lagging from a 220-V, 60-Hz, single-phase source. By placing a bank of capacitors in parallel with the load, the resultant power factor is to be raised to 0.95 lagging. Find the net capacitance of the capacitor bank in μF that is required.
- 2.28 Three loads are connected in parallel across a single-phase source voltage of 240 V (rms).
 Load 1 absorbs 12 kW and 6.667 kvar;
 Load 2 absorbs 4 kVA at 0.96 p.f. leading;
 Load 3 absorbs 15 kW at unity power factor.
 Calculate the equivalent impedance, Z, for the three parallel loads, for two cases:
 (i) Series combination of R and X, and (ii) parallel combination of R and X.

2.29 Modeling the transmission lines as inductors, with $Sij = S_{ji}^*$, Compute S_{13} , S_{31} , S_{23} , S_{32} , and S_{G3} , in Figure 2.25. (*Hint*: complex power balance holds good at each bus, statisfying KCL.)



2.30 Figure 2.26 shows three loads connected in parallel across a 1000-V (rms), 60-Hz single-phase source.

Load 1: Inductive load, 125 kVA, 0.28 p.f. lagging

Load 2: Capacitive load, 10 kW, 40 kvar

Load 3: Resistive load, 15 kW

(a) Determine the total kW, kvar, kva, and supply power factor.

(b) In order to improve the power factor to 0.8 lagging, a capacitor of negligible resistance is connected in parallel with the above loads. Find the KVAR rating of that capacitor and the capacitance in μf .

Comment on the magnitude of the supply current after adding the capacitor.



Circuit for Problem 2.30



- **2.31** Consider two interconnected voltage sources connected by a line of impedance $Z = jx \Omega$, as shown in Figure 2.27.
 - (a) Obtain expressions for P_{12} and Q_{12} .
 - (b) Determine the maximum power transfer and the condition for it to occur.

FIGURE 2.27

Circuit for Problem 2.31

PW



2.32 In PowerWorld Simulator Problem 2.32 (see Figure 2.28) a 8 MW/4 Mvar load is supplied at 13.8 kV through a feeder with an impedance of $1 + j2 \Omega$. The load is compensated with a capacitor whose output, Q_{cap} , can be varied in 0.5 Mvar steps between 0 and 10.0 Mvar. What value of Q_{cap} minimizes the real power line losses? What value of Q_{cap} minimizes the MVA power flow into the feeder?

FIGURE 2.28 Source Voltage = 14.98 kV

Screen for Problem 2.32



- **PW 2.33** For the system from Problem 2.32, plot the real and reactive line losses as Q_{cap} is varied between 0 and 10.0 Mvar.
- **PW 2.34** For the system from Problem 2.32, assume that half the time the load is 10 MW/5 Mvar, and for the other half it is 20 MW/10 Mvar. What single value of Q_{cap} would minimize the average losses? Assume that Q_{cap} can only be varied in 0.5 Mvar steps.

SECTION 2.4

2.35 For the circuit shown in Figure 2.29, convert the voltage sources to equivalent current sources and write nodal equations in matrix format using bus 0 as the reference bus. Do not solve the equations.



Circuit diagram for Problems 2.35 and 2.36



- 2.36 For the circuit shown in Figure 2.29, write a computer program that uses the sources, impedances, and bus connections as input data to (a) compute the 2×2 bus admittance matrix Y, (b) convert the voltage sources to current sources and compute the vector of source currents into buses 1 and 2.
- 2.37 Determine the 4×4 bus admittance matrix and write nodal equations in matrix format for the circuit shown in Figure 2.30. Do not solve the equations.



2.38 Given the impedance diagram of a simple system as shown in Figure 2.31, draw the admittance diagram for the system and develop the 4×4 bus admittance matrix Y_{bus} by inspection.



2.39 (a) Given the circuit diagram in Figure 2.32 showing admittances and current sources at nodes 3 and 4, set up the nodal equations in matrix format. (b) If the parameters are given by: $Y_a = -j0.8$ S, $Y_b = -j4.0$ S, $Y_c = -j4.0$ S, $Y_d = -j8.0$ S, $Y_e = -j5.0$ S, $Y_f = -j2.5$ S, $Y_g = -j0.8$ S, $I_3 = 1.0/-90^\circ$ A, and $I_4 = 0.62/-135^\circ$ A, set up the nodal equations and suggest how you would go about solving for the voltages at the nodes.



Circuit diagram for Problem 2.39



SECTIONS 2.5 AND 2.6

- **2.40** A balanced three-phase 208-V source supplies a balanced three-phase load. If the line current I_A is measured to be 10 A and is in phase with the line-to-line voltage V_{BC} , find the per-phase load impedance if the load is (a) Y-connected, (b) Δ -connected.
- **2.41** A three-phase 25-kVA, 480-V, 60-Hz alternator, operating under balanced steadystate conditions, supplies a line current of 20 A per phase at a 0.8 lagging power factor and at rated voltage. Determine the power triangle for this operating condition.
- **2.42** A balanced Δ -connected impedance load with $(12 + j9) \Omega$ per phase is supplied by a balanced three-phase 60-Hz, 208-V source. (a) Calculate the line current, the total real and reactive power absorbed by the load, the load power factor, and the apparent load power. (b) Sketch a phasor diagram showing the line currents, the line-to-line source voltages, and the Δ -load currents. Assume positive sequence and use V_{ab} as the reference.
- **2.43** A three-phase line, which has an impedance of $(2 + j4) \Omega$ per phase, feeds two balanced three-phase loads that are connected in parallel. One of the loads is Y-connected with an impedance of $(30 + j40) \Omega$ per phase, and the other is Δ -connected with an impedance of $(60 j45) \Omega$ per phase. The line is energized at the sending end

from a 60-Hz, three-phase, balanced voltage source of $120\sqrt{3}$ V (rms, line-to-line). Determine (a) the current, real power, and reactive power delivered by the sendingend source; (b) the line-to-line voltage at the load; (c) the current per phase in each load; and (d) the total three-phase real and reactive powers absorbed by each load and by the line. Check that the total three-phase complex power delivered by the source equals the total three-phase power absorbed by the line and loads.

- **2.44** Two balanced three-phase loads that are connected in parallel are fed by a three-phase line having a series impedance of $(0.4 + j2.7) \Omega$ per phase. One of the loads absorbs 560 kVA at 0.707 power factor lagging, and the other 132 kW at unity power factor. The line-to-line voltage at the load end of the line is $2200\sqrt{3}$ V. Compute (a) the line-to-line voltage at the source end of the line, (b) the total real and reactive power losses in the three-phase line, and (c) the total three-phase real and reactive power supplied at the source equals the total three-phase complex power absorbed by the line and loads.
- 2.45 Two balanced Y-connected loads, one drawing 10 kW at 0.8 power factor lagging and the other 15 kW at 0.9 power factor leading, are connected in parallel and supplied by a balanced three-phase Y-connected, 480-V source. (a) Determine the source current. (b) If the load neutrals are connected to the source neutral by a zero-ohm neutral wire through an ammeter, what will the ammeter read?
- 2.46 Three identical impedances Z_Δ = 30/30° Ω are connected in Δ to a balanced three-phase 208-V source by three identical line conductors with impedance Z_L = (0.8 + j0.6) Ω per line. (a) Calculate the line-to-line voltage at the load terminals. (b) Repeat part (a) when a Δ-connected capacitor bank with reactance (-j60) Ω per phase is connected in parallel with the load.
- **2.47** Two three-phase generators supply a three-phase load through separate three-phase lines. The load absorbs 30 kW at 0.8 power factor lagging. The line impedance is $(1.4 + j1.6) \Omega$ per phase between generator G1 and the load, and $(0.8 + j1) \Omega$ per phase between generator G2 and the load. If generator G1 supplies 15 kW at 0.8 power factor lagging, with a terminal voltage of 460 V line-to-line, determine (a) the voltage at the load terminals, (b) the voltage at the terminals of generator G2, and (c) the real and reactive power supplied by generator G2. Assume balanced operation.
- **2.48** Two balanced Y-connected loads in parallel, one drawing 15 kW at 0.6 power factor lagging and the other drawing 10 kVA at 0.8 power factor leading, are supplied by a balanced, three-phase, 480-volt source. (a) Draw the power triangle for each load and for the combined load. (b) Determine the power factor of the combined load and state whether lagging or leading. (c) Determine the magnitude of the line current from the source. (d) Δ -connected capacitors are now installed in parallel with the combined load. What value of capacitive reactance is needed in each leg of the Δ to make the source power factor unity? Give your answer in Ω . (e) Compute the magnitude of the current in each capacitor and the line current from the source.
- 2.49 Figure 2.33 gives the general Δ-Y transformation. (a) Show that the general transformation reduces to that given in Figure 2.16 for a balanced three-phase load. (b) Determine the impedances of the equivalent Y for the following Δ impedances: Z_{AB} = j10, Z_{BC} = j20, and Z_{CA} = -j25 Ω.



2.50 Consider the balanced three-phase system shown in Figure 2.34. Determine $v_1(t)$ and $i_2(t)$. Assume positive phase sequence.



- 2.51 A three-phase line with an impedance of (0.2 + j1.0) Ω/phase feeds three balanced three-phase loads connected in parallel.
 Load 1: Absorbs a total of 150 kW and 120 kvar; Load 2: Delta connected with an impedance of (150 j48) Ω/phase; Load 3: 120 kVA at 0.6 p.f. leading. If the line-to-neutral voltage at the load end of the line is 2000 V (rms), determine the magnitude of the line-to-line voltage at the source end of the line.
- **2.52** A balanced three-phase load is connected to a 4.16-kV, three-phase, four-wire, grounded-wye dedicated distribution feeder. The load can be modeled by an impedance of $Z_{\rm L} = (4.7 + j9) \Omega$ /phase, wye-connected. The impedance of the phase

conductors is $(0.3 + j1) \Omega$. Determine the following by using the phase A to neutral voltage as a reference and assume positive phase sequence:

(a) Line currents for phases A, B, and C.

(b) Line-to-neutral voltages for all three phases at the load.

(c) Apparent, active, and reactive power dissipated per phase, and for all three phases in the load.

(d) Active power losses per phase and for all three phases in the phase conductors.

CASE STUDY QUESTIONS

- **A.** What is a microgrid?
- **B.** What is an island in an interconnected power system?
- **C.** Why is a microgrid designed to be able to operate in both grid-connected and standalone modes?
- **D.** When operating in the stand-alone mode, what control features should be associated with a microgrid?

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