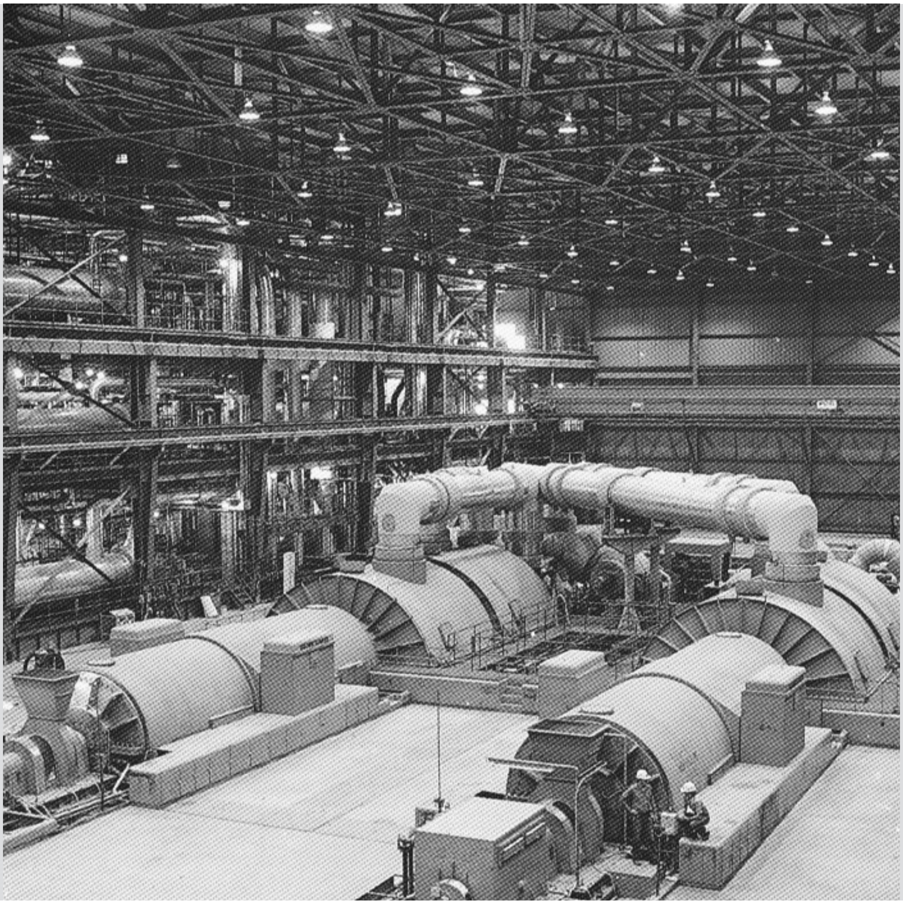
11 Transient Stability



1300-MW generating unit consisting of a cross-compound steam turbine and two 722-MVA synchronous generators (Courtesy of American Electric Power.)

Power system stability refers to the ability of a power system to move from one steady-state operating point following a disturbance to another steady-state operating point, without generators losing synchronism or having unacceptable voltage magnitude and frequency deviations [1]. There are three types of power system stability: steady-state, transient, and dynamic.

Steady-state stability, discussed in Chapter 5, involves slow or gradual changes in operating points. Steady-state stability studies, which are usually performed with a power-flow computer program (Chapter 6), ensure that phase angles across

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transmission lines are not too large, that bus voltages are close to nominal values, and that generators, transmission lines, transformers, and other equipment are not overloaded.

Transient stability, the main focus of this chapter, involves major disturbances such as loss of generation, line-switching operations, faults, and sudden load changes. Following a disturbance, synchronous machine frequencies undergo transient deviations from synchronous frequency (60 Hz in North America, 50 Hz in many other locations), and machine power angles change. The objective of a transient stability study is to determine whether or not the machines will return to synchronous frequency with new steady-state power angles. Changes in power flows and bus voltages are also of concern.

Elgerd [2] gives an interesting mechanical analogy to the power system transient stability program. As shown in Figure 11.1, a number of masses representing synchronous machines are interconnected by a network of elastic strings representing transmission lines. Assume that this network is initially at rest in steady-state, with the net force on each string below its break point, when one of the strings is cut, representing the loss of a transmission line. As a result, the masses undergo transient oscillations and the forces on the strings fluctuate. The system will then either settle down to a new steady-state operating point with a new set of string forces, or additional strings will break, resulting in an even weaker network and eventual system collapse. That is, for a given disturbance, the system is either transiently stable or unstable.

Transient stability studies are solved using a time-domain simulation for a specific disturbance that at each time point involves a solution of algebraic equations representing (primarily) the network power balance constraints, and differential equations representing the generator and sometimes the load dynamics. There are two main approaches for solving the system at each time point—either simultaneous in which the algebraic and differential equations are solved simultaneously, or partitioned in the program, alternately solves the algebraic and differential equations [15]. Both predisturbance, disturbance, and postdisturbance computations are performed. The program output includes power angles and frequencies of synchronous machines, bus voltages, and power flows versus time. Large-scale system studies can often involve many thousand algebraic equations and sometimes more than 100,000 differential equations.

Sometimes the transient stability is determined during the first swing of machine power angles following a disturbance. During the first swing, which typically lasts about 1 second, the mechanical output power and the internal voltage of a generating unit are often assumed constant. However, where multiswings lasting several seconds are of concern, models of turbine-governors and excitation systems (for example, see Figures 12.3 and 12.5) as well as more detailed machine models

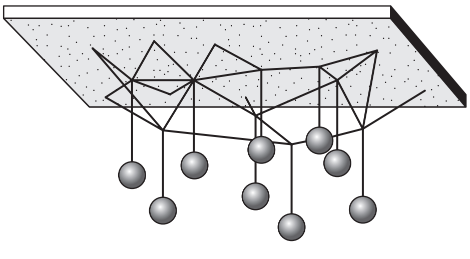

FIGURE 11.1

Mechanical analog of power system transient stability [2] (Based on Elgerd, Electric Energy Systems Theory: An Introduction, 1982. McGraw-Hill.)

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and sometimes dynamic load models can be employed to obtain accurate transient stability results over the longer time period.

Dynamic stability involves an even longer time period, typically several minutes. It is possible for controls to affect dynamic stability even though transient stability is maintained. The action of turbine-governors, excitation systems, tap-changing transformers, and controls from a power system dispatch center can interact to stabilize or destabilize a power system several minutes after a disturbance has occurred.

To simplify transient stability studies, the following assumptions are commonly made:

1. Only balanced three-phase systems and balanced disturbances are considered. Therefore, only positive-sequence networks are employed.
2. Deviations of machine frequencies from synchronous frequency (60 Hz) are small, and dc offset currents and harmonics are neglected. Therefore, the network of transmission lines, transformers, and impedance loads is essentially in steady-state; and voltages, currents, and powers can be computed from algebraic power-flow equations.

In Section 11.1 introduces the swing equation, which determines synchronous machine rotor dynamics. In Section 11.2, a simplified model of a synchronous machine and a Thévenin equivalent of a system consisting of lines, transformers, loads, and other machines are given. Then, the equal-area criterion; that gives a direct method for determining the transient stability of one machine connected to a system equivalent is presented in Section 11.3. Numerical integration techniques for solving swing equations step by step are covered in Section 11.4, and they are used in Section 11.5 to determine multimachine stability. Section 11.6 introduces a more detailed synchronous generator model, while Section 11.7 discusses how wind turbines are modeled in transient stability studies. Finally, Section 11.8 discusses design methods for improving power system transient stability.

CASE STUDY

The following case study provides an overview of the various issues involved in power system restoration following a blackout [11]. Restoration involves regulatory, economic, and technical issues. The case study focuses mainly on the technical issues. Most outages involve only a portion of a power system that can be restored with assistance from neighboring grids. In this case, *top-down restoration* is typically applied, where tie-lines to neighboring grids can be used to first energize the high-voltage grid, followed by energizing the subtransmission system, which can begin to restore some system load and supply auxiliary power at power plants to bring generation back on line in the blacked-out portion. In the case of a widespread blackout, there may be no help from neighboring grids, and *bottom-up restoration*

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must be applied, where restoration begins at preselected generating units (called "black-start units") that do not require any external power sources to start up. In order to ensure that facilities and personnel are prepared, restoration plans and procedures must be developed and coordinated among the various parties that own and operate the generators, the transmission system, and the distribution systems. Restoration studies and analyses, including steady-state and dynamic analyses, are required. Technical issues include load-frequency control, voltage control, motor starting, self-excitation of black-start units, system stability, cold load pickup, and transient overvoltages. Development of thorough restoration plans and testing of those plans via simulation and drills will help to minimize disruption of service to loads and equipment damage following a blackout.

Down, but Not Out

By James Feltes and Carlos Grande-Moran

Modern power systems are highly reliable. They are operated to withstand the variability in system conditions that occur in the course of normal operations, including the daily changes in load levels, generation dispatch, and equipment availability. Modern power systems must also provide reliable service to load during unusual events, such as faults and the tripping of transmission lines or generating units. Extreme events can occur, however, that may lead to partial or total system blackouts. To recover from such catastrophic events, utilities thus have a responsibility to develop detailed plans and procedures for restoring the power system safely, efficiently, and as expeditiously as possible. This case study gives a brief overview of many issues involved in power system restoration.

Restoration challenges can be subdivided into three areas: regulatory, economic, and technical. Only a very brief overview of the first two will be given here, as our emphasis will be on the technical issues associated with the restoration of a power system following a partial or total blackout.

Regulatory issues are associated with directives from the authorities in charge of determining the required level of service reliability to be supplied by the utilities to end users. As part of this process, the regulatory authorities generally establish criteria and requirements for power system restoration. In the United States, standards and criteria are established by the North American Electric Reliability Corporation (NERC), the Nuclear Regulatory Commission, the NERC regional authorities, and the local independent system operator (ISO) or regional transmission operator (RTO). While the exact process is different in other parts of the world, the concepts are generally quite similar.

The NERC requirements are delineated in its emergency preparedness

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and operations (EOP) standards. The standards that cover the areas being discussed here are EOP-001 (*Emergency Operations Planning*) and EOP-005 (*System Restoration from Blackstart Resources*). EOP-001 covers the requirements to develop, maintain, and implement a set of plans to mitigate operating emergencies. The purpose of EOP-005 is to ensure plans, facilities, and personnel are prepared to enable system restoration from black-start resources so as to assure that reliability is maintained during restoration and a high priority is placed on restoring the interconnection.

Economic issues are handled differently in different regions of the United States and also throughout other parts of the world. In the traditional, vertically integrated utility structure, the utility was responsible for generation, transmission, and distribution and hence for all the system components required for restoration. In a market-based system, however, coordination among many parties is required, as different owners and operators for the generators, the transmission system, and the distribution systems are involved. Common to both structures is the need to set clear requirements and reimburse the various parties for fulfilling these requirements. This is often a difficult process. As with expenditures for life and medical insurance, we must spend money on something we hope never to have to actually use. It is also quite difficult to measure the quality of restoration services until it is too late and they are already being mobilized. We will not delve into these issues here other than to state that system restoration is a critical service, and the inducements for generation and transmission owners must be sufficient for them to not only offer these services but to diligently participate in the testing and training needed to implement them successfully. Below we discuss the main emphasis of this article: the technical issues that affect power system restoration.

General Philosophy

Most outages involve only a portion of the power system that can be restored with assistance from neighboring power grids. While such restoration can be a complex process, the restoration generally occurs in a relatively straightforward manner. First, the tie lines from the outside power system to the blacked-out area are energized. These ties are the starting points used to energize the transmission high-voltage grid. With significant circuits in the transmission network energized, it is relatively straightforward to pick up the subtransmission system, start to restore some system load, and supply auxiliary power to selected power plants to bring generation back online. This is an example of a *top-down restoration*, where the highvoltage grid is energized first and then used to energize the lower-voltage networks.

In the case of a widespread blackout, however, there may be no help from neighboring systems. In this case, system restoration must begin from preselected generating units that have the ability to start themselves, that is, that do not require any external power sources. These units, called black-start units (BSUs) or black-start resources (BSRs), are then used as the kernels with which to start the restoration process. This can be considered a *bottom-up restoration*, starting from

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an individual generating unit and emanating outward toward the critical system load. Of course, to speed the restoration process, this black-start sequence will probably occur using several generating units simultaneously and independently; these independent islands of generation and load will later be synchronized so as to restore the original power system.

Restoration can, of course, be accomplished by combining the approaches described above. In the case of a total blackout, it is important to decide how quickly to energize the high-voltage system. As will be discussed in more detail below, energizing the high-voltage system is more difficult and usually requires that large generating units be online. If a high-voltage overlay can be successfully established, however, it will generally lead to faster restoration of the loads.

Restoration Plans and Procedures

Restoration plans and procedures must be developed, clearly delineating the actions to be undertaken and the responsibilities of each of the parties. These procedures must meet established reliability criteria and be able to be implemented by system operators at a central control center, a local control center, or both.

Restoration plans define the sequence of steps and cranking paths needed to restore power to critical loads and generation facilities from BSRs. Ideally, the BSRs are able to start quickly and energize secure transmission paths. A backbone network is formed so that critical load can be restored—in particular, stabilizing loads for the BSRs and auxiliary loads at power plants without black-start capability.

The restoration plan focuses on restoring the key system elements (generating stations, transmission lines, substations, and loads) that facilitate restoration of additional facilities. In particular, flexibility is important because major equipment may be out of service and not be available to assist in the restoration process. For example, restoration of a generation plant or substation with multiple transmission outlets is preferable to restoration of a location with a single path. This also illustrates the need for simplicity: the more complex a plan is, the more chances there are that a key component or action cannot be performed when or in the manner required.

In order to be able to fully restore the system, the restoration plan must be able to reenergize the transmission system to major generating facilities in a timely manner and also reach interconnection points with neighboring systems to form a larger and usually more stable grid. The supply of off-site power to nuclear power plant auxiliary systems is also a priority for those systems with nuclear generation.

The restoration plans and procedures must also address a myriad of other topics, including:

- Staffing and communication requirements;
- The training of control center staff and field staff needed to perform switching actions;

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- Communication protocols among control centers and transmission and generation facility operators;
- The requirements of the BSRs, including technical issues such as voltage and frequency control capabilities and managerial issues such as maintenance and adequate fuel supply capability; and
- The testing of the plans, including the starting of BSRs and drills testing the communication and coordination of the various power system operators.

Restoration Studies and Analysis

Each transmission operator is required to have plans, procedures, and resources available to restore the electric system following a partial or total shutdown of the system. The standards also require each transmission operator and balancing authority to verify their restoration procedure by actual testing or through simulation.

Restoration testing is a complex process. The testing of BSUs requires starting and running the units for a limited period of time, processes that involve straightforward procedures. The coordination of the test period, however, is complicated by considerations of staffing requirements, environmental emission restrictions, and the requirements of the overall system. The testing of line energizations is more complex, as it involves the deenergization of parts of the transmission system so they can be connected to the BSU. This must be accomplished without any adverse impact on loads, which may not always be possible. Restoration plans that require the energization of load at a particular step in the plan cannot be tested beyond that step because it is never acceptable to submit loads to outage and pickup as part of a test. In addition to any field testing performed, simulation is usually required to verify and validate the plan. A study including both steady-state and transient analysis is therefore performed.

The restoration plans must document the various cranking paths. For example, the plans show the number and switching sequence of transmission elements involved, including the initial switching requirements between each BSU and the unit to be started (i.e., its next-start unit).

In the case of a total system outage, system restoration must begin from the BSUs, with restoration of the power system proceeding outward toward critical system loads. As the BSUs themselves can only supply a small fraction of the system load, these units must be used to help start larger units that need their station service loads to be supplied by outside power sources. Full restoration of system load can only occur when these larger units can come online. The restoration plan following a system blackout should therefore include self-starting units that can be used to black-start large, steam turbine-driven plants located electrically close to these units. As mentioned above, another objective for

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many systems is the supply of auxiliary power to nuclear power stations in need of off-site power to supply critical station service loads. Other priority loads may include military facilities, law enforcement facilities, hospitals and other public health facilities, and communication facilities.

The typical black-start scenario includes the self-starting unit or units, the transmission lines that will transport the power supplied to the large motor loads in the power plant to be black-started, and at least three transformer units. These would include the generator step-up transformers of the black-start generating unit, the generator step-up transformers of the next-start unit involved (e.g., a large steam turbine unit), and one or more auxiliary transformers serving motor control centers at the next-start plant. The transmission lines used for the black start may be either overhead lines or high-voltage underground cables. The load to be black-started includes very large induction motors, ranging from a few hundred horsepower to several thousands of horsepower as well as plant lighting and small-motor loads.

The key concerns are the control of voltage and frequency. Both voltage and frequency must be kept within a tight band around nominal values to guard against damage to equipment and to ensure restoration progress. Any equipment failure will severely hinder restoration and may require starting over with a revised plan. System protection operations can also occur if voltage or frequency strays outside acceptable ranges, again with the potential to set back or stop the restoration process. The following sections give an overview of several of the technical concerns that must be addressed.

Steady-State Concerns

The black-start plan describes the steps that the transmission operators need to take to restore the isolated power system from the BSU. This includes sequentially energizing transformers, transmission lines, and, potentially, shunt compensation and load pickup to supply power to the next-start unit auxiliary loads, allowing their associated units to begin operation. Once larger generating units are available, higher-voltage lines can be energized, again in a step-by-step sequence, to supply power to major substations where load can be picked up and further interconnections made until the grid is fully restored.

The steady-state analysis of this isolated power system includes:

- Voltage control and steady-state overvoltage (Ferranti effect) analysis;
- Capability of the BSUs to absorb reactive power (vars) produced by the charging capacitance of the transmission system;
- Step-by-step simulation of the black-start plan being tested to ensure its feasibility and compliance with required operational limits;
- Verification of the robustness of the tested black-start plan to ensure its ability to compensate

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for the unavailability of key components to be used in the plan; and

- Demonstration of generation and load-matching capability.

Voltage control analysis determines the voltage reference set point of the black-starting generating unit and the off-nominal tap setting for all transformers that are part of the plan. This ensures proper control of voltage and provides the needed terminal voltage to start up the large induction motor loads at the black-started plant. Transformer tap settings that are appropriate for normal conditions that generally include significant current flows may result in high system voltages under the lightly loaded black-start condition. Since most taps on generator step-up transformers and station auxiliary transformers cannot be changed under load, the selection of transformer taps must be a balance between the needs of the black-start period and normal operation, when the power system is supplying a significant amount of load.

Load flow simulations can be used to calculate the receiving end bus voltage of the transmission lines when the black-starting unit energizes the unloaded generator, step-up transformer, and transmission lines. The charging current generated by an unloaded transmission line will result in a rise in voltage along the line. This is particularly true when underground cables are used as they have significantly more charging capacitance. The charging requirements can be large enough to result in the BSUs absorbing reactive power. There could be, under extreme conditions, the potential for self-excitation, which is discussed later.

The steady-state analysis of a black-start plan should include a step-by-step simulation of the plan to verify its compliance with required operational limits for voltage control and power flows. The robustness of the plan for the loss of a system component is also valuable knowledge because the events leading to the blackout could result in some equipment unavailability during the restoration period. Generally, thermal overloads are not a restoration issue because the system is lightly loaded. This may become a concern, however, as restoration progresses and load is picked up.

Dynamic Studies

Once the steady-state analysis has been completed, a dynamic analysis of the restoration plan is conducted. The dynamic analysis starts from an initial steady-state operating point representing a step in the plan. This initial system operating condition is usually obtained from the system steady-state analysis. One key simulation initially represents the isolated power system and then simulates the start-up of the largest induction motor load at the next-start generating unit. This verifies that the voltage supply is strong enough to start the motor and also that the voltage dip will not stall or cause the motor contactors of running motors to drop out.

The importance of accuracy in equipment modeling must be emphasized. The effect of the controls of an individual unit is generally not very

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significant under normal operation because a large number of units are sharing the control of system voltage and frequency. Both of these quantities are controlled solely by the BSU during the initial restoration period, however. The modeling of the generator, excitation system, and speed governor is therefore very important. The modeling of equipment that does not generally operate under normal conditions, such as over- and under-excitation limiters, can also be important. Governor modeling must take into account whether the machine is operated in an isochronous or droop control mode, as will be discussed later. The accuracy of the dynamic modeling parameters of any large motors to be started are also important to motor-starting simulations.

The dynamic analysis of a black-start plan includes some or all of the following functions:

- Load frequency control;
- Voltage control;
- Large induction motor starting;
- Motor-starting sequence assessment;
- Self-excitation assessment;
- System stability; and
- Transient overvoltages.

Because frequency may deviate significantly from its nominal value, the effect of frequency variation on system impedances must be modeled.

BSUs

As we have already explained, BSUs are units that do not require off-site power to start. Generally, these fall into four categories:

- **Hydroelectric units.** These units can be designed for black-start capability and have fast primary frequency response characteristics.
- **Diesel generator sets.** Diesel sets usually require only battery power and can be started very quickly. They are small in size and useful only for supplying the power needed to start larger units. They generally cannot be used to pick up any significant transmission system elements.
- **Aeroderivative gas turbine generator sets.** This type of gas turbine typically requires only local battery power to start. These units can usually be started using remote commands and can pick up load quickly.
- **Larger gas turbines operating in a simple cycle mode or steam turbine units.** These units are not in themselves black-start capable but are coupled with on-site diesel generator sets to make the plant a black-start source. The diesels are started and used to energize plant auxiliary buses and start either a gas turbine or a steam turbine. A gas turbine is generally quicker to bring online. The time to restart and available ramping capability are generally functions of how long the unit was off-line.

Load Frequency Control

When only a portion of the system is lost and is being restored using tie lines to a larger power system, load frequency

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control is not generally a large concern. The outside system generally has the capacity to absorb changes in load without significant frequency deviations.

When restoration without external resources is required, however, load frequency control is of critical importance. During the restoration process, the black-start generating unit will typically be used to pick up large induction motors associated with a larger power plant such as boiler feed pumps and forced-draft and induced-draft fan motors. The frequency of the black-start system will be controlled by the speed governor of the turbine driving the black-start synchronous generator.

Standard practice for units operating in parallel in multi-machine power systems under normal conditions is to operate all turbine speed governors in a droop-governing mode. This provides a stable sharing of the electric system load among all units. The proportional characteristic of the droop speed governor control, however, results in a steady-state frequency error remaining in the system. Automatic generation control (AGC) has the form of a pure integral controller and will follow the primary frequency control action of the speed governors to remove this undesirable steady-state frequency error. Typical steady-state regulating droop (R) for speed governors is 5% using a system frequency base of 50 Hz or 60 Hz and a power base equal to the turbine MW rating.

During a black-start event, AGC will not be operating, however, it is imperative that system frequency regain its scheduled value following the start-up of motors or pickup of other loads. This frequency control should be automatic, since the crew in charge of the BSU will be operating under extreme emergency conditions, which can lead to undesirable operating errors. The automation of the frequency control process can therefore be carried out by the prime-mover speed governor of the BSU, operated in a constant frequency or isochronous control mode. In this pure integral control mode, the steady-state frequency error is zero because of the resetting characteristic of the pure integral control. Most if not all modern diesel engines, gas turbines, and hydraulic turbines are furnished with digital speed governors with which a selection of either a droop or isochronous operating mode can be carried out by means of a simple change in command.

Once the system has more than one generating unit online, all speed governors should be operated in a droop control mode, unless it is decided in the restoration plan that one of the largest units should operate in isochronous control to maintain the control of the system frequency. As noted above, AGC would be disabled under this extreme condition.

In summary, the preferred control mode for speed governors associated with BSUs is isochronous or constant frequency control. When additional units are added, the preferred control mode for speed governors is droop control mode. In some cases, it may be preferable to keep one large unit in isochronous control mode. Units should not be operated

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in parallel with more than one unit in isochronous control mode.

Voltage Control

Control of voltage is obtained through the generator's excitation system. The excitation system must be operated in automatic control with the automatic voltage regulator (AVR) in service. The system voltage will be a function of the generator terminal voltage. The generator scheduled voltage may therefore need to be adjusted throughout the restoration process as load is picked up, and also coordinated with any changes in transformer tap positions. Such adjustments should be an integral part of the restoration plan. The changes in voltage that will occur with the starting of large motors or the pickup of large blocks of load require that the excitation system respond in a rapid, well-tuned manner.

Motor Starting

Motor starting is a concern during black-start restoration. A BSU's primary function is generally to start up the auxiliary load of a larger next-start unit. This auxiliary load is made up of lighting and motor load used, for the most part, in the start-up of steam generators and fuel systems. The motor load is made up of a large number of small- and medium-size motors and a few large motors ranging anywhere from several hundred horsepower to several thousand horsepower. Fuel and feedwater pump motors and forced- and induced-draft fan motors belong to this large-horsepower group. It is this latter group that presents the greatest challenge to the reactive power resources available in any well-designed black-start plan.

The method used for starting up these large motors is often a hard start, which applies full line-to-line voltage across the motor terminals. Occasionally, motors may also be soft-started by applying a reduced voltage during the starting period. It is therefore extremely important to properly identify the motor-starting method since this will greatly affect the depth of the dip in voltage resulting from the black-start process.

Accurate motor data are vital for conducting dynamic studies to verify the viability of a given black-start process. The information needed to establish the dynamic model for the large induction motors participating in the black-start process includes the inertia of the motor plus its mechanical load, the starting or locked rotor torque, the starting or locked rotor current and associated power factor, the pull-out torque, the full-load torque, and the full-load current and its associated power factor. All of these data should be at rated voltage and frequency. From these motor performance data, parameters for the stator and rotor circuits are estimated. The dynamic model for these large induction motors should include both inertial and rotor circuit flux dynamics. This dynamic model must closely match the speed-torque characteristic of the motors, particularly at starting, pull-out, and full-load operating points. In addition, it

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is important to include the mechanical load damping effect in the inertial model of the mechanical load, which for most centrifugal pumps and fans follows a quadratic speed-torque characteristic.

The motor-starting sequence is another variable that must be verified in any black-start process. The feasibility of plant start-up can be tested by dynamically simulating the various motor-starting sequences. The sequence must also accommodate the start-up requirements of the plant, which may require certain motors to be started before others.

The voltage dip caused by starting these large induction motors must be accurately quantified. This is because the motors already online have magnetic contactors that open at approximately 80% of the nominal bus voltage. IEEE Standard 399-1997 recommends a minimum terminal voltage of 80% of rated voltage. Occasionally there may be magnetic contactors that can hold their contacts with voltages as low as 70% of the rated value; the number of cycles for which this operating condition can be sustained is low, however. In addition, the life expectancy of the insulation of the stator and rotor windings is reduced as a result of the large currents circulating through these windings. In such situations, the motor manufacturer must be consulted to avoid motor damage, such as shorted turns or even catastrophic motor failure. Undervoltage protection settings should also be verified to avoid the opening of circuit protection caused by undervoltage relay action.

The accelerating time period required by an induction motor depends in great measure on the combined inertia of the motor and its mechanical load. The longer the accelerating period, the higher the heating experienced by the stator and rotor windings. When accelerating periods last a few tens of seconds, motor manufacturer data on allowed motor heating should be consulted to avoid a significant loss of useful operating life of the winding insulating material.

Self-Excitation

As noted above, energization of a transmission line or cable will result in a rise in voltage along the line or cable due to charging currents. The charging requirements can be large enough to result in the BSUs absorbing reactive power. There is the potential for self-excitation if the charging current is large relative to the size of the generating unit. The result can be an uncontrolled rise in voltage that could result in equipment failure. Such an undesirable operating condition may occur when the effective charging capacitive reactance of the transmission system used in the black-start operation, as seen by the BSU, is less than the q-axis generator reactance X_q. In generating units with no negative field current capability, self-excitation cannot be controlled by the excitation system, and thus the machine terminal voltage rises almost instantaneously for cases where the effective capacitive reactance is less than the d-axis reactance X_d. Generator excitation

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systems with negative field current capability delay but do not prevent the onset of self-excitation. It is worth noting that most generating units installed in the last 40 years do not have negative field current capability. It is therefore extremely important to verify the reactive power capability of the BSU when operated at a leading power factor.

Self-excitation can also occur from the load end for the inadvertent loss of supply resulting from the opening of a transmission line or cable at the sending end that leaves the line connected to a large motor or a group of motors.

System Stability

Power system stability was defined in 2004 by the IEEE/CIGRÉ Joint Task Force on Stability Terms and Definitions as "the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact." This simply means that the power system must be able to survive a disturbance and return to a sustainable operating point without a significant loss of equipment.

Power system stability can be further subdivided into:

- **Rotor angle stability:** the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance
- **Voltage stability:** the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance
- **Frequency stability:** the ability of a power system to maintain steady frequency following a severe system disturbance resulting in a significant imbalance between generation and load

As discussed above, voltage and frequency control present the greatest concerns during restoration. Angular stability is generally not a major concern in the early stages of the restoration process. When the system is being restored from a total blackout, angular stability is assessed only when more than one generating unit is used in the black-start plan. Even when there are multiple units connected in the early stages of a restoration plan, the system is operating in a weakened state and stable performance is not expected for all system contingencies. Under normal conditions, there are generally multiple transmission paths between groups of generators such that a fault and trip of one of these paths does not result in instability. During restoration, however, there may only be one strong path, and hence a fault and outage of that path would cause instability. This period of exposure to possible but unlikely events cannot limit the restoration, however. It is simply a stage the system must pass through to reach a more robust operating condition. The exposure to events that could result in instability can be used as one of the criteria with which to rank restoration alternatives.

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Cold Load Pickup

The purpose of the restoration process is, of course, to restore power supply to the loads and allow them to operate as they did prior to the outage. But the characteristics of the load immediately after reenergization may be quite different than the characteristics exhibited prior to the outage.

If the load has been deenergized for several hours or more, the inrush current upon reenergizing the load can be eight to ten times the normal load current. The magnitude and duration of the inrush current that flows when a feeder is reenergized after a prolonged outage is a function of the type of load served by the feeder. This could include: lighting; motors; and thermostatically controlled loads such as air conditioners, refrigerators, freezers, furnaces, and electric hot water heaters.

There are various components of the load that contribute to the total inrush current. One example is the component due to the filaments of incandescent lights. The resistance of the filament is very low until it warms to its operating temperature. This low resistance results in a very high inrush current—up to ten times the normal current. This high current flows for a short period, approximately one-tenth of a second.

Another component of the inrush current is the starting of motors when the load is picked up. When a motor starts, the current drawn will typically be five to six times normal, until the motor reaches its operating speed. This may take as long as several seconds for large, industrial-type motors.

A third component of inrush current is thermostatically controlled loads, which turn on and off automatically to hold temperature to a desired preset value. Under normal operating conditions, approximately one-third of these loads will likely be connected at any instant in time. But after a lengthy interruption of service, they will all have their thermostat contacts closed, waiting to run as soon as power is restored. As a result, these thermostatically controlled loads will be perhaps three times greater than they normally would be for the first half hour or so after being energized. Most thermostatically controlled loads also contain small, single-phase motors, which will draw five or six times running current until they are accelerated up to running speed in perhaps one-half of a second. This results in the initial current drawn by some thermostatically controlled loads being as high as 15 times normal current for the first one-half of a second following energization.

A summary of the magnitude and duration of the inrush for some of the various types of loads is shown in Figure 1.

Transient Overvoltages

Restoration of the power system is performed through a series of switching actions to sequentially reenergize system components. Energizing equipment during restoration conditions can result in higher overvoltages than during times of normal operation. These overvoltages can lead to equipment failure or damage that

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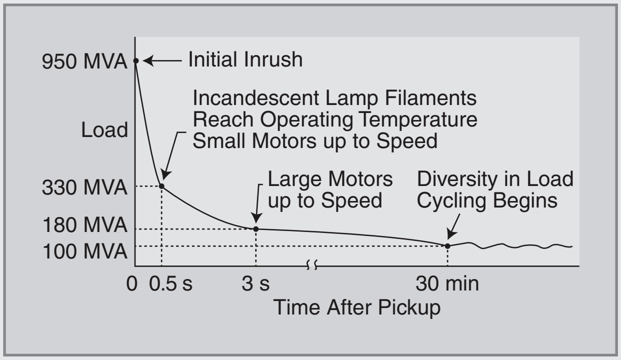

Figure 1 Load variation following cold load pickup

may hinder the successful implementation of the restoration plan.

Transient overvoltages include temporary overvoltages, switching surges, and lightning surges. Lightning surges, while an important design consideration, are usually not a concern that affects the restoration of a power system.

Switching surges are the transient overvoltages that immediately follow the opening or closing of a circuit breaker or other switching device. Switching surges have high-frequency components (from 100 Hz to 10 kHz) that decay quickly, typically within two to three cycles of the power frequency, and are followed by a normal steady-state voltage. Switching surges typically contain only one, or just a few, voltage peaks that are of interest, as shown in Figure 2. This sample waveform is from a simulation of the energizing of a typical overhead line.

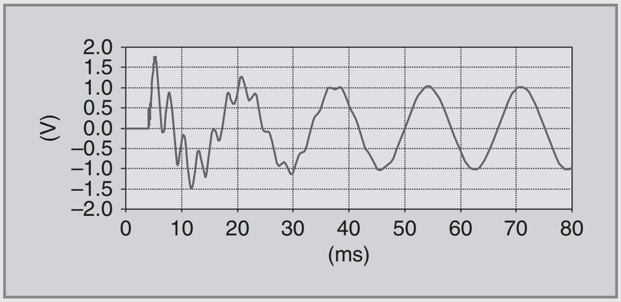

Figure 2 Switching surge from an electromechanical transients program simulation of the energizing of a typical overhead line. The voltage is shown per unit of the line's nominal voltage rating.

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The magnitude and wave shape of the switching surge depends on the angle of the power frequency source voltage wave at the instant of circuit breaker closure. This requires that many simulations be performed with various closing times to obtain a statistical distribution of the overvoltage results. Surge arresters are effective in limiting the peak of the switching surges.

Temporary overvoltages (TOVs) include many types of events for which the voltage transient lasts longer than the surges discussed above, exceeding the rated value for three cycles or, potentially, significantly longer. TOVs encompass power frequency phenomena such as the Ferranti rise on an open-ended line or cable and the overvoltage on an unfaulted phase during a single line-to-ground fault.

TOVs can also follow switching surges. For example, a TOV can result from switching circuits that saturate the core of a power transformer, for example, when cables and transformers are energized together. The harmonic-rich transformer inrush currents can interact with the harmonic resonances of the power system. The resonant frequencies are a function of the series inductance associated with the system's short-circuit strength and the shunt capacitances of cables and lines. Higher inductances (a property of relatively weak systems, such as those often occurring during restoration) and higher capacitances (such as those due to long cables) yield lower resonant frequencies and a higher chance of TOVs.

Figure 3 shows an example of a TOV taken from a simulation of the energization of a large transformer. Like switching surges, this type of TOV can be dependent on the circuit breaker closing times. In contrast to

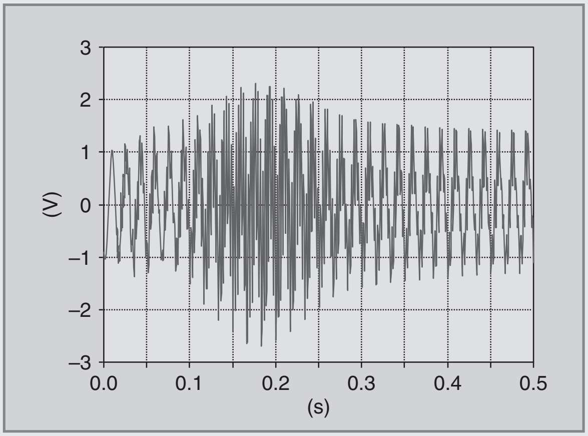

Figure 3 An example of a TOV; voltage is expressed per unit of nominal peak phase voltage

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switching surges with one predominant peak, TOVs can have hundreds of peaks, all of about the same magnitude, if the TOV lasts for several seconds.

The expected TOV magnitude and duration is often a major concern for surge arresters. Metal-oxide varistor-type surge arresters have little effect on TOVs that are below about 1.6 per unit. Silicon carbide-type surge arresters are not affected by TOV levels below the 60-Hz spark-over level. If the TOV repeatedly exceeds the spark-over level, however, then the multiple discharges may result in excessive energy absorption and consequent arrester failure.

A Black-Start Example

In this section, we present a simulated black start. The scenario is one where a fast-starting, gas turbine-driven generating unit is used as a BSR to start up a combined-cycle power plant. The black-start system consists of the BSU's step-up transformer, underground high-voltage (HV) cables that connect the HV substation at the BSU to the HV substation at the combined-cycle plant, and the generator step-up and auxiliary transformers at the combined-cycle plant. Both the taps of transformers with tap-adjustment capability and the voltage reference set point of the BSU were chosen to ensure that terminal voltages at the large induction motor units used in the black start were close to their nominal values.

The black-start plan begins with the across-the-line starting of a 2500-hp motor. The motor performance during the starting period is shown in Figure 4. Terminal voltage, motor reactive power, electrical torque, and motor slip are shown. Note the dip in motor terminal voltage and that the demand for reactive power rises during the period following the lowest voltage at the motor terminals. The motor's air gap torque increases significantly during the acceleration period, as expected, to overcome the mechanical load torque that opposes developed electromagnetic torque.

The dynamic response of the BSU during the starting of this large induction motor is shown in Figure 5. The performance of the excitation system is shown as it works to control the BSU's terminal voltage. Note the fast response and large field forcing applied to pull up the machine terminal voltage from the dip caused by the large reactive power demand imposed by the starting motor. The unit also sees a significant voltage rise caused by the rapid reduction in reactive power as the motor locks in to its operating speed. Electric power demand also increases during this period as the motor accelerates and moves toward its steady-state operating point.

Summary

Restoration actions involve very unusual conditions, especially for local generation used as a BSR. Important considerations for assuring that restoration plans are realizable include the ability to operate in islanded

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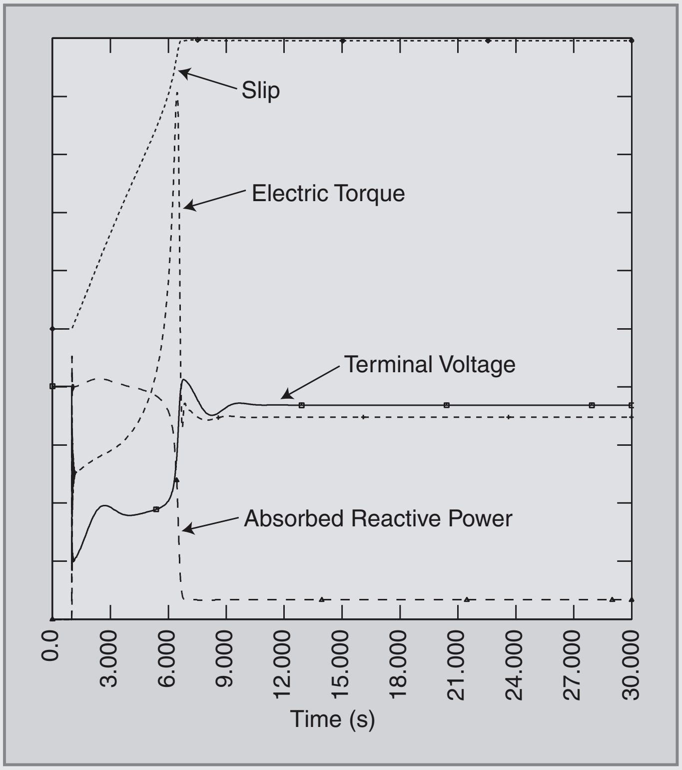

Figure 4 Motor performance during the starting period: motor terminal voltage, reactive power, electric torque, and slip

conditions with stable frequency and voltage control, the availability of synchronizing equipment at key substations to permit paralleling of separate sections, and the validity of assumptions used to assess the ability of synchronous generators to operate at the unusual points of their capability required during the restoration period.

Restoration actions performed to recover from a blackout may vary from those determined in the restoration studies. Restoration plans are based on a given set of assumptions, such as the available transmission, the amount of cold load to be picked up, and the many other conditions discussed in this case study. Although actual conditions could differ from these assumptions, restoration studies provide value by demonstrating the logic behind particular steps being taken, such as the reasoning behind the choice and sequence of operator actions and the

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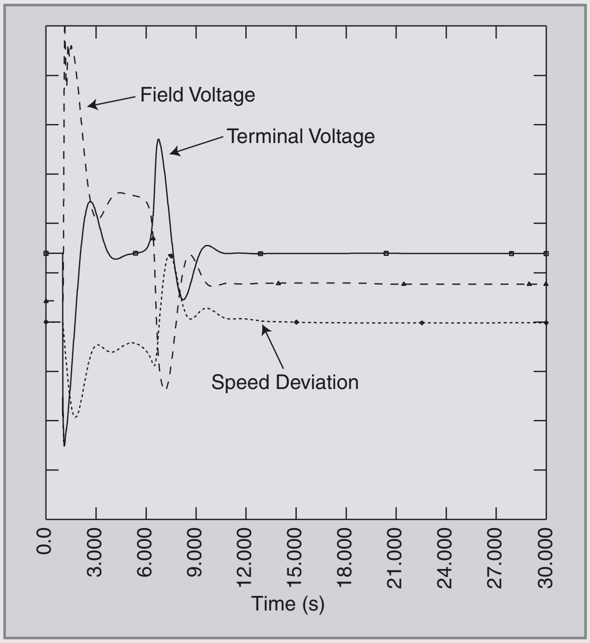

Figure 5 Dynamic response of the BSU as the large induction motor is started: field voltage, terminal voltage, and speed

expected results of those actions. With this understanding, the operating staff will be able to adapt to differences in the actual versus the assumed conditions.

This case study has described restoration operations and the studies that should be part of a restoration planning process. In particular, it attempted to describe technical issues such as system dynamics and control aspects of the black-start process. This overview should be helpful to utility staff involved in the development of restoration plans. Development of thorough restoration plans and the testing of those plans through simulation and drills will help to minimize disruption of service to loads and the risk of damage to equipment following partial or total power system blackouts.

For Further Reading

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Biographies

James Feltes is with Siemens Power Technologies International (Siemens PTI), Schenectady, New York.

Carlos Grande-Moran is with Siemens Power Technologies International (Siemens PTI), Schenectady, New York. ■

11.1 THE SWING EQUATION

Consider a generating unit consisting of a three-phase synchronous generator and its prime mover. The rotor motion is determined by Newton's second law, given by

$$J\alpha_m(t) = T_m(t) - T_e(t) = T_a(t) \tag{11.1.1}$$

where J = total moment of inertia of the rotating masses, kg-m^2
α_m = rotor angular acceleration, rad/s^2
T_m = mechanical torque supplied by the prime mover minus the retarding torque due to mechanical losses, N-m
T_e = electrical torque that accounts for the total three-phase electrical power output of the generator, plus electrical losses, N-m
T_a = net accelerating torque, N-m

Also, the rotor angular acceleration is given by

$$\alpha_m(t) = \frac{d\omega_m(t)}{dt} = \frac{d^2\theta_m(t)}{dt^2} \tag{11.1.2}$$

$$\omega_m(t) = \frac{d\theta_m(t)}{dt} \tag{11.1.3}$$

where ω_m = rotor angular velocity, rad/s
θ_m = rotor angular position with respect to a stationary axis, rad

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T_m and T_e *are* positive for generator operation. In steady-state T_m equals T_e, the accelerating torque T_a is zero, and, from (11.1.1), the rotor acceleration α_m is zero, resulting in a constant rotor velocity called *synchronous speed.* When T_m is greater than T_e, T_a is positive and α_m is therefore positive, resulting in increasing rotor speed. Similarly, when T_m is less than T_e, the rotor speed is decreasing.

It is convenient to measure the rotor angular position with respect to a synchronously rotating reference axis instead of a stationary axis. Accordingly,

$$\theta_m(t) = \omega_{msyn}t + \delta_m(t) \tag{11.1.4}$$

where ω_{msyn} = synchronous angular velocity of the rotor, rad/s
δ_m = rotor angular position with respect to a synchronously rotating reference, rad

Using (11.1.2) and (11.1.4), (11.1.1) becomes

$$J\frac{d^2\theta_m(t)}{dt^2} = J\frac{d^2\delta_m(t)}{dt^2} = T_m(t) - T_e(t) = T_a(t) \tag{11.1.5}$$

It is also convenient to work with power rather than torque, and to work in per-unit rather than in actual units. Accordingly, multiply (11.1.5) by $\omega_m(t)$ and divide by S_{rated}, the three-phase voltampere rating of the generator:

$$\frac{J\omega_m(t)}{S_{rated}}\frac{d^2\delta_m(t)}{dt^2} = \frac{\omega_m(t)T_m(t) - \omega_m(t)T_e(t)}{S_{rated}}$$

$$= \frac{p_m(t) - p_e(t)}{S_{rated}} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{11.1.6}$$

where $p_{mp.u.}$ = mechanical power supplied by the prime mover minus mechanical losses, per unit
$p_{ep.u.}$ = electrical power output of the generator plus electrical losses, per unit

Finally, it is convenient to work with a normalized inertia constant, called the H constant, which is defined as

$$H = \frac{\text{stored kinetic energy at synchronous speed}}{\text{generator voltampere rating}}$$

$$= \frac{\frac{1}{2}J\omega_{msyn}^2}{S_{rated}} \quad \text{joules/VA or per unit-seconds} \tag{11.1.7}$$

The H constant has the advantage that it falls within a fairly narrow range, normally between 1 and 10 p.u.-s, whereas J varies widely, depending on generating unit size and type. Solving (11.1.7) for J and using in (11.1.6),

$$2H\frac{\omega_m(t)}{\omega_{msyn}^2}\frac{d^2\delta_m(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{11.1.8}$$

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Defining per-unit rotor angular velocity.

$$\omega_{\text{p.u.}}(t) = \frac{\omega_m(t)}{\omega_{m\text{syn}}} \tag{11.1.9}$$

Equation (11.1.8) becomes

$$\frac{2\text{H}}{\omega_{m\text{syn}}}\omega_{\text{p.u.}}(t)\frac{d^2\delta_m(t)}{dt^2} = p_{m\text{p.u.}}(t) - p_{e\text{p.u.}}(t) = p_{a\text{p.u.}}(t) \tag{11.1.10}$$

For a synchronous generator with P poles, the electrical angular acceleration α, electrical radian frequency ω, and power angle δ are

$$\alpha(t) = \frac{\text{P}}{2}\alpha_m(t) \tag{11.1.11}$$

$$\omega(t) = \frac{\text{P}}{2}\omega_m(t) \tag{11.1.12}$$

$$\delta(t) = \frac{\text{P}}{2}\delta_m(t) \tag{11.1.13}$$

Similarly, the synchronous electrical radian frequency is

$$\omega_{\text{syn}} = \frac{\text{P}}{2}\omega_{m\text{syn}} \tag{11.1.14}$$

The per-unit electrical frequency is

$$\omega_{\text{p.u.}}(t) = \frac{\omega(t)}{\omega_{\text{syn}}} = \frac{\frac{2}{\text{P}}\omega(t)}{\frac{2}{\text{P}}\omega_{\textbf{syn}}} = \frac{\omega_m(t)}{\omega_{m\text{syn}}} \tag{11.1.15}$$

Therefore, using (11.1.13–11.1.15), (11.1.10) can be written as

$$\frac{2\text{H}}{\omega_{\text{syn}}}\omega_{\text{p.u.}}(t)\frac{d^2\delta(t)}{dt^2} = p_{m\text{p.u.}}(t) - p_{e\text{p.u.}}(t) = p_{a\text{p.u.}}(t) \tag{11.1.16}$$

Frequently (11.1.16) is modified to also include a term that represents a damping torque anytime the generator deviates from its synchronous speed, with its value proportional to the speed deviation

$$\begin{aligned} &2\text{H}/\omega_{\text{syn}}\omega_{\text{p.u.}}(t)(d^2\delta(t)/(dt^2)) \\ &\quad = p_{m\text{p.u.}}(t) - p_{e\text{p.u.}}(t) - \text{D}/\omega_{\text{syn}}(d\delta(t)/(dt)) \\ &\quad = p_{a\text{p.u.}}(t) \end{aligned} \tag{11.1.17}$$

where D is either zero or a relatively small positive number with typical values between 0 and 2. The units of D are per-unit power divided by per-unit speed deviation.

Equation (11.1.17), called the per-unit *swing equation*, is the fundamental equation that determines rotor dynamics in transient stability studies. Note that it is nonlinear due to $p_{e\text{p.u.}}(t)$ which is shown in Section 11.2 to be a nonlinear function

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of δ. Equation (11.1.17) is also nonlinear due to the $\omega_{\text{p.u.}}(t)$ term. However, in practice the rotor speed does not vary significantly from synchronous speed during transients. That is, $\omega_{\text{p.u.}}(t) \simeq 1.0$, which is often assumed in (11.1.17) for hand calculations.

Equation (11.1.17) is a second-order differential equation that can be rewritten as two first-order differential equations. Differentiating (11.1.4), and then using (11.1.3) and (11.1.12) through (11.1.14), result in

$$\frac{d\delta(t)}{dt} = \omega(t) - \omega_{\text{syn}} \tag{11.1.18}$$

Using (11.1.18) in (11.1.17),

$$\frac{2\text{H}}{\omega_{\text{syn}}}\omega_{\text{p.u.}}(t)\frac{d\omega(t)}{dt} = p_{m\text{p.u.}}(t) - p_{e\text{p.u.}}(t) - \text{D}/\omega_{\text{syn}}\frac{d\delta(t)}{dt} = p_{a\text{p.u.}}(t) \tag{11.1.19}$$

Equations (11.1.18) and (11.1.19) are two first-order differential equations.

EXAMPLE 11.1

Generator per-unit swing equation and power angle during a short circuit

A three-phase, 60-Hz, 500-MVA, 15-kV, 32-pole hydroelectric generating unit has an H constant of 2.0 p.u.-s and D = 0. (a) Determine ω_{syn} and $\omega_{m\text{syn}}$. (b) Give the per-unit swing equation for this unit. (c) The unit is initially operating at $p_{m\text{p.u.}} = p_{e\text{p.u.}} = 1.0$, $\omega = \omega_{\text{syn}}$, and $\delta = 10°$ when a three-phase-to-ground bolted short circuit at the generator terminals causes $p_{e\text{p.u.}}$ to drop to zero for $t \geqslant 0$. Determine the power angle 3 cycles after the short circuit commences. Assume $p_{m\text{p.u.}}$ remains constant at 1.0 per unit. Also assume $\omega_{\text{p.u.}}(t) = 1.0$ in the swing equation.

SOLUTION

a. For a 60-Hz generator,

$$\omega_{\text{syn}} = 2\pi 60 = 377 \text{ rad/s}$$

and, from (11.1.14), with P = 32 poles,

$$\omega_{m\text{syn}} = \frac{2}{\text{P}}\omega_{\text{syn}} = \left(\frac{2}{32}\right)377 = 23.56 \quad \text{rad/s}$$

b. From (11.1.16), with H = 2.0 p.u.-s,

$$\frac{4}{2\pi 60}\omega_{\text{p.u.}}(t)\frac{d^2\delta(t)}{dt^2} = p_{m\text{p.u.}}(t) - p_{e\text{p.u.}}(t)$$

c. The initial power angle is

$$\delta(0) = 10° = 0.1745 \quad \text{radian}$$

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Also, from (11.1.17), at $t = 0$,

$$\frac{d\delta(0)}{dt} = 0$$

Using $p_{mp.u.}(t) = 1.0$, $p_{ep.u.} = 0$, and $\omega_{p.u.}(t) = 1.0$, the swing equation from (b) is

$$\left(\frac{4}{2\pi 60}\right)\frac{d^2\delta(t)}{dt^2} = 1.0 \quad t \geqslant 0$$

Integrating twice and using the above initial conditions,

$$\frac{d\delta(t)}{dt} = \left(\frac{2\pi 60}{4}\right)t + 0$$

$$\delta(t) = \left(\frac{2\pi 60}{8}\right)t^2 + 0.1745$$

$$\text{At } t = 3 \text{ cycles} = \frac{3 \text{ cycles}}{60 \text{ cycles/second}} = 0.05 \text{ second,}$$

$$\delta(0.05) = \left(\frac{2\pi 60}{8}\right)(0.05)^2 + 0.1745$$

$$= 0.2923 \text{ radian} = 16.75°$$

EXAMPLE 11.2

Equivalent swing equation: two generating units

A power plant has two three-phase, 60-Hz generating units with the following ratings:

Unit 1: 500 MVA, 15 kV, 0.85 power factor, 32 poles, $H_1 = 2.0$ p.u.-s, $D = 0$

Unit 2: 300 MVA, 15 kV, 0.90 power factor, 16 poles, $H_2 = 2.5$ p.u.-s, $D = 0$

a. Give the per-unit swing equation of each unit on a 100-MVA system base.

b. If the units are assumed to "swing together," that is, $\delta_1(t) = \delta_2(t)$, combine the two swing equations into one equivalent swing equation.

SOLUTION

a. If the per-unit powers on the right-hand side of the swing equation are converted to the system base, then the H constant on the left-hand side must also be converted. That is,

(*Continued*)

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$$H_{new} = H_{old} \frac{S_{old}}{S_{new}} \quad \text{per unit}$$

Converting H_1 from its 500-MVA rating to the 100-MVA system base,

$$H_{1new} = H_{1old} \frac{S_{old}}{S_{new}} = (2.0)\left(\frac{500}{100}\right) = 10 \quad \text{p.u.-s}$$

Similarly, converting H_2,

$$H_{2new} = (2.5)\left(\frac{300}{100}\right) = 7.5 \quad \text{p.u.-s}$$

The per-unit swing equations on the system base are then

$$\frac{2H_{1new}}{\omega_{syn}} \omega_{1p.u.}(t) \frac{d^2\delta_1(t)}{dt^2} = \frac{20.0}{2\pi 60} \omega_{1p.u.}(t) \frac{d^2\delta_1(t)}{dt^2}$$

$$= p_{m1p.u.}(t) - p_{e1p.u.}(t)$$

$$\frac{2H_{2new}}{\omega_{syn}} \omega_{2p.u.}(t) \frac{d^2\delta_2(t)}{dt^2} = \frac{15.0}{2\pi 60} \omega_{2p.u.}(t) \frac{d^2\delta_2(t)}{dt^2} = p_{m2p.u.}(t) - p_{e2p.u.}$$

b. Letting:

$$\delta(t) = \delta_1(t) = \delta_2(t)$$

$$\omega_{p.u.}(t) = \omega_{1p.u.}(t) = \omega_{2p.u.}(t)$$

$$p_{mp.u.}(t) = p_{m1p.u.}(t) + p_{m2p.u.}(t)$$

$$p_{ep.u.}(t) = p_{e1p.u.}(t) + p_{e2p.u.}(t)$$

and adding the above swing equations

$$\frac{2(H_{1new} + H_{2new})}{\omega_{syn}} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2}$$

$$= \frac{35.0}{2\pi 60} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t)$$

When transient stability studies involving large-scale power systems with many generating units are performed, computation time can be reduced by combining the swing equations of those units that swing together. Such units, which are called *coherent machines*, usually are connected to the same bus or are electrically close, and they are usually remote from the network disturbances under study.

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11.2 SIMPLIFIED SYNCHRONOUS MACHINE MODEL AND SYSTEM EQUIVALENTS

Figure 11.2 shows a simplified model of a synchronous machine, called the classical model, that can be used in transient stability studies. As shown, the synchronous machine is represented by a constant internal voltage E' behind its direct axis transient reactance X_d'. This model is based on the following assumptions:

1. The machine is operating under balanced three-phase positive-sequence conditions.
2. Machine excitation is constant.
3. Machine losses, saturation, and saliency are neglected.

In transient stability programs, more detailed models can be used to represent exciters, losses, saturation, and saliency. However, the simplified model reduces model complexity while maintaining reasonable accuracy for some stability calculations.

Each generator in the model is connected to a system consisting of transmission lines, transformers, loads, and other machines. To a first approximation the system can be represented by an "infinite bus" behind a system reactance. An infinite bus is an ideal voltage source that maintains constant voltage magnitude, constant phase, and constant frequency.

Figure 11.3 shows a synchronous generator connected to a system equivalent. The voltage magnitude V_{bus} and 0° phase of the infinite bus are constant. The phase angle δ of the internal machine voltage is the machine power angle with respect to the infinite bus.

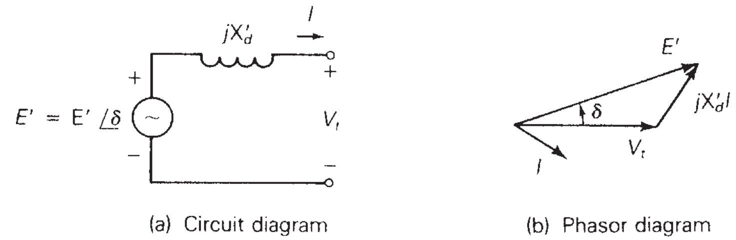

FIGURE 11.2

Simplified synchronous machine model for transient stability studies

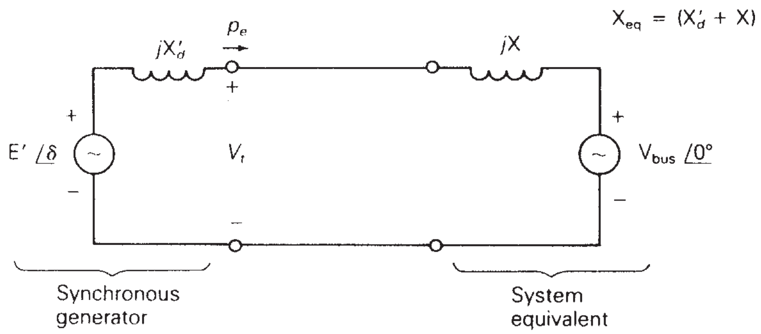

FIGURE 11.3

Synchronous generator connected to a system equivalent

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The equivalent reactance between the machine internal voltage and the infinite bus is $X_{eq} = (X'_d + X)$. From (6.7.3), the real power delivered by the synchronous generator to the infinite bus is

$$p_e = \frac{E'V_{bus}}{X_{eq}} \sin\delta \tag{11.2.1}$$

During transient disturbances both E' and V_{bus} are considered constant in (11.2.1). Thus p_e is a sinusoidal function of the machine power angle δ.

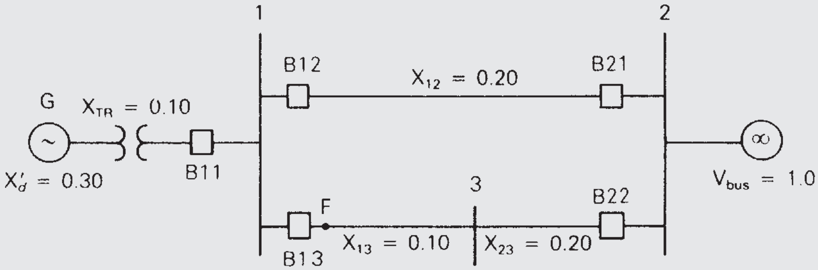
EXAMPLE 11.3

Generator internal voltage and real power output versus power angle

Figure 11.4 shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine (a) the internal voltage of the generator and (b) the equation for the electrical power delivered by the generator versus its power angle δ.

FIGURE 11.4

Single-line diagram for Example 11.3



SOLUTION

a. The equivalent circuit is shown in Figure 11.5, from which the equivalent reactance between the machine internal voltage and infinite bus is

$$X_{eq} = X'_d + X_{TR} + X_{12} \| (X_{13} + X_{23})$$

$$= 0.30 + 0.10 + 0.20 \| (0.10 + 0.20)$$

$$= 0.520 \quad \text{per unit}$$

The current into the infinite bus is

$$I = \frac{P}{V_{bus}(\text{p.f.})} \underline{/-\cos^{-1}(\text{p.f.})} = \frac{(1.0)}{(1.0)(0.95)} \underline{/-\cos^{-1} 0.95}$$

$$= 1.05263 \underline{/-18.195^\circ} \quad \text{per unit}$$

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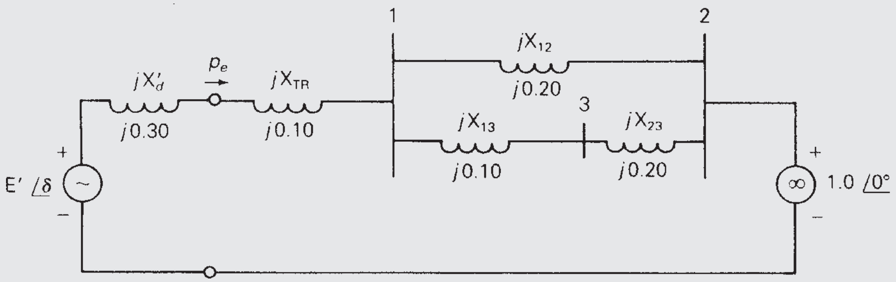

FIGURE 11.5

Equivalent circuit for Example 11.3

and the machine internal voltage is

$$E' = E'\underline{/\delta} = V_{bus} + jX_{eq}I$$
$$= 1.0\underline{/0^\circ} + (j0.520)(1.05263\underline{/-18.195^\circ})$$
$$= 1.0\underline{/0^\circ} + 0.54737\underline{/71.805^\circ}$$
$$= 1.1709 + j0.5200$$
$$= 1.2812\underline{/23.946^\circ} \quad \text{per unit}$$

b. From (11.2.1),

$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin\delta = 2.4628 \sin\delta \quad \text{per unit}$$

11.3 THE EQUAL-AREA CRITERION

Consider a synchronous generating unit connected through a reactance to an infinite bus. Plots of electrical power p_e and mechanical power p_m versus power angle δ are shown in Figure 11.6. p_e is a sinusoidal function of δ, as given by (11.2.1).

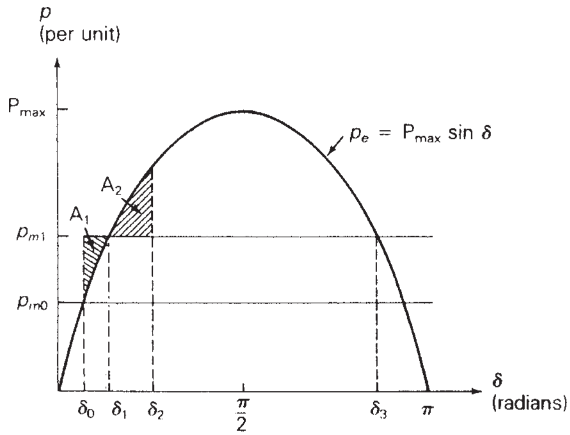

FIGURE 11.6

p_e and p_m versus δ

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Suppose the unit is initially operating in steady-state at $p_e = p_m = p_{m0}$ and $\delta = \delta_0$, when a step change in p_m from p_{m0} to p_{m1} occurs at $t = 0$. Due to rotor inertia, the rotor position cannot change instantaneously. That is, $\delta_m(0^+) = \delta_m(0^-)$; therefore, $\delta(0^+) = \delta(0^-) = \delta_0$ and $p_e(0^+) = p_e(0^-)$. Since $p_m(0^+) = p_{m1}$ is greater than $p_e(0^+)$, the acceleration power $p_a(0^+)$ is positive and, from (11.1.16), $(d^2\delta)/(dt^2)(0^+)$ is positive. The rotor accelerates and δ increases. When δ reaches δ_1, $p_e = p_{m1}$ and $(d^2\delta)/(dt^2)$ becomes zero. However, $d\delta/dt$ is still positive and δ continues to increase, overshooting its final steady-state operating point. When δ is greater than δ_1, p_m is less than p_e, p_a is negative, and the rotor decelerates. Eventually, δ reaches a maximum value δ_2 and then swings back toward δ_1. Using (11.1.16), which has no damping, δ would continually oscillate around δ_1. However, damping due to mechanical and electrical losses causes δ to stabilize at its final steady-state operating point δ_1. Note that if the power angle exceeded δ_3, then p_m would exceed p_e and the rotor would accelerate again, causing a further increase in δ and loss of stability.

One method for determining stability and maximum power angle is to solve the nonlinear swing equation via numerical integration techniques using a digital computer. This method, which is applicable to multimachine systems, is described in Section 11.4. However, there is also a direct method for determining stability that does not involve solving the swing equation; this method is applicable for one machine connected to an infinite bus or for two machines. The method, called the *equal-area criterion*, is described in this section.

In Figure 11.6, p_m is greater than p_e during the interval $\delta_0 < \delta < \delta_1$, and the rotor is accelerating. The shaded area A_1 between the p_m and p_e curves is called the accelerating area. During the interval $\delta_1 < \delta < \delta_2$, p_m is less than p_e, the rotor is decelerating, and the shaded area A_2 is the decelerating area. At both the initial value $\delta = \delta_0$ and the maximum value $\delta = \delta_2$, $d\delta/dt = 0$. The equal-area criterion states that $A_1 = A_2$.

To derive the equal-area criterion for one machine connected to an infinite bus, assume $\omega_{\text{p.u.}}(t) = 1$ in (11.1.16), giving

$$\frac{2H}{\omega_{\text{syn}}}\frac{d^2\delta}{dt^2} = p_{m\text{p.u.}} - p_{e\text{p.u.}} \tag{11.3.1}$$

Multiplying by $d\delta/dt$ and using

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

(11.3.1) becomes

$$\frac{2H}{\omega_{\text{syn}}}\left(\frac{d^2\delta}{dt^2}\right)\left(\frac{d\delta}{dt}\right) = \frac{H}{\omega_{\text{syn}}}\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = (p_{m\text{p.u.}} - p_{e\text{p.u.}})\frac{d\delta}{dt} \tag{11.3.2}$$

Multiplying (11.3.2) by dt and integrating from δ_0 to δ,

$$\frac{H}{\omega_{\text{syn}}}\int_{\delta_0}^{\delta} d\left[\frac{d\delta}{dt}\right]^2 = \int_{\delta_0}^{\delta}(p_{m\text{p.u.}} - p_{e\text{p.u.}})d\delta$$

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or

$$\frac{H}{\omega_{syn}}\left[\frac{d\delta}{dt}\right]^2\Bigg|_{\delta_0}^{\delta} = \int_{\delta_0}^{\delta}(p_{mp.u.} - p_{ep.u.})\,d\delta \tag{11.3.3}$$

The above integration begins at δ_0 where $d\delta/dt = 0$, and continues to an arbitrary δ. When δ reaches its maximum value, denoted δ_2, $d\delta/dt$ again equals zero. Therefore, the left-hand side of (11.3.3) equals zero for $\delta = \delta_2$ and

$$\int_{\delta_0}^{\delta_2}(p_{mp.u.} - p_{ep.u.})d\delta = 0 \tag{11.3.4}$$

Separating this integral into positive (accelerating) and negative (decelerating) areas, results in the equal-area criterion

$$\int_{\delta_0}^{\delta_1}(p_{mp.u.} - p_{ep.u.})d\delta + \int_{\delta_1}^{\delta_2}(p_{mp.u.} - p_{ep.u.})d\delta = 0$$

or

$$\int_{\delta_0}^{\delta_1}\underbrace{(p_{mp.u.} - p_{ep.u.})d\delta}_{A_1} = \int_{\delta_1}^{\delta_2}\underbrace{(p_{ep.u.} - p_{mp.u.})d\delta}_{A_2} \tag{11.3.5}$$

In practice, sudden changes in mechanical power usually do not occur, since the time constants associated with prime mover dynamics are on the order of seconds. However, stability phenomena similar to that described above can also occur from sudden changes in electrical power, due to system faults and line switching. The following three examples are illustrative.

EXAMPLE 11.4

Equal-area criterion: transient stability during a three-phase fault

The synchronous generator shown in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3, when a temporary three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 1, shown as point F in Figure 11.4. Three cycles later the fault extinguishes by itself. Due to a relay misoperation, all circuit breakers remain closed. Determine whether stability is or is not maintained and determine the maximum power angle. The inertia constant of the generating unit is 3.0 per unit-seconds on the system base. Assume p_m remains constant throughout the disturbance. Also assume $\omega_{p.u.} = (t) = 1.0$ in the swing equation.

SOLUTION

Plots of p_e and p_m versus δ are shown in Figure 11.7. From Example 11.3 the initial operating point is $p_e(0^-) = p_m = 1.0$ per unit and $\delta(0^+) = \delta(0^-) = \delta_0 = 23.95° =$

(Continued)

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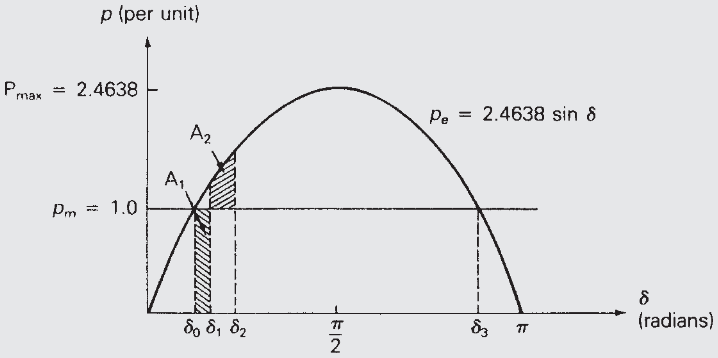
FIGURE 11.7

p–δ plot for Example 11.4



0.4179 radian. At $t = 0$, when the short circuit occurs, p_e instantaneously drops to zero and remains at zero during the fault since power cannot be transferred past faulted bus 1. From (11.1.16), with $\omega_{\text{p.u.}}(t) = 1.0$,

$$\frac{2H}{\omega_{\text{syn}}}\frac{d^2\delta(t)}{dt^2} = p_{m\text{p.u.}} \qquad 0 \leqslant t \leqslant 0.05 \quad \text{s}$$

Integrating twice with initial condition $\delta(0) = \delta_0$ and $\dfrac{d\delta(0)}{dt} = 0$,

$$\frac{d\delta(t)}{dt} = \frac{\omega_{\text{syn}}\, p_{m\text{p.u.}}}{2H}t + 0$$

$$\delta(t) = \frac{\omega_{\text{syn}}\, p_{m\text{p.u.}}}{4H}t^2 + \delta_0$$

At $t = 3$ cycles $= 0.05$ second,

$$\delta_1 = \delta(0.05 \text{ s}) = \frac{2\pi 60}{12}(0.05)^2 + 0.4179$$

$$= 0.4964 \text{ radian} = 28.44°$$

The accelerating area A_1, shaded in Figure 11.7, is

$$A_1 = \int_{\delta_0}^{\delta_1} p_m d\delta = \int_{\delta_0}^{\delta_1} 1.0 d\delta = (\delta_1 - \delta_0) = 0.4964 - 0.4179 = 0.0785$$

At $t = 0.05$ s the fault extinguishes and p_e instantaneously increases from zero to the sinusoidal curve in Figure 11.7. δ continues to increase until the decelerating area A_2 equals A_1. That is,

$$A_2 = \int_{\delta_1}^{\delta_2} (p_{\max} \sin\delta - p_m) d\delta$$

$$= \int_{0.4964}^{\delta_2} (2.4638 \sin\delta - 1.0) d\delta = A_1 = 0.0785$$

Integrating,

$$2.4638[\cos(0.4964) - \cos\delta_2] - (\delta_2 - 0.4964) = 0.0785$$

$$2.4638 \cos\delta_2 + \delta_2 = 2.5843$$

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The above nonlinear algebraic equation can be solved iteratively to obtain $\delta_2 = 0.7003$ radian $= 40.12°$

Since the maximum angle δ_2 does not exceed $\delta_3 = (180° - \delta_0) = 156.05°$, stability is maintained. In steady-state, the generator returns to its initial operating point $p_{ess} = p_m = 1.0$ per unit and $\delta_{ss} = \delta_0 = 23.95°$.

Note that as the fault duration increases, the risk of instability also increases. The *critical clearing time*, denoted t_{cr}, is the longest fault duration allowable for stability.

To see this case modeled in PowerWorld Simulator, open case Example 11_4 (see Figure 11.8). Then select **Add-Ons, Transient Stability**, which displays the Transient Stability Analysis Form. Notice that in the Transient Stability Contingency Elements list, a fault is applied to bus 1 at $t = 0$ s and cleared at $t = 0.05$ s (three cycles later). To see the time variation in the generator angle (modeled at bus 4 in PowerWorld Simulator), click the **Run Transient Stability** button. When the simulation is finished, a graph showing this angle automatically appears, as

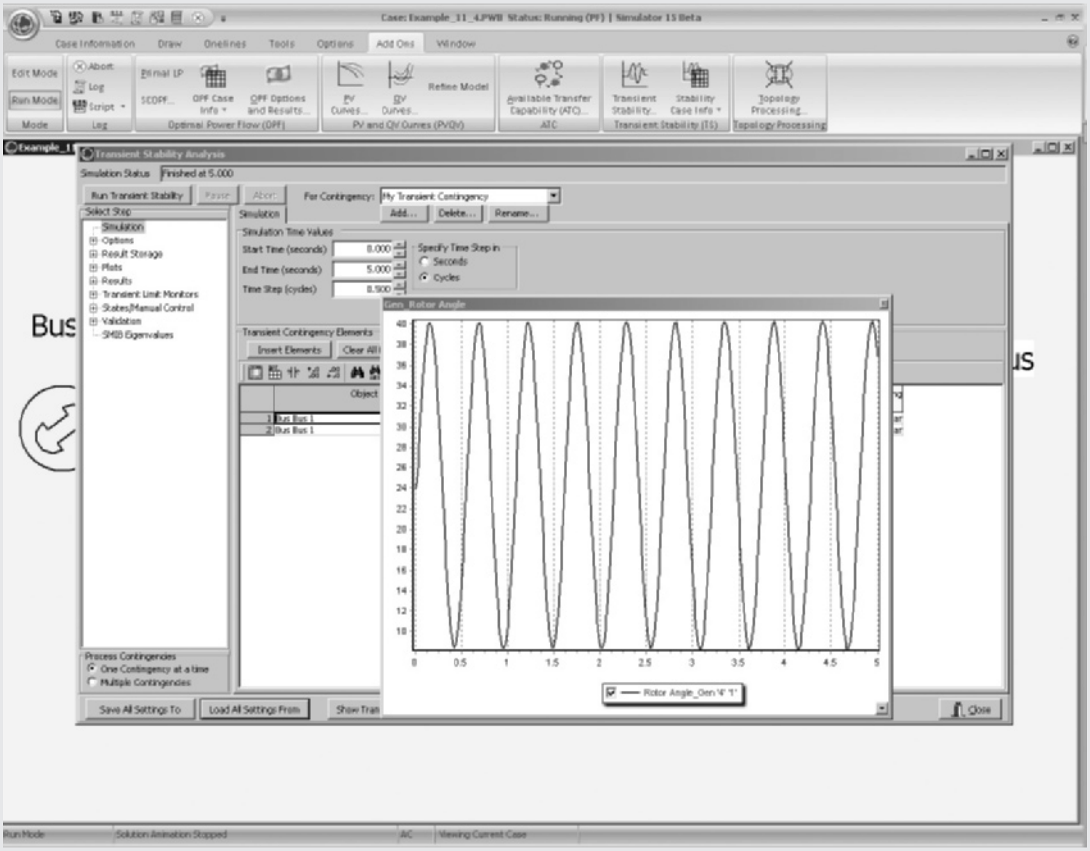

FIGURE 11.8

Variation in $\delta(t)$ without Damping

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shown in the figure. More detailed results are also available by clicking on **Results** in the list on the left side of the form. To rerun the example with a different fault duration, modify the Time (second) field in the Transient Contingency Elements list, and then again click the **Run Transient Stability** button.

Notice that because this system is modeled without damping (i.e., D = 0), the angle oscillations do not damp out with time. To extend the example, right-click on the Bus 4 generator on the one-line diagram and select **Generator Information Dialog**. Then click on the **Stability, Machine Models** tab to see the parameters associated with the GENCLS model (i.e., a classical model—a more detailed machine model is introduced in Section 11.6). Change the D field to 1.0, select **OK** to close the dialog, and then rerun the transient stability case. The results are as shown in Figure 11.9. While the inclusion of damping did not significantly alter the maximum for $\delta(t)$, the magnitude of the angle oscillations is now decreasing with time. For convenience this modified example is contained in PowerWorld Simulator case Example 11_4b.

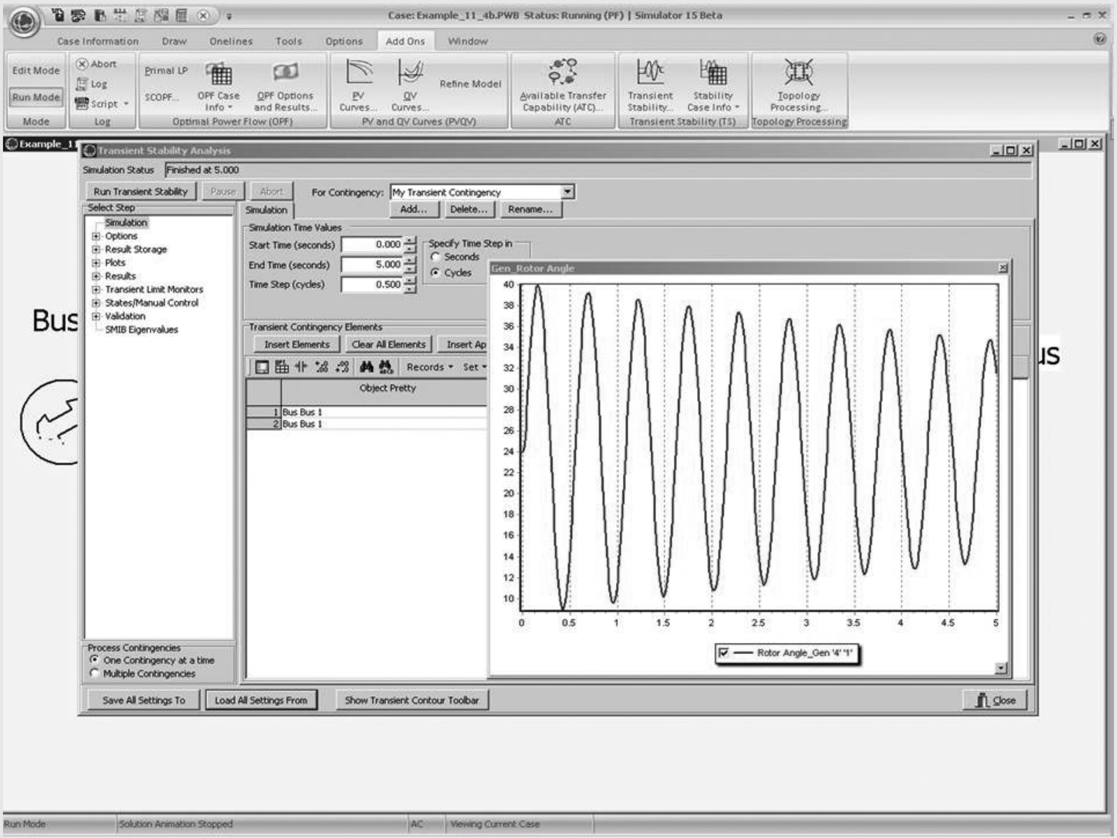

FIGURE 11.9

Variation in $\delta(t)$ with Damping

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EXAMPLE 11.5

Equal-area criterion: critical clearing time for a temporary three-phase fault

Assuming the temporary short circuit in Example 11.4 lasts longer than 3 cycles, calculate the critical clearing time.

SOLUTION

The p–δ plot is shown in Figure 11.10. At the critical clearing angle, denoted δ_{cr}, the fault is extinguished. The power angle then increases to a maximum value $\delta_3 = 180° - \delta_0 = 156.05° = 2.7236$ radians, which gives the maximum decelerating area. Equating the accelerating and decelerating areas,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} p_m\, d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{max} \sin\delta - p_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} 1.0\, d\delta = \int_{\delta_{cr}}^{2.7236} (2.4638 \sin\delta - 1.0)\, d\delta$$

Solving for δ_{cr},

$$(\delta_{cr} - 0.4179) = 2.4638[\cos\delta_{cr} - \cos(2.7236)] - (2.7236 - \delta_{cr})$$

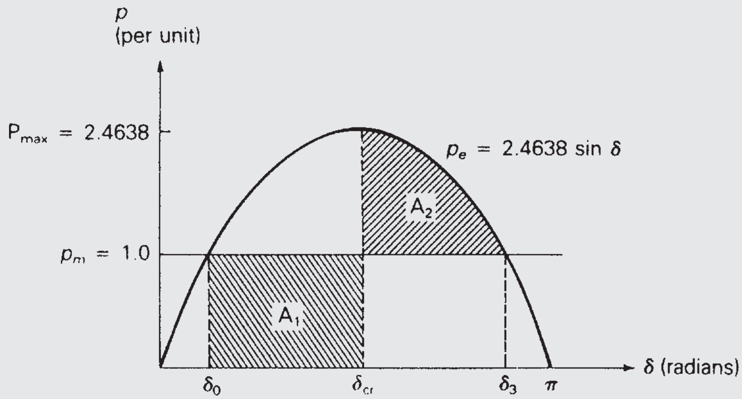
$$2.4638 \cos\delta_{cr} = +0.05402$$

$$\delta_{cr} = 1.5489 \text{ radians} = 88.74°$$

From the solution to the swing equation given in Example 11.4,

$$\delta(t) = \frac{\omega_{syn} p_{mp.u.}}{4H} t^2 + \delta_0$$

FIGURE 11.10 p–δ plot for Example 11.5



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Solving

$$t = \sqrt{\frac{4H}{\omega_{\text{syn}} p_{m\text{p.u.}}}(\delta(t) - \delta_0)}$$

Using $\delta(t_{\text{cr}}) = \delta_{\text{cr}} = 1.5489$ and $\delta_0 = 0.4179$ radian,

$$t_{\text{cr}} = \sqrt{\frac{12}{(2\pi 60)(1.0)}(1.5489 - 0.4179)}$$
$$= 0.1897 \text{ s} = 11.38 \text{ cycles}$$

If the fault is cleared before $t = t_{\text{cr}} = 11.38$ cycles, stability is maintained. Otherwise, the generator goes out of synchronism with the infinite bus; that is, stability is lost.

To see a time-domain simulation of this case, open Example 11_5 in PowerWorld Simulator (see Figure 11.11). Again select **Add-Ons, Transient Stability**

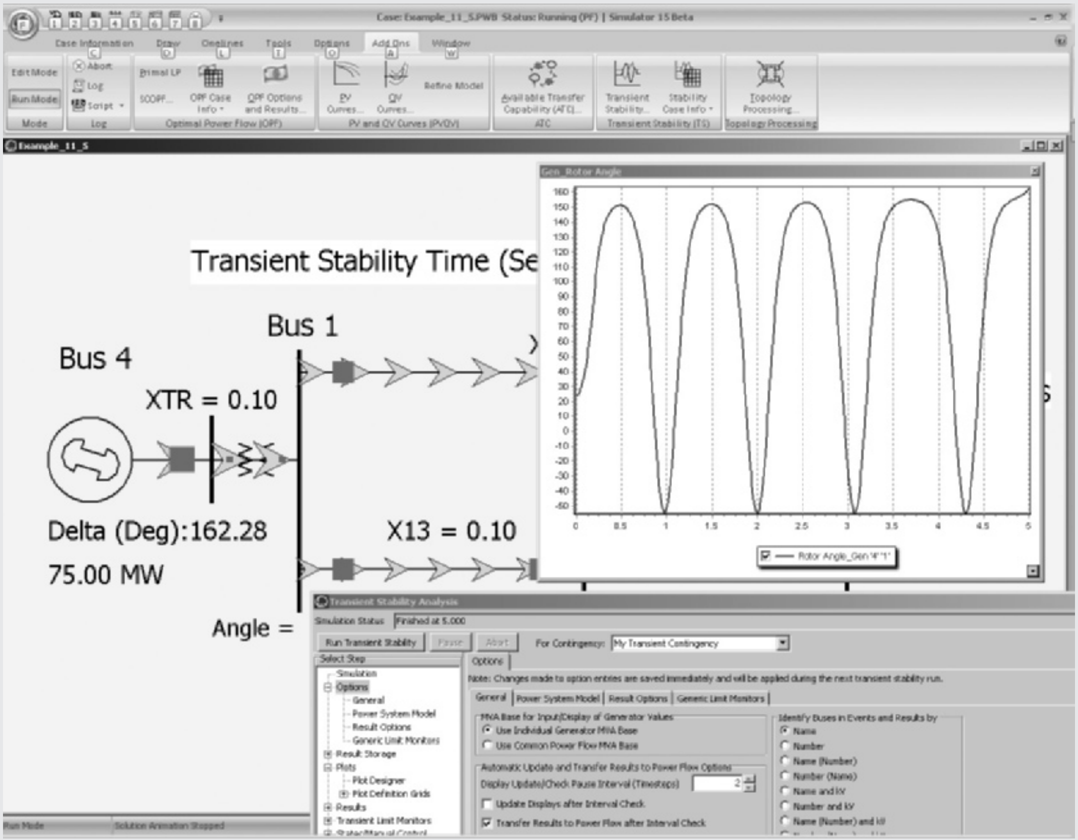

FIGURE 11.11

Variation in $\delta(t)$ for Example 11.5

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to view the Transient Stability Analysis Form. In order to better visualize the results on a PowerWorld one-line diagram, there is an option to transfer the transient stability results to the one-line every n timesteps. To access this option, select the **Options** page from the list on the left side of the display, then the **General** tab, then check the **Transfer Results to Power Flow after Interval Check** field. With this option checked, click on the **Run Transient Stability**, which will run the case with a critical clearing time of 0.1895 seconds. The plot is set to dynamically update as well. The final results are shown in Figure 11.11. Because the oneline is reanimated every n time-steps (4 in this case), a potential downside to this option is it takes longer to run. Uncheck the option to restore full solution speed.

EXAMPLE 11.6

Equal-area criterion: critical clearing angle for a cleared three-phase fault

The synchronous generator in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3 when a permanent three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 3. The fault is cleared by opening the circuit breakers at the ends of line 1–3 and line 2–3. These circuit breakers then remain open. Calculate the critical clearing angle. As in previous examples, H = 3.0 p.u.-s, $p_m = 1.0$ per unit and $\omega_{\text{p.u.}} = 1.0$ in the swing equation.

SOLUTION

From Example 11.3, the equation for the prefault electrical power, denoted p_{e1} here, is $p_{e1} = 2.4638 \sin \delta$ per unit. The faulted network is shown in Figure 11.12 (a), and the Thévenin equivalent of the faulted network, as viewed from the generator internal voltage source, is shown in Figure 11.12(b). The Thévenin reactance is

$$X_{\text{Th}} = 0.40 + 0.20 \| 0.10 = 0.46666 \quad \text{per unit}$$

and the Thévenin voltage source is

$$V_{\text{Th}} = 1.0\angle 0° \left[\frac{X_{13}}{X_{13} + X_{12}}\right] = 1.0\angle 0° \frac{0.10}{0.30}$$
$$= 0.33333\angle 0° \quad \text{per unit}$$

From Figure 11.12(b), the equation for the electrical power delivered by the generator to the infinite bus during the fault, denoted p_{e2}, is

$$p_{e2} = \frac{E'V_{\text{Th}}}{X_{\text{Th}}} \sin \delta = \frac{(1.2812)(0.3333)}{0.46666} \sin \delta = 0.9152 \sin \delta \quad \text{per unit}$$

(*Continued*)

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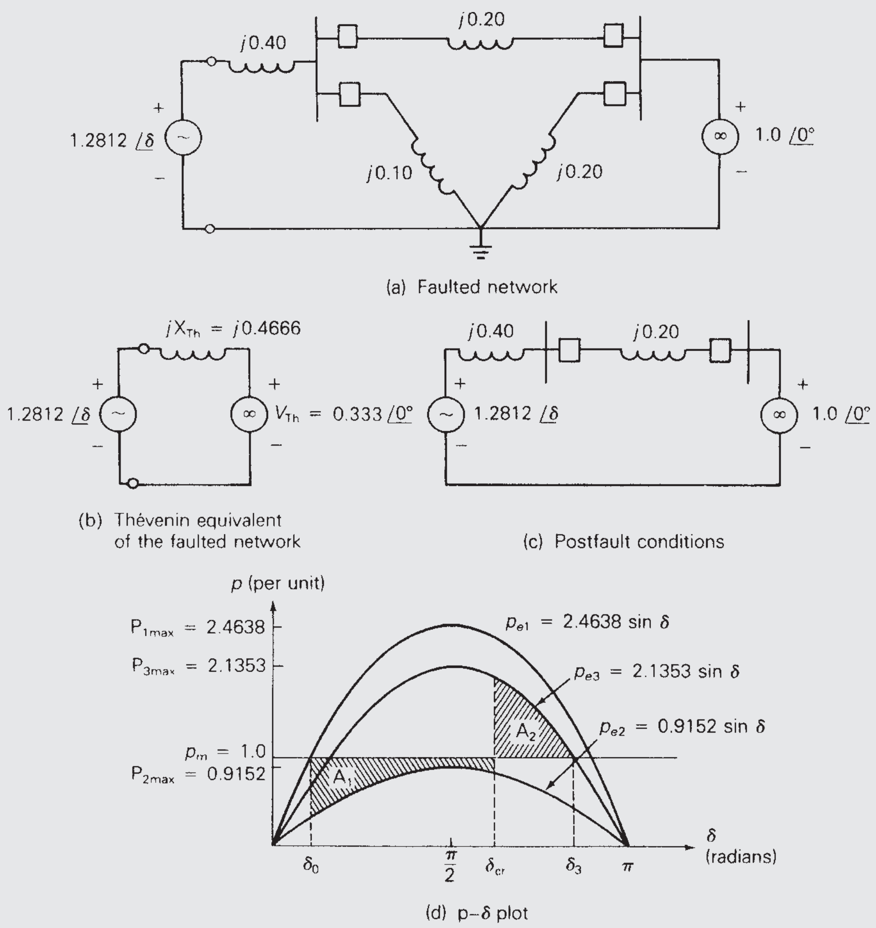
FIGURE 11.12
Example 11.6



The postfault network is shown in Figure 11.12(c), where circuit breakers have opened and removed lines 1–3 and 2–3. From this figure, the postfault electrical power delivered, denoted p_{e3}, is

$$p_{e3} = \frac{(1.2812)(1.0)}{0.60} \sin \delta = 2.1353 \sin \delta \quad \text{per unit}$$

The p–δ curves as well as the accelerating area A_1 and decelerating area A_2 corresponding to critical clearing are shown in Figure 11.12(d). Equating A_1 and A_2,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (p_m - P_{2max} \sin \delta) d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3max} \sin \delta - p_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin \delta) d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin \delta - 1.0) d\delta$$

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Solving for δ_{cr},

$$(\delta_{cr} - 0.4179) + 0.9152(\cos\delta_{cr} - \cos 0.4179)$$
$$= 2.1353(\cos\delta_{cr} - \cos 2.6542) - (2.6542 - \delta_{cr})$$
$$-1.2201\cos\delta_{cr} = 0.4868$$
$$\delta_{cr} = 1.9812 \text{ radians} = 111.5°$$

If the fault is cleared before $\delta = \delta_{cr} = 111.5°$, stability is maintained. Otherwise, stability is lost. To see this case in PowerWorld Simulator open case Example 11_6.

11.4 NUMERICAL INTEGRATION OF THE SWING EQUATION

The equal-area criterion is applicable to one machine and an infinite bus or to two machines. For multimachine stability problems, however, numerical integration techniques can be employed to solve the swing equation for each machine.

Given a first-order differential equation

$$\frac{dx}{dt} = f(x) \tag{11.4.1}$$

one relatively simple integration technique is Euler's method [1], illustrated in Figure 11.13. The integration step size is denoted Δt. Calculating the slope at the beginning of the integration interval, from (11.4.1),

$$\frac{dx_t}{dt} = f(x_t) \tag{11.4.2}$$

The new value $x_{t+\Delta t}$ is calculated from the old value x_t by adding the increment Δx,

$$x_{t+\Delta t} = x_t + \Delta x = x_t + \left(\frac{dx_t}{dt}\right)\Delta t \tag{11.4.3}$$

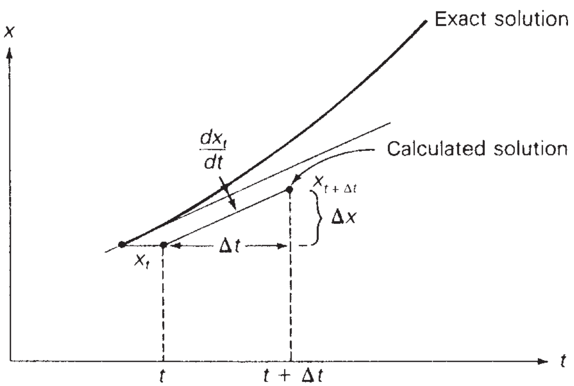

FIGURE 11.13
Euler's method

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As shown in the figure, Euler's method assumes that the slope is constant over the entire interval Δt. An improvement can be obtained by calculating the slope at both the beginning and end of the interval, and then averaging these slopes. The modified Euler's method is illustrated in Figure 11.14. First, the slope at the beginning of the interval is calculated from (11.4.1) and used to calculate a preliminary value $\tilde{x}$ given by

$$\tilde{x} = x_t + \left(\frac{dx_t}{dt}\right)\Delta t \tag{11.4.4}$$

Next, the slope at $\tilde{x}$ is calculated:

$$\frac{d\tilde{x}}{dt} = f(\tilde{x}) \tag{11.4.5}$$

Then, the new value is calculated using the average slope:

$$x_{t+\Delta t} = x_t + \frac{\left(\frac{dx_t}{dt} + \frac{d\tilde{x}}{dt}\right)}{2}\Delta t \tag{11.4.6}$$

Now apply the modified Euler's method to calculate machine frequency ω and power angle δ. Letting x be either δ or ω, the old values at the beginning of the interval are denoted δ_t and ω_t. From (11.1.17) and (11.1.18), the slopes at the beginning of the interval are

$$\frac{d\delta_t}{dt} = \omega_t - \omega_{\text{syn}} \tag{11.4.7}$$

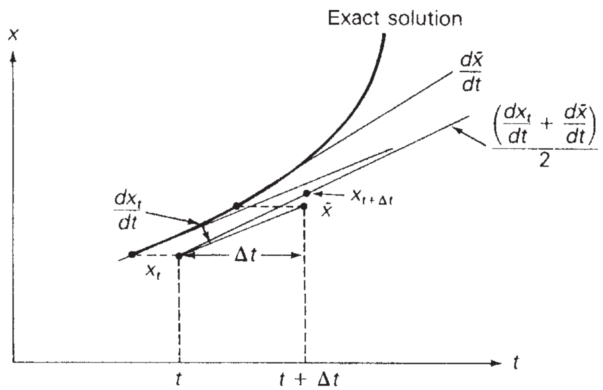
$$\frac{d\omega_t}{dt} = \frac{p_{a\text{p.u.}t}\omega_{\text{syn}}}{2\text{H}\omega_{\text{p.u.}t}} \tag{11.4.8}$$

where $p_{a\text{p.u.}t}$ is the per-unit accelerating power calculated at $\delta = \delta_t$ and $\omega_{\text{p.u.}t} = \omega_t/\omega_{\text{syn}}$. Applying (11.4.4), preliminary values are

$$\tilde{\delta} = \delta_t + \left(\frac{d\delta_t}{dt}\right)\Delta t \tag{11.4.9}$$

$$\tilde{\omega} = \omega_t + \left(\frac{d\omega_t}{dt}\right)\Delta t \tag{11.4.10}$$

FIGURE 11.14 Modified Euler's method



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Next, the slopes at $\tilde{\delta}$ and $\tilde{\omega}$ *are* calculated, again using (11.1.17) and (11.1.18):

$$\frac{d\tilde{\delta}}{dt} = \tilde{\omega} - \omega_{\text{syn}} \tag{11.4.11}$$

$$\frac{d\tilde{\omega}}{dt} = \frac{\tilde{p}_{a\text{p.u.}}\omega_{\text{syn}}}{2\text{H}\tilde{\omega}_{\text{p.u.}}} \tag{11.4.12}$$

where $\tilde{p}_{a\text{p.u.}}$ is the per-unit accelerating power calculated at $\delta = \tilde{\delta}$, and $\tilde{\omega}_{\text{p.u.}} = \tilde{\omega}/\omega_{\text{syn}}$. Applying (11.4.6), the new values at the end of the interval are

$$\delta_{t+\Delta t} = \delta_t + \frac{\left(\frac{d\delta_t}{dt} + \frac{d\tilde{\delta}}{dt}\right)}{2}\Delta t \tag{11.4.13}$$

$$\omega_{t+\Delta t} = \omega_t + \frac{\left(\frac{d\omega_t}{dt} + \frac{d\tilde{\omega}}{dt}\right)}{2}\Delta t \tag{11.4.14}$$

This procedure, given by (11.4.7) through (11.4.13), begins at $t = 0$ with specified initial values δ_0 and ω_0, and continues iteratively until $t = T$, a specified final time.

In addition to Euler's method, there are many other numerical integration techniques, such as Runge-Kutta, Picard's method, and Milne's predictor-corrector method [1]. Comparison of the methods shows a trade-off of accuracy versus computation complexity. The Euler method is a relatively simple method to compute, but requires a small step size Δt for accuracy. Some of the other methods can use a larger step size for comparable accuracy, but the computations are more complex.

To see this case in PowerWorld Simulator, open case Example 11_7, which plots both the generator angle and speed. However, rather than showing the speed in radians per second, Hertz is used. Also, in addition to plotting the angle and speed versus time, the case includes a "phase portrait" in which the speed is plotted as a function of the angle. Numeric results are available by clicking on the **Results** page. The results shown in PowerWorld differ slightly from those in the table because PowerWorld uses a more exact second order integration method.

EXAMPLE 11.7

Euler's method: computer solution to swing equation and critical clearing time

Verify the critical clearing angle determined in Example 11.6, and calculate the critical clearing time by applying the modified Euler's method to solve the swing equation for the following two cases:

Case 1 The fault is cleared at $\delta = 1.95$ radians $= 112°$ (which is less than δ_{cr})

Case 2 The fault is cleared at $\delta = 2.09$ radians $= 120°$ (which is greater than δ_{cr})

(*Continued*)

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For calculations, use a step size $\Delta t = 0.01$ s, and solve the swing equation from $t = 0$ to $t = T = 0.85$ s.

SOLUTION

Equations (11.4.7) through (11.4.14) are solved by a computer program written in BASIC. From Example 11.6, the initial conditions at $t = 0$ are

$$\delta_0 = 0.4179 \text{ rad}$$
$$\omega_0 = \omega_{\text{syn}} = 2\pi 60 \text{ rad/s}$$

Also, the H constant is 3.0 p.u.-s, and the faulted accelerating power is

$$p_{a\text{p.u.}} = 1.0 - 0.9152 \sin \delta$$

The postfault accelerating power is

$$p_{a\text{p.u.}} = 1.0 - 2.1353 \sin \delta \quad \text{per unit}$$

The computer program and results at 0.02 s printout intervals are listed in Table 11.1. As shown, these results agree with Example 11.6, since the system is stable for Case 1 and unstable for Case 2. Also from Table 11.1, the critical clearing time is between 0.34 and 0.36 s.

Case 1 Stable			Case 2 Unstable		
Time s	Delta rad	Omega rad/s	Time s	Delta rad	Omega rad/s
0.000	0.418	376.991	0.000	0.418	376.991
0.020	0.426	377.778	0.020	0.426	377.778
0.040	0.449	378.547	0.040	0.449	378.547
0.060	0.488	379.283	0.060	0.488	379.283
0.080	0.541	379.970	0.080	0.541	379.970
0.100	0.607	380.599	0.100	0.607	380.599
0.120	0.685	381.159	0.120	0.685	381.159
0.140	0.773	381.646	0.140	0.773	381.646
0.160	0.870	382.056	0.160	0.870	382.056
0.180	0.975	382.392	0.180	0.975	382.392
0.200	1.086	382.660	0.200	1.086	382.660
0.220	1.202	382.868	0.220	1.202	382.868
0.240	1.321	383.027	0.240	1.321	383.027
0.260	1.443	383.153	0.260	1.443	383.153
0.280	1.567	383.262	0.280	1.567	383.262
0.300	1.694	383.370	0.300	1.694	383.370
0.320	1.823	383.495	0.320	1.823	383.495
0.340	1.954	383.658	0.340	1.954	383.658
	Fault Cleared		0.360	2.090	383.876
0.360	2.076	382.516		Fault Cleared	
0.380	2.176	381.510	0.380	2.217	382.915
0.400	2.257	380.638	0.400	2.327	382.138
0.420	2.322	379.886	0.420	2.424	381.546
0.440	2.373	379.237	0.440	2.511	381.135
0.460	2.413	378.674	0.460	2.591	380.902
0.480	2.441	378.176	0.480	2.668	380.844

Program Listing

```
10 REM EXAMPLE 13.7
20 REM SOLUTION TO SWING EQUATION
30 REM THE STEP SIZE IS DELTA
40 REM THE CLEARING ANGLE IS DLTCLR
50 DELTA+.01
60 DLTCLR = 1.95
70 J = 1
80 PMAX = .9I52
90 PI = 3.1415927 #
I00 T = 0
110 X1 =.4179
I20 X2 = 2*PI*60
130 LPRINT "TIME DELTA OMEGA"
140LPRINT"srad rad/s"
150 LPRINT USING "#####.###"; T;XI;X2
160 FORK = 1 TO 86
170 REM LINE 180 IS EQ (13.4.7)
I80 X3 = X2 − (2*PI*60)
190 IF J = 2 THEN GOTO 240
200 IF X1 > DLTCLR OR XI = DLTCLR THEN
   PMAX = 2.I353
210 IF X1 > DLTCLR OR XI = DLTCLR THEN
   LPRINT "FAULT CLEARED"
220 IF XI > DLTCLR OR XI = DLTCLR THEN
   J = 2
```

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0.500	2.460	377.726	0.500	2.746	380.969	230 REM LINES 240 AND 250 ARE EQ (13.4.8)
0.520	2.471	377.307	0.520	2.828	381.288	240 X4=I-PMAX*SIN(XI)
0.540	2.473	376.900	0.540	2.919	381.824	250 X5 = X4 * (2 * PI * 60) * (2 * PI * 60)/(6 * X2)
0.560	2.467	376.488	0.560	3.022	382.609	260 REM LINE 270 IS EQ (13.4.9)
0.580	2.453	376.056	0.580	3.145	383.686	270 X6 = X1 +X3* DELTA
0.600	2.429	375.583	0.600	3.292	385.111	280 REM LINE 290 IS EQ (I3.4.10)
0.620	2.396	375.053	0.620	3.472	386.949	290 X7 = X2 + X5* DELTA
0.640	2.351	374.446	0.640	3.693	389.265	300 REM LINE 3 10 IS EQ (13.4.1 1)
0.660	2.294	373.740	0.660	3.965	392.099	3I0 X8 = X7-2*PI*60
0.680	2.221	372.917	0.680	4.300	395.426	320 REM LINES 330 AND 340 ARE EQ (13.4.12)
0.700	2.130	371.960	0.700	4.704	399.079	330X9 = I-PMAX*SIN (X6)
0.720	2.019	370.855	0.720	5.183	402.689	340 X10 = X9 * (2 * PI * 60) * (2 * PI * 60)/(6 * X7)
0.740	1.884	369.604	0.740	5.729	405.683	350 REM LINE 360 IS EQ (13.4.13)
0.760	1.723	368.226	0.760	6.325	407.477	360 X1 = X1 + (X3 + X8) * (DELTA/2)
0.780	1.533	366.773	0.780	6.941	407.812	370 REM LINE 380 IS EQ (I3.4.14)
0.800	1.314	365.341	0.800	7.551	406.981	380 X2 = X2 + (X5 + X10) * (DELTA/2)
0.820	1.068	364.070	0.820	8.139	405.711	390 T = K* DELTA
0.840	0.799	363.143	0.840	8.702	404.819	400Z = K/2
0.860	0.516	362.750	0.860	9.257	404.934	4I0M = INT (Z)
						420 IF M = Z THEN LPRINT USING "#####.###"; T;XI;X2
						430 NEXT K
						440 END

TABLE 11.1

Computer calculation of swing curves for Example 11.7

11.5 MULTIMACHINE STABILITY

The numerical integration methods discussed in Section 11.4 can be used to solve the swing equations for a multimachine stability problem. However, a method is required for computing machine output powers for a general network. Figure 11.15 shows a general N-bus power system with M synchronous machines. Each machine is the same as that represented by the simplified model of Figure 11.2, and the internal machine voltages are denoted E_1', E_2',…,E_M'. The M machine terminals are connected

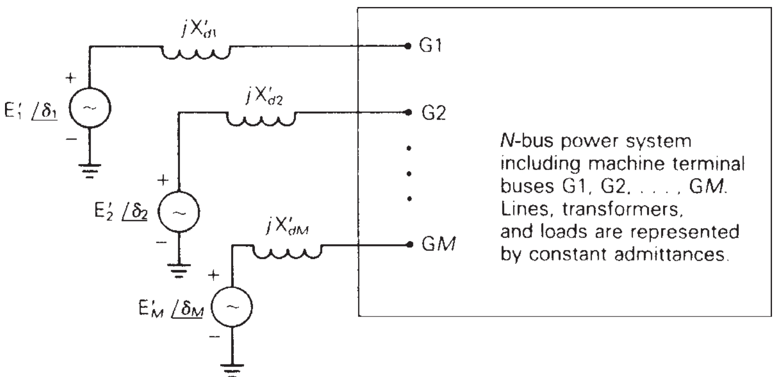

FIGURE 11.15

N-bus power-system representation for transient stability studies

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to system buses denoted Gl, G2,..., *GM* in Figure 11.15. All loads are modeled here as constant admittances. Writing nodal equations for this network,

$$\begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{12}^{\mathrm{T}} & \boldsymbol{Y}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{E} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \tag{11.5.1}$$

where

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \text{ is the } N \text{ vector of bus voltages} \tag{11.5.2}$$

$$\boldsymbol{E} = \begin{bmatrix} E_1' \\ E_2' \\ \vdots \\ E_M' \end{bmatrix} \text{ is the } M \text{ vector of machine voltages} \tag{11.5.3}$$

$$\boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} \begin{matrix} \text{is the } M \text{ vector of machine currents} \\ \text{(these are current sources)} \end{matrix} \tag{11.5.4}$$

$$\begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{12}^{\mathrm{T}} & \boldsymbol{Y}_{22} \end{bmatrix} \text{ is an } (N + M) \times (N + M) \text{ admitance matrix} \tag{11.5.5}$$

The admittance matrix in (11.5.5) is partitioned in accordance with the *N* system buses and *M* internal machine buses, as follows:

$\boldsymbol{Y}_{11}$ is $N \times N$
$\boldsymbol{Y}_{12}$ is $N \times M$
$\boldsymbol{Y}_{22}$ is $M \times M$

$\boldsymbol{Y}_{11}$ is similar to the bus admittance matrix used for power flows in Chapter 6, except that load admittances and inverted generator impedances are included. That is, if a load is connected to bus n, then that load admittance is added to the diagonal element Y_{11nn}. Also, $(1/jX'_{dn})$ is added to the diagonal element $Y_{11\mathrm{G}n\mathrm{G}n}$.

$\boldsymbol{Y}_{22}$ is a diagonal matrix of inverted generator impedances; that is,

$$\boldsymbol{Y}_{22} = \begin{bmatrix} \frac{1}{j\mathrm{X}'_{d1}} & & & 0 \\ & \frac{1}{j\mathrm{X}'_{d2}} & & \\ & & \ddots & \\ 0 & & & \frac{1}{j\mathrm{X}'_{dM}} \end{bmatrix} \tag{11.5.6}$$

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Also, the kmth element of $\boldsymbol{Y}_{12}$ is

$$Y_{12km} = \begin{cases} \dfrac{-1}{jX'_{dn}} & \text{if } k = \text{G}n \text{ and } m = n \\ 0 & \text{otherwise} \end{cases} \quad (11.5.7)$$

Writing (11.5.1) as two separate equations,

$$\boldsymbol{Y}_{11}\boldsymbol{V} + \boldsymbol{Y}_{12}\boldsymbol{E} = \boldsymbol{0} \quad (11.5.8)$$

$$\boldsymbol{Y}_{12}^{\mathrm{T}}\boldsymbol{V} + \boldsymbol{Y}_{22}\boldsymbol{E} = \boldsymbol{I} \quad (11.5.9)$$

Assuming $\boldsymbol{E}$ is known, (11.5.8) is a linear equation in $\boldsymbol{V}$ that can be solved either iteratively or by Gauss elimination. Using the Gauss-Seidel iterative method given by (7.2.9), the kth component of $\boldsymbol{V}$ is

$$V_k(i+1) = \frac{1}{Y_{11kk}}\left[-\sum_{n=1}^{M} Y_{12kn}E_n - \sum_{n=1}^{k-1} Y_{11kn}V_n(i+1) - \sum_{n=k+1}^{N} Y_{11kn}V_n(i)\right] \quad (11.5.10)$$

After $\boldsymbol{V}$ is computed, the machine currents can be obtained from (11.5.9). That is,

$$\boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} = \boldsymbol{Y}_{12}^{T}\boldsymbol{V} + \boldsymbol{Y}_{22}\boldsymbol{E} \quad (11.5.11)$$

The (real) electrical power output of machine n is then

$$p_{en} = \mathrm{Re}[E_n I_n^*] \qquad n = 1,2,\ldots,M \quad (11.5.12)$$

The computation procedure for solving a transient stability problem alternately solves the swing equations representing the machines and the above algebraic power-flow equations representing the network. Use the modified Euler method of Section 11.4 to solve the swing equations and the Gauss-Seidel iterative method to solve the power-flow equations. The procedure is outlined in the following 11 steps.

TRANSIENT STABILITY COMPUTATION PROCEDURE

STEP 1 Run a prefault power-flow program to compute initial bus voltages V_k, $k = 1, 2,\ldots,N$, initial machine currents I_n, and initial machine electrical power outputs p_{en}, $n = 1, 2,\ldots,M$. Set machine mechanical

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power outputs, $p_{mn} = p_{en}$. Set initial machine frequencies, $\omega_n = \omega_{syn}$. Compute the load admittances.

STEP 2 Compute the internal machine voltages:

$$E_n = E_n \angle \delta_n = V_{Gn} + (jX'_{dn})I_n \quad n = 1, 2, \ldots, M$$

where V_{Gn} and I_n are computed in Step 1. The magnitudes E_n will remain constant throughout the study. The angles δ_n *are* the initial power angles.

STEP 3 Compute Y_{11}. Modify the $(N \times N)$ power-flow bus admittance matrix by including the load admittances and inverted generator impedances.

STEP 4 Compute Y_{22} from (11.5.6) and Y_{12} from (11.5.7).

STEP 5 Set time $t = 0$.

STEP 6 Is there a switching operation, change in load, short circuit, or change in data? For a switching operation or change in load, modify the bus admittance matrix. For a short circuit, set the faulted bus voltage [in (11.5.10)] to zero.

STEP 7 Using the internal machine voltages $E_n = E_n \angle \delta_n$, $n = 1, 2, \ldots, M$, with the values of δ_n at time t, compute the machine electrical powers p_{en} at time t from (11.5.10) to (11.5.12).

STEP 8 Using p_{en} computed in Step 7 and the values of δ_n and ω_n at time t, compute the preliminary estimates of power angles δ_n and machine speeds $\tilde{\omega}_n$ at time $(t + \Delta t)$ from (11.4.7) to (11.4.10).

STEP 9 Using $E_n = E_n \angle \tilde{\delta}_n$, $n = 1, 2, \ldots, M$, compute the preliminary estimates of the machine electrical powers p_{en} at time $(t + \Delta t)$ from (11.5.10) to (11.5.12).

STEP 10 Using $\tilde{p}_{en}$ computed in Step 9, as well as $\tilde{\delta}_n$ and $\tilde{\omega}_n$ computed in Step 8, compute the final estimates of power angles δ_n and machine speeds ω_n at time $(t + \Delta t)$ from (11.4.11) to (11.4.14).

STEP 11 Set time $t = t + \Delta t$. Stop if $t > T$. Otherwise, return to Step 6.

An important transient stability parameter is the step size (time step), Δt, used in the numerical integration. Because the time required to solve a transient stability problem varies inversely with the time step, a larger value would be preferred. However, if too large a value is chosen, then the solution accuracy may suffer, and for some integration methods, such as Euler's, the solution can experience numeric instability. To see an example of numeric instability, re-do the PowerWorld Simulator Example 11_7, except change the time step to 0.02 seconds. A typical time step for commercial transient stability simulations is 1/2 cycle (0.00833 seconds for a 60 Hz system).

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EXAMPLE 11.8

Modifying power-flow Y_{bus} for application to multimachine stability

Consider a transient stability study for the power system given in Example 6.9, with the 184-Mvar shunt capacitor of Example 6.14 installed at bus 2. Machine transient reactances are $X'_{d1} = 0.05$ and $X'_{d2} = 0.025$ per unit on the system base. Determine the admittance matrices $\boldsymbol{Y}_{11}$, $\boldsymbol{Y}_{22}$, and $\boldsymbol{Y}_{12}$.

SOLUTION

From Example 6.9, the power system has $N = 5$ buses and $M = 2$ machines. The second row of the 5×5 bus admittance matrix used for power flows is calculated in Example 6.9. Calculating the other rows in the same manner, results in

$$Y_{bus} = \begin{bmatrix} (3.728 - j49.72) & 0 & 0 & 0 & (-3.728 + j49.72) \\ 0 & (2.68 - j26.46) & 0 & (-0.892 + j9.92) & (-1.784 + j19.84) \\ 0 & 0 & (7.46 - j99.44) & (-7.46 + j99.44) & 0 \\ 0 & (-0.892 + j9.92) & (-7.46 + j99.44) & (11.92 - j148.) & (-3.572 + j39.68) \\ (-3.728 + j49.72) & (-1.784 + j19.84) & 0 & (-3.572 + j39.68) & (9.084 - j108.6) \end{bmatrix} \text{per unit}$$

To obtain $\boldsymbol{Y}_{11}$, $\boldsymbol{Y}_{bus}$ is modified by including load admittances and inverted generator impedances. From Table 6.1, the load at bus 3 is $P_{L3} - jQ_{L3} = 0.8 + j0.4$ per unit and the voltage at bus 3 is $V_3 = 1.05$ per unit. Representing this load as a constant admittance,

$$Y_{\text{load }3} = \frac{P_{L3} - jQ_{L3}}{V_3^2} = \frac{0.8 - j0.4}{(1.05)^2} = 0.7256 - j0.3628 \quad \text{per unit}$$

Similarly, the load admittance at bus 2 is

$$Y_{\text{load }2} = \frac{P_{L2} - jQ_{L2}}{V_2^2} = \frac{8 - j2.8 + j1.84}{(0.959)^2} = 8.699 - j1.044$$

where V_2 is obtained from Example 6.14 and the 184-Mvar (1.84 per unit) shunt capacitor bank is included in the bus 2 load.

The inverted generator impedances are: for machine 1 connected to bus 1,

$$\frac{1}{jX'_{d1}} = \frac{1}{j0.05} = -j20.0 \quad \text{per unit}$$

and for machine 2 connected to bus 3,

$$\frac{1}{jX'_{d2}} = \frac{1}{j0.025} = -j40.0 \quad \text{per unit}$$

(Continued)

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To obtain $\boldsymbol{Y}_{11}$, add $(1/jX'_{d1})$ to the first diagonal element of $\boldsymbol{Y}_{\text{bus}}$, add $\boldsymbol{Y}_{\text{load2}}$ to the second diagonal element, and add $Y_{\text{load 3}} + (1/jX'_{d2})$ to the third diagonal element. The 5 × 5 matrix $\boldsymbol{Y}_{11}$ is then

$$\boldsymbol{Y}_{11} = \begin{bmatrix} (3.728 - j69.72) & 0 & 0 & 0 & (-3.728 - j49.72) \\ 0 & (11.38 - j29.50) & 0 & (-0.892 + j9.92) & (-1.784 + j19.84) \\ 0 & 0 & (8.186 - j139.80) & (-7.46 + j99.44) & 0 \\ 0 & (-0.892 + j9.92) & (-7.46 + j99.44) & (11.92 - j148.) & (-3.572 - j39.68) \\ (-3.728 + j49.72) & (-1.784 + j19.84) & 0 & (-3.572 + j39.68) & (9.084 - j108.6) \end{bmatrix} \text{ per unit}$$

From (11.5.6), the 2 × 2 matrix $\boldsymbol{Y}_{22}$ is

$$\boldsymbol{Y}_{22} = \begin{bmatrix} \dfrac{1}{jX'_{d1}} & 0 \\ 0 & \dfrac{1}{jX'_{d2}} \end{bmatrix} = \begin{bmatrix} -j20.0 & 0 \\ 0 & -j40.0 \end{bmatrix} \text{ per unit}$$

From Figure 6.2, generator 1 is connected to bus 1 (therefore, bus G1 = 1 and generator 2 is connected to bus 3 (therefore G2 = 3). From (11.5.7), the 5 × 2 matrix $\boldsymbol{Y}_{12}$ is

$$\boldsymbol{Y}_{12} = \begin{bmatrix} j20.0 & 0 \\ 0 & 0 \\ 0 & j40.0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ per unit}$$

To see this case in PowerWorld Simulator, open case Example 11_8. To see the $\boldsymbol{Y}_{11}$ matrix entries, first display the Transient Stability Analysis Form, and then select **States/Manual Control, Transient Stability Ybus**. By default, this case is set to solve a self-clearing fault at bus 4 that extinguishes itself after three cycles (0.05 s). Both generators are modeled with H = 5.0 p.u.-s and D = 1.0 p.u.

For the bus 4 fault, Figure 11.16 shows the variation in the rotor angles for the two generators with respect to a 60 Hz synchronous reference frame. The angles are increasing with time because neither of the generators is modeled with a governor, and there is no infinite bus. While it is clear that the generator angles remain together, it is very difficult to tell from Figure 11.16 the exact variation in the angle differences. Therefore transient stability programs usually report angle differences, either with respect to the angle at a specified bus or with respect to the average of all the generator angles. The latter is shown in Figure 11.17 which displays the results from the PowerWorld Simulator Example 11_8A case.

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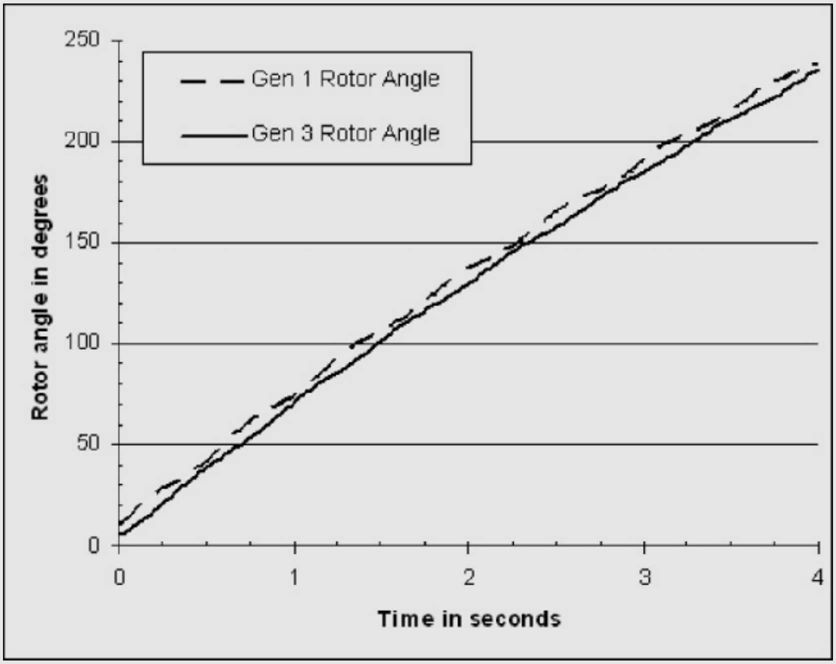

FIGURE 11.16

Variation in the rotor angles with respect to a synchronous speed reference frame

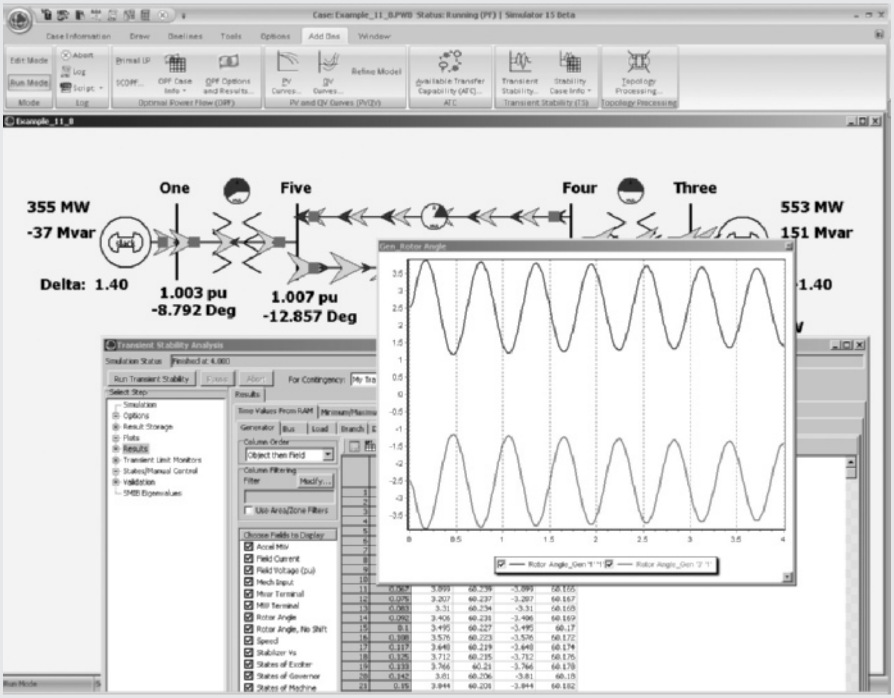

FIGURE 11.17

Relative variation of the rotor angles

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EXAMPLE 11.9

Stability results for 37 bus, 9 generator system

PowerWorld Simulator case Example 11_9 demonstrates a transient stability solution using the 37 bus system introduced in Chapter 6 with the system augmented to include classical models for each of the generators. By default, the case models a transmission line fault on the 69 kV line between bus 44 (PEACH69) and bus 14 (REDBUD69) with the fault at the bus 44 end of the line. The fault is cleared after 0.1 seconds by opening this transmission line. The results from this simulation are shown in Figure 11.18, with the largest generator angle variation occurring (not surprisingly) at the bus 44 generator. Notice that during and initially after the fault, the bus 44 generator's angle increases relative to all the other angles in the system. The critical clearing time for this fault is about 0.262 seconds.

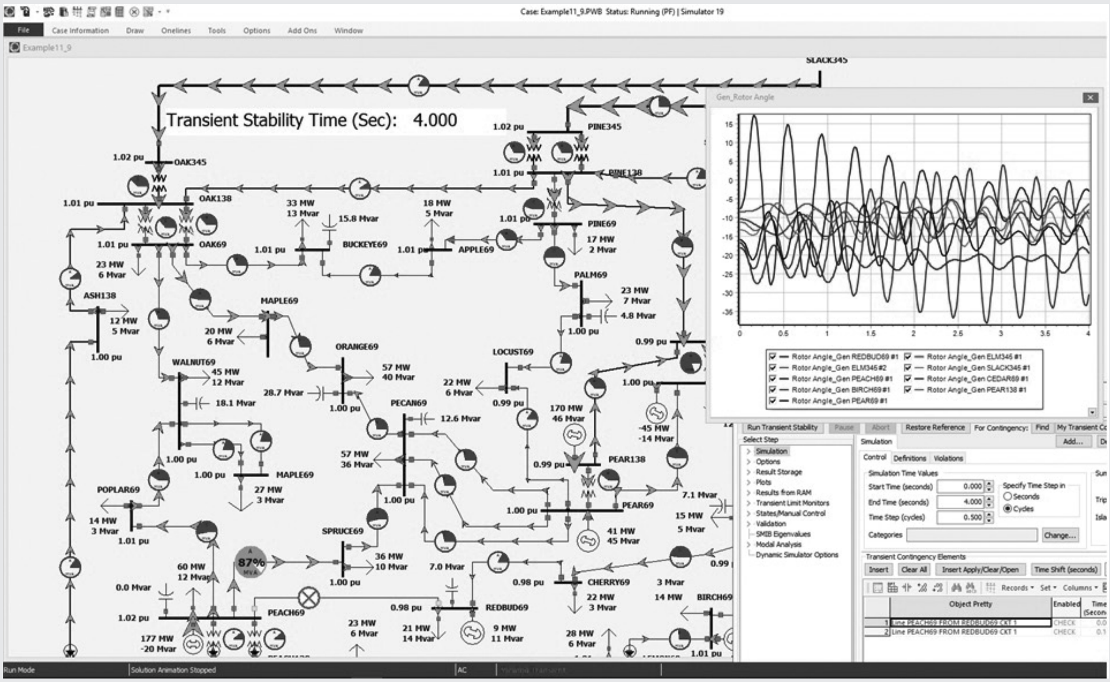

FIGURE 11.18

Rotor Angles for Example 11.9 case

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11.6 A TWO-AXIS SYNCHRONOUS MACHINE MODEL

While the classical model for a synchronous machine provides a useful mechanism for introducing transient stability concepts, it is only appropriate for the most basic of system studies. Also, it is usually not coupled with the exciter and governor models that are introduced in the next chapter. In this section a more realistic synchronous machine model is introduced.

The analysis of more detailed synchronous machine models requires that each machine model be expressed in a frame of reference that rotates at the same speed as its rotor. The standard approach is to use a *d*-*q* reference frame in which the major "direct" (*d*) axis is aligned with the rotor poles, and the quadrature (*q*) axis leads the direct axis by 90°. The rotor angle δ is then defined as the angle by which the *q*-axis leads the network reference frame (see Figure 11.19). The equation for transforming the network quantities to the *d*-*q* reference frame is given by (11.6.1) and from the *d*-*q* reference frame by (11.6.2),

$$\begin{bmatrix} V_r \\ V_i \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \tag{11.6.1}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix} \tag{11.6.2}$$

where the terminal voltage in the network reference frame is $V_T = V_r + jV_i$. A similar conversion is done for the currents.

Numerous different transient stability models exist for synchronous machines, most of which are beyond the scope of this text. The two-axis model, which

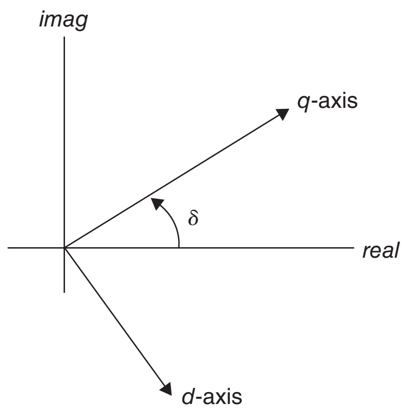

FIGURE 11.19

Reference frame transformations

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models the dynamics associated with the synchronous generator field winding and one damper winding, while neglecting the faster subtransient damper dynamics and stator transients, provides a nice compromise. For accessibility, machine saturation is not considered. With the two-axis model, the electrical behavior of the generator is represented by two algebraic equations and two differential equations

$$E'_q = V_q + R_a I_q + X'_d I_d \tag{11.6.3}$$

$$E'_d = V_d + R_d I_d - X'_q I_q \tag{11.6.4}$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{do}}\left(-E'_q - (X_d - X'_d)I_d + E_{fd}\right) \tag{11.6.5}$$

$$\frac{dE'_d}{dt} = \frac{1}{T'_{qo}}\left(-E'_d + (X_q - X'_q)I_q\right) \tag{11.6.6}$$

where $V_d + jV_q$ and $I_d + jI_q$ are the generator's terminal voltage and current shifted into the generator's reference frame, and E_{fd} is proportional to the field voltage. The per unit electrical torque, T_{elec} is then

$$T_e = V_d I_d + V_q I_q + R_a(I_d^2 + I_q^2) \tag{11.6.7}$$

While $p_e = T_e\omega_{\text{p.u.}}$, it is often assumed that $\omega_{\text{p.u.}} = 1.0$ [10] with the result being an assumption that $p_e = T_e$. When (11.6.5) and (11.6.6) are combined with generator mechanical equations presented in (11.1.18) and (11.1.19), substituting (11.6.7) for $p_{e\text{p.u.}}$, the result is a synchronous generator model containing four first-order differential equations.

The initial value for δ can be determined by noting that in steady-state, δ is the same as the angle of the internal voltage [10],

$$E = V_T + jX_q I \tag{11.6.8}$$

Hence the initial value of δ is the angle on E. Once δ has been determined, (11.6.2) is used to transfer the generator terminal voltage and current into the generator's reference frame, and then (11.6.3), (11.6.4), and (11.6.5) (assuming the left-hand side is zero) are used to determine the initial values of E'_q, E'_d, and E_{fd}. In this chapter the field voltage, E_{fd}, will be assumed constant (the use of the generator exciters to control the field voltage is a topic covered in the next chapter).

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EXAMPLE 11.10

Two-Axis Model Example

For the system from Example 11.3, with the synchronous generator modeled using a two-axis model, determine (a) the initial conditions, and then (b) use PowerWorld Simulator to determine the critical clearing time for the Example 11.6 fault (three phase fault at bus 3, cleared by opening lines 1–3 and 2–3). Assume H = 3.0 per unit-seconds, $R_a = 0$, $X_d = 2.1$, $X_q = 2.0$, $X_d' = 0.3$, $X_q' = 0.5$, all per unit using the 100 MVA system base.

SOLUTION

a. From Example 11.3, the current out of the generator is

$$I = 1.0526\angle{-18.20°} = 1 - j0.3288$$

which gives a generator terminal voltage of

$$V_T = 1.0\angle 0° + (j0.22)(1.0526\angle{-18.20°}) = 1.0946\angle 11.59° = 1.0723 + j0.220$$

From (11.6.8),

$$E = 1.0946\angle 11.59° + (j2.0)(1.052\angle{-18.2°}) = 2.814\angle 52.1°$$

$$\rightarrow \delta = 52.1°$$

Using (11.6.2) gives

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

and

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

Then, solving (11.6.3), (11.6.4), and (11.6.5) gives

$$E_q' = 0.8326 + (0.3)(0.9909) = 1.1299$$
$$E_d' = 0.7107 - (0.5)(0.3553) = 0.5330$$
$$E_{fd} = 1.1299 + (2.1 - 0.3)(0.9909) = 2.9135$$

b. Open PowerWorld Simulator case Example 11_10 (see Figure 11.20). **Select Add-Ons, Transient Stability** to view the Transient Stability Analysis Form. Initially the bus 3 fault is set to clear at 0.05 seconds. Select **Run Transient Stability** to create the results shown in Figure 11.20. In comparing

(*Continued*)

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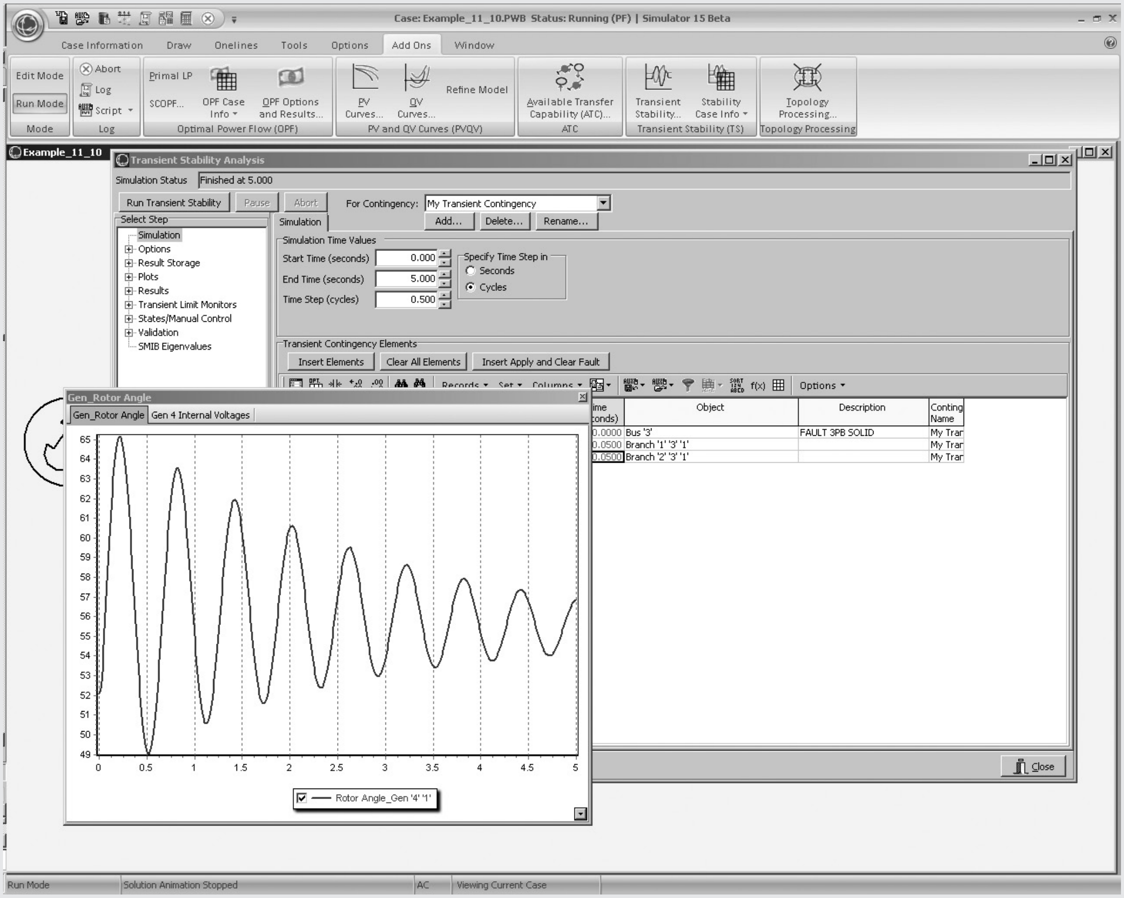

FIGURE 11.20

Variation in generator 4 rotor angle with a fault clearing time of 0.05 seconds

these results with those from Example 11.4, notice that while the initial value of δ is different, the initial angle increase of about 13° is similar to the increase of 16° in Example 11.4. A key difference between the two is the substantial amount of damping in the two-axis model case. This damping arises because of the explicit modeling of the field and damper windings with the two-axis model. The critical clearing time can be determined by gradually increasing the clearing time until the generator loses synchronism. This occurs at about 0.30 seconds, with the almost critically cleared angle shown in Figure 11.21. Since there are now two additional state variables for generator 4, E'_q, and E'_d, their values can also be shown. This is done in Figure 11.22, again for the 0.30 second clearing time.

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FIGURE 11.21

Variation in generator 4 rotor angle with a fault clearing time of 0.30 seconds

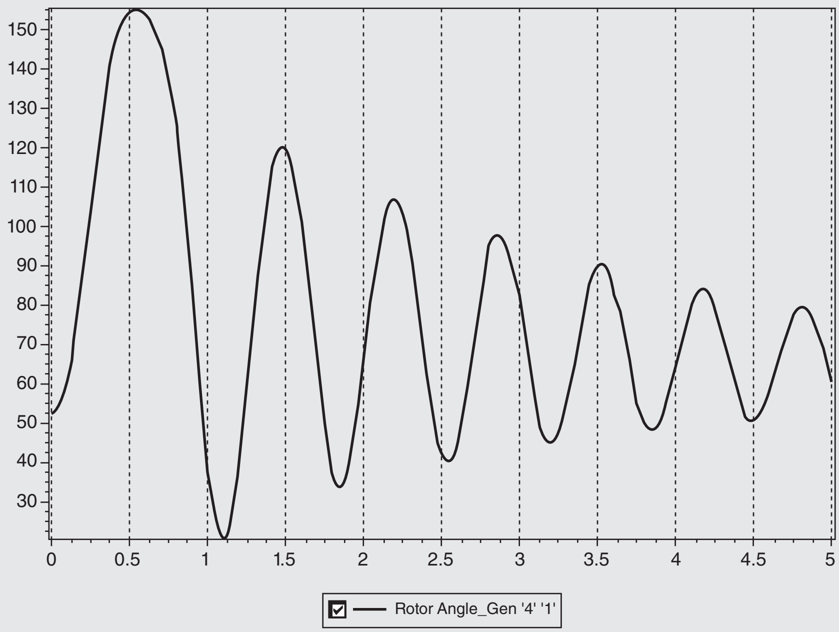
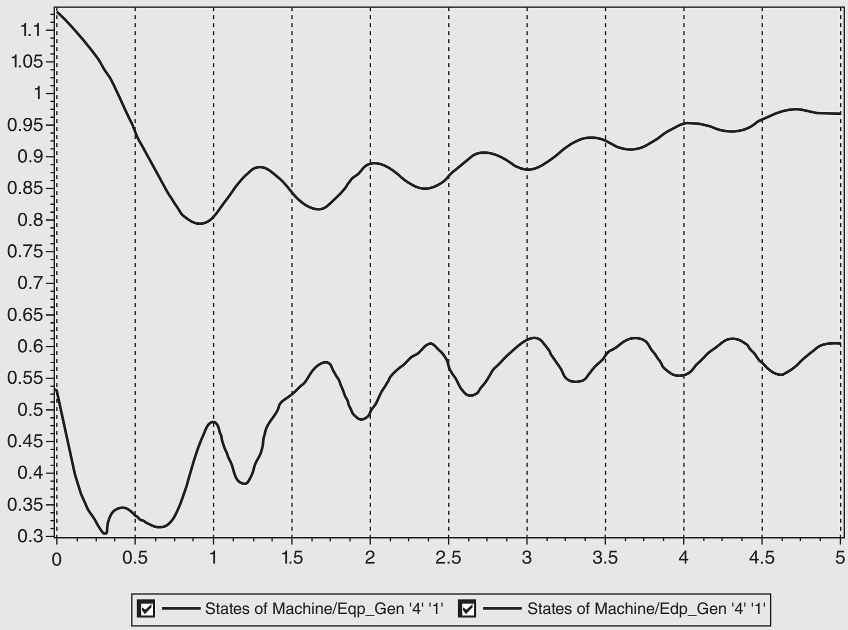


FIGURE 11.22

Variation in generator 4 E_q' and E_d' with a fault clearing time of 0.30 seconds



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11.7 WIND TURBINE MACHINE MODELS

As wind energy continues its rapid growth, wind turbine models need to be included in transient stability analysis. As was introduced in Chapter 6, there are four main types of wind turbines that must be considered. Model types 1 and 2 are based on an induction machine models. As is the case with a synchronous machine, the stator windings of the induction machine are connected to the rest of the electric network. However, rather than having a dc field winding on the rotor, the ac rotor currents are induced by the relative motion between the rotating magnetic field setup by the stator currents, and the rotor. Usually the difference between the per unit synchronous speed, n_s, and the per unit rotor speed, n_r, is quantified by the slip (S), defined (using the standard motor convention), as

$$S = \frac{n_s - n_r}{n_s} \tag{11.7.1}$$

From (11.7.1), it is clear that if the machine were operating at synchronous speed, its slip would be 0, with positive values when it is operating as a motor and negative values when it is operating as a generator. Expressing all values in per unit, the mechanical equation for an induction machine is

$$\frac{dS}{dt} = \frac{1}{2H}(T_m - T_e) \tag{11.7.2}$$

where H is the inertia constant, T_m is the mechanical torque, and T_e the electrical torque, defined in (11.7.10).

The simplified electric circuit for a single-cage induction machine is shown in Figure 11.23, using the generator convention in which current out of the machine is assumed to be positive. Similar to what is done for synchronous machines, an induction machine can be modeled as an equivalent voltage behind the stator resistance and a transient reactance X'. Referring to Figure 11.23, the values used in this representation are

$$X' = X_a + \frac{X_1 X_m}{X_1 + X_m} \tag{11.7.3}$$

where X' is the apparent reactance seen when the rotor is locked (i.e., slip is 1),

$$X = X_a + X_m \tag{11.7.4}$$

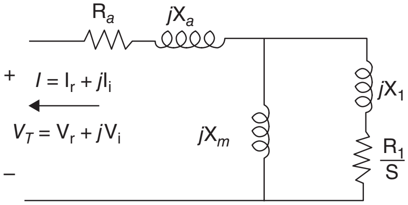
X is the synchronous reactance, and

$$T'_0 = \frac{(X_1 + X_m)}{\omega_0 R_1} \tag{11.7.5}$$

is the open-circuit time constant for the rotor. Also, X_a is commonly called the leakage reactance.

FIGURE 11.23

Equivalent circuit for a single cage induction machine



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Electrically the induction machine is modeled using two algebraic and two differential equations. However, in contrast to synchronous machines, because the reactances of induction machines do not depend upon the rotor position values, they are specified in the network reference frame. The equations are

$$V_r = E'_r - R_a I_r + X' I_i \tag{11.7.6}$$

$$V_i = E'_i - R_a I_i - X' I_r \tag{11.7.7}$$

$$\frac{dE'_r}{dt} = \omega_o S E'_i - \frac{1}{T'_o}\left((E'_r - (X - X')I_i\right) \tag{11.7.8}$$

$$\frac{dE'_i}{dt} = -\omega_o S E'_r - \frac{1}{T'_o}\left((E'_i + (X - X')I_r\right) \tag{11.7.9}$$

The induction machine electric torque is then given by

$$T_e = (E'_r I_i + E'_i I_i)/\omega_o \tag{11.7.10}$$

and the terminal real power injection by

$$P_e = (V_r I_r + V_i I_i) \tag{11.7.11}$$

The transient stability initial conditions are determined by setting (11.7.8) and (11.7.9) to zero, and then using the power flow real power injection and terminal voltage as inputs to solve (11.7.6), (11.7.7), (11.7.8), (11.7.9), and (11.7.11) for the other variables. The Newton-Raphson approach (Section 6.3) is commonly used. Since the induction machine reactive power injection will not normally match the power flow value, the difference is modeled by including a shunt capacitor whose susceptance is determined to match the initial power flow conditions. The reactive power produced by the machine is given by

$$Q_e = (-V_r I_i + V_i I_r) \tag{11.7.12}$$

with the value negative since induction machines consume reactive power.

EXAMPLE 11.11

Induction Generator Example

For the system from Example 11.3, assume the synchronous generator is replaced with an induction generator and shunt capacitor in order to represent a wind farm with the same initial real and reactive power output as in Example 11.3. The induction generator parameters are $H = 0.9$ per unit-seconds, $R_a = 0.013$, $X_a = 0.067$, $X_m = 3.8$, $R_1 = 0.0124$, $X_1 = 0.17$ (all per unit using the 100 MVA system base). This system is modeled in PowerWorld Simulator case Example 11_11. (a) Use the previous equations to verify the initial conditions of $S = -0.0111$, $E'_r = 0.9314$, $E'_i = 0.4117$, $I_r = 0.7974$, $I_i = 0.6586$. (b) Plot the terminal voltage for the fault sequence from Example 11.6.

(Continued)

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SOLUTION

a. Using (11.7.3) to (11.7.5) the values of X', X, and T_0' are determined to be 0.2297, 3.867, per unit and 0.85 seconds respectively. With $V_T = 1.0723 + j0.220$ and $P_e = 1.0$ per unit, (11.7.6) and (11.7.7) are verified as

$$V_r = 0.9314 - (0.013)\ (0.7974) + (0.2297(0.6586) = 1.0723$$
$$V_i = 0.4117 - (0.013)\ (0.6586) - (0.2297(0.7974) = 0.2200$$

And (11.7.8), (11.7.9) as

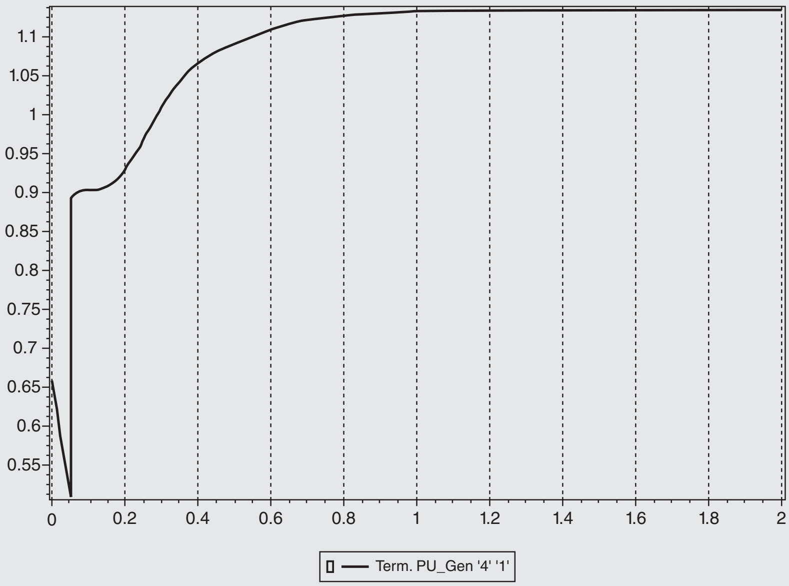
$$\frac{dE_r'}{dt} = 2\pi 60(-0.0111)(0.4117) - \frac{1}{0.85}(0.9314 - (3.637)(0.6586)) = 0.0$$
$$\frac{dE_i'}{dt} = -2\pi 60(-0.0111)(0.9314) - \frac{1}{0.85}(0.4117 + (3.637)(0.7974)) = 0.0$$
$$P_e = (1.0723)(0.7974) + (0.220)(0.6586) = 1.000$$
$$Q_e = -(1.0723)(0.6586) + (0.220)(0.7974) = -0.531$$

You can see the initial values in PowerWorld Simulator by first displaying the **States/Manual Control** page of the Transient Stability Analysis form, which initializes the transient stability. Then from the one-line diagram view the Generator Information Dialog for the generator at bus 4, and select **Stability, Terminal and State, Terminal Values**. Because the generator in the power flow is producing 57.2 Mvar, and the induction machine is consuming 53.1 Mvar, a shunt capacitor that produces 110.3 Mvar with a 1.0946 terminal voltage must be modeled.

b. Figure 11.24 plots the terminal voltage for the three cycle fault.

FIGURE 11.24

Example 11.11 Generator 4 voltage magnitude for a fault clearing time of 0.05 seconds



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Both the Type 1 and 2 wind turbine models utilize induction generators, but whereas the Type 1 models have a conventional squirrel cage rotor with fixed rotor resistance, the Type 2 models are wound rotor induction machines that utilize a control system to vary the rotor resistance. The reason for this is to provide a more steady power output from the wind turbine during wind variation. From (11.7.5) it is clear that increasing this external resistance has the effect of decreasing the open circuit time constant. The inputs to the rotor resistance control system are turbine speed and electrical power output, while the output is the external resistance that is in series with R_1 from Figure 11.22. Figure 11.25 plots the variation in the real power output for the Example 11.11 generator as a function of speed for the original rotor resistance of 0.0124 and for a total rotor resistance of 0.05 per unit. With a total resistance of 0.05, the operating point slip changes to about –0.045, which corresponds to a per-unit speed of 1.045.

Most new wind turbines are either Type 3 or Type 4. Type 3 wind turbines are used to represent doubly-fed asynchronous generators (DFAGs), also sometimes referred to as doubly-fed induction generators (DFIGs). A DFAG consists of a traditional wound rotor induction machine, but with the rotor windings connected to the ac network through an ac-dc-ac converter—the machine is “doubly-fed” through both the stator and rotor windings (see Figure 11.26). The advantages of this arrangement are that it allows for separate control of both the real and reactive power (like a synchronous machine), and the ability to transfer power both ways through the rotor converter allows for a much wider speed range. Because the stator is directly connected to the ac grid, the rotor circuit converter need only be sized to about 30% of the machine’s rated capacity. Another consequence of this design is the absence of an electrical coupling with the mechanical equation such as was seen with (11.1.10) for the synchronous machines and in (11.7.2) for the Type 1 and 2 induction machine models.

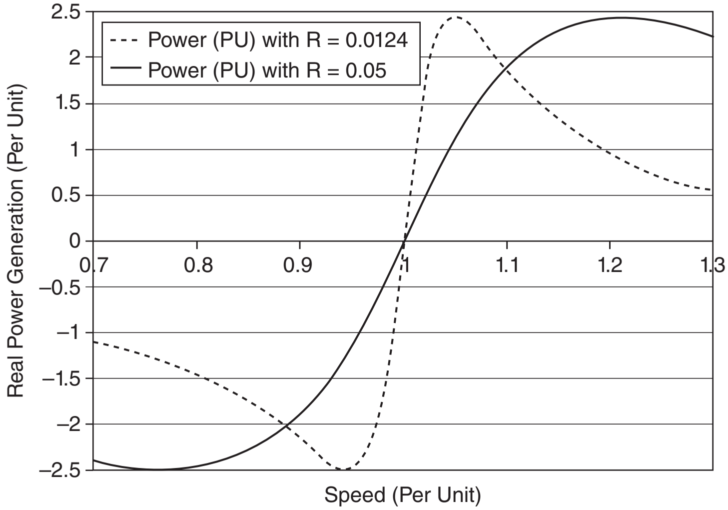

FIGURE 11.25

Effect of varying external resistance on an induction machine torque-speed curve

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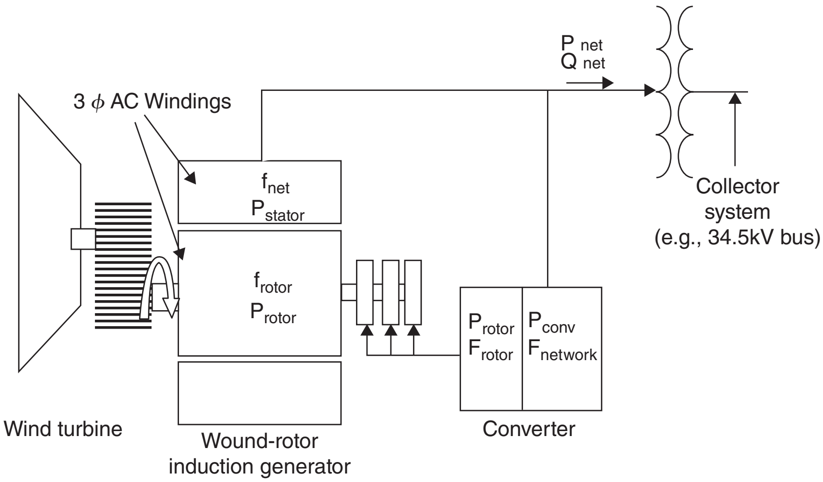
FIGURE 11.26

Doubly-fed asynchronous generator components



From a transient stability perspective, the DFAG dynamics are driven by the converter, with the result that the machine can be well approximated as a voltage-source converter (VSC). A VSC can be approximated as a synthesized current injection in parallel with an effective reactance, X_{eq}, (Figure 11.27) in which the current in phase with the terminal voltage, I_p, and the reactive power current, I_q, can be controlled independently. Low and high voltage current management is used to limit these values during system disturbances. With a terminal voltage angle of 0, the current injection on the network reference is

$$I_{sorc} = (\mathrm{I_p} + j\mathrm{I_q})\ (1\angle\theta) \tag{11.7.13}$$

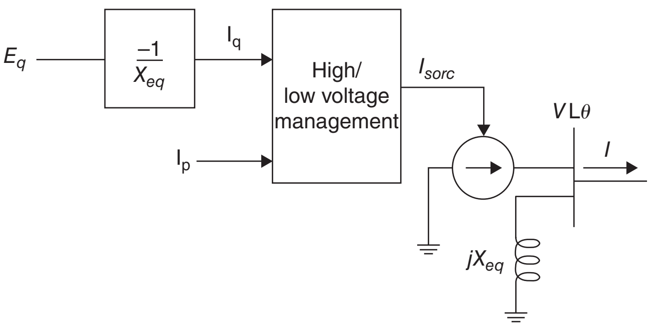
And the reactive voltage is

$$E_q = -\mathrm{I_q} X_{eq} \tag{11.7.14}$$

Type 4 wind turbines utilize a completely asynchronous design in which the full output of the machine is connected to the ac network through an ac-dc-ac converter (see Figure 11.28). Because the converter completely decouples the electric generator from the rest of the network, there is considerable freedom in selecting the electric

FIGURE 11.27

Type 3 DFAG model circuit diagram



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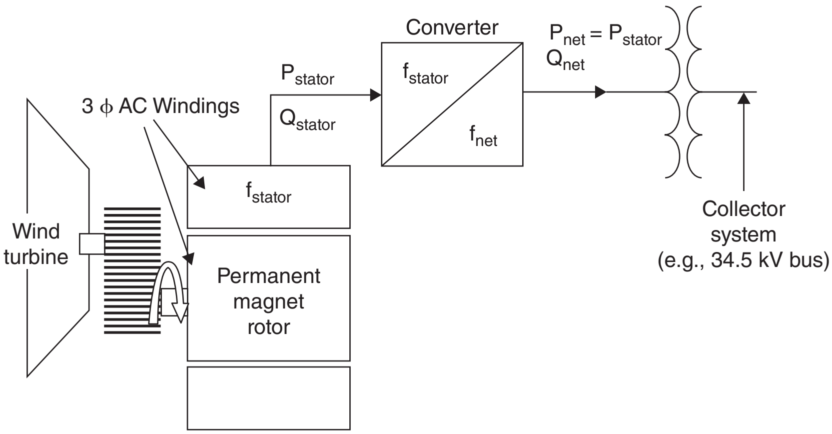

FIGURE 11.28

Type 4 full converter components

machine type: for example, a conventional synchronous generator, a permanent magnet synchronous generator, or even a squirrel cage induction machine.

From a transient stability perspective, electrically, the Type 4 model is similar to the Type 3 in that it can also be represented as a VSC. The key difference is lack of the effective reactance, with I_p and I_q being the direct control variables for the Type 4 model. As is the case with a DFAG, there is no electrical coupling with the turbine dynamics.

EXAMPLE 11.12

Doubly-Fed Asynchronous Generator Example

For the system from Example 11.3, assume the synchronous generator is replaced with a Type 3 DFAG generator in order to represent a wind farm with the initial current into the infinite bus set to 1.0 (unity power factor). The DFAG reactance $X_{eq} = 0.8$ per unit using a 100 MVA system base. Determine the initial values for I_p, I_q, and E_q.

SOLUTION

With $I = 1.0$ and an impedance of $j0.12$ between the machine's terminal and the infinite bus, the terminal voltage is

$$V_T = 1.0 + (1.0)(j0.22) = 1.0 + j0.22 = 1.0239\underline{/12.41°}$$

The amount supplied by I_{sorc} is I plus the amount modeled as going into X_{eq}

$$I_{sorc} = I - \frac{V_T}{jX_{eq}} = 1.00 + \frac{1.0 + j0.220}{j0.8}$$

$$I_{sorc} = 1.275 - j1.25$$

(*Continued*)

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The values of I_p and I_q are then calculated by shifting these values backward by the angle of the terminal voltage

$$I_p + jI_q = (1.275 - j1.25) * (1\underline{/12.41°}) = 0.977 - j1.495$$

And then

$$E_q = -(-1.495)(0.8) = 1.196$$

You can see the initial values in PowerWorld Simulator by opening case Example 11_12 and then displaying the **States/Manual Control** page of the Transient Stability Analysis form and selecting **Reset to Start Time** which initializes the transient stability. Then from the one-line diagram view the Generator Information Dialog for the generator at bus 4, and select **Stability, Terminal and State, Terminal Values.**

11.8 DESIGN METHODS FOR IMPROVING TRANSIENT STABILITY

Design methods for improving power system transient stability include the following:

1. Improved steady-state stability
   - **a.** Higher system voltage levels
   - **b.** Additional transmission lines
   - **c.** Smaller transmission-line series reactances
   - **d.** Smaller transformer leakage reactances
   - **e.** Series capacitive transmission-line compensation
   - **f.** Static var compensators and flexible ac transmission systems (FACTS)
2. High-speed fault clearing
3. High-speed reclosure of circuit breakers
4. Single-pole switching
5. Larger machine inertia, lower transient reactance
6. Fast responding, high-gain exciters
7. Fast valving
8. Braking resistors

These design methods are discussed in the following paragraphs.

1. Increasing the maximum power transfer in steady-state can also improve transient stability, allowing for increased power transfer through the unfaulted portion of a network during disturbances. Upgrading voltage on existing transmission or opting for higher voltages on new

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transmission increases line loadability (5.5.6). Additional parallel lines increase power-transfer capability. Reducing system reactances also increases power-transfer capability. Lines with bundled phase conductors have lower series reactances than lines that are not bundled. Oversized transformers with lower leakage reactances also help. Series capacitors reduce the total series reactances of a line by compensating for the series line inductance.

2. High-speed fault clearing is fundamental to transient stability. Standard practice for EHV systems is 1-cycle relaying and 2-cycle circuit breakers, allowing for fault clearing within 3 cycles (0.05 s). Ongoing research is presently aimed at reducing these to one-half cycle relaying and 1-cycle circuit breakers.

3. The majority of transmission-line short circuits are temporary, with the fault arc self-extinguishing within 5–40 cycles (depending on system voltage) after the line is deenergized. High-speed reclosure of circuit breakers can increase postfault transfer power, thereby improving transient stability. Conservative practice for EHV systems is to employ high-speed reclosure only if stability is maintained when reclosing into a permanent fault with subsequent reopening and lockout of breakers.

4. Since the majority of short circuits are single line-to-ground, relaying schemes and independent-pole circuit breakers can be used to clear a faulted phase while keeping the unfaulted phases of a line operating, thereby maintaining some power transfer across the faulted line. Studies have shown that single line-to-ground faults are self-clearing even when only the faulted phase is deenergized. Capacitive coupling between the energized unfaulted phases and the deenergized faulted phase is, in most cases, not strong enough to maintain an arcing short circuit [5].

5. Inspection of the swing equation, (11.1.16), shows that increasing the per-unit inertia constant H of a synchronous machine reduces angular acceleration, thereby slowing down angular swings and increasing critical clearing times. Stability is also improved by reducing machine transient reactances, which increases power-transfer capability during fault or postfault periods [see (11.2.1)]. Unfortunately, present-day generator manufacturing trends are toward lower H constants and higher machine reactances, which are a detriment to stability.

6. Modern machine excitation systems with fast thyristor controls and high amplifier gains (to overcome generator saturation) can rapidly increase generator field excitation after sensing low terminal voltage during faults. The effect is to rapidly increase internal machine voltages during faults, thereby increasing generator output power during fault and postfault periods. Critical clearing times are also increased [6].

7. Some steam turbines are equipped with fast valving to divert steam flows and rapidly reduce turbine mechanical power outputs. During faults near the generator, when electrical power output is reduced, fast valving action acts to balance mechanical and electrical power, providing

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reduced acceleration and longer critical clearing times. The turbines are designed to withstand thermal stresses due to fast valving [7].

8. In power systems with generation areas that can be temporarily separated from load areas, braking resistors can improve stability. When separation occurs, the braking resistor is inserted into the generation area for a second or two, preventing or slowing acceleration in the generation area. Shelton et al. [8] describe a 3-GW-s braking resistor.

PROBLEMS

SECTION 11.1

11.1 A three-phase, 60-Hz, 500-MVA, 11.8-kV, 4-pole steam turbine-generating unit has an H constant of 5 p.u.-s. Determine: (a) ω_{syn} and ω_{msyn}; (b) the kinetic energy in joules stored in the rotating masses at synchronous speed; (c) the mechanical angular acceleration α_m and electrical angular acceleration α if the unit is operating at synchronous speed with an accelerating power of 500 MW.

11.2 Calculate J in kg-m^2 for the generating unit given in Problem 11.1.

11.3 Generator manufacturers often use the term WR^2, which is the weight in pounds of all the rotating parts of a generating unit (including the prime mover) multiplied by the square of the radius of gyration in feet. WR^2/32.2 is then the total moment of inertia of the rotating parts in slug-fl^2. (a) Determine a formula for the stored kinetic energy in ft-lb of a generating unit in terms of WR^2 and rotor angular velocity ω_m. (b) Show that

$$H = \frac{2.31 \times 10^{-4} \text{WR}^2 (\text{rpm})^2}{S_{rated}} \quad \text{per unit-seconds}$$

where S_{rated} is the voltampere rating of the generator, and rpm is the synchronous speed in r/min. Note that 1 ft-lb = 746/550 = 1.356 joules, (c) Evaluate H for a three-phase generating unit rated 750 MVA, 3600 r/min, with WR^2 = 4,000,000 lb-ft^2.

11.4 The generating unit in Problem 11.1 is initially operating at $p_{mp.u.} = p_{ep.u.} =$ 0.7 per unit, $\omega = \omega_{syn}$, and $\delta = 12°$ when a fault reduces the generator electrical power output by 60%. Determine the power angle 3 five cycles after the fault commences. Assume that the accelerating power remains constant during the fault. Also assume that $\omega_{p.u.}(t) = 1.0$ in the swing equation.

11.5 How would the value of H change if a generator's assumed operating frequency is changed from 60 Hz to 55 Hz?

11.6 Repeat Example 11.1 except assume the number of poles is changed from 32 to 16, H is changed from 2.0 p.u.-s to 1.5 p.u.-s, and the unit is initially operating with an electrical and mechanical power of 0.5 p.u.

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SECTION 11.2

11.7 Given that for a moving mass $W_{kinetic} = 1/2\,Mv^2$, how fast would a 80,000 kg diesel locomotive need to go to equal the energy stored in a 60-Hz, 100-MVA, 60-Hz, 2-pole generator spinning at synchronous speed with an H of 3.0 p.u.-s?

11.8 The synchronous generator in Figure 11.4 delivers 0.8 per-unit real power at 1.05 per-unit terminal voltage. Determine: (a) the reactive power output of the generator; (b) the generator internal voltage; and (c) an equation for the electrical power delivered by the generator versus power angle δ.

11.9 The generator in Figure 11.4 is initially operating in the steady-state condition given in Problem 11.8 when a three-phase-to-ground bolted short circuit occurs at bus 3. Determine an equation for the electrical power delivered by the generator versus power angle δ during the fault.

11.10 For the five bus system from Example 6.9, assume the transmission lines and transformers are modeled with just their per unit reactance (e.g., neglect their resistance and B shunt values). If bus one is assumed to be an infinite bus, what is the equivalent (Thévenin) reactance looking into the system from the bus three terminal? Neglect any impedances associated with the loads.

11.11 Repeat Problem 11.10, except assume there is a three-phase-to-ground bolted short circuit at bus five.

SECTION 11.3

11.12 The generator in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3 when circuit breaker B12 inadvertently opens. Use the equal-area criterion to calculate the maximum value of the generator power angle δ. Assume $\omega_{p.u.}(t) = 1.0$ in the swing equation.

11.13 The generator in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3 when a temporary three-phase-to-ground short circuit occurs at point F. Three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to determine the maximum value of the power angle δ.

11.14 If breakers B13 and B22 in Problem 11.13 open later than 3 cycles after the fault commences, determine the critical clearing time.

11.15 Building upon Problem 11.11, assume a 60 Hz nominal system frequency, that the bus fault actually occurs on the line between buses five and two but at the bus two end, and that the fault is cleared by opening breakers B21 and B52. Again, neglecting the loads, assume that the generator at bus three is modeled with the classical generator model having a per unit value (on its 800 MVA base) of $X'_d = 0.24$, and H = 3 p.u.-s. Before the fault occurs the generator is delivering 300 MW into the infinite bus at unity power factor (hence its terminal voltage is not 1.05 as was assumed in Example 6.9). Further, assume the fault is cleared after 3 cycles.

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Determine: (a) the initial generator one power angle, (b) the power angle when the fault is cleared, and (c) the maximum value of the power angle using the equal area criteria.

11.16 Analytically determine whether there is a critical clearing time for Problem 11.15.

SECTION 11.4

11.17 Consider the first order differential equation, $\frac{dx_1}{dt} = -x_2$, with an initial value $x(0) = 10$. With an integration step size of 0.1 seconds, determine the value of $x(0.5)$ using (a) Euler's method, (b) the modified Euler's method.

11.18 The following set of differential equations can be used to represent that behavior of a simple spring-mass system, with $x_1(t)$ the mass's position and $x_2(t)$ its velocity:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = -x_1$$

For the initial condition of $x_1(0) = 1.0$, $x_2(0) = 0$, and a step size 0.1 seconds, determine the values $x_1(0.3)$ and $x_2(0.3)$ using (a) Euler's method, (b) the modified Euler's method.

11.19 A 60 Hz generator is supplying 400 MW (and 0 Mvar) to an infinite bus (with 1.0 per unit voltage) through two parallel transmission lines. Each transmission line has a per unit impedance (100 MVA base) of $0.09j$. The per unit transient reactance for the generator is $0.0375j$, the per unit inertia constant for the generator (H) is 20 seconds, and damping is 0.1 per unit (all with a 100 MVA base). At time = 0, one of the transmission lines experiences a balanced three-phase short to ground one third (1/3) of the way down the line from the generator to the infinite bus. (a) Using the classical generator model, determine the prefault internal voltage magnitude and angle of the generator. (b) Express the system dynamics during the fault as a set of first order differential equations. (c) Using Euler's method, determine the generator internal angle at the end of the second timestep. Use an integration step size of one cycle.

PW **11.20** Open PowerWorld Simulator case Problem 11_20. This case models the Example 11.4 system with damping at the bus 1 generator, and with a line fault midway between buses 1 and 2. The fault is cleared by opening the line. Determine the critical clearing time for this fault.

PW **11.21** Open PowerWorld Simulator case Problem 11_21. This case models the Example 11.4 system with damping at the bus 1 generator, and with a line fault midway between buses 2 and 3. The fault is cleared by opening the line. Determine the critical clearing time (to the nearest 0.01 second) for this fault.

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SECTION 11.5

11.22 Consider the six-bus power system shown in Figure 11.29, where all data are given in per-unit on a common system base. All resistances as well as transmission-line capacitances are neglected. (a) Determine the 6 × 6 per-unit bus admittance matrix $\boldsymbol{Y}_{\text{bus}}$ suitable for a power-flow computer program. (b) Determine the per-unit admittance matrices $\boldsymbol{Y}_{11}$, $\boldsymbol{Y}_{12}$, and $\boldsymbol{Y}_{22}$ given in (11.5.5), which are suitable for a transient stability study.

11.23 Modify the matrices $\boldsymbol{Y}_{11}$, $\boldsymbol{Y}_{12}$, and $\boldsymbol{Y}_{22}$ determined in Problem 11.22 for (a) the case when circuit breakers B32 and B51 open to remove line 3–5; and (b) the case when the load $P_{L3} + jQ_{L3}$ is removed.

PW **11.24** Open PowerWorld Simulator case Problem 11_24, which models the Example 6.9 with transient stability data added for the generators. Determine the critical clearing time (to the nearest 0.01 second) for a fault on the line between buses 4 and 5 at the bus 4 end which is cleared by opening the line.

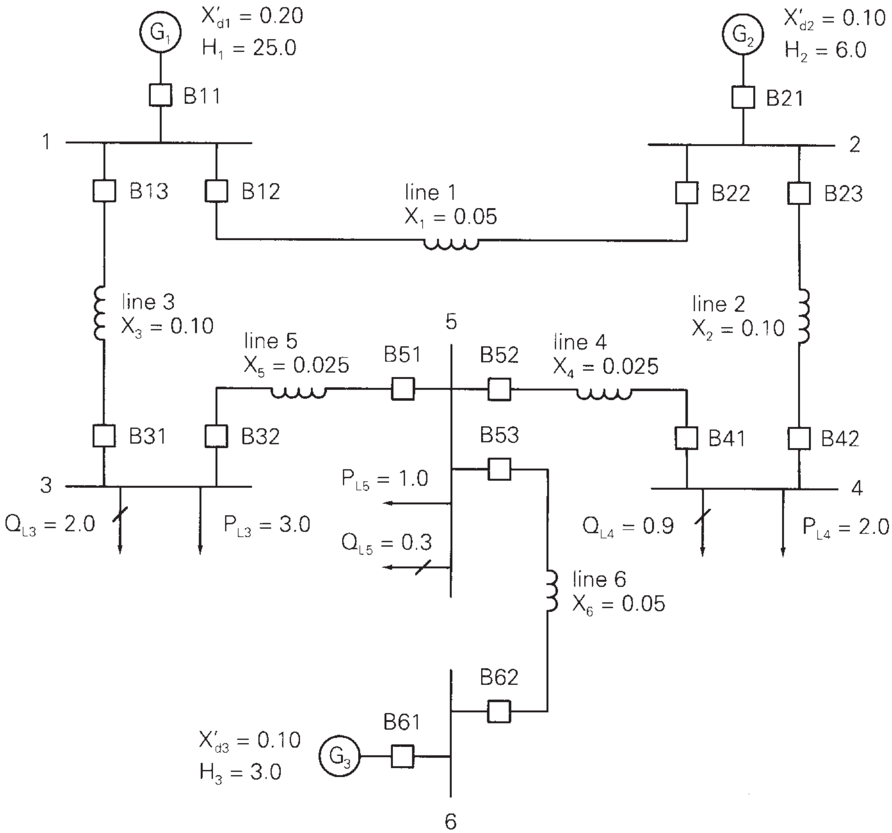

FIGURE 11.29 Single-line diagram of a six-bus power system (per-unit values are shown)

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PW **11.25** With PowerWorld Simulator using the Example 11_9 case determine the critical clearing time (to the closest 0.01 second) for a transmission line fault on the transmission line between bus 44 (PEACH69) and bus 14 (REDBUD69), with the fault occurring near bus 44.

SECTION 11.6

PW **11.26** PowerWorld Simulator case Problem 11_26 duplicates Example 11.10, except with the synchronous generator initially supplying 75 MW at unity power factor to the infinite bus. (a) Derive the initial values for δ, E_q', E_d' and E_{fd}. (b) Determine the critical clearing time for the Example 11.10 fault.

PW **11.27** PowerWorld Simulator case Problem 11_27 duplicates the system from Problem 11.24, except the generators are modeled using a two-axis model, with the same X_d' and H parameters as are in Problem 11.24. Compare the critical clearing time between this case and the Problem 11.24 case.

SECTION 11.7

PW **11.28** PowerWorld Simulator case Problem 11_28 duplicates Example 11.11 except the wind turbine generator is set so it is initially supplying 100 MW to the infinite bus at unity power factor. (a) Use the induction machine equations to verify the initial conditions of $S = -0.0129$, $E_r' = 0.8475$, $E_i' = 0.4230$, $I_r = 0.8433$, and $I_i = 0.7119$. (b) Plot the terminal voltage for the fault sequence from Example 11.6.

11.29 Redo Example 11.12 with the assumption the generator is supplying $100 + j10$ MVA to the infinite bus.

CASE STUDY QUESTIONS

a. What is a black-start generating unit? Name three types of black-start units. Are black-start units subject to self-excitation? If so, how?

b. In the case study, system stability is subdivided into what three categories? What is rotor angle stability?

c. Define cold-load pickup. What are the components of cold-load pickup?

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