# Kinematics for Constrained Continuum Robots Using Wavelet Decomposition

Ian Gravagne and Ian D. Walker
Dept. of Electrical and Computer Engineering
Clemson University
igravag@ces.clemson.edu, ianw@ces.clemson.edu

*Abstract*

Over the past several years, there has been a rapidly expanding interest in the study and construction of a new class of robot manipulators which utilize high degree of freedom, or continuous, backbone structures. In this paper, we consider some basic properties of these "continuum" or "hyper-redundant" robots. We base our analysis around remotely-driven, tendon-actuated manipulators such as the Rice/Clemson "Elephant's Trunk". We briefly discuss the kinematic model, before detailing how to approach the inverse kinematics for a planar continuum robot by decomposition into either a natural or a wavelet basis. We also examine how a wavelet decomposition method can help resolve redundancy in a planar continuum robot.

## Introduction

By observing manipulation methods in nature, one may eventually reach the conclusion that rigid-link, low degree of freedom devices should meet the majority of manipulative and locomotive needs. However, some creatures make use of alternative methods based on very high degree of freedom (HDOF) backbones, such as snakes, or continuous "trunk" or "tentacle" structures. These manipulators, a subset of a class termed *hyper-redundant*, exhibit unique capabilities including extremely enhanced maneuverability. Hyper-redundant manipulators have the potential to navigate extremely complex paths, and to suffer localized damage or faults while still maintaining a healthy degree of functionality. In principle, this makes them suitable for a variety of delicate and dangerous tasks where a traditional robot could not reach, or where failure of a traditional robot would completely paralyze all subsequent operations. Examples of such tasks are nuclear waste inspection and removal, and navigation or inspection of highly cluttered environments such as collapsed buildings.

In this paper we will concentrate on the fundamentals of a specific type of hyper-redundant robot, frequently referring to the Rice/Clemson "Elephant's Trunk"[2]. This is a type of remotely-actuated device which uses cables, or tendons generally, to transmit forces from a motor platform into the trunk itself. The salient feature of the Elephant's Trunk is that its high number of links (16), combined with the small size of each link, allow us to closely approximate it as a truly continuous backbone. Similar robots in our laboratory do in fact possess continuous backbones made of various materials, termed continuum robots. The following problems and theories apply to all of these robots. Using a continuous backbone model for the robot kinematics, we address the issue of how to approach inverse kinematics given physical properties of the robot.

## Background

Several researchers have worked in the area of hyper-redundant, including continuum and HDOF manipulators, for various reasons. In Japan, Hirose pioneered the development of snake-like robots, especially with regards to locomotion; an overview of his work exists in [1]. Also, Mochiyama et. al. have investigated the problem of controlling the shape of an HDOF rigid-link robot with two-degree-of-freedom joints using spatial curves[3-5]. For robots possessing continuous back-bones, a good overview exists in [11]. These authors plus Suzumori et. al.[12] have done significant work in flexible hydraulic micro-actuators for grippers, which are essentially small, flexible, 3-DOF manipulators. The primary body of work upon which we draw is that of Chirikjian and Burdick, who laid the foundations for the kinematic theory of hyper-redundant robots[6-10].

For the most part, research in hyper-redundant manipulators has consisted of creating and analyzing an "imaginary" or virtual continuous robot, giving it certain properties and behaviors, and then attempting to match or "fit" a real robot to the virtual one. However, there are some drawbacks to this method. Among these we find that the methods of spatial curve matching assume that one can exactly control the parameters of the real robots, which are still basically serial, rigid-link robots tending to be very heavy and expensive. Additionally, the total system complexity is high because not only must one account for the kinematics of the virtual continuum robot and the real robot simultaneously, but also the curve-fitting algorithm which connects the two. Unfortunately, the kinematic models for the virtual continuum robots do not correspond with any known devices in the laboratory, except in the planar case (which is primarily considered in this paper). However, much of the fundamental theory for the virtual models, such as [6-10], will help to describe the real robots.

Figure 1. The Rice/Clemson "Elephants Trunk".

**Kinematic Model**

Essentially, we will consider a robot such as the Elephants Trunk, which is really a HDOF robot, to have a continuous backbone. This is because its range of motion combined with its high number of links allow it to assume shapes which, for all practical purposes, appear continuous (figure 1). As such, continuum robots in general are essentially infinite-degree-of-freedom devices, controlled by applying forces or torques at periodic locations along the backbone. Since those forces act upon the robot via the tendons or cables, this usually means that a specific set of tendon lengths will not imply a unique pose for the robot. The kinematic model must reflect physical parameters such as the backbone stiffness profile, friction between the cables and the cable guides, external forces due to gravity, and minimum potential energy constraints. These are qualities which the kinematics of traditional rigid-link robots need not reflect.

In order to model continuum robots kinematics, we first take a "continuous" frame, i.e. a mathematical set of 3 orthogonal basis vectors which describe the robot's orientation at any point along its length. We take as our frame

$$Q(s)=\begin{bmatrix} c_a^2+s_a^2c_b & s_a s_b & -c_a s_a(1-c_b) \\ -s_a s_b & c_b & c_a s_b \\ -c_a s_a(1-c_b) & c_a s_b & s_a^2+c_a^2c_b \end{bmatrix}$$

where $c_a=\cos a(s), s_a=\sin a(s),$ etc. The frame evolves continuously because the various angles which comprise it also vary continuously, according to an independent parameter, usually denoted *s*. The parameter *s* usually is taken to vary between 0 and 1, and if the robot does not extend or contract, s represents the arc-length from the origin. The columns of *Q(s)* correspond to the orthogonal frame vector triple $(e_1,e_2,e_3)$. (The details for why this choice fits the physical problem at hand can be found in [15].) The quantities $a(s)$ and $b(s)$ vary continuously along the robot's length, and together determine the bending of the backbone. The second column of *Q(s)*, denoted $\underline{q}_2(s)$, is tangent to the backbone curve at all points, and thus the robot "grows" from the origin by integrating $\underline{q}_2(s)$ to get position,

$$\underline{x}(s) = \int_0^s \underline{q}_2(s)\,ds = \int_0^s \begin{bmatrix} s_a s_b & c_b & c_a s_b \end{bmatrix}^T ds \ .$$

Because $a(s)$ and $b(s)$ are functions (i.e. infinite-dimensional quantities) we must constrain the kinematics by accounting for physical realities, the most important of which is potential energy.

The backbone is taken to be a homogeneous flexible rod (or beam) which tends to return to a straight line. By observing how the frame *Q(s)* changes, we may quantify the spring energy stored in a section of backbone as

$$SE = \int_0^1 w(s)\left[\dot{a}^2(1-\cos b) + \tfrac{1}{2}\dot{b}^2\right] ds$$

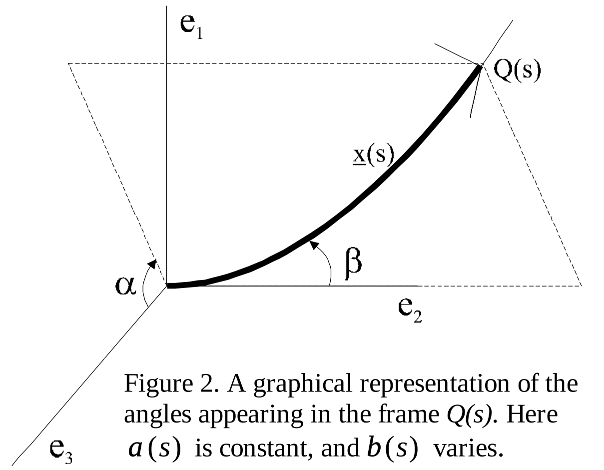


Figure 2. A graphical representation of the angles appearing in the frame *Q(s)*. Here $a(s)$ is constant, and $b(s)$ varies.

where *w(s)* weights the relative "bendability" of the section along its length, and $\dot{a}$, $\dot{b}$ are derivatives with respect to s.

With no other constraints, the methods of variational calculus yield a set of differential equations which describe $a(s)$ and $b(s)$,

$$\ddot{a}(1-\cos b) = -\frac{\dot{w}}{w}\dot{a}(1-\cos b) - \dot{a}\dot{b}(\sin b)$$

$$\ddot{b} = -\frac{\dot{w}}{w}\dot{b} + \dot{a}^2(\sin b).$$

Initial conditions for this system are

$$\dot{b}(0) = m_1 \qquad b(0) = 0$$
$$\dot{a}(0) = 0 \qquad a(0) = m_2$$

Unfortunately, this differential system contains a singularity at $b = 0$; however we will limit our discussion to the plane, where $a$ = *constant*, and $\dot{a} = 0$. With this constraint, we need only concern ourselves with the solution for $b(s)$. Furthermore, we assume that each section of the backbone consists of a material with no affinity for bending in any particular place, therefore $w(s)$ is constant and $\dot{w} = 0$. Thus, not accounting for the effects of static friction, we have the system

$$\ddot{b} = 0; \qquad \dot{b}(0) = m_1 \text{ and } b(0) = 0$$

which has the simple linear solution $b(s) = m_1 s$.

**Shape Control by Modal Decompositions**

To reiterate, several conditions have led to the derivation that $b(s)$ is linear: the backbone material bends uniformly, there is no bending out-of-plane, and there are no gravitational effects. Similarly, there must be limited frictional effects between the cables and the cable guides. While these constraints are rather limiting, if we are content to abide by them, we may begin to make some headway in the areas of shape and motion generation using a method first suggested in [6], known as "modal decomposition". (Further comments for the general non-planar robot with greatly relaxed constraints may be found in [15]). The work in [6] laid the foundations for modal decompositions, but tended to require that the continuum robot could bend into any number of "basis shapes" which tendon-driven continuum robots, at least, cannot achieve. However, using information gained by the kinematics analysis above, we may now put the method of modal decomposition to good use.

Fundamentally, modal decomposition performs the all-important task of mapping the infinite-dimensional quantity $b(s)$ into a finite-dimensional space, hopefully the same dimension as the number of degrees of freedom of the system (for rigid-link robots this would be the number of joints), or the number of actuators in the system. While $b(s)$ is a simple linear quantity, the mapping needs only one dimension to capture the function: that is the slope $m_1$. This reflects the fact that a manipulator constructed of one cable

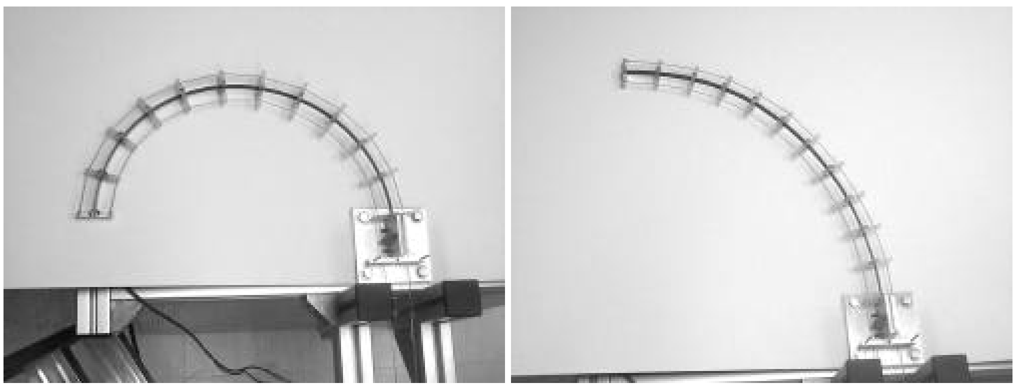

Fig 3. A continuum planar robot with one cable pair bends in a semi-circular arc (accounting for a bit of foreshortening in the image). The backbone consists of a piece of "spring" steel.

pair will have only one degree of freedom, and the linearity in $b(s)$ implies that the backbone will bend in a circular arc (see fig. 3).

If we serially "connect" *n* of these simple planar robots -- as in figure 1, where the Elephant's Trunk has 4 sections -- then the function $b(s)$ will appear continuous, piece-wise linear; alternatively its derivative $\dot{b}(s)$ will be discontinuous, piece-wise constant. The constants will be the sequence $\{m_1, m_2 ... m_n\}$. Splitting the robot backbone into *n* pieces is equivalent to "sampling" the arc length at *n+1* locations, $\{s_0, s_1, ..., s_n\}$ where $s_0 = 0$ and $s_n = 1$ by definition. Thus we may "decompose" $\dot{b}(s)$ into a finite sum of "box" functions,

$$\dot{b}(s) = \tfrac{1}{\sqrt{n}} \sum_{i=1}^{n} m_i f_i^b(s); \quad f_i^b(s) = \begin{Bmatrix} \sqrt{n}, & s \in [s_{i-1}, s_i] \\ 0, & \text{otherwise} \end{Bmatrix}.$$

The superscript 'b' on the basis functions $f_i^b(s)$ denotes the box functions, or the "natural basis set". Note that the natural basis functions have been scaled so they are orthonormal in the sense of an appropriate inner product [14]. Integrating yields

$$b(s) = \tfrac{1}{\sqrt{n}} \sum_{i=1}^{n} m_i \Phi_i^b(s); \quad \Phi_i^b(s) = \int_0^1 f_i^b(s) ds .$$

This "natural" basis decomposition, while elementary, provides a stepping stone into more complex decompositions which have greater utility. Because the natural basis coefficients, $\{m_1, m_2 ... m_n\}$, have physical meaning (they relate directly to the amount of cable displacement necessary to achieve a particular bend, see [15]), we must always transform back to the natural basis set somehow. Nevertheless, it is not a unique choice as we will now demonstrate.

The natural basis functions, being "boxes" of finite extent, each support only one section of a multi-section planar robot. In other words, these basis functions have well-defined spatial range, but very limited capacity to affect the entire robot's shape. Of more utility would be a basis set which strikes a

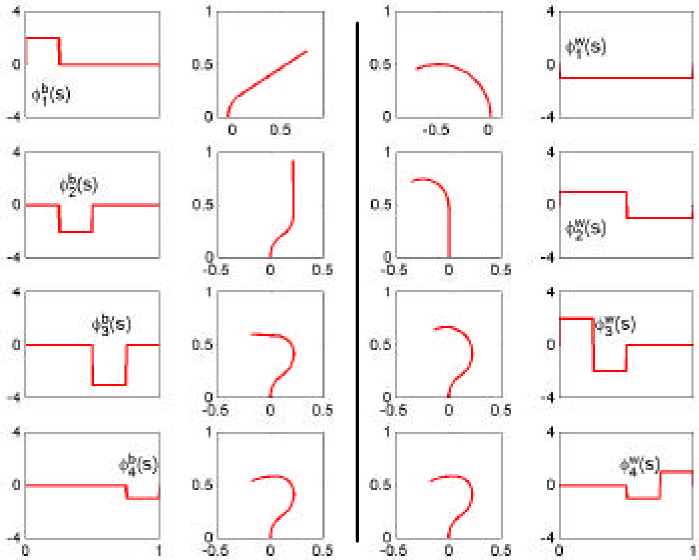

Figure 4. The center columns feature a continuum robot with 4 degrees of freedom ($n$=4), illustrating the cumulative effect of the natural and wavelet scaled basis functions, respectively. Note how both sets of bases result in the same robot pose.

balance, where certain coefficients have the power to change the shape of the whole robot, while others are more spatially limited for finer detail. This concept is known as "multi-resolution", and has been extensively studied in wavelet theory [14]. The basic concepts are the same, where we employ an orthogonal basis set to build up the piece-wise functions of interest,

$$b(s) = \sum_{i=1}^{n} a_i \Phi_i^w(s); \quad \Phi_i^w(s) = \int_0^1 f_i^w(s)ds$$

The new wavelet basis functions are pictured on the right in figure 4, and are known as the Haar wavelet basis set. We will not go into the details of its derivation here, but we note that these functions also are orthonormal. They exhibit the qualities of multiresolution, where $f_1^w(s)$ acts over the entire robot's backbone (with $a_1$ representing the average curvature). $f_2^w(s)$ provides rough detail (the ability to bend in more than one direction), while $f_3^w(s)$ and $f_4^w(s)$ take care of finer but more localized bending still, as illustrated in figure 4. This process can continue indefinitely, with the next level being $\{f_5^w(s), f_6^w(s), f_7^w(s), f_8^w(s)\}$ and so forth.

**Motion of the Backbone**

In matrix-vector notation, we may write the natural and wavelet decompositions as $b(s) = \frac{1}{\sqrt{n}} \underline{m}^T \underline{\Phi}^b(s) = \underline{a}^T \underline{\Phi}^w(s)$, with $\underline{m} = [m_1 ... m_n]^T$, etc. Given the backbone position,

$$\underline{x}(s) = \begin{bmatrix} \int_0^s \sin b(s)ds \\ \int_0^s \cos b(s)ds \end{bmatrix}$$

we note the standard Jacobian relationship for backbone velocity

$$\frac{d\underline{x}(s)}{dt} = \frac{\partial \underline{x}(s)}{\partial \underline{a}}\frac{d\underline{a}}{dt} \equiv \left[J(\underline{a},s)\right]\frac{d\underline{a}}{dt}; \quad \left\{ \begin{array}{l} J_{1,i} = \int_0^s \Phi_i^w(s)\cos(\underline{a}^T \underline{\Phi}(s))ds \\ J_{2,i} = -\int_0^s \Phi_i^w(s)\sin(\underline{a}^T \underline{\Phi}(s))ds \end{array} \right\}.$$

Traditionally only the end-effector velocities will matter, setting the upper limit on the integrals in the Jacobian to 1. Now, because of the nature of the wavelet bases, the first two columns of *J* determine the end-effector velocities due to the low-resolution coefficients which act on the whole backbone. If the robot is redundant, with more columns than rows, then some or all of the columns beyond the first two may be *thrown out* if their contributions are not desired, until *J* is square. This represents a form of redundancy resolution which does not require one to resort to a Jacobian pseudo-inverse, which is required when using the natural basis decomposition. Of course, any extra capabilities provided by the redundancy are lost as a result. If one desires to retain these capabilities, a pseudo-inverse with a gradient projection will work with either basis set.

For instance, we may quantify the "spring" energy in a continuum robot as $SE = \frac{1}{2}\int_0^1 \left\{\dot{b}(s)\right\}^2 ds$. Substituting the wavelet decomposition method in for $b(s)$ yields, accounting for orthonormality, $SE = \frac{1}{2}\underline{a}^T\underline{a}$. Thus, the gradient pointing in the direction of minimum energy (equivalent to minimum "bending") is simply $-\nabla_{\underline{a}}(SE) = -\underline{a}$, and the motion executing local minimum-bending behavior is

$$\frac{d\underline{a}}{dt} = \left[J^+\right]\frac{d\underline{x}(1)}{dt} + k\left[I_{n\times n} - J^+J\right]\underline{a}$$

where $k$ adjusts how strongly the robot will tend toward the minimum bending solution.

In the end, we must still relate the wavelet coefficients back to the natural basis coefficients in order to command the cable actuators to achieve the desired motions. Fortunately, the Fast Wavelet Transform (FWT) in matrix form is exactly the transformation needed [14]. For a 4-section robot (4 DOF or "bends" in the backbone), the transformation is

$$\underline{m} = T\underline{a}; \quad T = 2\begin{bmatrix} r^2 & r^2 & r & 0 \\ r^2 & r^2 & -r & 0 \\ r^2 & -r^2 & 0 & r \\ r^2 & -r^2 & 0 & -r \end{bmatrix}; \quad r = \frac{1}{\sqrt{2}}.$$

In general, the multiplier of "2" above is replaced with $\sqrt{n}$ and $T_{i,j} = \int_0^1 f_i^b(s) f_i^w(s)ds$. Excepting the multiplier, *T* is unitary and trivially inverted. In fact, for the case in which the problem dimension is reduced in the wavelet domain (i.e. throwing out columns of the Jacobian), the dimensions of $\underline{a}$ and $\underline{m}$ will not be equal. In that case, one may simply ignore the associated columns of *T*, and still be able to relate $\underline{a}$ to $\underline{m}$.

## Conclusions

The purpose of this work in general is to explore alternative types of manipulators based on flexible backbones. Especially in the case where these can be remotely actuated, they may ultimately provide manipulation solutions where traditional robots cannot reach, or become too heavy or unwieldy to use in difficult environments. In this paper specifically, we address the issues of how an appropriate kinematic model can provide a natural method for reducing the problem dimension via a decomposition transformation known as "modal analysis". We also illustrate how this decomposition is not unique, and may be improved upon. Further work remains in analyzing the behavior of continuum robots in non-planar situations (where the differential system mentioned earlier exhibits a singularity), as well as in further classifying the effects of friction and backbone material. Experimental prototypes are under construction to further examine these properties.

The authors gratefully acknowledge the support of NSF grant CMS-9796328 and DOE contract DE-FG07-97ER14830. Thanks to Brian Bunnell and Michael Hannan for help constructing the robot prototypes.

**References**

1. S. Hirose, *Biologically Inspired Robots*. Oxford University Press, 1993
2. I.D. Walker and M.W. Hannan, "A Novel Elephant's Trunk Robot", to appear at 1999 IEEE/ASME Intl. Conf. on Advanced Intelligent Mechatronics, Atlanta, GA
3. H. Mochiyama and H. Kobayashi, "The Shape Jacobian of a Manipulator with Hyper Degrees of Freedom," Proceedings 1999 IEEE\ Int'l Conf. on Robotics and Automation (ICRA), Detroit MI, May 1999, pp. 2837-2842
4. H. Mochiyama and H. Kobayashi, "Shape Correspondence between a Spatial Curve and a Manipulator with Hyper Degrees of Freedom," Proceedings 1998 IEEE/RSJ Int'l Conf. on Intelligent Robots and Systems, Victoria, Canada, October 1998, pp. 161-166
5. H. Mochiyama, E. Shimemura and H. Kobayashi, "Direct Kinematics of Manipulators with Hyper Degrees of Freedom and Serret-Frenet Formula," Proceedings 1998 IEEE Int. Conf. Robotics and Automation, Leuven, Belgium, May 1998, pp. 1653-1658
6. G.S. Chirikjian and J.W. Burdick, "A Modal Approach to Hyper-Redundant Manipulator Kinematics," IEEE Transactions on Robotics and Automation, vol. 10, no. 3, June 1994, pp. 343-353
7. G.S. Chirikjian and J.W. Burdick, "Kinematically Optimal Hyper-Redundant Manipulator Configurations," IEEE Trans. Robotics and Automation, vol. 11, no. 6, Dec 1995, pp. 794-80
8. G.S. Chirikjian, "A General Numerical Method for Hyper-Redundant Manipulator Inverse Kinematics," Proc. IEEE Int. Conf. Robotics and Automation, Atlanta GA, May 1993, pp. 107-112
9. G.S. Chirikjian and J.W. Burdick, "Kinematically Optimal Hyper-Redundant Manipulator Configurations," Proc. IEEE Int. Conf. Robotics and Automation, Nice, France, May 1992, pp. 415-420
10. G.S. Chirikjian and J.W. Burdick, "An Obstacle Avoidance Algorithm for Hyper-Redundant Manipulators," Proc. IEEE Int. Conf. Robotics and Automation, Cincinatti, OH, May 1990, pp. 625-631
11. G. Robinson and J.B.C. Davies, "Continuum Robots -- A State of the Art," Proc. IEEE Int. Conf. Robotics and Automation, Detroit, MI, May 1999, pp. 2849-2854
12. K. Suzumori, S. Ilkura, and H. Tanaka, "Development of Flexible Microactuator and Its Applications to Robotic Mechanisms," Proc IEEE Int. Conf. Robotics and Automation, Sacramento, CA, April 1991, pp. 1622-1627
13. Robert Weinstock, *Calculus of Variations with Applications for Physics and Engineering*, Dover Pulications, NY, 1974
14. C.S. Burrus, R.A. Gopinath, and H. Guo, *Introduction to Wavelets and Wavelet Transforms*,' Prentice Hall, NJ, 1998
15. I. Gravagne, ECE Internal Report "Fundamentals of Continuum Tendon-Driven Robots", Aug 1999; contact author.