Engineering Mathematics for Graduate Students: To Teach or Not To Teach?

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Abstract

This paper discusses when and why it may be appropriate for engineering faculty to teach a graduate-level survey course in applied mathematics. An example of one such course is given, along with a discussion of the specific topics and overall educational themes of the course.

Introduction

Much attention has been paid at all levels of the undergraduate engineering curriculum to the teaching and comprehension of first principles in mathematics, especially the types of mathematical tools that are relevant to engineers. However, at the graduate level there has been less discussion, perhaps owing to a widely divergent set of opinions among faculty regarding what should be taught and with how much depth. At the largest universities, graduate engineering mathematics skills can often be honed in the mathematics, applied mathematics, or statistics departments. On the other hand, smaller universities (or large ones with smaller engineering and science programs, as at Baylor) often provide few or no viable options for engineering graduate students, leaving the engineering departments to decide whether to create and staff a general-purpose mathematics course. This paper argues that, even if an engineering program can only teach one survey course in the broad area of applied mathematics, there will be student benefit. An example of one such course, EGR5302 “Engineering Analysis,” shared between the engineering departments at Baylor, is given along with details about the curriculum. The paper concludes with a discussion about the relative pros and cons of including or excluding certain subjects in a graduate mathematics survey course.

Comments on Undergraduate Preparation

It is common that undergraduate engineering programs require a fairly rigorous diet of mathematics: several levels of calculus, ordinary and partial differential equations, linear algebra, complex variables and, of course, mathematics embedded into the engineering curriculum itself. It can even be said that engineering students often have an affinity for mathematics, although not always. Nevertheless, there seem to be several factors working against the average student in regards to his or her mathematics preparation. The following are common themes that have surfaced in the author’s conversation about this topic with educators in engineering and mathematics:

- The crushing demands of the undergraduate engineering curriculum. It is sometimes lamented that more engineering students do not take elective upper-level mathematics courses, but time demands often force even mathematically savvy students to give up the idea of much additional mathematics training.

- The gnawing sense that, while interesting and sometimes relevant in class, additional mathematics study may not be of much benefit to one's job prospects or career performance.
- A tendency by mathematics faculty to focus upper-level courses on "pure mathematics." This is not a criticism, but a reflection of the need for mathematics instructors to serve first their primary audience, mathematics majors.
- The not uncommon possibility that a beginning engineering student is already "behind" in mathematics and must spend every spare credit-hour simply getting up to par.

On balance, the literature suggests that most educators believe undergraduate engineering students' mathematical moxie is generally weak, and many initiatives have been undertaken to strengthen their analytical and computational skills. Beyond the development of courses designed to address engineering mathematics, some other interesting techniques include:

- Encouraging mathematics faculty to offer interdisciplinary courses (and topics) that emphasize practical computation for scientists and engineers[5];
- Surveying students about their views on the practicality of their mathematics training with the ultimate goal of refining how calculus is taught[6];
- Developing methodologies based upon SAT/ACT scores and other metrics from early college mathematics courses to raise early warning flags if students' mathematics performance is likely to hinder their progress through engineering[2].
- Simply taking practical steps to encourage students to enroll in advanced mathematics courses through targeted advertising, accommodating scheduling of courses, and "carrots" such as awarding a minor in mathematics for one or two extra courses[5].

These initiatives are without doubt positive developments and almost sure to benefit undergraduate engineering students. But they beg the question, with so much focus on the state of undergraduate mathematics preparation, can we simply trust that graduate students are well- prepared? Furthermore, what exactly constitutes "well-prepared" when it comes to graduate engineering study?

Comments on Graduate Preparation

It is at the level of graduate study where opinions on student mathematical preparation diverge. There are probably several reasons for this, not the least of which is a very wide diversity of avenues for study and research in graduate school, which require a widely varying degree of mathematical ability[4,8,10,11]. Research degrees such as the M.S. and Ph.D. do not have standardized curricula, and a student's expected mathematical performance is highly dependent on the research advisor's needs and preferences. Areas like computational electromagnetics, control theory, digital signal processing, computational fluid dynamics, etc., frequently require advanced achievement in fundamental mathematics. This population of students, if they study at a university with a large enough mathematics or applied mathematics program, can frequently find good options within those departments.

On the other hand, that leaves two populations untreated: the students at schools with smaller mathematics programs (or faculty focused heavily on mathematics of a nature that is not helpful for engineers) and students pursuing research in areas that are less mathematical in nature.

At Baylor, until recently, graduate engineering students fell into the former category. Baylor's mathematics department is weighted largely toward "pure" mathematicians, and a faculty of dozens counts only four or five tenure-track individuals with an affinity for applied mathematics. (Of those three, two arrived within the last 6 years.) In the absence of specific classes accessible to non-mathematics graduate students, departments such as physics and engineering had to confront the question of whether and how to offer advanced mathematical coursework from within their programs. In the engineering departments, this question eventually gave rise to the class EGR 5302, a one-semester, three-credit-hour survey course in advanced mathematics taught by the author.

At the outset, it was evident that a single semester-long course could not possibly provide the depth many students will need in their various research endeavors. Therefore, it was decided that depth would not be the goal, but rather to develop a handful of broadly useful areas and results *with rigor*. In other words, if it is not possible to give students everything they need to know about any given area of mathematics, then at least they should be taught disciplined, rigorous mathematics that will serve to spur intellectual maturation and to hone the skills that will undergird future independent mathematical study. The various areas that were selected for coverage, and the objectives in teaching those topics, are given next.

A Survey Course in Graduate Engineering Mathematics

It probably goes without saying that you can't please everyone when designing a mathematics survey course at any level. Nevertheless, there is evidence that intensive survey-style courses are among the bet options for graduate-level mathematics preparation[13]. The author felt there were four principal areas that would serve a wide variety of students, and also provide good fodder for teaching discipline in mathematics. These topics were discussed (and generally agreed upon) by the faculty of the Electrical and Computer Engineering Department, but students from all graduate engineering programs have taken the course.

1.) *Basic Functional (or Real) Analysis*. Notes and handouts for topics in this area were derived from the first three chapters Kreyczig's text, *Introductory Functional Analysis with Applications*[9], along with some material from chapter 7 on spectral theory. A summary includes:
   - Metrics and metric spaces
   - Norms and normed vector spaces
   - Inner products and Hilbert spaces
   - The concepts of open/closed, complete, convergence and compactness
   - The generalized notions of linear operators.

The focus in this section is very much on the notion of proof and logical, water-tight argumentation. Students from all majors and backgrounds were found to be extremely weak in proof technique and a great deal of effort is expended in this section teaching the "ins and outs" of proof. But overall, the chosen topics are the foundation for a tremendous edifice of modern mathematics and useful in their own right.

Throughout this section, frequent reference is made through examples and homework problems to the stalwarts of engineering mathematics: Laplace, Fourier and Hilbert transforms (and their inverses). These are excellent exemplars of bounded finite- and infinite-dimensional linear operators with various interesting properties, and students already have some familiarity with them.

2.) *Linear Algebra*. Here the focus is squarely on two subjects, eigenvalue theory and over/underdetermined equations. Notes and handouts are derived from Brogan's *Modern Control Theory*[1], chapters 6 and 7 on simultaneous linear equations and eigenvalues/eigenvectors.

- Gram-Schmidt expansion (which the students now know in its more general form from section 1)
- Underdetermined and overdetermined linear equations
- Projections and the projection operator (also seen before as an example of a linear operator on a Hilbert space)
- Existence of eigenvalues, eigenvectors and generalized eigenvectors
- Spectral decomposition and invariance
- The Singular Value Decomposition and its application to non-square matrix operators

Here there is more overlap with material students may have seen as undergraduates. Nevertheless, it is often through a second exposure that real understanding emerges, and the concepts from section 1 help students to see the bigger picture, e.g. that matrices are specific examples of linear operators, that matrix eigenvalues form the discrete spectrum of a bounded linear operator, etc. Again, emphasis is retained on proof and logical development.

3.) Variational Calculus. The method of first variations is taught using notes derived from several excellent paperback works on the subject[3,12,15]. The variational operator is examined in the context of optimization, and then used in conjunction with Hamilton's principle to derive the ordinary and partial differential equations for various linear and nonlinear dynamical systems, including electrical and mechanical examples. Careful attention is paid to the details, e.g. the existence of the variational operator, its boundedness, continuity issues, and boundary conditions.

4.) Applications of Graph Theory. This might not seem an obvious choice for an applied mathematics survey course, but in fact the fundamentals of graph theory, in conjunction with operator theory, bring into focus a very unified and generalized view of what previously seemed unrelated. Now it is understood that finding the steady-state dynamic response of a circuit and the vibrational modes of a bridge are essentially the same problem (see Figure 1); that the methods of Galerkin and Finite Elements are immediately derived from the concepts of inner products on infinite-dimensional Hilbert spaces; and that the natural laws of energy and thermodynamics often frame exactly the needed constraints to solve an underdetermined set of linear equations. Class notes in this section are derived from various sources, notably the first three chapters of *Introduction to Applied Mathematics*[14], by Gil Strang. (Students are encouraged to purchase this text.)

Three important threads are woven throughout all sections of the course.

Proof. Proof technique and proof standards are, in the author's experience, an area of extreme deficiency for new graduate students. Certainly it is unlikely that any graduate's career will involve "doing proofs;" however, it requires no great leap of imagination to foresee that sound reasoning and logical argumentation (often of a highly technical nature) can only be beneficial for students' future scholastic and professional occupations. A great deal of effort is expended in the course on standards, especially the difference between "show" and "prove." Particularly troublesome are the notions of proof by induction, use of the contrapositive, and proper use of contradiction. Class notes are supplemented with handouts and internet URLs about proof.

Modeling. Engineering educators have long recognized the need for continuous exposure to modeling techniques[7]. "Modeling" is somewhat of an amorphous term and as difficult to teach as it is to define. However, in the context of EGR5302, modeling simply means to associate the mathematical material with a non-mathematical context: a graph network with its associated electrical circuit; a partial linear differential equation with a beam or string; the idea of isometric transformations with examples of lossy and lossless compression algorithms. In the analysis section, the concepts of metrics and norms support some particularly interesting discussion as various examples are examined to see whether they actually meet the mathematical requires of metrics and norms: the Weight Watchers ® "point" system; credit scores (based on estimates of the Fair Isaac Corporation's proprietary algorithms); common indexes from finance such as the Relative Strength Index (RSI) and the Money Flow Index (MFI); and of course the relation of norms and metrics to common engineering measurement problems. Modeling problems also serve the important purpose of linking abstract mathematical concepts to concrete physical systems and phenomena.

Mathematical Literacy. Lastly, there is also a tertiary objective, to develop what is sometimes termed "mathematical literacy." The term suggests qualitative characteristics that indicate progress, such as the ability to accurately use common mathematical phrases and terms ("Because this matrix is not full rank, we cannot calculate a pseudoinverse directly but must first diagonalize to find the non-zero singular values..."); the ability to read and write the kind of mathematical notation that is common to advanced study, and a general comfort level with mathematically-intensive literature. (Sometimes research articles are introduced into the homework, such as an assignment to reproduce in MATLAB the algorithm of J. Cadzow on minimum infinity-norm solutions to underdetermined matrix equalities.)

From an organizational point of view, the class is lectured approximately 150 minutes per week, with a homework assignment due almost every period. The speed and intensity of the assignments result in what is essentially "immersion mathematics," with the purpose of inducing at least a marginal mathematical literacy in much the same way as language immersion is known to do. (Students jocularly refer to the class as "Adult Math" or "Grad Student Boot Camp," the latter of which also alludes to an immersion learning technique!)

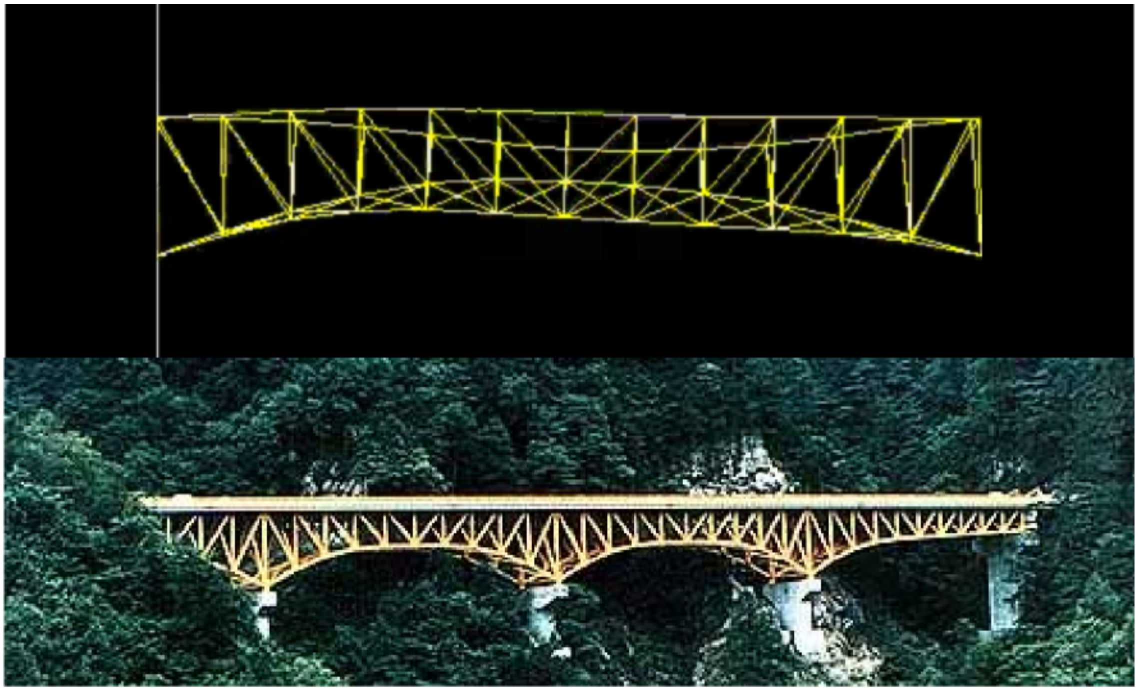

Figure 1. A homework assignment uses MATLAB to model the vibrational modes of the "Mameyaki II" open-spandrel truss bridge in Japan. Lower image from http://attila.sdsu.edu/~tubongba/mameyaki_truss.jpg.

Without extensive longitudinal studies, it is impossible to know the lasting impact of this kind of course. However, some qualitative feedback is available through student commentary.

- "I finally understand some things that I saw before but never understood." (A reference, presumably, to that student's undergraduate classes.)
- "This was the first time I actually learned WHY... these things work." (Discussing linear transforms.)
- "Now I can read research papers and understand them."

Shortcomings were also evident:

- "Make it two semesters!"
- "You can't assume we all know MATLAB." (A subject for another ASEE paper!)
- "At times the class was utterly demoralizing... [A] bit of lightening of the difficulty level would give... encouragement."

Conclusions and Developments

A course such as this is a great deal of work to teach, needless to say. And, it may be said, there is no guarantee that what is learned will be utilized in a given student's graduate study or career. However, the author would argue that the benefits of a high-intensity applied mathematics survey course transcend simply what is learned. In the author's opinion, sound methods of proof and argument are critically important in the academic and professional careers of engineers; lapses in this area can have very detrimental consequences. As well, there is value in seeing how mathematical topics that previously seemed disparate are in fact highly interconnected –

branches on a tree, as it were. And, comments about the demoralizing pace notwithstanding, with hard-won knowledge comes the confidence to tackle difficult problems, a quality all research advisors want to see in their students.

In closing, it should be acknowledged that many important topics must be left out of this type of course. Among these is any treatment of statistical and probabilistic methods; their omission in EGR5302 is intentional and rooted in the realization that probability and/or statistics cannot be fairly treated in a survey-style course. The argument could be made that these deserve an equivalent sort of course all to themselves.

By way of recent developments, in the summer of 2008 the mathematics department at Baylor approved a 2-course sequence in applied mathematics, designed with both mathematics and non-mathematics students in mind. The first course is much like EGR5302, focused on fundamentals, proof, and broadly applicable mathematical skills; the second course is computationally oriented, focused on the role of computers in applied mathematics. EGR5302 will likely no longer be taught, but it served its purpose at Baylor and provided a firm foundation for many graduate students to build their research agendas.

References

1. Brogan, William; Modern Control Theory, 3rd ed., Prentice-Hall, 1991.
2. Buechler, Dale; "Mathematical Background Versus Success in Electrical Engineering," Proc. ASEE Annual Conf. and Exp., June 2004.
3. Ewing, George; Calculus of Variations with Applications, Dover, 1985 (paperback).
4. Fadel, Goerges; Wiecek, Margaret; "Integration Through Optimization: An Interdisciplinary Graduate-Level Course on Engineering Design and Mathematical Optimization," 1 1th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conf., Sept. 2006, pp. 2493-2505.
5. Graves, Elton; "How to Get Engineers to Enroll in Mathematics Courses," Proc. ASEE Annual Conf. and Exp., July 2006.
6. Graves, Elton; "The Usefulness of Mathematics as Seen by Engineering Seniors," ASEE Annual Conf. and Exp., June 2005.
7. High, Karen; Maase, Eric; "Active Problem-Solving in a Graduate Course on Modeling and Numerical Methods," ASEE Annual Conf. and Exp., June 2007.
8. Kevrekidis, Ioannis; "Matrices Are Forever: On Applied Mathematics and Computing In Chemical Engineering," Proc. Symp. Frontiers of Chemical Engineering Science, vol. 50, no. 24(B), March 1995, pp. 4005-4025.
9. Kreyszig, Erwin; Introductory Functional Analysis with Applications, Wiley & Sons, 1989.
10. Pena, Daniel; "Teaching Time Series to Graduate Engineering Students," Int. J. Continuing Engineering Education, vol. 4, 1994, pp. 43-49.
11. Petzold, L.R.; "Graduate Education in Computational Science and Engineering," SIAM Review, vol. 43, no. 1, March 2001, pp. 163-177.
12. Sagan, Hans; Introduction to the Calculus of Variations, Dover, 1992 (paperback).
13. Siegenthaler, Kenneth; "Advanced Mathematics Preparation for Graduate School of Undergraduate Science and Engineering Students," Proc. ASEE Annual Conf. and Exp., 2004, Session 3565.
14. Strang, Gil; Introduction to Applied Mathematics, Wellesley-Cambridge, 1986.
15. Weinstock, Robert; Calculus of Variations with Applications to Physics and Engineering, Dover, 1974 (paperback).

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