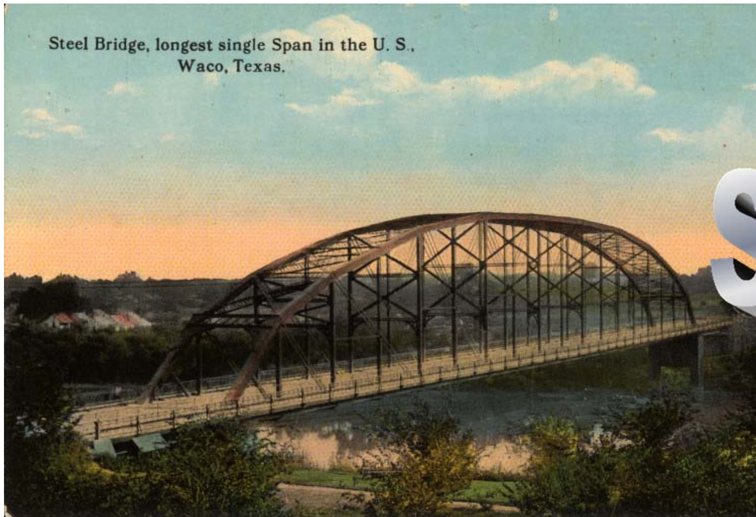

STATICS

EGR 1301
Introduction to Static Analysis

Presentation adapted from
Distance Learning / Online Instructional Presentation
Originally created by Mr. Dick Campbell

Presented by:
Departments of Engineering Baylor University

Baylor University
Engineering & Computer Science

Outline

- Understand the definition of Mechanics
- Learn the difference between static and dynamic analysis
- Understand the concept of force as a vector
- Separate vector into x and y components.
- Apply this concept to analyzing sums of forces ($\Sigma F=0$), as well as sum of moments ($\Sigma M=0$).
- Understand the importance of engineering analysis design.
- Apply these concepts to a foot bridge, wall hanging, and even biomechanics

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Mechanics

- **<u>Mechanics</u> -** the study of objects at rest or in motion, the effects of forces on a body, and the prediction motion.
- The fundamentals of Mechanics were formulated by Isaac Newton, using his three Laws:
  1. A body at rest or in constant motion remains in that state until acted upon by an external unopposed force.
  2. An unopposed force causes a mass to accelerate.
  3. Every force action has an equal and opposite reaction.

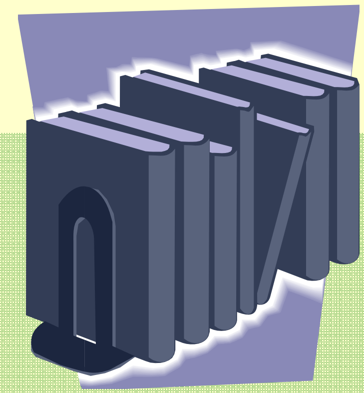
Mechanics

- Mechanics is divided into the study of Statics and Dynamics.

- Newton's 2nd Law is expressed as:

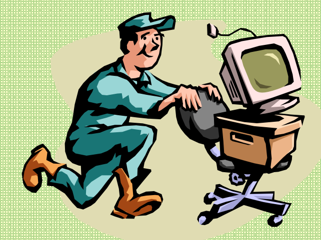
$$\sum F = ma$$

Statics vs. Dynamics



Static analysis $\sum F = 0$

- Therefore...$a = 0$
- In Statics, nothing is accelerating!
  - Newton's 1st Law
- Statics is the study of forces acting on a non-accelerating body, and the reaction of that body



Dynamic Analysis

- If an unopposed force acts: $\sum F \neq 0$
- Acceleration is proportional to the mass of the body
- Dynamics is the study of the motion of a body, both in translation and rotation.

The Example of Engineering Analysis

- Baylor Engineers in Africa, May 2005
  - 3 professors, 6 students
  - Kenya, Africa
  - Engineering services to Kenya’s poor population
- Foot Bridge Project
  - 40-m wide river
  - Analyze for safety and possible design improvements

Need For Bridge

- Village was divided, far side had trouble:
  - Taking their farm produce to market
  - Attending school
  - Getting medical care
- Situation
  - 5 miles to nearest bridge (20 mile round trip)
  - Several drownings per year

Need For Bridge

- Estimates of approximately 400 crossings per day
  - Saving 1,460,000 miles of walking per year
- Approximate cost: $5000
  - 1/3 ¢ per mile per year
  - Great impact at minimal cost

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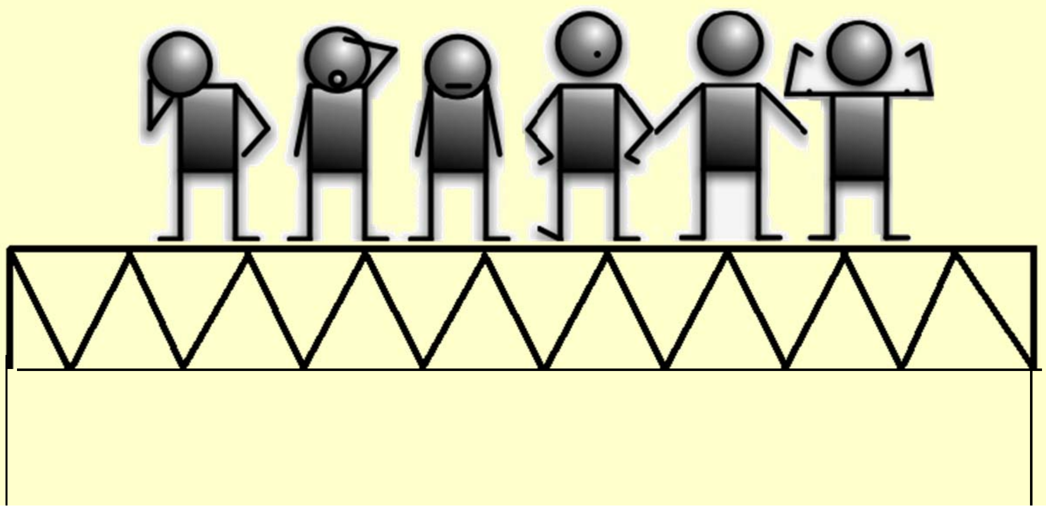
How Does Engineering Analysis Help?

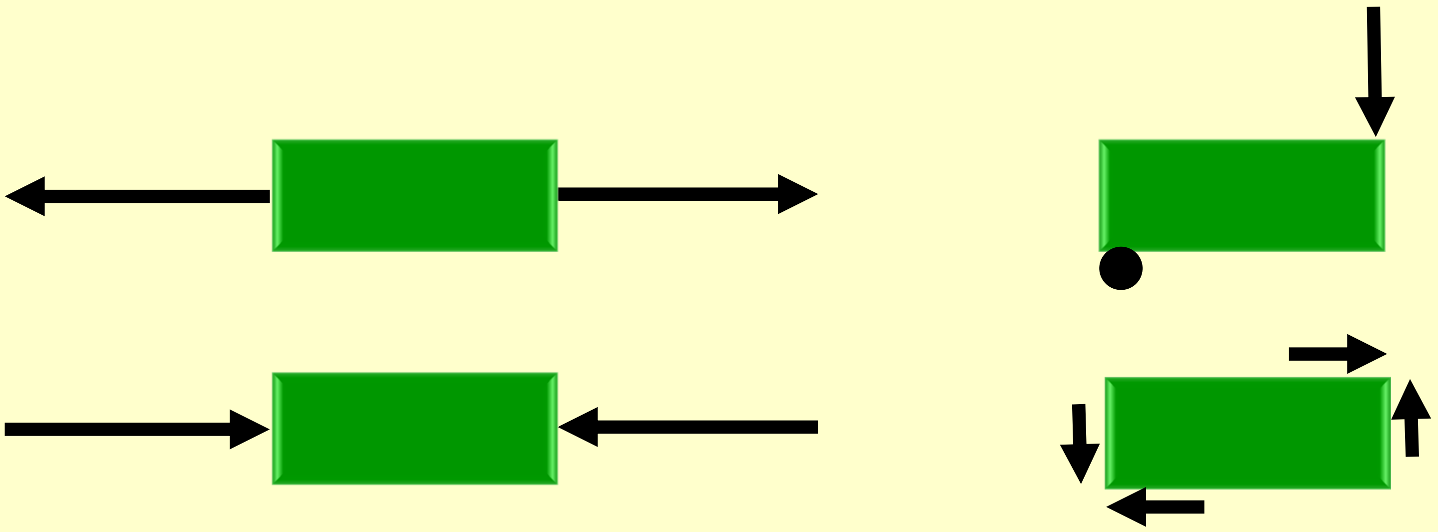
- Cable used is rated to withstand a maximum load in tension of 16,000 lbs.
  - How much can the bridge support?
  - How is cable failure considered in the design?
  - If six people (est. 1000 lbs.) stand in the center, what is the cable tension?
  - What is the "safety factor" (SF)?

$$S.F. = \frac{Rating}{Load}$$



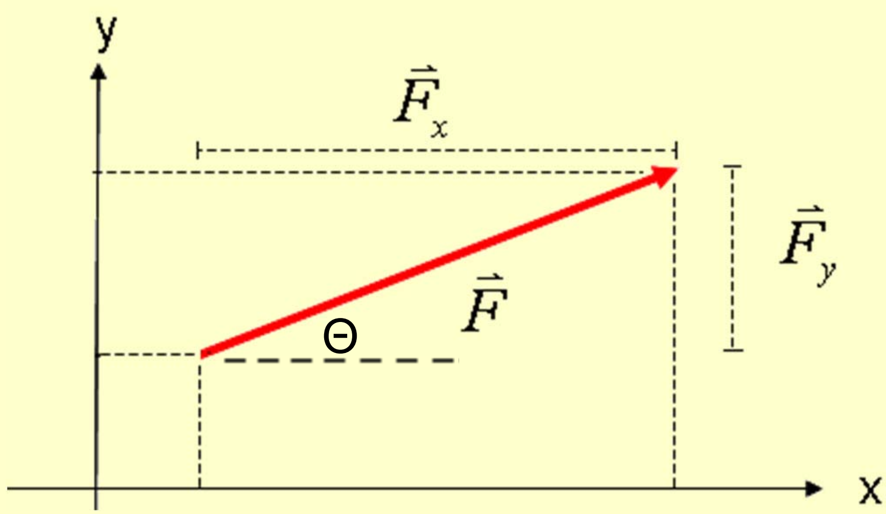
Types of Loading

- Tension – pull apart
- Compression – push together
- Moment – rotation
- Shear – distort shape



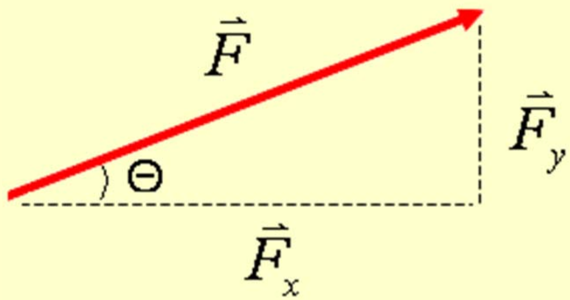
Force as a Vector

- Scalar
  - Has magnitude only (i.e. it's "just a number")
- Vector
  - Has magnitude *and* direction
  - In an x-y coordinate system, force may be broken down into "components"
    - X-component parallel to x-axis
    - Y-component parallel to y-axis



Force as a Vector

- Use Trigonometry to:
  - Calculate the magnitude and angle of the vector from the magnitude of its components (Pythagorean theorem & inverse tangent)
  - Calculate the magnitude of the components from the magnitude and angle of the vector (sine and cosine)



$$\Theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\left|\vec{F}\right| = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F * \sin(\Theta)$$

$$F_x = F * \cos(\Theta)$$

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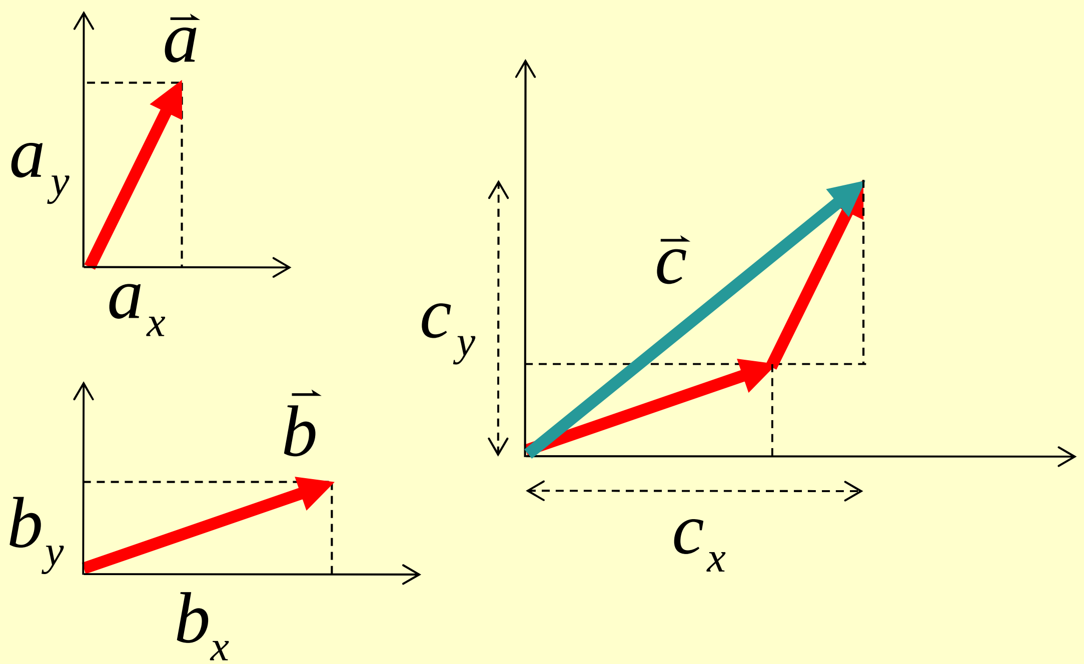
Adding Vectors

- Addition can be interpreted:
  - Graphically: move the “start” of **b** to the “tail” of **a**
  - Mathematically: add the components that lie in the same direction



$$\vec{c} = \vec{a} + \vec{b}$$

$$c_x = a_x + b_x$$

$$c_y = a_y + b_y$$

Figure 2.6

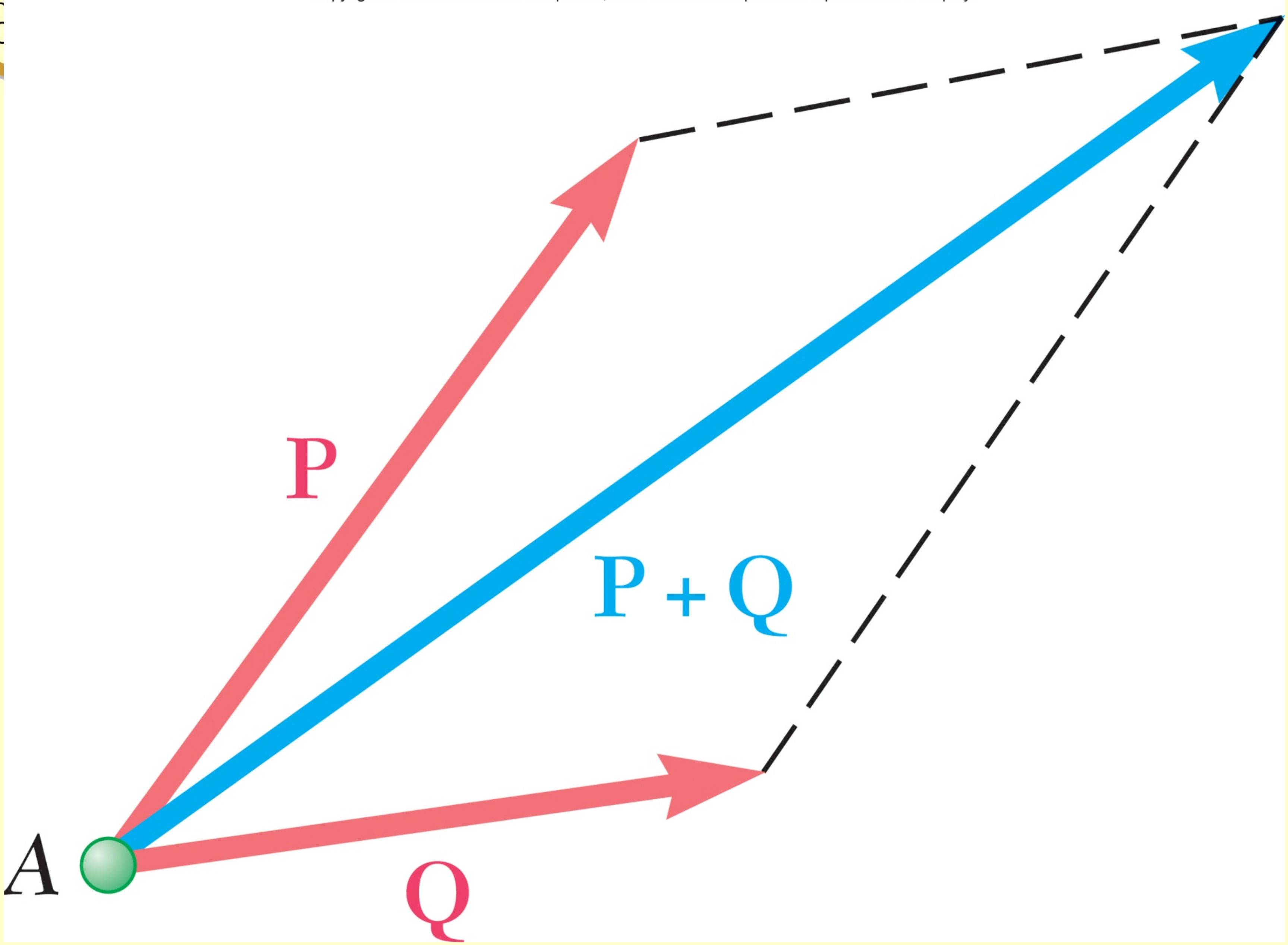
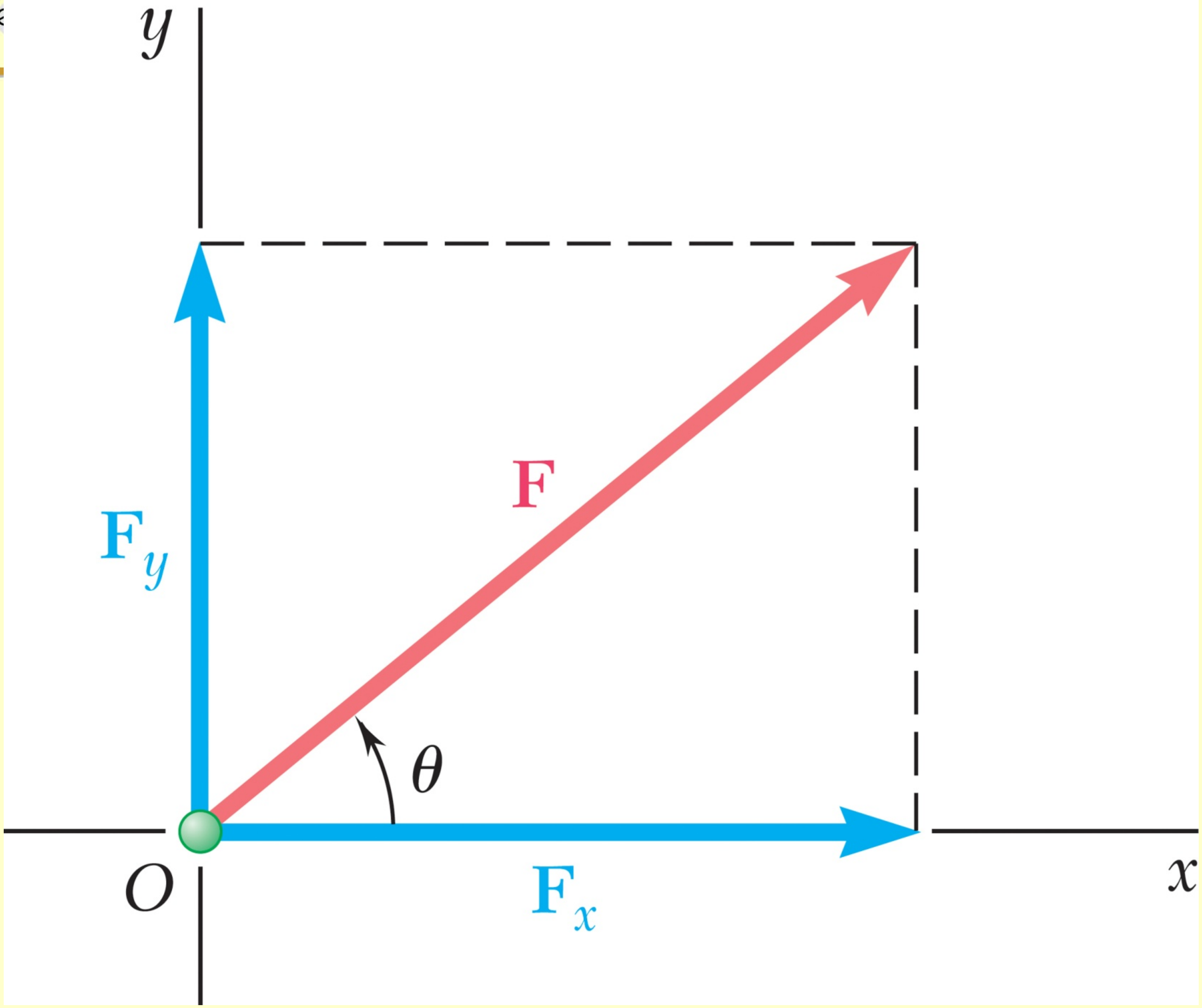
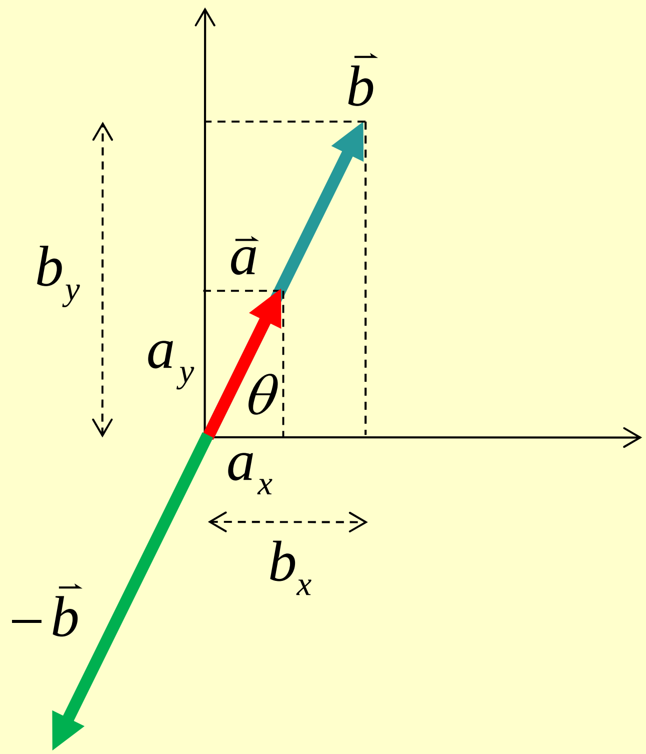


Figure 2.18



Multiplying a Vector by a Scalar

- Multiplying by a scalar means:
  - Changing the magnitude of the vector
  - But not changing direction
  - Except for negative numbers



$$\vec{b} = c \cdot \vec{a}$$

where c is a scalar
(i.e. "just a number")

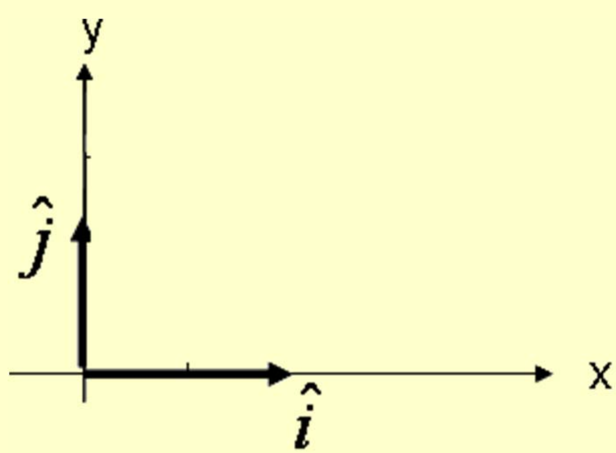
$$\left|\vec{b}\right| = c \cdot \left|\vec{a}\right|$$

$$b_x = c \cdot |a| \cdot \cos(\theta)$$

$$b_y = c \cdot |a| \cdot \sin(\theta)$$

Unit Vectors

- î and ĵ indicate direction of vector components
  - î has magnitude of 1 unit in the x-direction
  - ĵ has magnitude of 1 unit in the y-direction
- When a vector component is multiplied by î or ĵ
  - Magnitude of the vector component remains unchanged
  - The direction of the vector component is defined as parallel to the x- or y-axis

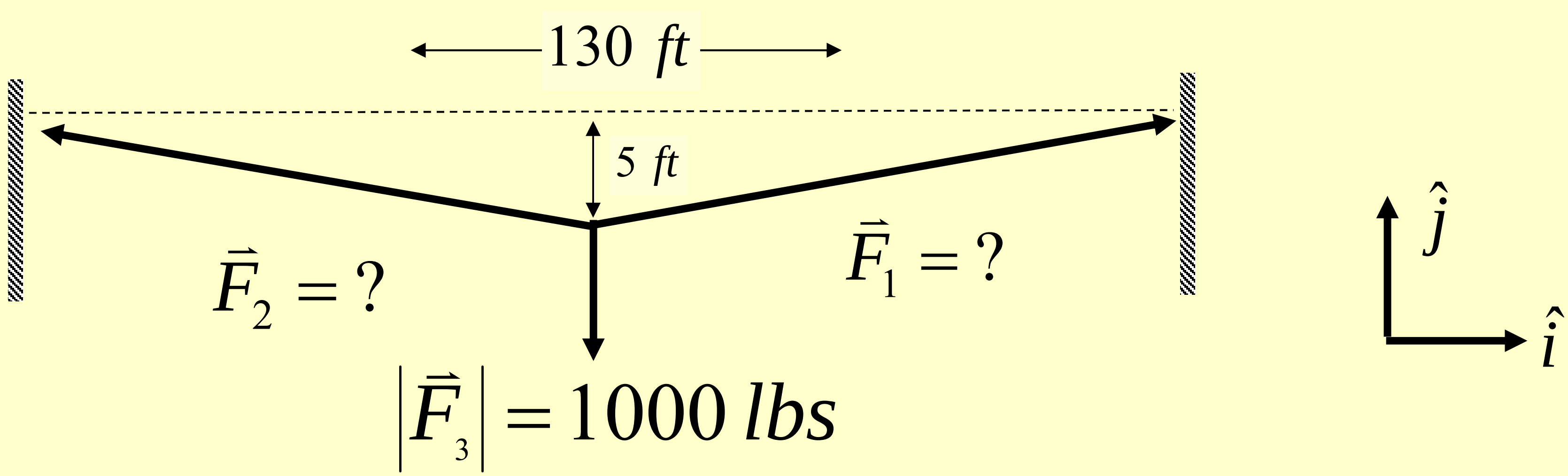


Can now write a vector as:

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

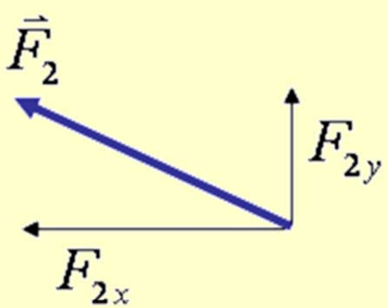
Force Vectors & Static Analysis of Foot Bridge

- Simplifying assumptions
  - Loaded with six persons (approx. 1000 lbs) at the center
  - Cables are straight
  - Neglect the weight of the bridge

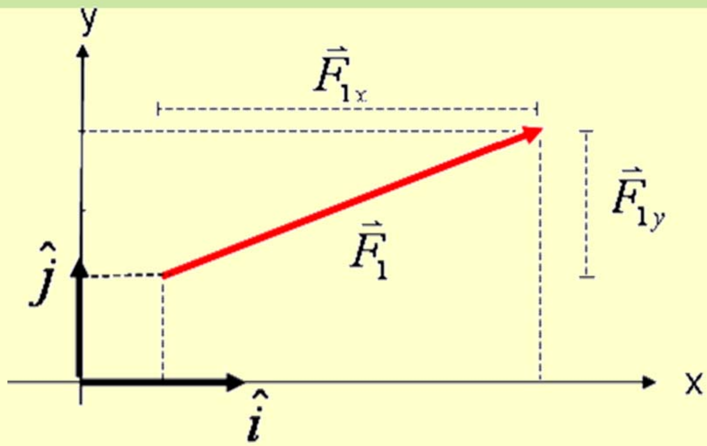


Setting Up the Analysis

- Draw a sketch of the forces as vectors
- Write each force vector in terms of *i* and *j* components
  - Components perpendicular to each other can be treated separately
- Invoke Newton's 1st Law
  - Sum of *i* components=zero (x direction)
  - Sum of *j* components = zero (y direction)
- Solve the two equations with two unknowns



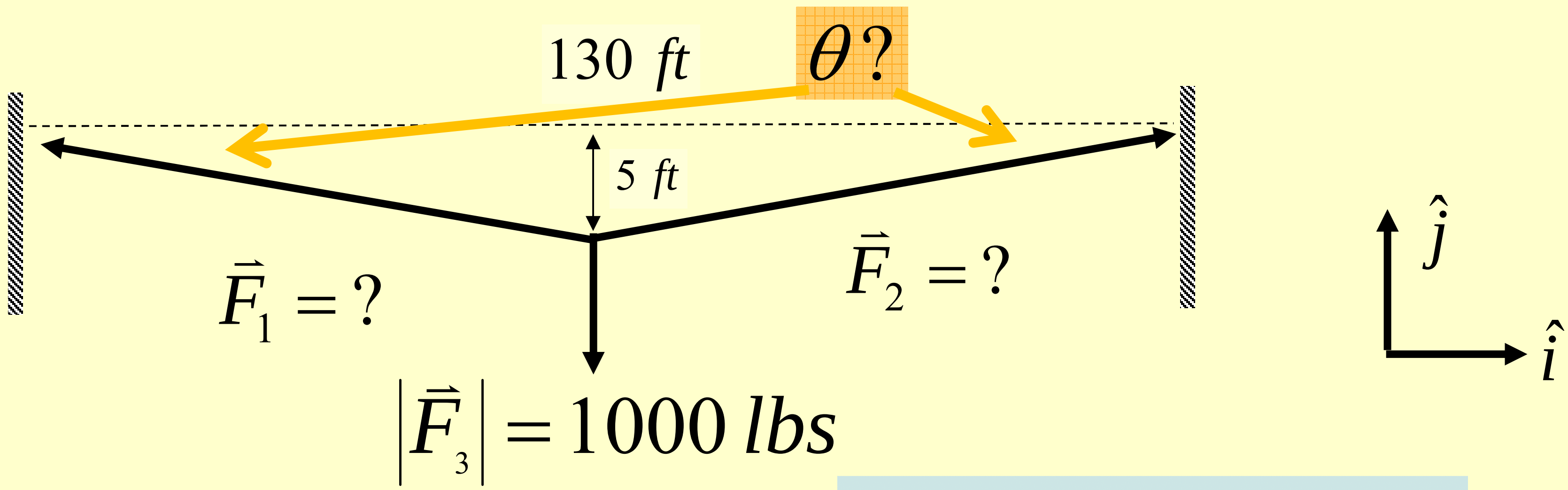
$$\vec{F}_2 = F_{2x}\hat{i} + F_{2y}\hat{j}$$



$$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j}$$

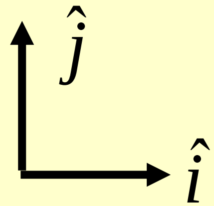
Static Analysis of the Foot Bridge's Cable

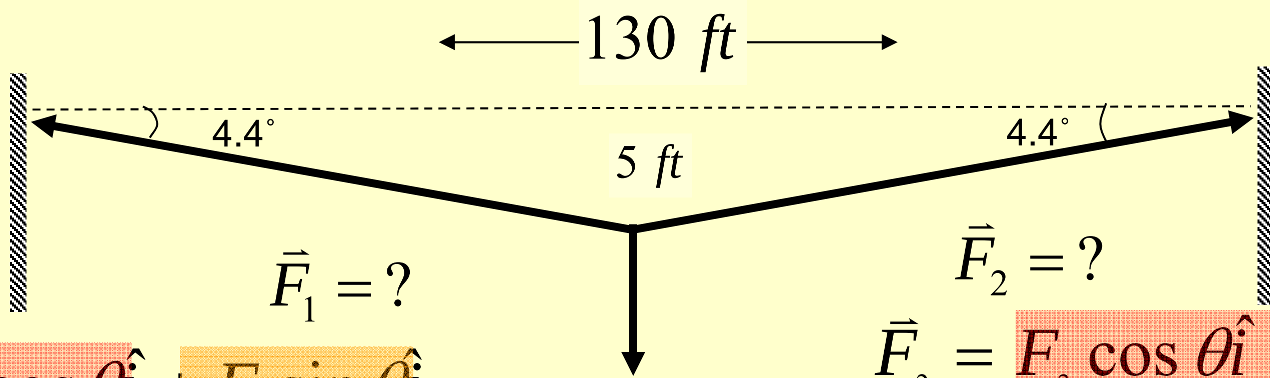
- Resolve the three forces into î, ĵ components
- Identify our unknowns (i.e., F_1 and F_2)
- Set up summation equations in î, ĵ directions
  - Solve these two equations for the unknowns



$$\theta = (\tan^{-1} \tfrac{5}{65}) = 4.4°$$

Static Analysis of the Foot Bridge's Cable





$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$

$$\vec{F}_1 = -F_1 \cos\theta \hat{i} + F_1 \sin\theta \hat{j}$$

$$\vec{F}_2 = F_2 \cos\theta \hat{i} + F_2 \sin\theta \hat{j}$$

$$\left|\vec{F}_3\right| = 1000\, lbs$$

$$\vec{F}_3 = 0 \cdot lbs \cdot \hat{i} - 1000 \cdot lbs \cdot \hat{j}$$

$$\Sigma F\hat{i} = -F_1 \cos\theta \hat{i} + F_2 \cos\theta \hat{i} + 0 \cdot lbs \cdot \hat{i} = 0$$

$$\Sigma F\hat{j} = F_1 \sin\theta \hat{j} + F_2 \sin\theta \hat{j} - 1000 \cdot lbs \cdot \hat{j} = 0$$

Static Analysis of the Foot Bridge's Cable

$$\Sigma F\hat{i} = -F_1 \cos\theta\hat{i} + F_2 \cos\theta\hat{i} + 0 \cdot lbs \cdot \hat{i} = 0$$
$$\Sigma F\hat{j} = F_1 \sin\theta\hat{j} + F_2 \sin\theta\hat{j} - 1000 \cdot lbs \cdot \hat{j} = 0$$

Rearranging and substituting in angles:

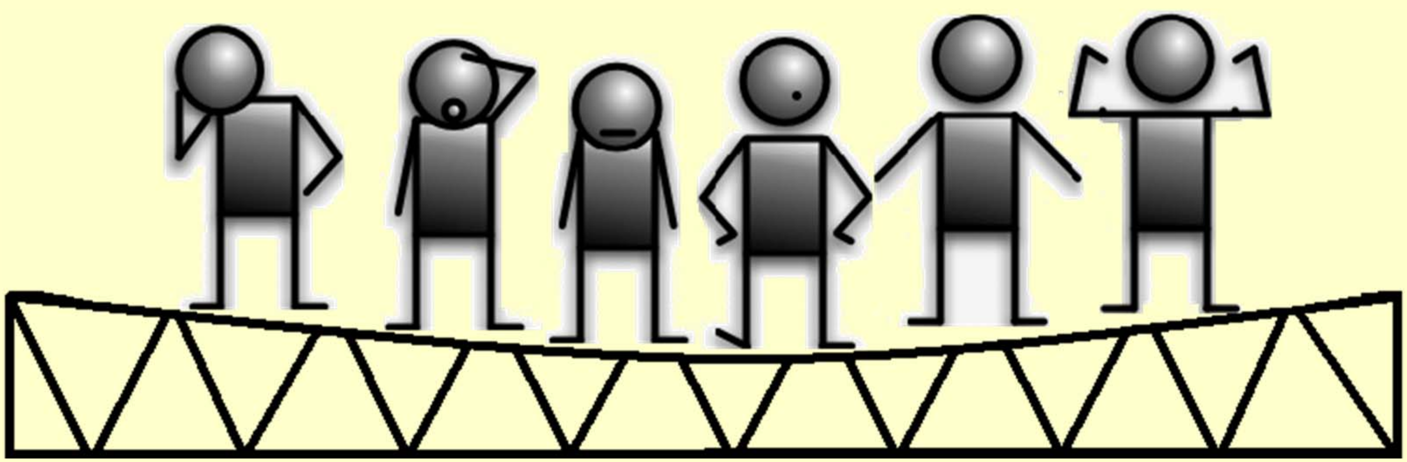
$$F_1 \cos(4.4) = F_2 \cos(4.4), \qquad F_1 = F_2$$
$$F_1 \sin(4.4) + F_2 \sin(4.4) = 1000 \cdot lbs$$
$$F_1 = F_2 = 6527 \cdot \text{lbs}$$

Safety Factor (Cable Strength = 16,000 lbs):

$$S.F. = \frac{16000 \cdot \text{lbs}}{6527 \cdot \text{lbs}} = 2.45$$

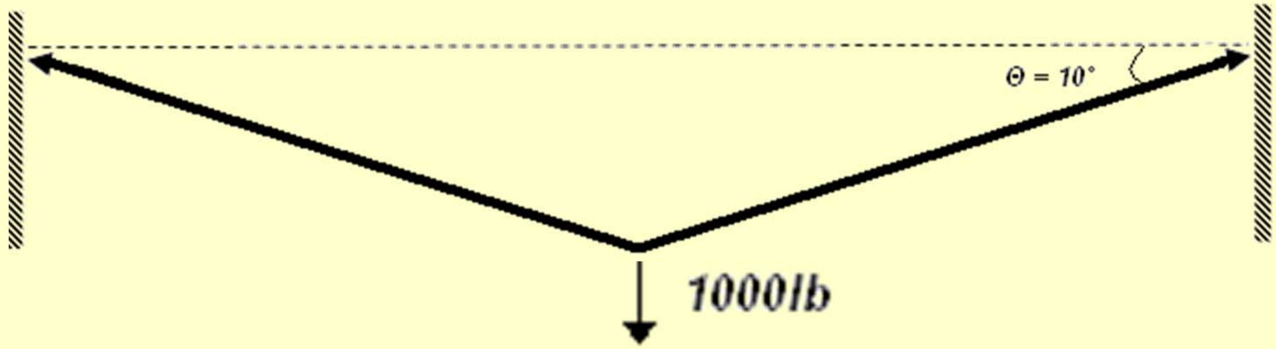
Engineering is an Exercise in Trade-Offs!

- If we allow the bridge to have larger sag, what will happen to the tension in the cables?
- What will happen to the required anchors for the cables at the two ends of the bridge?
- What is the disadvantage of having larger sag in the bridge?



Static Analysis of the Foot Bridge's Cable

What happens to the Safety Factor if we increase the sag? Let : $\theta = 10^o$



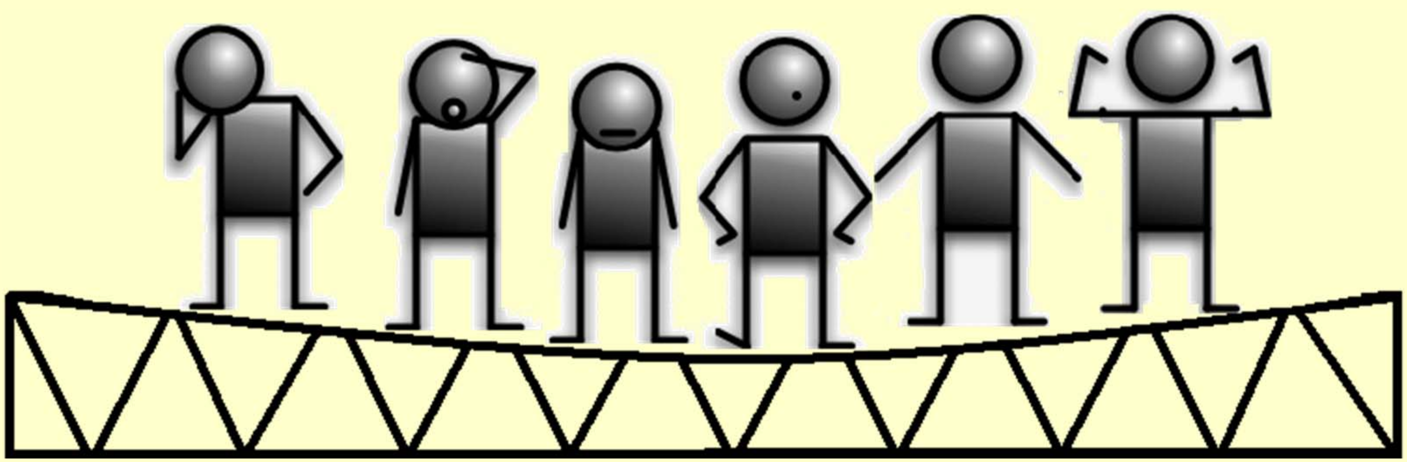
Then: $2F_1 \sin(10) = 1000lbs$

$$F_1 = F_2 = 2879\ lbs$$

$$S.F. = \frac{16000\ \text{lbs}}{2879\ \text{lbs}} = 5.56$$

Engineering is an Estimate

- What did our model (sketch) not take into account?
- How will that affect our calculations, in particular, the safety factor?
- Does our safety factor (2.45) now seem too low, too high, or about right?
- How do you determine what constitutes a “good” safety factor?



What About the Weight of the Bridge?

- The wood decking was built using 3-1"x 6" planks laid side by side.
  - The bridge span is 130 feet.
  - The wood's density is 30 lbs/ft^3.
  - The weight of the cable and hangers is approx. 112 lbs.
  - Can you calculate the weight of the decking?
- What does this do to our safety factor?

$$\rho = \frac{m}{V}$$

$$V = (3)(1 \cdot in)(6 \cdot in)\left(\frac{1 \cdot ft}{12 \cdot in}\right)^2 (130 \cdot ft) = 16.25 \cdot ft^3$$

$$m = \rho V = \left(\frac{30 \cdot lbs}{ft^3}\right)(16.25 \cdot ft^3) = 488 \cdot lbs$$

$$488 \cdot lbs + 112 \cdot lbs = 600 \cdot lbs$$

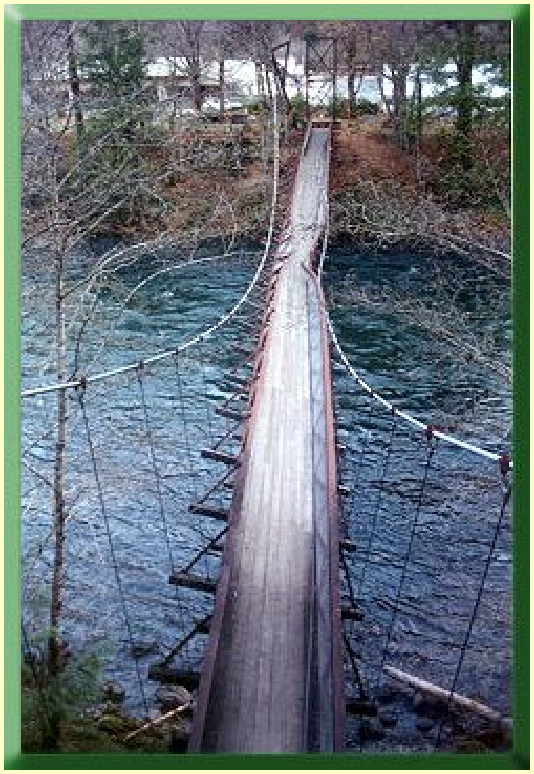
Static Analysis of the Foot Bridge's Cable

What happens to the Safety Factor if we include the weight of the bridge? Let: $\theta = 10^o$ and $W = 1000 + 600\text{lbs}$

Then: $$2F_1 \sin(10) = 1600 \cdot lbs$$

$$F_1 = F_2 = 4607 \cdot \text{lbs}$$

$$S.F. = \frac{16000 \cdot \text{lbs}}{4607 \cdot \text{lbs}} = 3.47$$



Suspension Bridges

- How does a suspension bridge like the Golden Gate provide a way to achieve the goals of lower cost and convenience of use?

- What are the disadvantages in the suspension bridge design?

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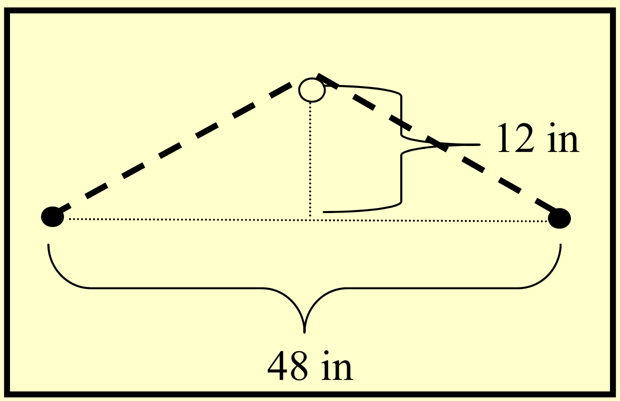
Engineering Disasters:
Tacoma Narrows Bridge

http://archive.org/details/SF121

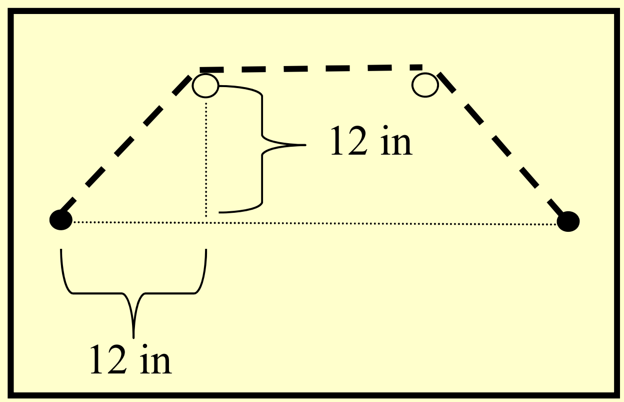
Picture Frame Sample Problem

What is the best way to hang a 40 lb mirror on a wall? How much tension is in the hanger wire in each case?

Option 1: One Hanger in Wall

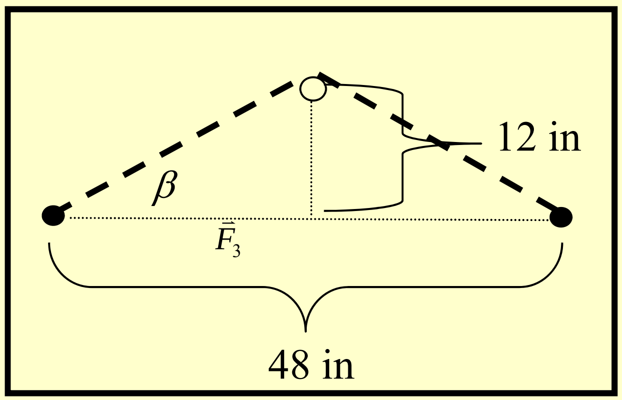


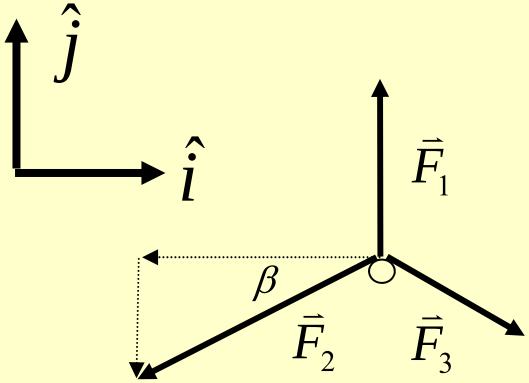
Option 2: Two Hangers in Wall



Picture Frame Sample Problem

Option 1: One Hanger in Wall





$$\vec{F}_1 = 0 \cdot lbs \cdot \hat{i} + 40 \cdot lbs \cdot \hat{j}$$

$$\beta = \tan^{-1}\left(\frac{12}{24}\right) = 26.6^\circ$$

$$\vec{F}_2 = -F_2 \cdot \cos\beta \cdot \hat{i} - F_2 \cdot \sin\beta \cdot \hat{j}$$

$$\vec{F}_3 = F_3 \cdot \cos\beta \cdot \hat{i} - F_3 \cdot \sin\beta \cdot \hat{j}$$

$$\sum F_{\hat{i}} = 0 - F_2 \cdot \cos\beta + F_3 \cdot \cos\beta = 0$$

$$F_2 = F_3$$

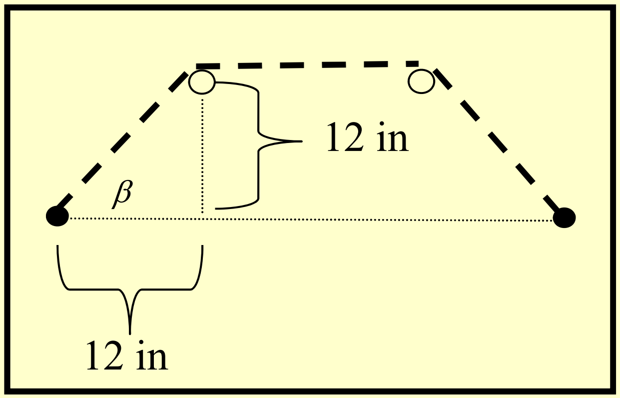
$$\sum F_{\hat{j}} = 40 \cdot lbs - F_2 \cdot \sin\beta - F_3 \cdot \sin\beta = 0$$

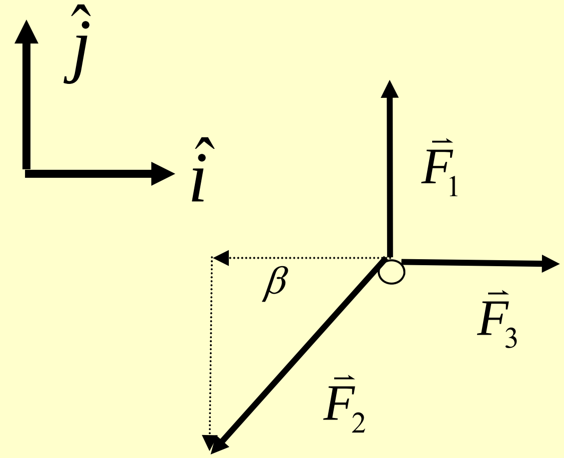
$$40 \cdot lbs = 2F_2 \cdot \sin\beta$$

$$F_2 = F_3 = \frac{40 \cdot lbs}{2 \cdot \sin(26.6)} = 44.7 \cdot lbs$$

Picture Frame Sample Problem

Option 2: Two Hangers in Wall





$$\vec{F}_1 = 0 \cdot lbs \cdot \hat{i} + 20 \cdot lbs \cdot \hat{j}$$

$$\beta = \tan^{-1}\left(\frac{12}{12}\right) = 45^\circ$$

$$\vec{F}_2 = -F_2 \cdot \cos\beta \cdot \hat{i} - F_2 \cdot \sin\beta \cdot \hat{j}$$

$$\vec{F}_3 = F_3 \cdot \hat{i} + 0 \cdot \hat{j}$$

$$\sum F_{\hat{i}} = 0 - F_2 \cdot \cos\beta + F_3 = 0$$

$$F_2 \cdot \cos\beta = F_3$$

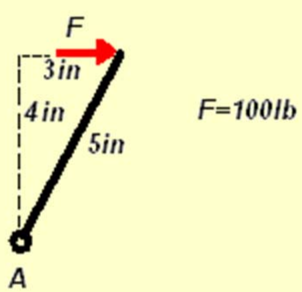
$$\sum F_{\hat{j}} = 20 \cdot lbs - F_2 \cdot \sin\beta = 0$$

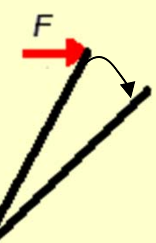
$$F_2 = \frac{20 \cdot lbs}{\sin\beta} = 28.3 \cdot lbs$$

$$F_3 = 28.3 \cdot lbs \cdot \cos\beta = 20.0 \cdot lbs$$

Moments

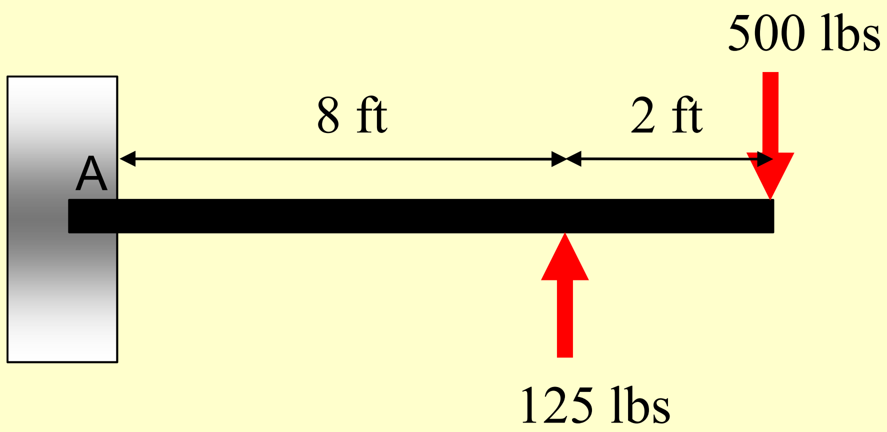
- Moments occur at a given point and are caused by a force that causes rotation about a point or an axis.
- A moment is equal to the Force times the perpendicular distance from the force to the point that is being evaluated.
- Force **F** causes a clockwise rotation if unopposed about point A in the picture below.
- 4in is the perpendicular distance and 100lb is the force in the picture, so the Moment **M** caused by this force is: ***M=-(100lb x 4in) = -400 lb-in***
- The moment is negative because it is clockwise.
- Be careful with units, Moments can be measured in lb-in, lb- ft, N-m, or other units.



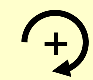


Moments Example 1

Calculate the moment at point "A".



Remember the sign convention for moments:



M_1 = -(8 ft)(125 lbs) = -1000 lb-ft

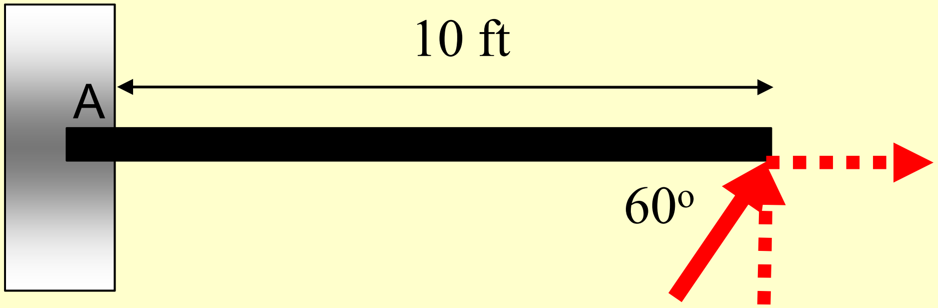
M_2 = +(10 ft)(500 lbs) = 5000 lb-ft

$M_A = M_1 + M_2$ = 4000 lb-ft

Moments Example 2

Calculate the moment at point "A".

Remember the sign convention for moments:



$$F_x = 200 \cdot lbs \cdot \cos(60) = 100 \cdot lbs$$

$$F_y = 200 \cdot lbs \cdot \sin(60) = 173.2 \cdot lbs$$

$$M_{Fx} = +(100 \cdot lbs)(0 \cdot ft) = 0 \cdot lbs \cdot ft$$

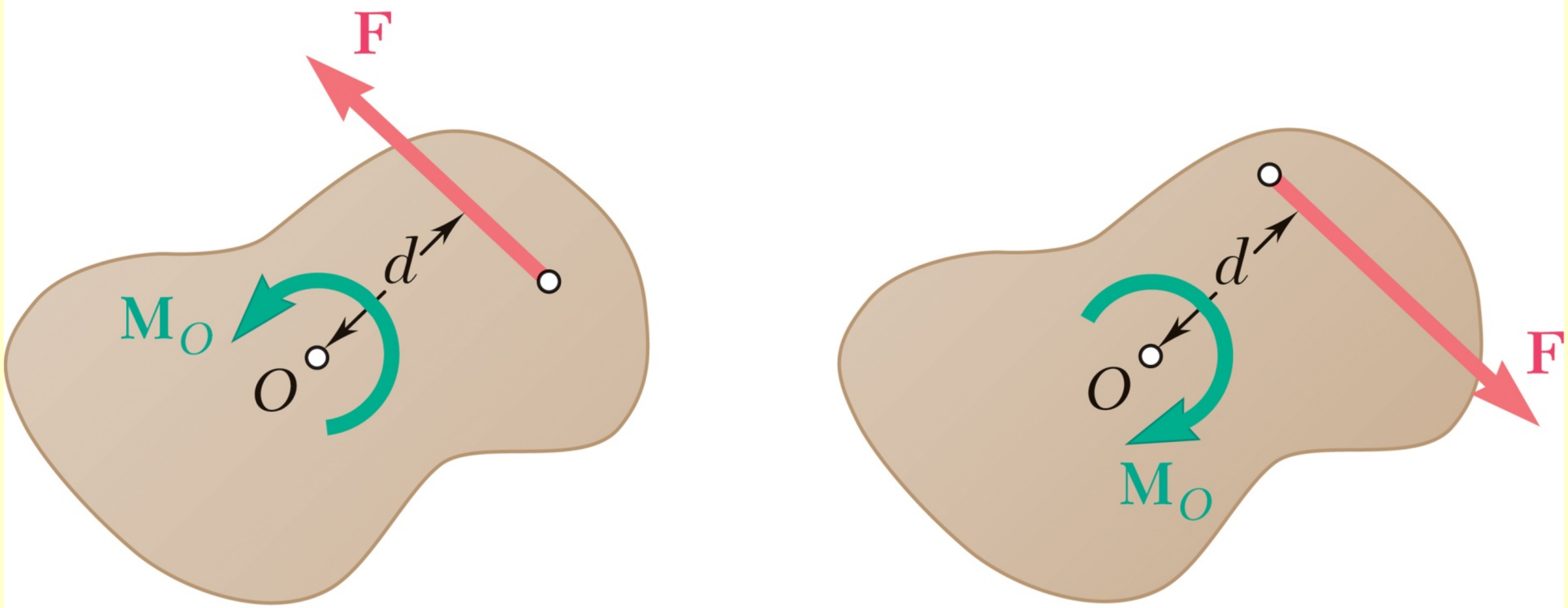
$$M_{Fy} = -(173.2 \cdot lbs)(10 \cdot ft) = -1732 \cdot lbs \cdot ft$$

$$M_A = -(173.2 \cdot lbs)(10 \cdot ft) + (100 \cdot lbs)(0 \cdot ft) = -1732 \cdot lbs \cdot ft$$

Figure 3.13

Moment of a Force About a Point

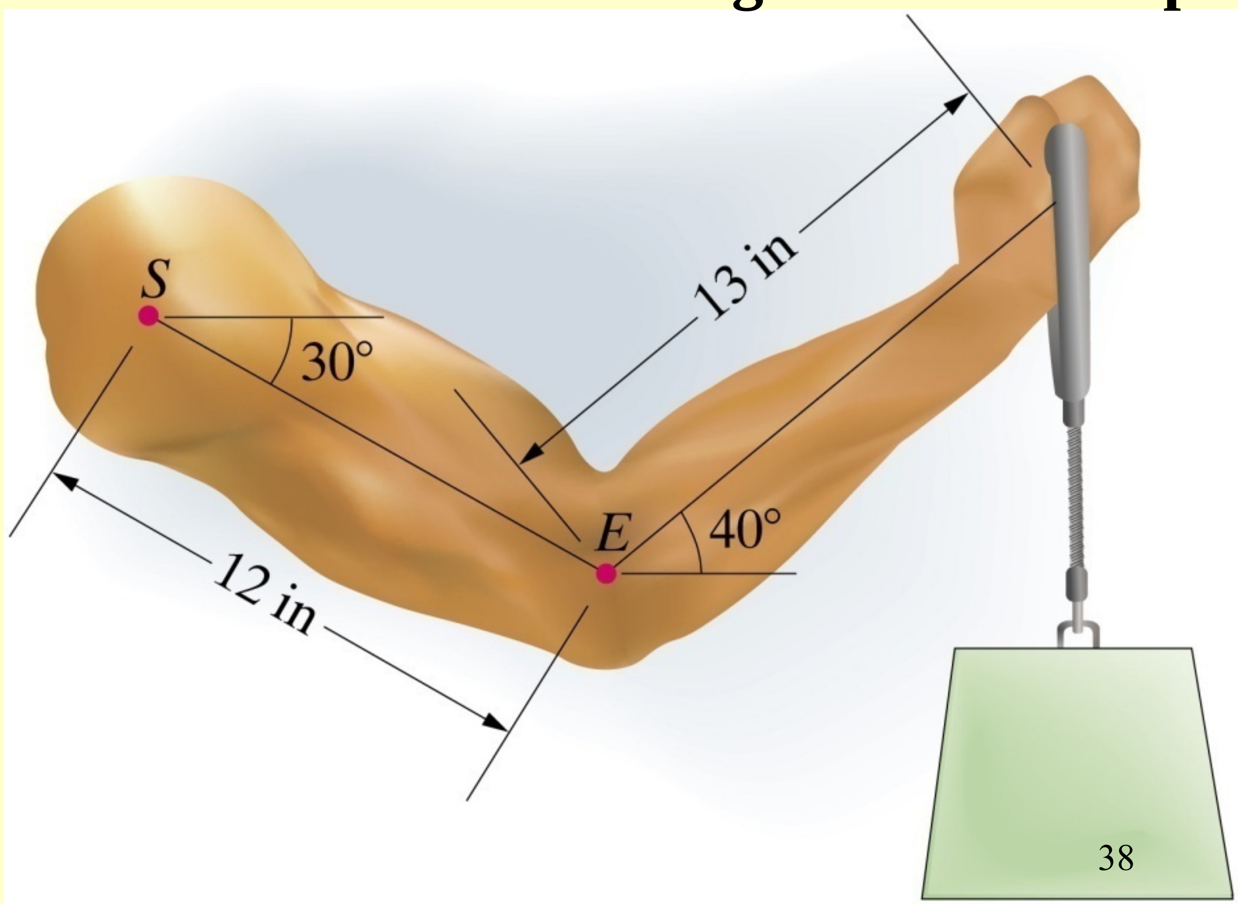
Product of a Force and the Perpendicular Distance to the Point



(a) $\mathbf{M}_O = +Fd$ (b) $\mathbf{M}_O = -Fd$

Example Problem

- **The moment exerted about point E by the weight is 300 lb-in. What moment does the weight exert about point S?**



Equilibrium of A Particle

$$\sum \vec{F} = 0$$

$$\sum \overrightarrow{F_x} = 0$$

$$\sum \overrightarrow{F_y} = 0$$

$$\sum M_O = 0$$