Problem 2. The positive-sequence one-line diagram for a network is shown below. The ground ties at busses 1 and 4 represent the subtransient impedances of machines. Prefault voltages are all 1.0pu.

a. Use the definition \( z_{jk} = \frac{\partial v_j}{\partial i_k} \) \( I_k \) \( I_m = 0, m \neq k \) to fill in column 1 of the \( Z \) matrix.

Now, a solidly-grounded three-phase fault occurs at bus 1.

b. Compute the fault current

c. Use the fault current and \( Z \) matrix terms to compute the voltages at busses 2 and 3.

d. Find the magnitude of the current flowing in the line connecting busses 2 and 3.

\[
V_2 = V_3 + j0.12 \left( \frac{0.167}{2} \right) = j0.0334
\]

\[
I_2 = \frac{V_2}{Z_{11} - j0.05} = \frac{1}{j0.0417} = \frac{1}{j0.0417} = j2.40A
\]

\[
V_{za} = V_{za} - Z_{21} I_{1a} = 1.0 - (j0.0334)(-j2.4) = 1.0 - 0.800 = 0.200 V
\]

\[
V_{3a} = V_{3a} - Z_{31} I_{1a} = 1.0 - (j0.0167)(-j2.4) = 1.0 - 0.401 = 0.600 V
\]

\[
\frac{V_2 - V_3}{j0.2} = \frac{0.2 - 0.6}{j0.2} = j2.0
\]
A 30MVA, 12kV generator is connected to a delta-grounded wye transformer. The generator and transformer are isolated and not connected to a "power grid." Impedances are given on equipment bases.

A single-phase to ground fault, with zero impedance, suddenly appears on phase a of the 69kV transformer terminal. Find the resulting a-b-c generator currents (magnitude in amperes and phase). Regarding reference angle, assume that the pre-fault phase a voltage on the transformer's 69kV bus has angle \( \theta = 0 \).

\[
I_B = \frac{30 \times 10^6}{\frac{12 \times 10^3}{\sqrt{3}}} = \frac{10 \times 10^6}{6928} = 1443A
\]

Generator is connected GY through a \( j0.5 \) ohm grounding reactor

\[
Z_B = \frac{j(\beta)}{30} = \frac{144}{30} = 4.8\Omega
\]

So, \( j0.5\Omega = j0.1042\Omega \)

For \( + \) sequence, \( \ominus \) sequence like \( + \), but source voltage is zero.

\[
\begin{align*}
Z_{1z} &= j(0.18 + 0.05) = j0.23 \\
Z_{2z} &= \frac{2V_1}{jI_z} = j0.18I_z = j0.18 \\
\end{align*}
\]

For \( \ominus \) sequence

\[
Z^{(\ominus)} = j0.05, \quad Z^{(\ominus)}_{1z} = 0
\]

Now, for the fault current,

\[
I_{F}^{Za} = \frac{3V_{za}}{Z^{(\ominus)} + Z^{(\ominus)}_{2z} + Z^{(\ominus)}_{1z} + 3Z_{F}^{\infty}} = \frac{3/0}{j0.05 + 2j(0.23)} = -j5.88\text{pu}
\]

For a single phase fault, we know that

\[
I_{F}^{K0} = I_{F}^{K1} = I_{F}^{K2} = \frac{1}{3} I_{F}^{K_a_1}. \quad \text{So} \quad I_{F}^{Z0} = I_{F}^{Z1} = I_{F}^{Z2} = \frac{-j5.88}{3} = -j1.960\text{pu}
\]
Test 9, 11/5/04, continued

Now, $\Theta$ and $\Omega$ sequence currents are the same at the generator (except for phase shift) because we have a simple series path. Furthermore, the zero sequence current does not flow through the transformer. Thus, on the generator side, we have ANSI shifts

$$I_{gen,1}^F = -j1.960 \left( \frac{1}{-30^\circ} \right) = 1.960 / -120^\circ$$
$$I_{gen,2}^F = -j1.960 \left( \frac{1}{+30^\circ} \right) = 1.960 / -60^\circ$$
$$I_{gen,0}^F = 0.$$

Converting to abc

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ 1.960 / -120^\circ \\ 1.960 / -60^\circ \end{bmatrix} = \begin{bmatrix} 1.960 / -120^\circ + 1.960 / -60^\circ \\ 1.960 / -120^\circ + 1.960 / -60^\circ \\ 1.960 / -120^\circ + 1.960 / -60^\circ \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = 1.960 \begin{bmatrix} 1 / -120^\circ + 1 / -60^\circ \\ 1 / -120^\circ + 1 / -60^\circ \\ 0 \end{bmatrix} = 1.960 \begin{bmatrix} \sqrt{3} / 400 \\ \sqrt{3} / 400 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -j3.39 \\ -j3.39 \\ 0 \end{bmatrix}.$$  In amps, $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -j4892 \\ -j4892 \\ 0 \end{bmatrix}$

Note: In the general case, you use the off-diagonal $\Omega$ elements. Doing the problem in the way...
\[ V_{1,0}^F = V_{1,0}^{\text{pre}} - Z_{12,0} I_{2,0}^F = 0 \]
\[ V_{1,1}^F = V_{1,1}^{\text{pre}} - Z_{12,1} I_{2,1}^F = 1.0 - j0.18 (-j1.960) \]
\[ = 1.0 - 0.353 = 0.647 \]
\[ V_{1,2}^F = V_{1,2}^{\text{pre}} - Z_{12,2} I_{2,2}^F = -j0.18 (-j1.960) = -0.353 \]

Then, using the network

You can see these currents match the previous ones. And so on,
Impedance already given on system base.

- Like above, but with voltage sources turned off (i.e., shorted through)

\[ Z_{\text{zz}}^+ = Z_{\text{zz}}^- = \left( j0.20 + j0.08 \right) || \left( j0.12 + j0.08 + j0.20 \right) = \left( j0.28 \right) || j0.40 = j0.1647 \text{pu} \]

\[ I_{zf} = \frac{120}{j0.1647} = -j6.07 \text{pu} \]

- \[ Z_{\text{zz}}^0 = j0.08 \] (there is no path to ground going right)

\[ I_{zf} = \frac{360}{Z_{\text{zz}}^0 + Z_{\text{zz}}^+ + Z_{\text{zz}}^- + 3Z_{\text{ff}}^0} = \frac{3}{j(0.08 + 0.1647 + 0.1647)} \]

\[ I_{zf} = \frac{3}{j0.409} = -j7.33 \text{pu} \]
One-line diagram for Prob. 6.15.

"Net +30°" is the positive sequence phase shift with respect to the faulted point.

Neg Seq. Same, except gen voltages are 0.
One-line diagram for Prob. 6.15.

For $Z_{44}$, turn off the sources (0/J0).

\[ Z_{44} = (j1.4) || (j0.1653 + j0.837) || (j1.207) \]
\[ = j1.4 || (j0.1653 + j0.494) = j1.4 || j0.659 \]

\[ Z_{44} = \boxed{j0.448} \]

Any \[ Z_{44} = \frac{V_I}{I_4} = \frac{V_4}{I_4} = Z_{44} \cdot \frac{V_4}{V_4} \]

\[ Z_{44} = Z_{44} \cdot \frac{V_1}{V_4} = j0.448 \left( \frac{j1.0}{j1.0 + j.0.4} \right) = \boxed{j0.320} \]
One-line diagram for Prob. 6.15.

\[
Z_{54} = Z_{44} \cdot \frac{V_5}{V_4} = Z_{44} \cdot \frac{j0.494}{j0.494 + j0.1653} = Z_{44} \cdot 0.749
\]

\[
Z_{54} = j0.448(0.749) = \boxed{j0.336}
\]

\[
I_{4,1}^F = \frac{V_{4,1}^{pre}}{Z_{44,1} + Z_{F}^0} = \frac{1/0}{j0.448 + 1/0} = \boxed{-j2.34/2.23}
\]

\[
V_{4,1} = 0 \quad (\text{bolted} \ 3\phi)
\]

\[
V_{4,1}^F = V_{4,1}^{pre} - Z_{4,1}I_{4,1}^F = 1/0 - (j0.320)(-j2.34)
\]

\[
V_{4,1}^F = 0.286/0
\]

\[
V_{5,1}^F = V_{5,1}^{pre} - Z_{54,1}Z_{4,1}^F = 1/0 - (j0.321)(-j2.23)
\]

\[
V_{5,1}^F = \boxed{0.251/0}
\]

\[
I_{4,1}^F = \frac{V_{4,1}^F}{Z_{F}^0} = \frac{0.286}{0.1653} = \boxed{-j0.715}
\]

\[
I_{4,1}^F = \frac{V_{5,1}^F}{0.1653} = \frac{0.251}{0.1653} = \boxed{-j1.519}
\]

\[
\text{Sum} = -j2.23 \quad \text{(Need more precision usually)}
\]
One-line diagram for Prob. 6.15.

Volts and Amps at gen #1 terminals?

Ignoring "Net 30,"

\[ V_{F1} = 0.236 \angle 0 \]
\[ I_{F1} = 0 \]
\[ I_{F12} = 0 \]

Now, include the "Net 30"

\[ V_{F11} = 0.286 \angle -30 \]  \[ V_{14a} = 0.286 \left( \frac{13800}{\sqrt{3}} \right) \angle -30 \]  \[ = \frac{2279}{\sqrt{3}} \angle -30 \text{ Volt (Line-to-Neutral)} \]  
\[ I_{14a} = 0.715 \angle -120 \]  \[ = \frac{2991}{\sqrt{3}} \angle -120 \text{ Amps} \]

Rated Machine Amps

\[ I_{\text{BASE}} = \frac{20 \text{ MVA}/\sqrt{3}}{13.8 \text{ kV}/\sqrt{3}} = 837 \text{ A} \]
One-line diagram for Prob. 6.15.

Zero Seq Network

\[ Z_{44,0} = j 0.40 \left[ (j 0.434 + j 0.517 + j 0.333) \right] = j 0.40 \left[ j 1.284 \right] = j 0.305 \]

\[ I_{kA} = \frac{3 V_{kA}}{Z_{kk1} + Z_{kk2} + Z_{kk3}} = \frac{3 \times 250}{j 0.448 + j 0.448 + j 0.305} = \frac{750}{j 2.50} \]

\[ = -j 2.50 \text{ A} \]
Now, get the 012 voltages.

One-line diagram for Prob. 6.15.

\[
I_{4,0}^F = I_{4,1}^F = I_{4,2}^F = \frac{I_{4,0}^F}{3} = -j0.833A
\]

\[
\begin{bmatrix}
V_{4,0}^F \\
V_{4,1}^F \\
V_{4,2}^F
\end{bmatrix} = \begin{bmatrix}
V_{4,0}^{pre} = 0 \\
V_{4,1}^{pre} = 1(0) \\
V_{4,2}^{pre} = 0
\end{bmatrix} - \begin{bmatrix}
0.305 & 0 & 0 \ 
0 & 0.448 & 0 \ 
0 & 0 & 0.448
\end{bmatrix} \begin{bmatrix}
-j0.833 \\
-j0.833 \\
-j0.833
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 - 0.254 = -0.254 \\
1 - 0.373 = 0.627 \\
0 - 0.373 = -0.373
\end{bmatrix}
\]

Check \( V_{4,0}^F = 0 \)

\( = V_{4,0}^F + V_{4,1}^F + V_{4,2}^F \)

\( = (-0.254) + 0.627 - 0.373 \)

\( = 0 \)

\[
\begin{bmatrix}
V_{1,0}^F \\
V_{1,1}^F \\
V_{1,2}^F
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} - \begin{bmatrix}
0 & 0 & z_{4,1} \\
0 & 0.320 & 0 \\
0 & 0 & 0.320
\end{bmatrix} \begin{bmatrix}
-j0.833 \\
-j0.833 \\
-j0.833
\end{bmatrix}
\]

\( z_{4,0} = 0 \) because \( \Delta V_{1,0} = 0 \)

\( V_{1,0}^F = 0 - 0 = 0 \)

\( V_{1,1}^F = 1 - (j0.32)(j0.833) = 0.733 \) \( \text{Page 19 of 21} \)

\( V_{1,2}^F = 0 - (j0.32)(-j0.833) \)

\( = 0.267 \)
One-line diagram for Prob. 6.15.

\[
\begin{bmatrix}
V_{5,0}^F \\
V_{5,1}^F \\
V_{5,2}^F
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix}
\frac{Z_{5,0,1}}{Z_{5,0,0}} & 0 \\
0 & \frac{Z_{5,0,2}}{Z_{5,0,0}}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
-j0.833
\end{bmatrix}
\]

For \( Z_{5,0} = Z_{4,0} \cdot \frac{V_5}{V_4} \)

\[
Z_{5,0} = Z_{4,0} \cdot \frac{V_5}{V_4}
\]

\[
= j0.305 \begin{bmatrix}
0.850 \\
0.434 + 0.850
\end{bmatrix}
\]

\[
Z_{5,0} = j0.202.
\]

So,

\[
V_{5,0}^F = 0 - (j0.202)(-j0.833) = -0.1683
\]

\[
V_{5,1}^F = 1 - (0.336)(-j0.833) = 0.720
\]

\[
V_{5,2}^F = 0 - (0.336)(-j0.833) = -0.280
\]
Now, the branch currents feeding the fault:

\[ I_{F_{\text{14,0}}} = \frac{V_{F_{\text{14,0}}} - V_{F_{\text{4,0}}}}{\text{broken path}} = 0. \]

But

\[ I_{F_{\text{4,0}}} = \frac{0.635 \cdot 0.40}{0.40 - (-0.254)} \]

\[ = -j0.635 \]

\[ I_{F_{\text{14,1}}} = \frac{V_{F_{\text{14,1}}} - V_{F_{\text{4,1}}}}{j0.40} = 0.733 - 0.629 \]

\[ = -j0.265 \]

\[ \text{Check Sum} \]

\[ I_{F_{\text{14,0}}} + I_{F_{\text{54,0}}} = \sqrt{0.635^2 - j0.1975^2} = -j0.833 \]

\[ j0.635 \text{ (from the left)} \]

\[ I_{F_{\text{14,1}}} + I_{F_{\text{54,1}}} = -j0.265 - j0.563 = -j0.828 \text{ (OK)} \]

\[ I_{F_{\text{14,2}}} + I_{F_{\text{54,2}}} = -j0.265 - j0.563 = -j0.828 \text{ (OK)} \]
Reflecting $I_4^F$ sequence components with the Net 30:

$F$

$I_{14,0} = 0$

$I_{14,1} = -j0.265 \cdot 1/30 = 0.265/\angle-120^\circ$

$I_{14,2} = -j0.265 \cdot 1/30 = 0.265/\angle60^\circ$

$V_{1,0}^F = 0$

$V_{1,1}^F = 0.733 \cdot 1/30 = 0.733/\angle30^\circ$

$V_{1,2}^F = -0.267 \cdot 1/30 = 0.267/\angle-150^\circ$
Stevenson Problem 6.15, Phase A to Ground Fault at Bus #4

Enter Polar Form 012 Currents at Gen #1, Compute the ABC Currents

Enter Polar Form 012 Voltages at Gen #1, Compute the ABC Voltages