# Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc 

## Report Details

Choose any of the solar stations, but take your measurements only when the short circuit current is at least 3.5 A . The weather forecast from www.weatherunderground.com can help you plan your schedule.

Your report should include graphs of I versus V, and P versus V. Both actual data and Excel approximations should be plotted together. When plotting with Excel, be sure to use the "scatter plot" option so that the non-uniform spacing between voltage points on the x -axis show correctly. You should also work out numerical values for Equations (1) - (10) for the day and time of your measurements.

## Overview

Incident sunlight can be converted into electricity by photovoltaic conversion using a solar panel. A solar panel consists of individual cells that are large-area semiconductor diodes, constructed so that light can penetrate into the region of the p-n junction. The junction formed between the ntype silicon wafer and the p-type surface layer governs the diode characteristics as well as the photovoltaic effect. Light is absorbed in the silicon, generating both excess holes and electrons. These excess charges can flow through an external circuit to produce power.


Figure 1. Equivalent Circuit of a Solar Cell
Diode current $A\left(e^{B V}-1\right)$ comes from the standard I-V equation for a diode, plotted above. From Figure 1, it is clear that the current I that flows to the external circuit is

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$I=I_{S c}-A\left(e^{B V}-1\right)$. If the solar cell is open circuited, then all of the $I_{S c}$ flows through the diode and produces an open circuit voltage of about $0.5-0.6 \mathrm{~V}$. If the solar cell is short circuited, then no current flows through the diode, and all of the $I_{S c}$ flows through the short circuit.

Since the $\mathrm{V}_{\text {oc }}$ for one cell is approximately $0.5-0.6 \mathrm{~V}$, then individual cells are connected in series as a "solar panel" to produce more usable voltage and power output levels. Most solar panels are made to charge 12 V batteries and consist of 36 individual cells (or units) in series to yield panel $\mathrm{V}_{\mathrm{Oc}} \approx 18-20 \mathrm{~V}$. The voltage for maximum panel power output is usually about 1617 V . Each $0.5-0.6 \mathrm{~V}$ series unit can contain a number of individual cells in parallel, thereby increasing the total panel surface area and power generating capability.


Figure 2. I-V Characteristics of Solar Panel
On a clear day, direct normal solar insolation (i.e., incident solar energy) is approximately $1 \mathrm{~kW} / \mathrm{m}^{2}$. Since solar panel efficiencies are approximately $14 \%$, a solar panel will produce about 140 W per square meter of surface area when facing a bright sun. High temperatures reduce panel efficiency. For 24/7 power availability, solar power must be stored in deepdischarge batteries that contain enough energy to power the load through the nighttime and overcast days. On good solar days in Austin, you can count on solar panels producing about 1 kWH of energy per square meter.

An everyday use of solar power is often seen in school zone and other LED flashing signs, where TxDOT and municipal governments find them economical when conventional electric service is not readily available or when the monthly minimum electric fees are large compared to the monthly kWH used. Look for solar panels on top of these signs, and also note their orientation.

## The Solar Panels on ENS Rooftop

The ENS rooftop is equipped with six pairs of commercial " 12 V class" panels, plus one larger " 24 V class" commercial panel. The panels are:

- three pair of British Petroleum BP585, (mono-crystalline silicon, laser grooved, each panel 85 W , voltage at maximum power $=18.0 \mathrm{~V}$, current at maximum power $=4.7 \mathrm{~A}$, open circuit voltage $=22.3 \mathrm{~V}$, short circuit current $=5.0 \mathrm{~A}$ ). These three pairs are connected to ENS212 stations 17, 18, and 19.
- two pair of Solarex SX85U (now BP Solar) (polycrystalline silicon, each panel 85W, voltage at maximum power $=17.1 \mathrm{~V}$, current at maximum power $=5.0 \mathrm{~A}$, open circuit voltage $=21.3 \mathrm{~V}$, short circuit current $=5.3 \mathrm{~A}$ ). These two pairs are connected to ENS212 stations 15 and 16.
- one pair of Photowatt PW750-80 (multi-crystalline cells, each panel 80 W , voltage at maximum power $=17.3 \mathrm{~V}$, current at maximum power $=4.6 \mathrm{~A}$, open circuit voltage $=$ 21.9 V , short circuit current $=5.0 \mathrm{~A}$ ). This pair is connected to ENS212 station 21.
- one British Petroleum BP3150U, 150W panel (multicrystalline), open circuit voltage $=$ 43.5 V , short circuit current $=4.5 \mathrm{~A}$. This is connected to ENS212 station 20.

Each of the seven stations is wired to ENS212 and has an open circuit voltage of approximately 40 V and a short circuit current of approximately 5 A . The I-V and P-V characteristics for one of the panel pairs is shown in Figure 3. The I-V curve fit equation for Figure 3 is

$$
I(V)=5.34-0.00524\left(e^{0.1777 V}-1\right)
$$

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Figure 3. I-V and P-V Characteristics for One of the Panel Pairs (data points taken by loading the panel pair with a variable load resistor)

## Maximum Power

As seen in bottom figure of Figure 3, panels have a maximum power point. Maximum power corresponds to $\mathrm{V}_{\mathrm{m}}$ and $\mathrm{I}_{\mathrm{m}}$ in Figure 2. Because solar power is relatively expensive (approx. \$45 per watt for the panels, plus the same amount for batteries and electronics), it is important to operate panels at their maximum power conditions. Unfortunately, $\mathrm{V}_{\mathrm{m}}, \mathrm{I}_{\mathrm{m}}$, and the Thevenin
equivalent resistance vary with light level. DC-DC converters are often used to "match" the load resistance to the Thevenin equivalent resistance of the panel to maximize the power drawn from the panel. These "smart" converters (often referred to as "tracking converters") also charge the storage batteries in such a way as to maximize battery life.

## Sun Position, Panel Orientation, and PV Harvest Prediction - The Big 10 Equations

 Ideally, a solar panel should track the sun so that the incident solar rays are perpendicular to the panel surface, thus maximizing the capture of solar energy. However, because of high wind loads, most panels are fixed in position. Often, panel tilt (with respect to horizontal) is adjusted seasonally. Orientation of fixed panels should be carefully chosen to capture the most energy for the year, or for a season.The position of the sun in the sky varies dramatically with hour and season. Sun zenith angle $\theta_{\text {sun }}^{\text {zenith }}$ is expressed in degrees from vertical. Sun azimuth $\phi_{\text {sun }}^{\text {azimuth }}$ is expressed in degrees from true north. Sun zenith and azimuth angles are illustrated in Figure 4.


Figure 4. Sun Zenith and Azimuth Angles
Sun position angles are available in many references, and with different levels of complexity. Some of the following equations were taken from the University of Oregon Solar Radiation Monitoring Laboratory (http://solardat.uoregon.edu/SolarRadiationBasics.html):

Sun declination angle (in degrees) is

$$
\begin{align*}
\delta & =23.45 \sin (B), \text { where } \\
B & =\frac{360}{365}(n-81) \text { degrees }, \text { and } \\
n & =\text { day of year (i.e., } 1,2,3, \ldots, 364,365) . \tag{1}
\end{align*}
$$

| First Day of | n | First Day of | n |
| :---: | :---: | :---: | :---: |
| January | 1 | July | $182+$ |
| February | 32 | August | $213+$ |
| March | $60+$ | September | $244+$ |
| April | $91+$ | October | $274+$ |
| May | $121+$ | November | $305+$ |
| June | $152+$ | December | $335+$ |

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Equation of time (in decimal minutes) is

$$
\begin{equation*}
E_{q t}=9.87 \sin (2 B)-7.53 \cos (B)-1.5 \sin (B) . \tag{2}
\end{equation*}
$$

Solar time (in decimal hours) is

$$
\begin{equation*}
T_{\text {solar }}=T_{\text {local }}+\frac{E_{q t}}{60}+\frac{\left(\text { Longitude }_{\text {timezone }}-\text { Longitude }_{\text {local }}\right)}{15} \tag{3}
\end{equation*}
$$

where

- $T_{\text {local }}$ is local standard time in decimal hours,
- Longitude timezone is the longitude at the eastern edge of the time zone (e.g., $90^{\circ}$ for Central Standard Time).
(Note - in the Solar_Data_Analyzer Excel spreadsheet program, (Longitude ${ }_{\text {timezone }}-$ Longitude $_{\text {local }}$ ) is entered as "Longitude shift (deg)." At Austin, with Longitude $_{\text {local }}=97.74^{\circ}$, the longitude shift is $\left(90^{\circ}-97.74^{\circ}\right)=-7.74^{\circ}$.

Hour angle (in degrees) is

$$
\begin{equation*}
H=15 \bullet\left(12-T_{\text {solar }}\right) \tag{4}
\end{equation*}
$$

Cosine of the zenith angle is

$$
\begin{equation*}
\cos \left(\theta_{\text {sun }}^{\text {zenith }}\right)=\sin (L) \sin (\delta)+\cos (L) \cos (\delta) \cos (H), \tag{5}
\end{equation*}
$$

where $L$ is the latitude of the location.
Solar azimuth comes from the following calculations. Using the formulas for solar radiation on tilted surfaces, consider vertical surfaces directed east and south. The fraction of direct component of solar radiation on an east-facing vertical surface is

$$
\begin{equation*}
f_{V E}=\cos (\delta) \sin (H) \tag{6}
\end{equation*}
$$

The fraction of direct component of solar radiation on a south-facing vertical surface is

$$
\begin{equation*}
f_{V S}=-\sin (\delta) \cos (L)+\cos (\delta) \sin (L) \cos (H) \tag{7}
\end{equation*}
$$

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Equations (6) and (7) correspond to the projections, on the horizontal plane, of a vector pointing toward the sun. By examining Figure $4, \phi_{\text {sun }}^{\text {azimuth }}$ can be found as follows:

$$
\text { If } f_{V E} \geq 0, \phi_{\text {sun }}^{\text {azimuth }}=\cos ^{-1}\left(\frac{-f_{V S}}{\sqrt{f_{V E}^{2}+f_{V S}^{2}}}\right) \text { degrees, }
$$

$$
\begin{equation*}
\text { If } f_{V E}<0, \phi_{s u n}^{\text {azimuth }}=180+\cos ^{-1}\left(\frac{f_{V S}}{\sqrt{f_{V E}^{2}+f_{V S}^{2}}}\right) \text { degrees. } \tag{8}
\end{equation*}
$$

As a check, all components of the sun radiation should account for the total, i.e.

$$
\sqrt{f_{V E}^{2}+f_{V S}^{2}+\cos ^{2}\left(\theta_{\text {sun }}^{\text {zenith }}\right)}=1 .
$$

Illustrations of seasonal and daily sun positions for Austin are shown in Figures 5a and 5b.
An example of the step-by-step calculations for 3 pm (i.e., 15.00 decimal hours) on October $25^{\text {th }}$ in Austin follows.

| Input | $\mathrm{n}=298^{\text {th }}$ day |
| :---: | :---: |
| Compute | $\mathrm{B}=214.0^{\circ}$ |
| Compute | $\delta=-13.11^{\circ}$ |
| Compute | $E_{q t}=16.21$ decimal minutes |
| Input | ${\text { Longitude }=97.74^{\circ}}^{\text {Input }}$ |
| Longitude shift $=-7.74^{\circ}$ |  |
| Input | $T_{\text {local }}=15.00$ decimal hours |
| Compute | $T_{\text {solar }}=14.75$ decimal hours |
| Compute | $H=-41.25^{\circ}$ |
| Input | ${\text { Latitude }(\mathrm{L})=30.29^{\circ}}^{\substack{\circ}}$Compute |
| Compute | $\theta_{\text {sun }}^{\text {zenith }}=58.81^{\circ}$ |
| Compute | $f_{V E}=-0.6421$ |
| Compute | $f_{V S}=0.5651$ |



Figure 5a. Sun Position for Winter and Spring Seasons in Austin (note - solar noon in Austin occurs at approximately 12:30pm CST)


Figure 5b. Sun Position for Summer and Fall Seasons in Austin (note - solar noon in Austin occurs at approximately 12:30pm CST)

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## Definitions from www.weatherground.com

| Twilight | This is the time before sunrise and after sunset where it is still light outside, but the sun is not in the sky. |
| :---: | :---: |
| Civil Twilight | This is defined to be the time period when the sun is no more than 6 degrees below the horizon at either sunrise or sunset. The horizon should be clearly defined and the brightest stars should be visible under good atmospheric conditions (i.e. no moonlight, or other lights). One still should be able to carry on ordinary outdoor activities. |
| Nautical <br> Twilight | This is defined to be the time period when the sun is between 6 and 12 degrees below the horizon at either sunrise or sunset. The horizon is not defined and the outline of objects might be visible without artificial light. Ordinary outdoor activities are not possible at this time without extra illumination. |
| Astronomical Twilight | This is defined to be the time period when the sun is between 12 and 18 degrees below the horizon at either sunrise or sunset. The sun does not contribute to the illumination of the sky before this time in the morning, or after this time in the evening. In the beginning of morning astronomical twilight and at the end of astronomical twilight in the evening, sky illumination is very faint, and might be undetectable. |
| Length Of Day | This is defined to be the time of Actual Sunset minus the time of Actual Sunrise. The change in length of daylight between today and tomorrow is also listed when available. |
| Length Of Visible Light | This is defined to be the time of Civil Sunset minus the time of Civil Sunrise. |
| Altitude (or Elevation) | First, find your azimuth. Next, the Altitude (or elevation) is the angle between the Earth's surface (horizon) and the sun, or object in the sky. Altitudes range from $-90^{\circ}$ (straight down below the horizon, or the nadir) to $+90^{\circ}$ (straight up above the horizon or the Zenith) and $0^{\circ}$ straight at the horizon. |
| Azimuth | The azimuth (az) angle is the compass bearing, relative to true (geographic) north, of a point on the horizon directly beneath the sun. The horizon is defined as an imaginary circle centered on the observer. This is the 2-D, or Earth's surface, part of calculating the sun's position. As seen from above the observer, these compass bearings are measured clockwise in degrees from north. Azimuth angles can range from $0-359^{\circ} .0^{\circ}$ is due geographic north, $90^{\circ}$ due east, $180^{\circ}$ due south, and 360 due north again. |


| Hour Angle of <br> the Sun | The Solar Hour Angle of the Sun for any local location on the Earth is <br> zero ${ }^{\circ}$ when the sun is straight overhead, at the zenith, and negative before <br> local solar noon and positive after solar noon. In one 24-hour period, the <br> Solar Hour Angle changes by 360 degrees (i.e. one revolution). |
| :--- | :--- |
| Mean Anomaly <br> of the Sun | The movement of the Earth around the Sun is an ellipse. However, if the <br> movement of the Earth around the Sun were a circle, it would be easy to <br> calculate its position. Since, the Earth moves around the sun about one <br> degree per day, (in fact, it's 1/365.25 of the circle), we say the Mean <br> Anomaly of the Sun is the position of the Earth along this circular path. <br> The True Anomaly of the Sun is the position along its real elliptical path. |
| Obliquity | Obliquity is the angle between a planet's equatorial plane and its orbital <br> plane. |
| Right <br> Ascension of <br> the Sun | The Celestial Sphere is a sphere where we project objects in the sky. We <br> project stars, the moon, and sun, on to this imaginary sphere. The Right <br> Ascension of the Sun is the position of the sun on our Celestial Sphere |
| Solar Noon <br> (and Solar <br> Time) | Solar Time is based on the motion of the sun around the Earth. The <br> apparent sun's motion, and position in the sky, can vary due to a few things <br> such as: the elliptical orbits of the Earth and Sun, the inclination of the axis <br> of the Earth's rotation, the perturbations of the moon and other planets, and <br> of course, your latitude and longitude of observation. Solar Noon is when <br> the sun is at the highest in the sky, and is defined when the Hour Angle is <br> $0^{\circ}$. Solar Noon is also the midpoint between Sunrise and Sunset. |
| Sun <br> Declination | The Declination of the sun is how many degrees North (positive) or South <br> (negative) of the equator that the sun is when viewed from the center of the <br> earth. The range of the declination of the sun ranges from approximately <br> $+23.5^{\circ}$ (North) in June to -23.5 (South) in December. |

## Panel Orientation and Solar Incident Angle

Unless there are obstructions, panels should face due south (i.e., have an azimuth angle of $180^{\circ}$ ). Recommended panel tilt angles (above horizontal) are latitude $+15^{\circ}$ in winter, and latitude $-15^{\circ}$ in summer. In Austin, with latitude $=30^{\circ}$, these recommendations correspond to $45^{\circ}$ in winter, and $15^{\circ}$ in summer. If no seasonal adjustments are made, then the best fixed panel tilt angle is latitude (i.e., $30^{\circ}$ in Austin). The tilt angles of our panels are adjusted twice each year, at the spring and fall equinoxes. Our tilt angles are $20^{\circ}$ in summer, and $45^{\circ}$ in winter.


Figure 6. Panel Tilt Angle


All panels atop ENS have azimuth angle $=190^{\circ}$
View Facing Front of ENS Panels (i.e., looking toward north) (Note - areas shown are for individual panels, so for a pair, double the values shown)

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Panels atop ENS

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The angle between the rays of the sun and a vector perpendicular to the panel surface is known as the angle of incidence $\left(\beta_{\text {incident }}\right)$. The cosine of $\beta_{\text {incident }}$ is found by first expressing a unit vector pointed toward the sun, and a unit vector perpendicular to the panel surface, and then taking the dot product of the two unit vectors. When $\cos \left(\beta_{\text {incident }}\right)=1$, then the sun's rays are perpendicular to the panel surface, so that maximum incident solar energy is captured. The expressions follow.

Considering Figure 4, the unit vector pointed toward the sun is

$$
\hat{\boldsymbol{a}}_{\text {sun }}=\left\lfloor\sin \theta_{\text {sun }}^{\text {zenith }} \cos \phi_{\text {sun }}^{\text {azimuth }} \hat{\boldsymbol{a}}_{\boldsymbol{x}}+\left\lfloor\sin \theta_{\text {sun }}^{\text {zenith }} \sin \phi_{\text {sun }}^{\text {azimuth }} \hat{\boldsymbol{a}}_{\boldsymbol{y}}-\left\lfloor\cos \theta_{\text {sun }}^{z e n i t h} \hat{\boldsymbol{a}}_{z} .\right.\right.\right.
$$

Considering Figure 6, the unit vector perpendicular to the panel surface is

$$
\hat{\boldsymbol{a}}_{\text {panel }}=\left\lfloor\sin \theta_{\text {panel }}^{\text {tilt }} \cos \phi_{\text {panel }}^{\text {azimuth }} \hat{\boldsymbol{a}}_{\boldsymbol{x}}+\left\lfloor\sin \theta_{\text {panel }}^{\text {tilt }} \sin \phi_{\text {panel }}^{\text {azimuth }}\right\rfloor \hat{\boldsymbol{a}}_{\boldsymbol{y}}-\left\lfloor\cos \theta_{\text {panel }}^{\text {tilt }}\right\rfloor \hat{\boldsymbol{a}}_{\boldsymbol{z}}\right.
$$

The dot product of the two unit vectors is then

$$
\begin{aligned}
& \cos \beta_{\text {incident }}=\hat{\boldsymbol{a}}_{\text {sun }} \bullet \hat{\boldsymbol{a}}_{\text {panel }}=\left\lfloor\sin \theta_{\text {sun }}^{\text {zenith }} \cos \phi_{\text {sun }}^{\text {azimuth }} \sin \theta_{\text {panel }}^{\text {tilt }} \cos \phi_{\text {panel }}^{\text {azimuth }}\right\rfloor \\
& \quad+\left\lfloor\sin \theta_{\text {sun }}^{\text {zenith }} \sin \phi_{\text {sun }}^{\text {azimuth }} \sin \theta_{\text {panel }}^{\text {tilt }} \sin \phi_{\text {panel }}^{\text {azimuth }}\right\rfloor+\left\lfloor\cos \theta_{\text {sun }}^{\text {zenith }} \cos \theta_{\text {panel }}^{\text {tilt }}\right\rfloor .
\end{aligned}
$$

Combining terms yields

$$
\begin{aligned}
& \cos \beta_{\text {incident }}=\sin \theta_{\text {sun }}^{\text {zenith }} \sin \theta_{\text {panel }}^{\text {tilt }}\left\lfloor\cos \phi_{\text {sun }}^{\text {azimuth }} \cos \phi_{\text {panel }}^{\text {azimuth }}+\sin \phi_{\text {sun }}^{\text {azimuth }} \sin \phi_{\text {panel }}^{\text {azimuth }}\right\rfloor \\
& \quad+\cos \theta_{\text {sun }}^{\text {zenith }} \cos \theta_{\text {panel }}^{\text {tilt }} .
\end{aligned}
$$

Simplifying the above equation yields the general case,

$$
\begin{equation*}
\cos \beta_{\text {incident }}=\sin \theta_{\text {sun }}^{z e n i t h} \sin \theta_{\text {panel }}^{\text {tilt }} \cos \left(\phi_{\text {sun }}^{\text {azimuth }}-\phi_{\text {panel }}^{\text {azimuth }}\right)+\cos \theta_{\text {sun }}^{z e n i t h ~} \cos \theta_{\text {panel }}^{\text {tilt }} \tag{9}
\end{equation*}
$$

Some special cases are

1. Flat panel (i.e., $\theta_{\text {panel }}^{\text {tilt }}=0$ ). Then,

$$
\cos \beta_{\text {incident }}=\cos \theta_{\text {sun }}^{z e n i t h ~} .
$$

2. Sun directly overhead (i.e., $\theta_{\text {zenith }}^{\text {sun }}=0$ ). Then,

# Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc <br> $\cos \beta_{\text {incident }}=\cos \theta_{\text {panel }}^{\text {tilt }}$. 

3. Equal azimuth angles (i.e., azimuth tracking, $\phi_{\text {sun }}^{\text {azimuth }}=\phi_{\text {panel }}^{\text {azimuth }}$ ). Then,

$$
\cos \beta_{\text {incident }}=\sin \theta_{\text {sun }}^{z e n i t h} \sin \theta_{\text {panel }}^{\text {tilt }}+\cos \theta_{\text {sun }}^{\text {zenith }} \cos \theta_{\text {panel }}^{\text {tilt }}=\cos \left(\theta_{\text {sun }}^{z e n i t h}-\theta_{\text {panel }}^{\text {tilt }}\right) .
$$

4. Sun zenith angle equals panel tilt angle (i.e., zenith tracking, $\theta_{\text {sun }}^{\text {zenith }}=\theta_{\text {panel }}^{\text {tilt }}$ ). Then,

$$
\cos \beta_{\text {incident }}=\sin ^{2} \theta_{\text {sun }}^{\text {zenith }} \cos \left(\phi_{\text {sun }}^{\text {azimuth }}-\phi_{\text {panel }}^{\text {azimuth }}\right)+\cos ^{2} \theta_{\text {sun }}^{\text {zenith }} .
$$

To illustrate the general case, consider the following example: 3 pm (standard time) in Austin on October 25. The sun position is

$$
\phi_{\text {sun }}^{\text {azimuth }}=228.7^{\circ}, \theta_{\text {sun }}^{\text {zenith }}=58.8^{\circ}, \text { so that } \hat{\boldsymbol{a}}_{\text {sun }}=-0.565 \hat{\boldsymbol{a}}_{\boldsymbol{x}}-0.643 \hat{\boldsymbol{a}}_{\boldsymbol{y}}-0.517 \hat{\boldsymbol{a}}_{z} \text {, }
$$

and the panel angles are

$$
\phi_{\text {panel }}^{\text {azimuth }}=190^{\circ}, \theta_{\text {panel }}^{\text {tilt }}=45^{\circ} \text {, so that } \hat{\boldsymbol{a}}_{\text {panel }}=-0.696 \hat{\boldsymbol{a}}_{\boldsymbol{x}}-0.1228 \hat{\boldsymbol{a}}_{\boldsymbol{y}}-0.707 \hat{\boldsymbol{a}}_{\boldsymbol{z}} .
$$

Evaluating the dot product yields $\cos \beta_{\text {incident }}=0.838$, so $\beta_{\text {incident }}=33.1^{\circ}$.

## Solar Radiation Measurements

The three most important solar radiation measurements for studying solar panel performance are global horizontal (GH), diffuse horizontal (DH), and direct normal (DN). GH is "entire sky," including the sun disk, looking straight up. DH is "entire sky," excluding the sun disk, looking straight up. DN is facing directly toward the sun. The units for $\mathrm{GH}, \mathrm{DH}$, and DN are $\mathrm{W} / \mathrm{m}^{2}$.

The direct measurement of DN requires a sun tracking device. The Sci Tek 2AP tracker takes DN, GH, and DH readings every five minutes using three separate thermocouple sensors. The DN sensor tracks and sees only the disk of the sun. The GH sensor points straight up and sees the entire sky with sun disk. The DH sensor points straight up, but a shadow ball blocks the disk of the sun, so that it sees entire sky minus sun disk.

Rotating shadowband pyranometers use one PV sensor, pointed straight up, to measure GH and DH every minute, and then save average values every 5 minutes. Once per minute, the shadow band swings over, and when the shadow falls on the sensor, the DH reading is taken. Using GH and DH , the rotating shadow-band pyranometer estimates DN .


Rotating Shadowband Pyranometers

Rotating shadow band pyranometers are simple in that they do not track the sun. Instead, they merely rotate a shadow band every minute across the PV sensor. When there is no shadow on the sensor, the sensor reads GH . When the shadow falls on the sensor, the sensor reads DH .

## Computing Incident Solar Power on a Panel Surface

To compute the incident solar power on a panel surface, we assume that the panel captures all of the diffuse horizontal $(\mathrm{DH})$ power, plus the fraction of $(\mathrm{GH}-\mathrm{DH})$ that is perpendicular to the panel surface.

$$
\begin{equation*}
P_{\text {incident }}=D H+\frac{(G H-D H)}{\cos \left(\theta_{\text {sun }}^{\text {zenith }}\right)} \bullet \cos \left(\beta_{\text {incident }}\right) \mathrm{W} / \mathrm{m}^{2} \tag{10}
\end{equation*}
$$

The above value, in $\mathrm{W} / \mathrm{m}^{2}$, is then multiplied by the panel surface area to yield total incident solar power $P_{\text {incident }}$. Multiplying by panel efficiency yields maximum expected electrical power output.

Because panels are rated at $1 \mathrm{~kW} / \mathrm{m} 2,(10)$ is also the estimated panel $W$ output per kW rated. Integrate over all hours of the day and divide by 1000 , and you get estimated $\mathbf{k W H}$ output per kW rated (i.e., the PV daily harvest).

To avoid serious overcorrection when the sun is near the horizon, ignore the $\cos \left(\theta_{\text {sun }}^{\text {zenith }}\right)$ term when $\theta_{\text {sun }}^{z e n i t h}>85^{\circ}$. For the 3 pm , October $25^{\text {th }}$ example, the readings are $G H=535 \mathrm{~W} / \mathrm{m}^{2}$, and $D H=38 \mathrm{~W} / \mathrm{m}^{2}$,

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$$
P_{\text {incident }}=\left[38+\frac{(535-38)}{\cos \left(58.9^{\circ}\right)} \bullet \cos \left(33.1^{\circ}\right)\right] \bullet A_{\text {panel }}=844 \bullet A_{\text {panel }} \mathrm{W} / \mathrm{m}^{2}
$$

which means that a PV panel or array would produce 844 W per kW rated power.


NREL Sci Tec Two-Axis Tracker

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## Computing Sunrise and Sunset Times

To reduce the volume of data stored and processed, it is best to start and stop solar data loggers just before sunrise and just after sunset. I developed the following empirical formulas for sunrise and sunset, which are applicable to north latitudes $25^{\circ}$ through $45^{\circ}$. The worst-case error is 6 minutes. To take into account the error and the fact that the formulas apply to the situation where one-half of the sun is above the horizon, start/stop 15 minutes before/after the computed values.

$$
\text { Sunrise }=A_{\text {rise }}+B \bullet \sin \left(\frac{2 \pi \bullet D O Y}{365}+\frac{C_{\text {rise }} \bullet \pi}{180}\right)+D \bullet \sin \left(\frac{4 \pi \bullet D O Y}{365}+\frac{E \bullet \pi}{180}\right),
$$

where Sunrise is the decimal minute of the day at the eastern edge of the time zone, and
$D O Y$ is day of the year (1 through 365, ignore leap year),

$$
A_{\text {rise }}=360, C_{\text {rise }}=94, D=9.0, E=20.1,
$$

$B=B_{0}+B_{1} \bullet L+B_{2} \bullet L^{2}$, where $L$ is latitude north,

$$
B_{0}=21.3, B_{1}=0.07, B_{2}=0.0371 .
$$

Sunset follows the same formula, but replace $A_{\text {rise }}$ with $A_{\text {set }}$, and $C_{\text {rise }}$ with $C_{\text {set }}$, where

$$
A_{\text {set }}=A_{\text {rise }}+720, C_{\text {set }}=C_{\text {rise }}-168 .
$$

To correct for locations west of the eastern edge, recall that the sun moves 15 degrees of longitude in one hour, which corresponds to 4 minutes delay per degree of longitude.

## The Experiment

Your assignment is to measure the I-V and P-V characteristics of a solar panel pair, plot the points, determine maximum power, estimate panel efficiency, and use the Excel Solver to approximate the I-V and P-V curves using

$$
I=I_{s c}-A\left(e^{B V}-1\right), P=V I=V \bullet\left[I_{s c}-A\left(e^{B V}-1\right)\right\rfloor,
$$

where the Solver estimates coefficients $\mathrm{I}_{\mathrm{Sc}}$, A , and B from your measured I-V data set. See the Appendix for a description of the Excel Solver.

## Experimental Procedure

You will need about 30 minutes to take the experimental data. Go to an available panel station, and check the short circuit current. Take your measurements when the short circuit current is at least 3.5 A (try for a sunny day, between 11:30am and 1:30pm. CDT (corresponding to solar noon, plus or minus 1 hour). (Note - weather site www.weatherunderground.com can help you make your plans for upcoming days.) Then, using the voltage at the panel (i.e., the left-most meter in the yellow solar panel interface box), and the panel ammeter (the rightmost meter), perform the following steps given below, recording and plotting your data on the experimental form and on the graph as you go:


Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc

Form and Graph for Recording and Plotting Your Readings as You Take Them (have this page signed by Dr. Grady before beginning your report)

Panel Station $=$ $\qquad$ Date and Time of Measurements= $\qquad$ , Sky Conditions = $\qquad$

| $\approx 2 \mathrm{~V}$ spacing for $\mathrm{V}>25 \mathrm{~V}$ | $\mathrm{V}_{\text {panel }}{ }^{*}$ | Ipanel* | $\begin{gathered} \text { P (i.e., } \mathrm{V}_{\text {panel }} \\ \left.\bullet \mathrm{I}_{\text {panel }}\right) \end{gathered}$ | Notes |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{V}_{\mathrm{Oc}}=$ |  |  | Open circuit condition |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| $\approx 5 \mathrm{~V}$ spacing for $\mathrm{V} \leq 25 \mathrm{~V}$ |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  | $\mathrm{I}_{\mathrm{Sc}}=$ |  | Short Circuit Condition |

* $\mathrm{V}_{\text {panel }}$ (i.e., at the panel) is the left-most meter in the yellow interface box, and $\mathrm{I}_{\text {panel }}$ is the right-most meter.



## Steps

1. Measure the panel pair's open circuit voltage, and record in the table and on the graph. The current is zero for this case.
2. Short the output terminals with one of the red shorting bar or with a wire. Measure the short circuit current and panel pair voltage. Record both and add the point to your graph. The panel voltage will be small for this condition.
3. Connect one of the "solar testers" (i.e., the heavy-duty variable resistor boxes with the large knobs) to the panel pair output terminals. You will use the variable rheostat and switch to sweep the entire I-V curve.

4. Beginning with the near open circuit condition, (i.e., maximum resistance), lower the solar tester resistance so that the panel pair voltage decreases from open circuit toward zero in steps of approximately 2 V between $25-40 \mathrm{~V}$, and in 5 V steps below 25 V . Record panel pair voltage and current at each step, and hand plot I versus V results as you go. If your points do not form a smooth curve, you may want to retake the outliers. Cloud movement can cause these variations.

The laboratory measurement portion of the experiment is now completed. Your graph should be fairly smooth and free of outlying points. You can now leave the lab bench.

Next, you will
5. Download Excel file EE362L_PV_Plots_Solver.xls from the course web page, and then enter your V and I values in Excel. Modify the plot command so that all the data for your experiment will be plotted. Plot I versus V points, and $\mathrm{P}=\mathrm{V} \bullet \mathrm{I}$ versus V points using the "scatter plot" option.
6. Visually estimate $\mathrm{V}_{\mathrm{m}}, \mathrm{I}_{\mathrm{m}}$, and $\mathrm{P}_{\max }$ (i.e., peak power conditions) from your plots.
7. Use the Excel Solver to compute coefficients $\mathrm{I}_{\mathrm{Sc}}$, A, and B from your I-V data. Modify the Solver command so that all your data will be included in the calculations.

## Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc

Superimpose the Solver equations on the I-V and P-V graphs of Step 5. See the Appendix for Solver instructions. Use your Solver graph to estimate $\mathrm{P}_{\text {max }}$.

Now, use the following steps to estimate panel pair efficiency:
8. Go to the class web page and download the Excel spreadsheet and solar data file

Solar_Data_Analyzer_EE462L.xls, and
SOLAR_DATA_through_XXX.zip, (XXX is the last data day in the zip)
Note - the 1-minute data averages are recorded by a shadow band tracker atop ETC and are updated daily on the web page while EE462L is being taught.
9. Display the data for your day (note - these data are given in Central Standard Time).
10. For the minute that best represents your time of measurements, work through the Big 10 equations.
11. Compare your Big 10 equations to the Solar_Data_Analyzer spreadsheet values for your day/minute.
12. For your day, use the Solar_Data_Analyzer spreadsheet as demonstrated in class to predict Method 1 daily kWh per installed kW for

- for fixed panel azimuth $=180$, and panel tilts 20, 30, and 40 degrees,
- for single axis tracking, azimuth $=180$, tilt $=20$ and 30 ,
- for two-axis tracking.

Interpret and comment on the results.

## Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc

Solar Data Analyzer. Prof. Mack Grady, U.T. Austin. Sept. 6, $2011 . \quad$ X



Page 22 of 25

# Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc 

## Appendix A: Using the Excel Solver to Curve-Fit Measured Data

The Excel Solver is not part of the "Typical User" installation. Check to see if the Solver is activated in your Excel installation by selecting "Tools," and then "Add-Ins." If Solver is checked, it is ready for use. Otherwise, check "Solver Add-In," and Excel will guide you through the steps. It will probably be necessary to insert your Excel installation CD rom.


To use Solver, refer to the following page. Enter your V and I data, and establish cells for I equation coefficients Isc, A, and B. Then, key-in the I equation shown previously to form a column of predicted currents, linking each cell to the Isc, A, and B cells. Next, establish a column of squared errors for current, and then one cell with sum of squared errors.


## Solar_Power_Experiment_and_Equations_Mack_Grady_May_08_2012.doc

Now, under "Tools," select "Solver." The following window will appear. Enter your "Target Cell" (the sum of squared errors cell), plus the "Changing Cells" that correspond to Isc, A, and $B$. It is for your starting values for $I_{s c}, A$, and $B$ are reasonable. You should probably use the $A$ and $B$ values shown above as your starting point. Use your own measured short circuit current for $\mathbf{I}_{\text {sc }}$.


Be sure to request "Min" to minimize the error, and then click "Solve."


If successful, click "OK" and then plot your measured I, and your estimated I, versus V to make a visual comparison between the measured and estimated currents. Use the scatter plot option to maintain proper spacing between voltage points on the x -axis.

If unsuccessful when curve fitting, try changing $\mathrm{I}_{\mathrm{Sc}}, \mathrm{A}$, or B , and re-try.

## Appendix B: National and State Solar Insolation Data

## Direct Solar Insolation Levels

(courtesy of Texas State Energy Conservation Office, www.infinitepower.org)


AVERAGE DIRECT NORMAL INSOLATION MAP LEGEND

| $\begin{gathered} \text { COLOR } \\ \text { KEY } \end{gathered}$ | $\begin{gathered} \text { per day } \\ \left(\mathrm{kWh} / \mathrm{m}^{2} \text {-day }\right) \end{gathered}$ | per YEAR |  |
| :---: | :---: | :---: | :---: |
|  |  | (MJ/m2) | (quads/100 mi') |
|  | $<3.0$ | <3,940 | $<1.0$ |
|  | 3.0-3.5 | 3,940-4,600 | 1.0-1.1 |
|  | 3.5-4.0 | 4,600-5,260 | 1.1-1.3 |
|  | 4.0-4.5 | 5,260-5,910 | 1.3-1.5 |
|  | 4.5-5.0 | 5,910-6,570 | 1.5-1.6 |
|  | 5.0-5.5 | 6,570-7,230 | 1.6-1.8 |
|  | 5.5-6.0 | 7,230-7,880 | 1.8-1.9 |
|  | 6.0-6.5 | 7,880-8,540 | 1.9-2.1 |
|  | 6.5-7.0 | 8,540-9,200 | 2.1-2.3 |
|  | $>7.0$ | >9,200 | $>2.3$ |


[^0]:    + add 1 for leap years

