TOPICS

1. **Power Definitions and Equations. Why?**
   As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.

2. **Three-Phase Power. Why?**
   Losses are minimized because there are no ground or neutral return currents. Also, three-phase machines are smaller and much more efficient than single-phase machines of the same power rating.

3. **Transformer Models. Why?**
   To move power over long distances and minimize I^2R losses, transmission line voltages must be boosted to much higher levels than voltages produced by generators or required by end-users.

4. **Per Unit System. Why ?**
   To make modeling and simulation of circuits having transformers much easier. A by-product is that per-unitized equipment parameters such as ratings and impedances fall into narrow and predictable ranges.

5. **Symmetrical Components. Why?**
   Greatly simplifies analysis of normal power system operation, as well as abnormal events such as unbalanced faults.

6. **Transmission Line Models. Why?**
   Necessary for designing and simulating power systems for loadflow, short-circuit, and stability purposes.

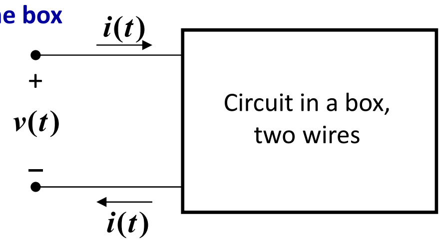
7. **Fault (Short-Circuit) Calculations and Voltage Sags. Why?**
   So that a power system can be properly protected by detecting and isolating faults within 0.1 second. 1 second is a very long time for a fault to exist - long enough to cause grid blackouts.

1. Power Definitions and Equations, cont.

As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.

Instantaneous power p(t) flowing into the box

$$v(t) = V\sin(\omega_o t + \delta),$$
$$i(t) = I\sin(\omega_o t + \theta)$$
$$p(t) = v(t) \bullet i(t)$$



$$p(t) = v(t) \bullet i(t) = V\sin(\omega_o t + \delta) \bullet I\sin(\omega_o t + \theta)$$

zero average

$$p(t) = VI\left[\frac{\cos(\delta-\theta) - \cos(2\omega_o t + \delta + \theta)}{2}\right]$$

peak

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt \quad = \frac{VI}{2}\cos(\delta-\theta) = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\delta-\theta)$$

rms

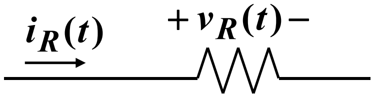
Average power P_{avg} flowing into the box

$$P_{avg} = V_{rms}I_{rms}\cos(\delta-\theta)$$

Power factor

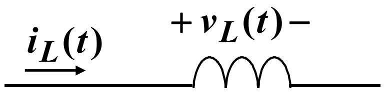
1. Power Definitions and Equations.

As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.



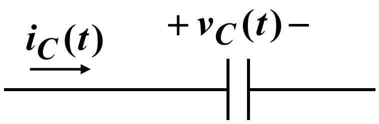
$$i_R(t) = \frac{v_R(t)}{R}$$

$i(t) = I\ sin(\omega t)$,
$v(t) = IR\ sin(\omega t)$,
$\therefore$ $i(t)$ in phase with $v(t)$



$$v_L(t) = L\frac{di(t)}{dt}$$

$i(t) = I\ sin(\omega t)$,
$v(t) = \omega L\ cos(\omega t)$,
$\therefore$ $i(t)$ lags $v(t)$ by 90°



$$i_C(t) = C\frac{dv(t)}{dt}$$

$v(t) = V\ sin(\omega t)$,
$i(t) = \omega C\ cos(\omega t)$,
$\therefore$ $i(t)$ leads $v(t)$ by 90°

1. Power Definitions and Equations, cont.

Thanks to Charles Steinmetz, Steady-State AC problems are greatly simplified with phasor analysis because differential equations are replaced by complex numbers

	Time Domain	Frequency Domain
Resistor	$i_R(t) = \frac{v_R(t)}{R}$	$Z_R = \frac{\tilde{V}_R}{\tilde{I}_R} = R$
Inductor	voltage leads current $v_L(t) = L\frac{di(t)}{dt}$	$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L} = j\omega L$
Capacitor	current leads voltage $i_C(t) = C\frac{dv(t)}{dt}$	$Z_C = \frac{\tilde{V}_C}{\tilde{I}_C} = \frac{1}{j\omega C}$

1. Power Definitions and Equations, cont.

Voltage and Current Phasors for R's, L's, C's

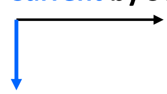
Resistor $Z_R = \dfrac{\tilde{V}_R}{\tilde{I}_R} = R, \quad \tilde{V}_R = R\tilde{I}_R$

Voltage and Current in phase

Q = 0

Inductor $Z_L = \dfrac{\tilde{V}_L}{\tilde{I}_L} = j\omega L, \quad \tilde{V}_L = j\omega L\tilde{I}_L$

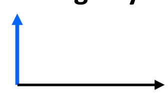
Voltage leads Current by 90°



Q > 0

Capacitor $Z_C = \dfrac{\tilde{V}_C}{\tilde{I}_C} = \dfrac{1}{j\omega C}, \quad \tilde{V}_C = \dfrac{\tilde{I}_C}{j\omega C}$

Current leads Voltage by 90°



Q < 0

1. Power Definitions and Equations, cont.

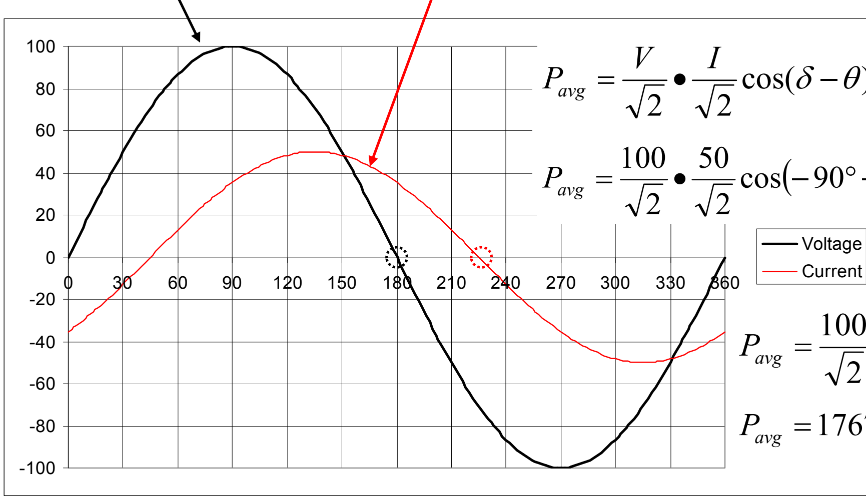
Converting Time Domain Waveforms to Phasor Domain

Using a cosine reference,

Voltage cosine has peak = 100V, phase angle = -90º

Current cosine has peak = 50A, phase angle = -135º

Phasors $\tilde{V} = \dfrac{100}{\sqrt{2}}\angle -90^\circ\, V, \quad \tilde{I} = \dfrac{50}{\sqrt{2}}\angle -135^\circ\, A$



$$P_{avg} = \frac{V}{\sqrt{2}} \bullet \frac{I}{\sqrt{2}} \cos(\delta - \theta)$$

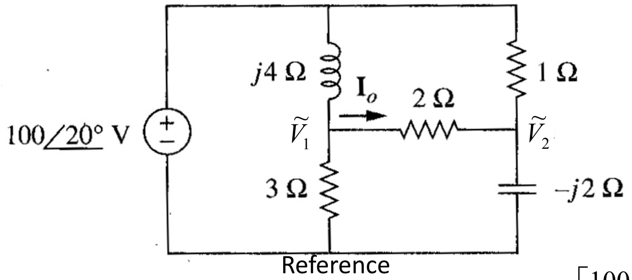
$$P_{avg} = \frac{100}{\sqrt{2}} \bullet \frac{50}{\sqrt{2}} \cos(-90^\circ - (-135^\circ))$$

$$P_{avg} = \frac{100}{\sqrt{2}} \bullet \frac{50}{\sqrt{2}} \cos(45^\circ)$$

$$P_{avg} = 1767W$$

1. Power Definitions and Equations, cont.

Circuit analysis using the Nodal Method. Write KCL equations at major nodes 1 and 2, and solve for phasor voltages V_1 and V_2.



$$\begin{bmatrix} \frac{1}{j4}+\frac{1}{3}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+1+\frac{1}{-j2} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{100\angle 20^\circ}{j4} \\ \frac{100\angle 20^\circ}{1} \end{bmatrix}$$

$$\tilde{V}_1 = \frac{\begin{bmatrix} \frac{100\angle 20^\circ}{j4} & -\frac{1}{2} \\ \frac{100\angle 20^\circ}{1} & \frac{1}{2}+1+\frac{1}{-j2} \end{bmatrix}}{D}$$

$$D = \left[\frac{1}{j4}+\frac{1}{3}+\frac{1}{2}\right] \bullet \left[\frac{1}{2}+1+\frac{1}{-j2}\right] - \left[-\frac{1}{2}\right] \bullet \left[-\frac{1}{2}\right]$$

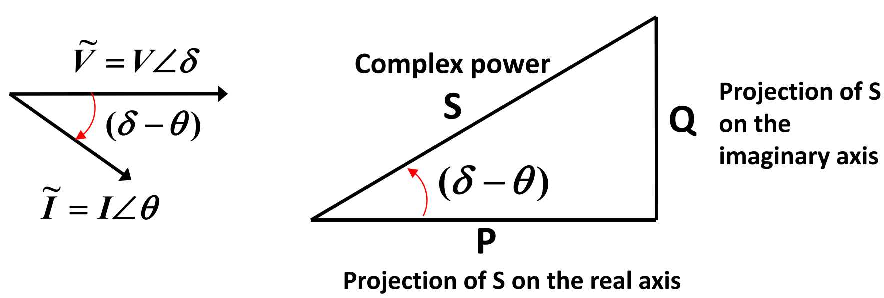
$$\tilde{V}_2 = \frac{\begin{bmatrix} -\frac{1}{2} & \frac{100\angle 20^\circ}{j4} \\ \frac{1}{2}+1+\frac{1}{-j2} & \frac{100\angle 20^\circ}{1} \end{bmatrix}}{D}$$

1. Power Definitions and Equations, cont.

Active power P_{avg} and reactive power Q form a power triangle

$$P_{avg} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} \cos(\delta - \theta), \quad Q = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} \sin(\delta - \theta),$$

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\tilde{I}\right]^* = \left[V\angle\delta\right] \bullet \left[I\angle\theta\right]^* = VI\angle(\delta - \theta)$$



$\cos(\delta - \theta)$ **is the power factor**

1. Power Definitions and Equations, cont.

Resistor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^* = \frac{V^2}{Z^*} = \frac{V^2}{R}$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^* = I^2 Z = I^2 R$$

Thus ⇒ $P = \frac{V^2}{R} = I^2 R, \; Q = 0$

Inductor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^* = \frac{V^2}{Z^*} = \frac{V^2}{-j\omega L} = j\frac{V^2}{\omega L}$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^* = I^2 Z = j\omega L I^2$$

Thus ⇒ $P = 0, \; Q = \frac{V^2}{\omega L} = \omega L I^2$

Inductor consumes reactive power

Capacitor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^* = \frac{V^2}{Z^*} = \frac{V^2}{\frac{1}{-j\omega C}} = -j\omega C V^2$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^* = I^2 \frac{1}{j\omega C} = -j\omega L I^2$$

Thus ⇒ $P = 0, \; Q = -\omega C V^2 = \frac{-I^2}{\omega C}$

Capacitor produces reactive power

Always use rms values of voltage and current in the above equations

1. Power Definitions and Equations, cont.

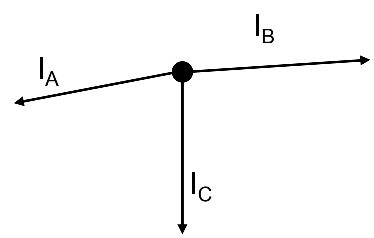
Question: Why is the sum of power out of a node = 0?
Answer: KCL and conservation of power

Question: What about reactive power Q?
Answer: It depends.

Question: Can you be a bit more specific?
Answer: Unlike P, there is no physical for Q to be conserved.

When voltage and current are not sinusoidal, then cross products of voltage and current exist and Q is not conserved.

But power systems are mostly sinusoidal, so as shown on the right with phasors, both P and Q are conserved.



$$\tilde{I}_A + \tilde{I}_B + \tilde{I}_C = 0$$

$$\tilde{V}\left(\tilde{I}_A + \tilde{I}_B + \tilde{I}_C\right) = 0$$

$$\tilde{V}\left(\tilde{I}_A + \tilde{I}_B + \tilde{I}_C\right)^* = 0$$

$$P_A + jQ_A + P_B + jQ_B + P_C + jQ_C = 0$$

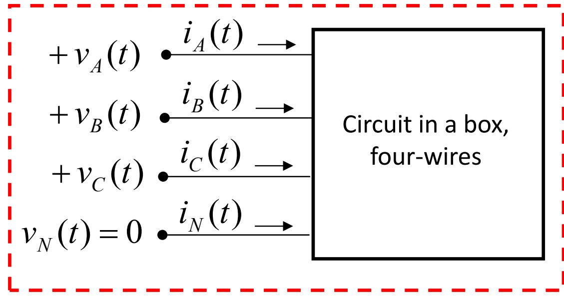
$$P_A + P_B + P_C = 0$$

$$Q_A + Q_B + Q_C = 0$$

2. Three-Phase Power.

Instantaneous power p(t) flowing into the box,

$$p_{ABC}(t) = v_A(t) \bullet i_A(t) + v_B(t) \bullet i_B(t) + v_C(t) \bullet i_C(t)$$



For a balanced system,

$$\begin{aligned} p_{ABC}(t) &= V_A \sin(\omega t + \delta) \bullet I_A \sin(\omega t + \theta) \\ &+ V_A \sin(\omega t + \delta - 120°) \bullet I_A \sin(\omega t + \theta - 120°) \\ &+ V_A \sin(\omega t + \delta + 120°) \bullet I_A \sin(\omega t + \theta + 120°). \end{aligned}$$

Combining terms,

$$\begin{aligned} p_{ABC}(t) &= V_A I_A \big[\sin(\omega t + \delta)\sin(\omega t + \theta) \\ &+ \sin(\omega t + \delta - 120°)\sin(\omega t + \theta - 120°) \\ &+ \sin(\omega t + \delta + 120°)\sin(\omega t + \theta + 120°)\big]. \end{aligned}$$

Trig. identity $\sin(\alpha)\sin(\beta) = \dfrac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$ **yields**

$$p_{ABC}(t) = \frac{V_A I_A}{2}\left[\cos(\delta - \theta) + \cos(2\omega t + \delta + \theta) + \cos(\delta - \theta) + \cos(2\omega t + \delta + \theta - 240°) + \cos(\delta - \theta) + \cos(2\omega t + \delta + \theta + 240°)\right],$$

2. Three-Phase Power, cont.

$$p_{ABC}(t) = 3\frac{V_A}{\sqrt{2}}\frac{I_A}{\sqrt{2}}\left[\cos(\delta - \theta) + \frac{\cos(2\omega t + \delta + \theta) + \cos(2\omega t + \delta + \theta - 240°) + \cos(2\omega t + \delta + \theta + 240°)}{3}\right].$$

Letting x = (2ωt+δ+Θ) and expanding the time varying term,

$$\cos(x) + \cos(x - 240°) + \cos(x + 240°) = \cos(x) + \cos(x + 120°) + \cos(x - 120°),$$

then expanding $\cos(x) + \cos(x + 120°) + \cos(x - 120°)$ **yields**

cancel each other

$$\cos(x) + \cos(x)\cos(120°) - \sin(x)\sin(120°) + \cos(x)\cos(120°) + \sin(x)\sin(120°),$$

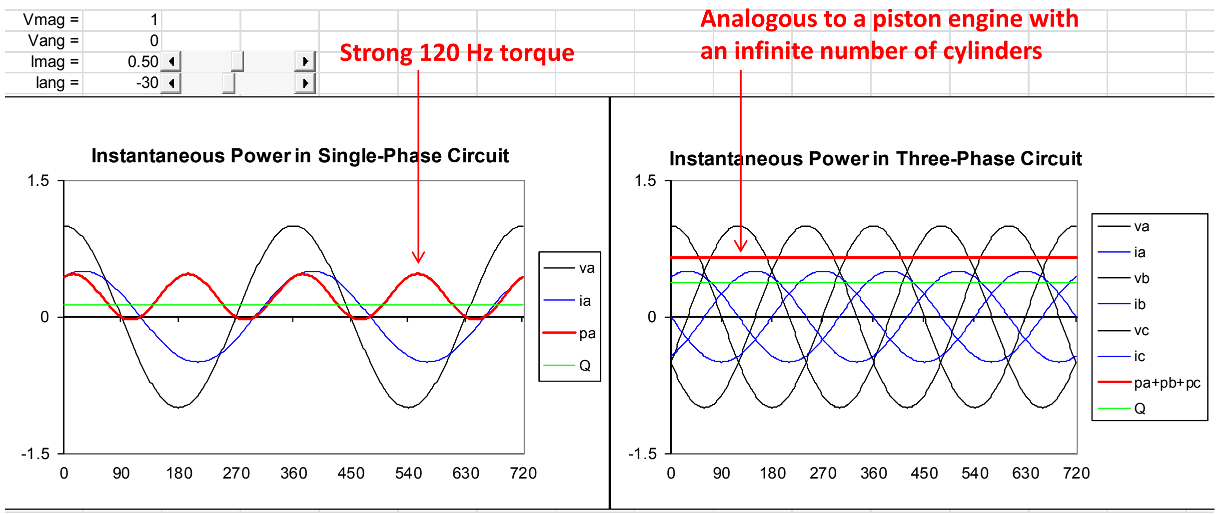
$$\cos(x) + \cos(x)\cos(120°) + \cos(x)\cos(120°),$$

$$\cos(x)\left[1 + \cos(120°) + \cos(120°)\right] = \cos(x)\left[1 - \frac{1}{2} - \frac{1}{2}\right] = 0,$$

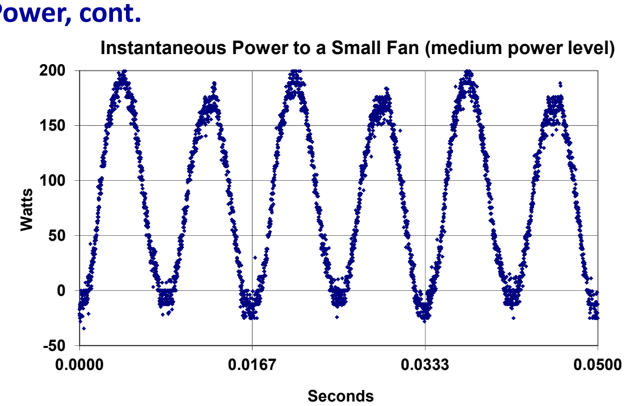
$$p_{ABC}(t) = 3\frac{V_A}{\sqrt{2}}\frac{I_A}{\sqrt{2}}\cos(\delta - \theta) = \boxed{3V_{Arms} I_{Arms}\cos(\delta - \theta)}$$

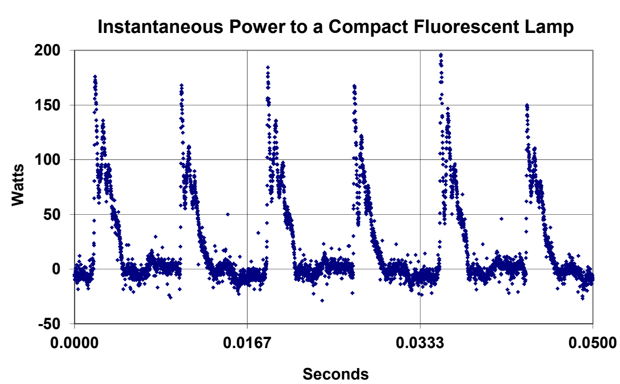
Instantaneous three-phase power is constant – thus, smooth running machines!

2. Three-Phase Power, cont.



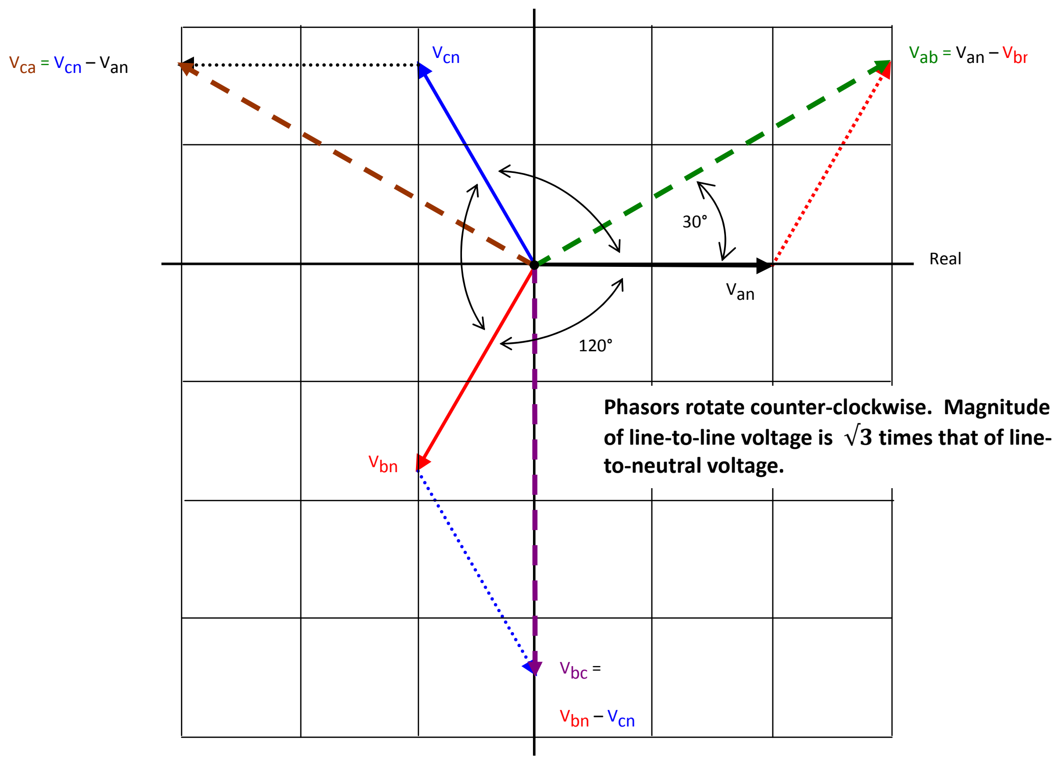
2. Three-Phase Power, cont.





2. Three-Phase Power, cont.

Balanced Three-Phase System

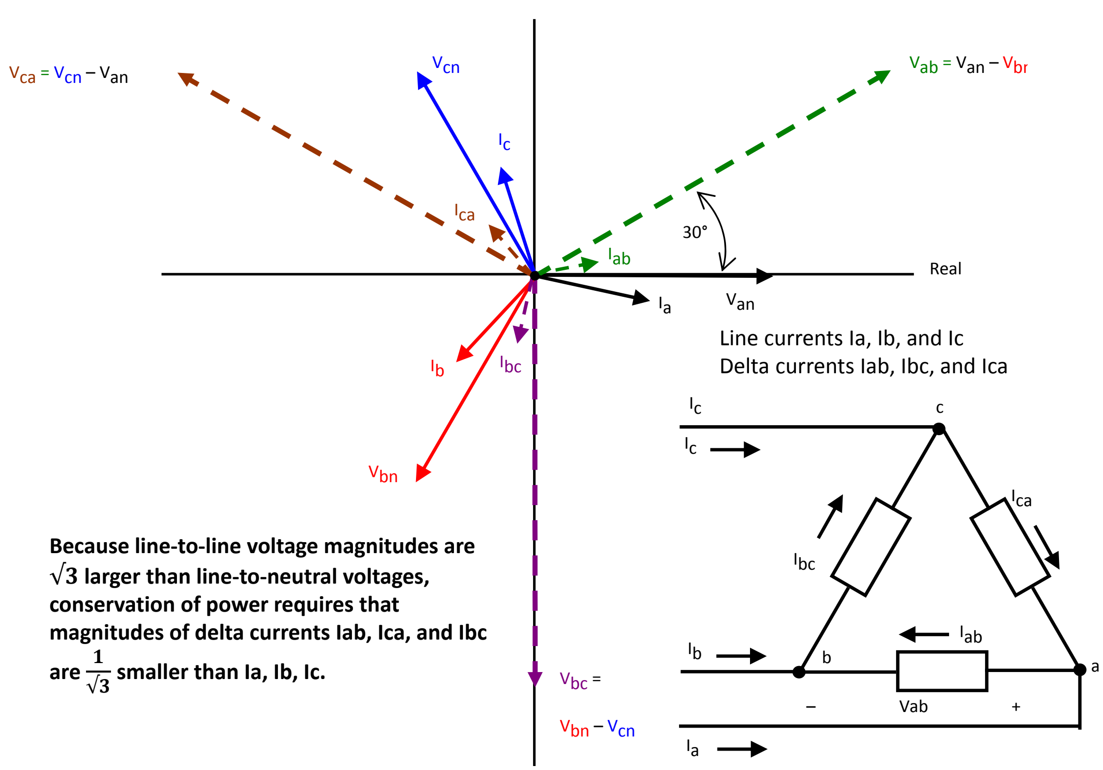


Phasors rotate counter-clockwise. Magnitude of line-to-line voltage is $\sqrt{3}$ times that of line-to-neutral voltage.

2. Three-Phase Power, cont.

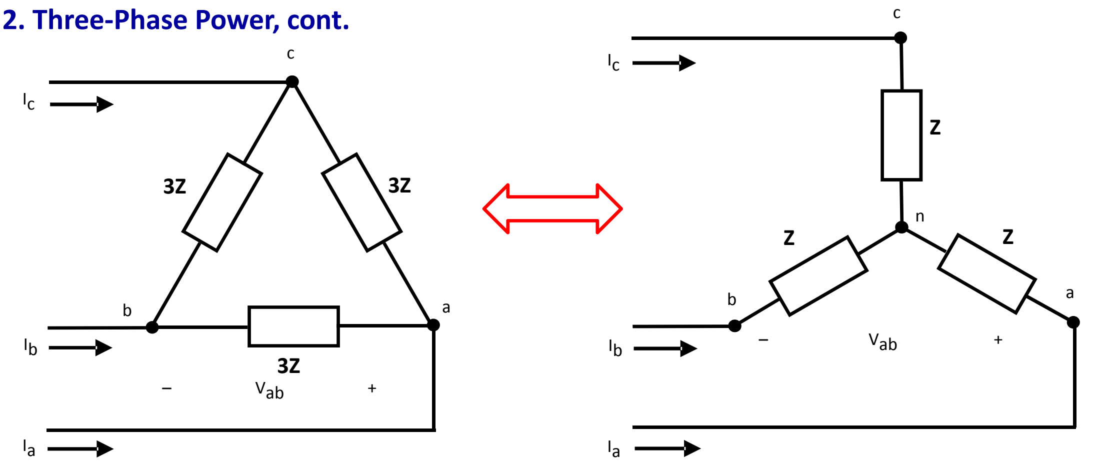
Balanced Three-Phase System

Line currents Ia, Ib, and Ic
Delta currents Iab, Ibc, and Ica



Because line-to-line voltage magnitudes are $\sqrt{3}$ larger than line-to-neutral voltages, conservation of power requires that magnitudes of delta currents Iab, Ica, and Ibc are $\frac{1}{\sqrt{3}}$ smaller than Ia, Ib, Ic.

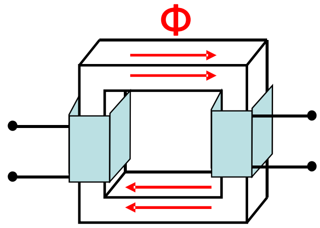
2. Three-Phase Power, cont.



Balanced three-phase systems, no matter if they are delta connected, wye connected, or a mix of wye and delta, are easier to solve if you follow these steps:

- **Convert the entire circuit to an equivalent wye with a grounded neutral.**
- **Draw the one-line diagram for phase a, recognizing that phase a has one third of the P and Q.**
- **Solve the one-line diagram for line-to-neutral voltages and line currents.**
- **If needed, compute line-to-neutral voltages and line currents for phases b and c using the ±120° relationships.**
- **If needed, compute line-to-line voltages and delta currents using the and ±30° relationships.**

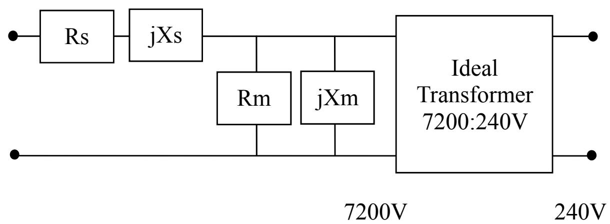
3. Transformer Models.



Turns ratio 7200:240

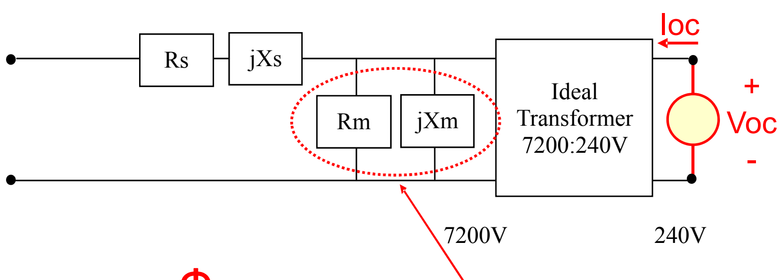
(30 : 1)

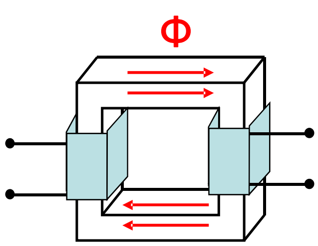
(approx. same amount of copper in each winding)



3. Transformer Models, cont.

Open Circuit Test





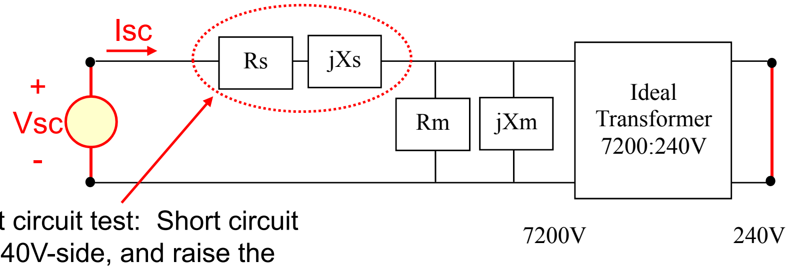
Turns ratio 7200:240

Open circuit test: Open circuit the 7200V-side, and apply 240V to the 240V-side. The winding currents are small, so the series terms are negligible.

$$R_m \parallel jX_m = \frac{\tilde{V}_{oc}}{\tilde{I}_{oc}}$$

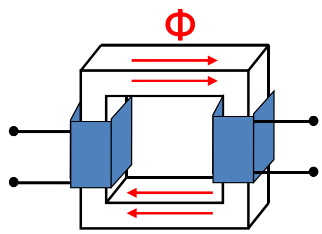
3. Transformer Models, cont.

Short Circuit Test



Short circuit test: Short circuit the 240V-side, and raise the 7200V-side voltage to a few percent of 7200, until rated current flows. There is almost no core flux so the magnetizing terms are negligible.

$$R_s + jX_s = \frac{\tilde{V}_{sc}}{\tilde{I}_{sc}}$$

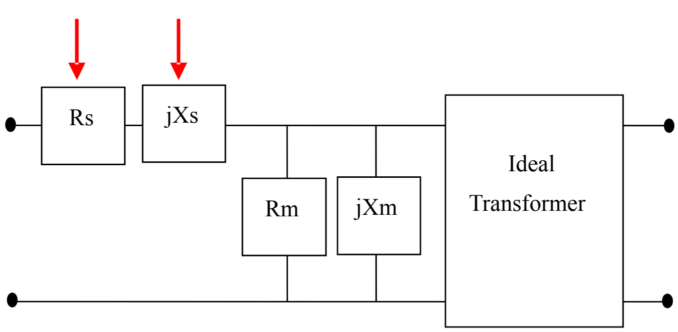


Turns ratio 7200:240

3. Transformer Models, cont.

X / R Ratios for Three-Phase Transformers

- 345kV to 138kV, X/R = 10
- Substation transformers (e.g., 138kV to 25kV or 12.5kV, X/R = 2, X = 12%
- 25kV or 12.5kV to 480V, X/R = 1, X = 5%
- 480V class, X/R = 0.1, X = 1.5% to 4.5%



3. Transformer Models, cont.

1. **Given the standard percentage values below for a 125kVA transformer, determine the R's and X's in the diagram, in Ω.**
2. **If the R's and X's are moved to the 240V side, compute the new Ω values.**
3. **If standard open circuit and short circuit tests are performed on this transformer, what will be the P's and Q's (Watts and VArs) measured in those tests?**

Single Phase Transformer. Percent values on transformer base.

Winding 1
kV = 7.2, kVA = 125

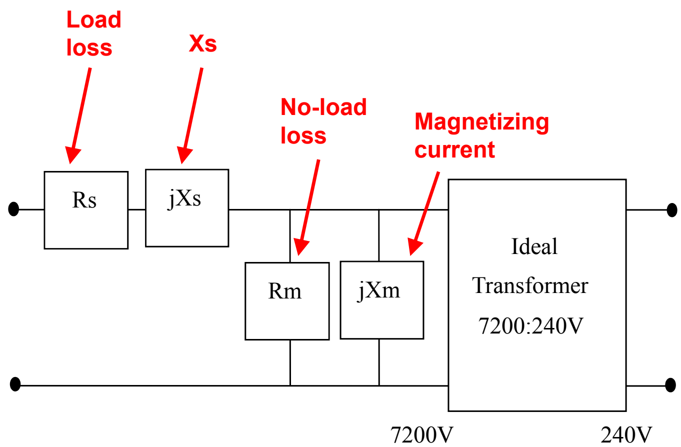
Winding 2
kV = 0.24, kVA = 125

%Imag = 0.5

%Load loss = 0.9

%No-load loss = 0.2

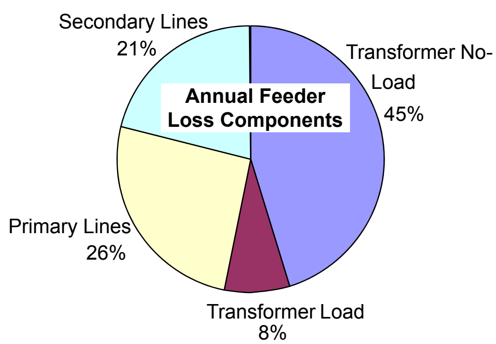
%Xs = 2.2



3. Transformer Models, cont.

EPRI Study, Distribution Feeder Loss Example

- Annual energy loss = 2.40%
- Largest component is transformer no-load loss (45% of the 2.40%)

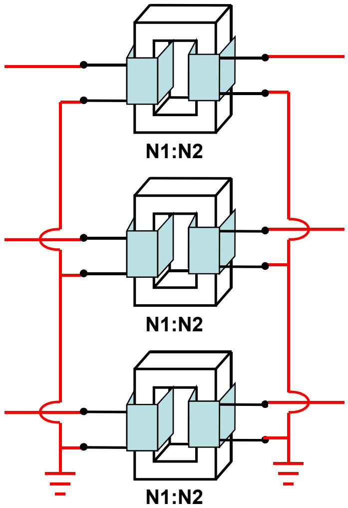


Modern Distribution Transformer:

- Load loss at rated load (I^2R in conductors) = 0.75% of rated transformer kW.
- No load loss at rated voltage (magnetizing, core steel) = 0.2% of rated transformer kW.
- Magnetizing current = 0.5% of rated transformer amperes

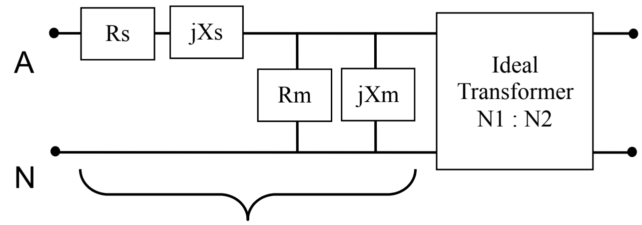
3. Transformer Models, cont.

A three-phase transformer can be three separate single-phase transformers, or one large transformer with three sets of windings



Y - Y

Wye-Equivalent One-Line Model

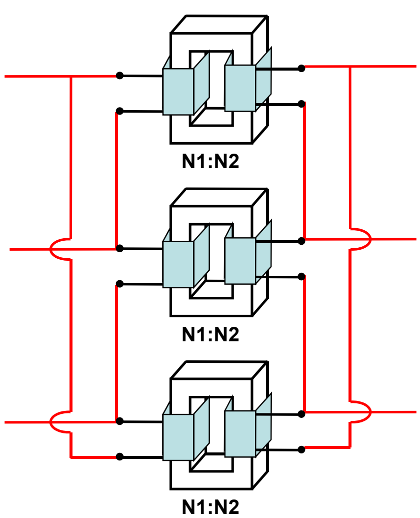


- **Reflect to side 2 using individual transformer turns ratio N1:N2**

Standard 345/138kV autotransformers, GY-GY, have a tertiary 12.5kV Δ winding to permit circulating 3rd harmonic current

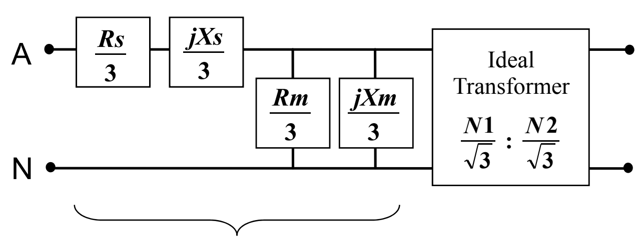
3. Transformer Models, cont.

For Modeling a Delta-Delta Connection, Convert the Transformer to Equivalent Wye-Wye



Δ - Δ

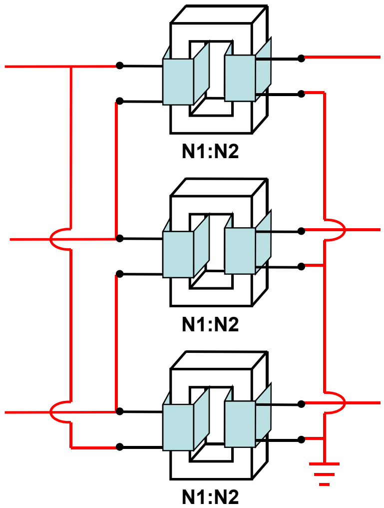
Wye-Equivalent One-Line Model



- **Convert side 1 impedances from delta to equivalent wye**
- **Then reflect to side 2 using individual transformer turns ratio N1:N2**

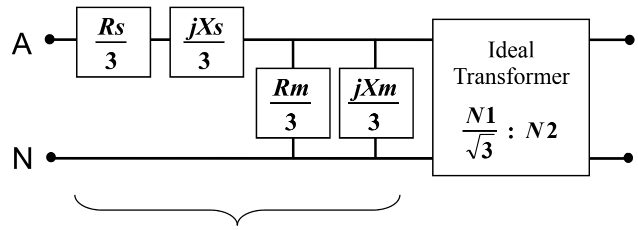
3. Transformer Models, cont.

For Modeling a Delta-Wye Connection, Convert the Transformer to Equivalent Wye-Wye



Δ - Y

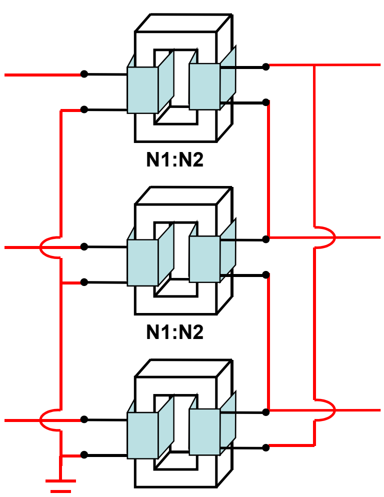
Wye-Equivalent One-Line Model



- Convert side 1 impedances from delta to wye
- Then reflect to side 2 using three-phase line-to-line turns ratio $N1 : \sqrt{3}N2$
- Has 30° degree phase shift due to line-to-neutral to line-to-line relationship. ANSI standard requires the transformer to be labeled such that high-voltage side leads the low-voltage side by 30° .

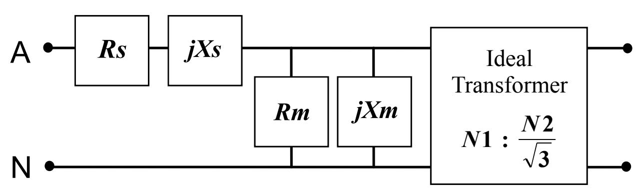
3. Transformer Models, cont.

For Modeling a Wye-Delta Connection, Convert the Transformer to Equivalent Wye-Wye



Y - Δ

Wye-Equivalent One-Line Model



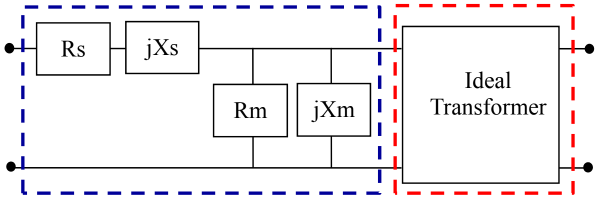
- Reflect to side 2 using three-phase bank line-to-line turns ratio $\sqrt{3}N1 : N2$
- Has 30° degree phase shift due to line-to-neutral to line-to-line relationship. ANSI standard requires the transformer to be labeled such that high-voltage side leads the low-voltage side by 30°

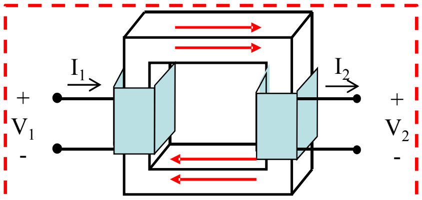
Thus, for all configurations, equivalent wye-wye transformer ohms can be reflected from one side to the other using the three-phase bank line-to-line turns ratio

4. Per Unit System.

Physical R's, L's. KCL, KVL.

Two equations describe the ideal transformer





Turns ratio N1 : N2

Faraday's Law

$$v_2(t) = v_1(t) \bullet \frac{N_2}{N_1},$$

Conservation of Power

$$v_1(t) \bullet i_1(t) = v_2(t) \bullet i_2(t)$$

From conservation of power,

$$v_2(t) \bullet i_2(t) = v_1(t) \bullet i_1(t)$$

$$i_2(t) = \frac{v_1(t)}{v_2(t)} \bullet i_1(t)$$

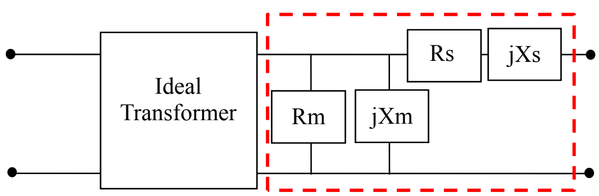
Merge Faraday's Law into conservation of power,

$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$

$$\frac{v_2(t)}{i_2(t)} = \frac{v_1(t)\frac{N_2}{N_1}}{i_1(t)\frac{N_1}{N_2}} = \frac{v_1(t)}{i_1(t)}\left[\frac{N_2}{N_1}\right]^2$$

4. Per Unit System, cont.

$$\frac{v_2(t)}{i_2(t)} = \frac{v_1(t)}{i_1(t)}\left[\frac{N_2}{N_1}\right]^2$$



Turns ratio N1 : N2

Moving the impedances from low-side to high-side AND multiplying them by the square of turns ratio

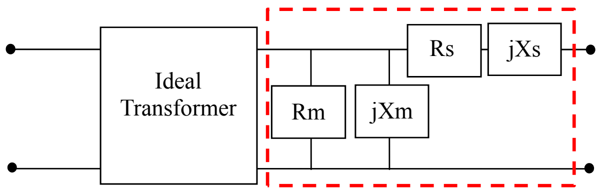
$$\left[\frac{N_2}{N_1}\right]^2$$

satisfies Faraday's Law and conservation of power, and provides an exact equivalent circuit for the transformer.

- Developers of the per unit system recognized that an N1:N2 ideal transformer could be replaced with a 1:1 ideal transformer if "base voltages" were used in circuit simulations, where **base voltages vary according to N1:N2 ratio**.
- To achieve conservation of power and have the same base power everywhere in a network, **base current must vary according to the inverse of N1:N2 ratio**.
- Thus, base voltage divided by base current varies according to $(N2:N1)^2$, which means that **transformer impedances "in per unit" are identical on both sides of the transformer, and the 1:1 ideal transformers can be excluded from circuit simulations**.

4. Per Unit System, cont.

$$\frac{v_2(t)}{i_2(t)} = \frac{v_1(t)}{i_1(t)} \left[\frac{N_2}{N_1} \right]^2$$



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Moving the impedances from low-side to high-side AND multiplying them by the square of turns ratio

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- **Developers of the per unit system recognized that an N1:N2 ideal transformer could be replaced with a 1:1 ideal transformer if "base voltages" were used in circuit simulations, where base voltages vary according to N1:N2 ratios.**
- **To achieve conservation of power and have the same base power everywhere in a network, base current must vary according to the inverse of N1:N2.**
- **Thus, base voltage divided by base current varies according to $(N2:N1)^2$, which means that transformer impedances "in per unit" are identical on both sides of the transformer, and the 1:1 ideal transformers are not needed in circuit simulations.**

4. Per Unit System, cont.

Single-Phase Procedure

- **Select an important bus in a system. Chose Vbase and Sbase values at that bus that fit the conditions. For example, 7200 Vrms, 240 Vrms, etc. And, 25 kVA, 100 kVA, etc.**
- **Calculate Ibase(rms) = Sbase / Vbase. Zbase = Vbase / Ibase = $(Vbase)^2$ / Sbase.**
- **If there are additional transformers in the circuit, adjust Vbase, Ibase, and Zbase for that voltage level, so that Vbase varies with turns ratio, Ibase varies inversely with turns ratio, and Zbase becomes the new Vbase/Ibase**
- **Replace transformers with their per unit internal impedances, expressed on the bases you have chosen.**
- **Convert all impedances to per unit.**
- **Convert all P and Q to per unit using Sbase.**
- **Solve the circuit using per unit values.**
- **Once solved, convert per unit answers to actual volts, amps, voltamps by multiplying by the appropriate base value.**

4. Per Unit System, cont.

Single-Phase Procedure. Converting Per Unit Impedances from Equipment Base to Your Chosen Base at their Bus

- Transformer impedances are given as per unit on the transformer base. Base voltage is rated rms voltage, base voltamps is rated voltamps.
- Zpu = Zactual / Zbase, so Zactual = Zpu • Zbase
- Zactual = Zpu_old • Zbase_old. Also, the same Zactual = Zpu_new • Zbase_new
- Thus, Zpu_new = Zpu_old • Zbase_old / Zbase_new
- Continuing,

$$Z_{pu,new} = Z_{pu,old}\left[\frac{V^2{}_{base,old}}{S_{base,old}}\right]\left[\frac{S_{base,new}}{V^2{}_{base,new}}\right]$$

$$\boxed{Z_{pu,new} = Z_{pu,old}\left[\frac{S_{base,new}}{S_{base,old}}\right]\left[\frac{V_{base,old}}{V_{base,new}}\right]^2}$$

4. Per Unit System, cont.

Extension to Three-Phase

- The key to three-phase per unit analysis is to recognize that transformer impedances reflect from one side to the other in proportion to squared *line-to-line* voltage.
- In your mind, convert the system to a three-phase wye-equivalent with line-to-neutral voltage. Per unit current is the current each wire, not the sum of all three. One wire carries one-third of the voltamps.
- Base voltage is line-to-line. Even so, it is helpful to remember its relationship to line-to-neutral.
- Base voltamperes is the sum of three phases.

$$S_{base,3\phi} = 3S_{base,1\phi} = 3V_{base,LN}I_{base} = 3\frac{V_{base,LL}}{\sqrt{3}}I_{base} = \boxed{\sqrt{3}\,V_{base,LL}I_{base}}$$

$$Z_{base,3\phi} = \frac{\left[V_{base,LN}\right]^2}{S_{base,1\phi}} = \frac{\left[\dfrac{V_{base,LL}}{\sqrt{3}}\right]^2}{\dfrac{S_{base,3\phi}}{3}} = \boxed{\frac{\left[V_{base,LL}\right]^2}{S_{base,3\phi}}}$$

4. Per Unit System, cont.

Converting Impedance from One Base to Another

$$Z_{base,3\phi} = \frac{\left[V_{base,LN}\right]^2}{S_{base,1\phi}} = \frac{\left[\frac{V_{base,LL}}{\sqrt{3}}\right]^2}{\frac{S_{base,3\phi}}{3}} = \frac{\left[V_{base,LL}\right]^2}{S_{base,3\phi}}$$

$$Z_{actual} = Z_{PU,old} \bullet Z_{base,old} = Z_{PU,new} \bullet Z_{base,new}$$

$$Z_{PU,new} = Z_{PU,old}\left[\frac{Z_{base,old}}{Z_{base,new}}\right] = Z_{PU,old}\left[\frac{V^2{}_{baseLL,3\phi,old}}{S_{base,3\phi,old}}\right]\left[\frac{S_{base,3\phi,new}}{V^2{}_{baseLL,3\phi,new}}\right]$$

$$Z_{PU,new} = Z_{PU,old}\left[\frac{V_{baseLL,3\phi,old}}{V_{baseLL,3\phi,new}}\right]^2\left[\frac{S_{base,3\phi,new}}{S_{base,3\phi,old}}\right]$$

4. Per Unit System, cont.

Zero-Sequence (i.e., Common Mode) Transformer Models

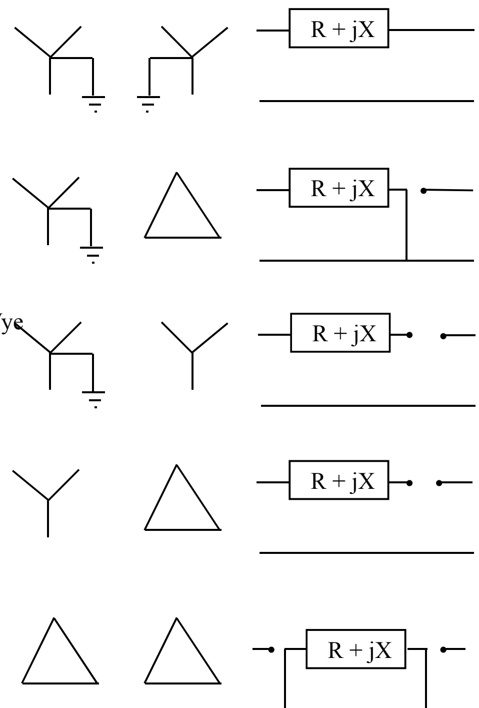
Grounded Wye - Grounded Wye

Grounded Wye - Delta

Grounded Wye - Ungrounded Wye

Ungrounded Wye - Delta

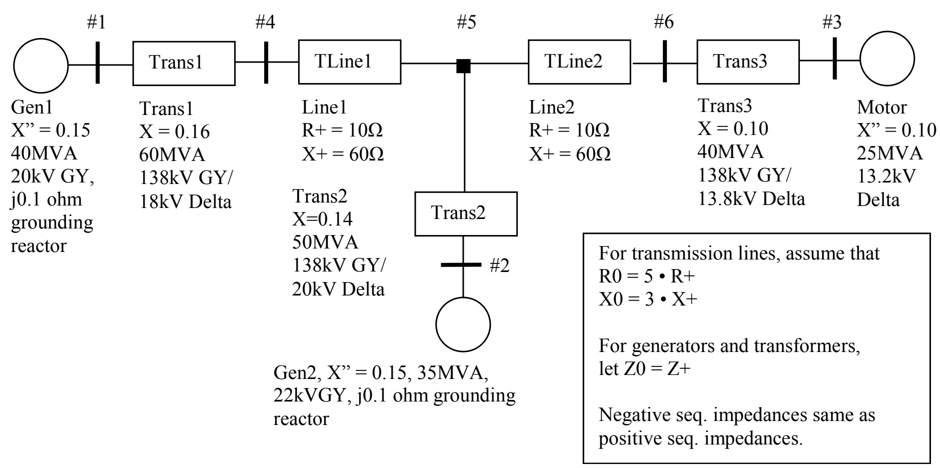
Delta - Delta



4. Per Unit System, cont.

Example Problem:

Information for a small power system is shown below. Per unit values are given on the equipment bases. Using a 138kV, 100MVA base in the transmission lines, draw the positive and negative sequence per unit diagrams. Assume that no current is flowing in the network, so that all generator and motor voltages are 1.0pu (positive sequence) in your final diagram.



5. Symmetrical Components.

Fortescue's Theorem: An unbalanced set of *N* related phasors can be resolved into *N* systems of phasors called the symmetrical components of the original phasors. For a three-phase system (i.e. *N* = 3), the three sets are:

- **Positive Sequence (indicated by "+" or "1")** - three phasors, equal in magnitude, 120° apart, with the same sequence (a-b-c) as the original phasors.
- **Negative Sequence ("–" or "2")** - three phasors, equal in magnitude, 120° apart, with the opposite sequence (a-c-b) of the original phasors.
- **Zero Sequence ("0")** - three identical phasors (i.e. equal in magnitude, with no relative phase displacement).

The original set of phasors

$$\tilde{V}_a = \tilde{V}_{a0} + \tilde{V}_{a1} + \tilde{V}_{a2}$$
$$\tilde{V}_b = \tilde{V}_{b0} + \tilde{V}_{b1} + \tilde{V}_{b2}$$
$$\tilde{V}_c = \tilde{V}_{c0} + \tilde{V}_{c1} + \tilde{V}_{c2}$$

Symmetrical Components and their relationship to one another

$$\tilde{V}_{a0} = \tilde{V}_{b0} = \tilde{V}_{c0}$$
$$\tilde{V}_{b1} = \tilde{V}_{a1} \bullet 1\angle -120^o \qquad \tilde{V}_{c1} = \tilde{V}_{a1} \bullet 1\angle +120^o$$
$$\tilde{V}_{b2} = \tilde{V}_{a2} \bullet 1\angle +120^o \qquad \tilde{V}_{c2} = \tilde{V}_{a2} \bullet 1\angle -120^o$$

5. Symmetrical Components, cont.

The symmetrical components of all a-b-c voltages are usually written in terms of the symmetrical components of phase a by defining complex number a as

$$a = 1\angle +120^o$$
$$a^2 = 1\angle +240^o = 1\angle -120^o$$
$$a^3 = 1\angle +360^o = 1\angle 0^o$$

Substituting into the previous equations for $\tilde{V}_a, \tilde{V}_b, \tilde{V}_c$ yields

$$\tilde{V}_a = \tilde{V}_{a0} + \tilde{V}_{a1} + \tilde{V}_{a2}$$
$$\tilde{V}_b = \tilde{V}_{a0} + a^2\tilde{V}_{a1} + a\tilde{V}_{a2}$$
$$\tilde{V}_c = \tilde{V}_{a0} + a\tilde{V}_{a1} + a^2\tilde{V}_{a2}$$

$$\Rightarrow \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix}$$

---

which in simplified form is

$$\tilde{V}_{abc} = T \bullet \tilde{V}_{012}, \tilde{V}_{012} = T^{-1} \bullet \tilde{V}_{abc} \ , \quad T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}, \quad T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

5. Symmetrical Components, cont.

Why do all this? Answer: to simplify most situations.

$$\tilde{V}_{012} = T^{-1} \bullet \tilde{V}_{abc},$$

$$\begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a^2 \end{bmatrix}\begin{bmatrix} \tilde{V}_{a} \\ \tilde{V}_{b} \\ \tilde{V}_{c} \end{bmatrix}$$

For balanced set, have only positive-sequence

$$\begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \tilde{V}_{a} \\ a^2\tilde{V}_{a} \\ a\tilde{V}_{a} \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1+a^2+a \\ 1+a^3+a^3 \\ 1+a^4+a^2 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1+a^2+a \\ 1+a^3+a^3 \\ 1+a+a^2 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = \tilde{V}_a\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Swap phases b and c (negative rotation), have only negative-sequence

$$\begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \tilde{V}_{a} \\ a\tilde{V}_{a} \\ a^2\tilde{V}_{a} \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1+a+a^2 \\ 1+a^2+a^4 \\ 1+a^3+a^3 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1+a^2+a \\ 1+a^2+a \\ 1+1+1 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \tilde{V}_a\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Common mode (a, b, c identical), have only zero-sequence

$$\begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \tilde{V}_{a} \\ \tilde{V}_{a} \\ \tilde{V}_{a} \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 3 \\ 1+a+a^2 \\ 1+a^2+a \end{bmatrix} = \frac{\tilde{V}_a}{3}\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \tilde{V}_a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

5. Symmetrical Components, cont.

Zero-Sequence (i.e., Common Mode) Transformer Models, again

Zero-sequence current can flow ONLY when there is a ground path

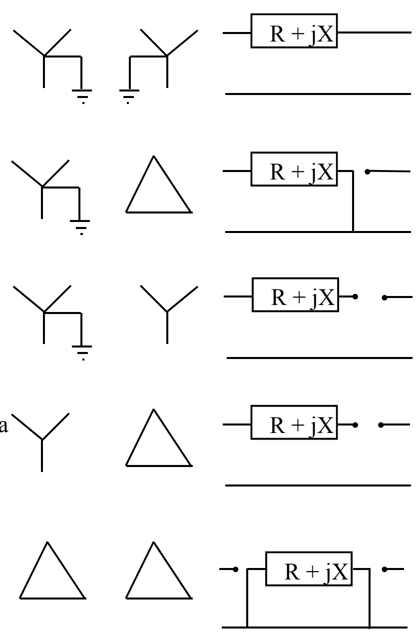
Grounded Wye - Grounded Wye

Grounded Wye - Delta

Grounded Wye - Ungrounded Wye

Ungrounded Wye - Delta

Delta - Delta



5. Symmetrical Components, cont.

Three-phase circuits can be solved in full a-b-c, or in using 0-1-2 sequence components.

For a balanced circuit, solve it in positive-sequence because negative- and zero-sequences are zero, so that the positive-sequence solution **IS** the phase A solution. Hence, the "one-line" diagram.

Consider the following a-b-c equations for a symmetric network. "Symmetric" means that self-impedances **S** for each phase are equal, and mutual-impedances **M** between phases are equal.

$$\tilde{V}_{abc} = Z_{abc} \bullet \tilde{I}_{abc} \quad \Rightarrow \quad \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} S & M & M \\ M & S & M \\ M & M & S \end{bmatrix} \begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \end{bmatrix}$$

Converting to 0-1-2

$$T\tilde{V}_{012} = Z_{abc} \bullet T\tilde{I}_{012}$$

$$T^{-1}T\tilde{V}_{012} = T^{-1}Z_{abc}T\tilde{I}_{012}$$

$$\tilde{V}_{012} = Z_{012}\tilde{I}_{012}, \text{ where } Z_{012} = T^{-1}Z_{abc}T$$

5. Symmetrical Components, cont.

$$Z_{012} = T^{-1}Z_{abc}T$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} S & M & M \\ M & S & M \\ M & M & S \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

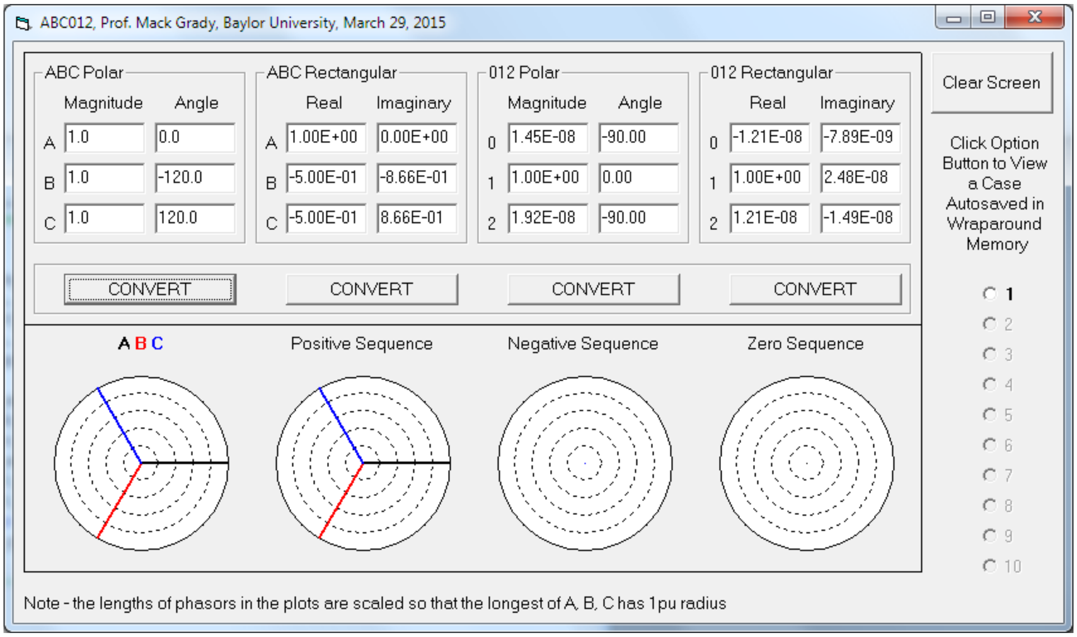
$$= \frac{1}{3}\begin{bmatrix} S+2M & S+2M & S+2M \\ S-M & a(S-M) & a^2(S-M) \\ S-M & a^2(S-M) & a(S-M) \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3(S+2M) & 0 & 0 \\ 0 & 3(S-M) & 0 \\ 0 & 0 & 3(S-M) \end{bmatrix}$$

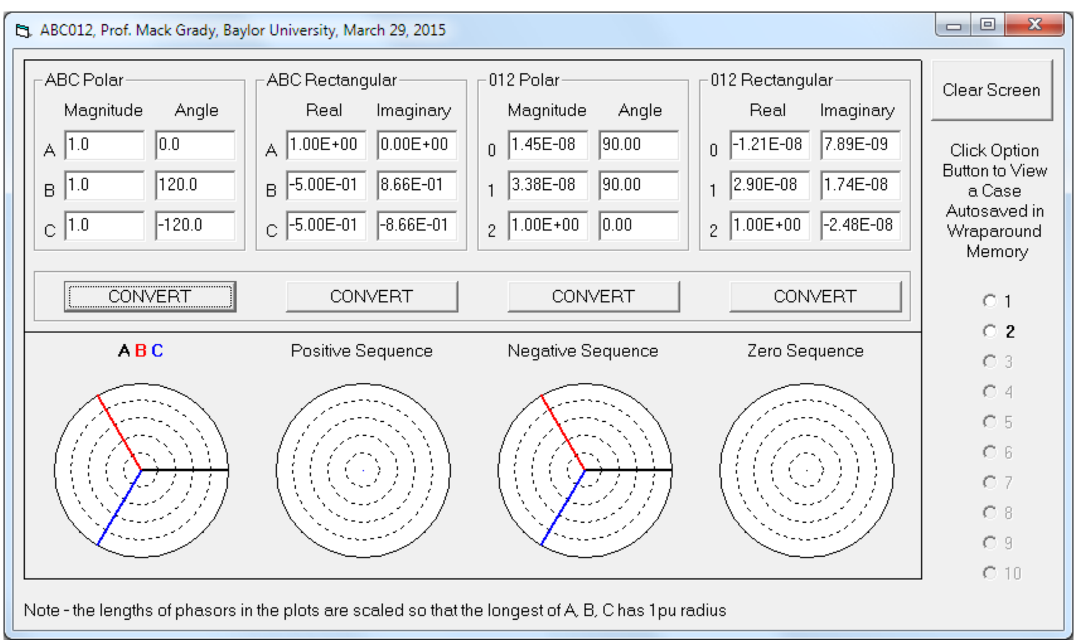
$$= \begin{bmatrix} (S+2M) & 0 & 0 \\ 0 & (S-M) & 0 \\ 0 & 0 & (S-M) \end{bmatrix} \Leftarrow$$

This is huge! If network impedances are balanced, then a coupled a-b-c network can be solved using three simple uncoupled 0-1-2 networks.

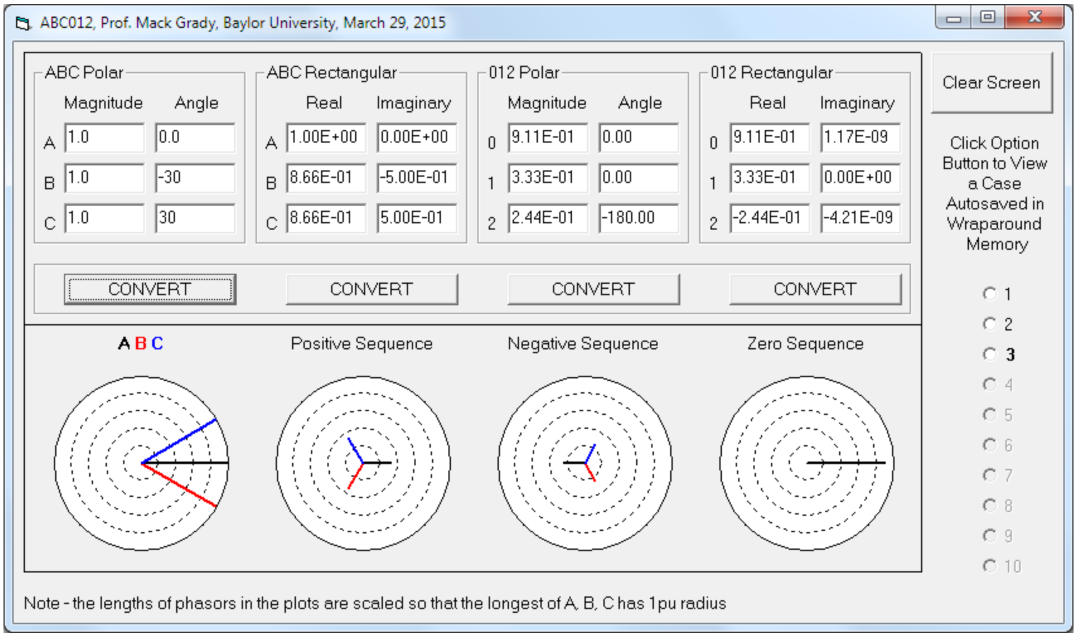
5. Symmetrical Components.



5. Symmetrical Components.

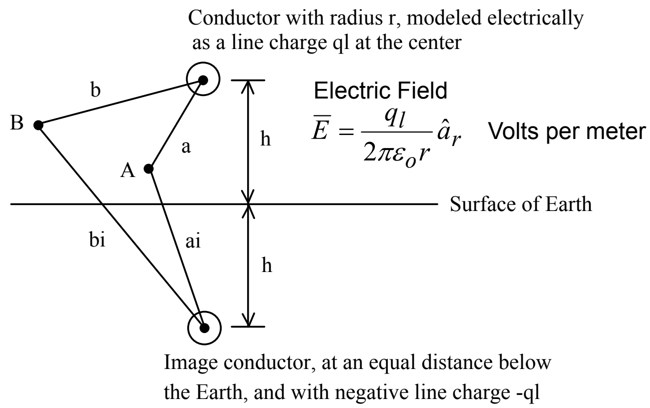


5. Symmetrical Components.



6. Transmission Line Models

Wire over Earth capacitance



$$V_{ab} = \int_{r=a}^{r=b} \overline{E}_\rho \bullet \hat{a}_r + \int_{r=ai}^{r=bi} \overline{E}_{\rho i} \bullet \hat{a}_r = \frac{q_l}{2\pi\varepsilon_o}\left[\ln\left(\frac{b}{a}\right) - \ln\left(\frac{bi}{ai}\right)\right] = \frac{q_l}{2\pi\varepsilon_o}\ln\left(\frac{b \bullet ai}{a \bullet bi}\right)$$

$$V_{rg} \approx \frac{q_l}{2\pi\varepsilon_o}\ln\left(\frac{2h}{r}\right)$$ for $h >> r$.

2π • 8.854 pF per meter length = 55.6 pF / meter

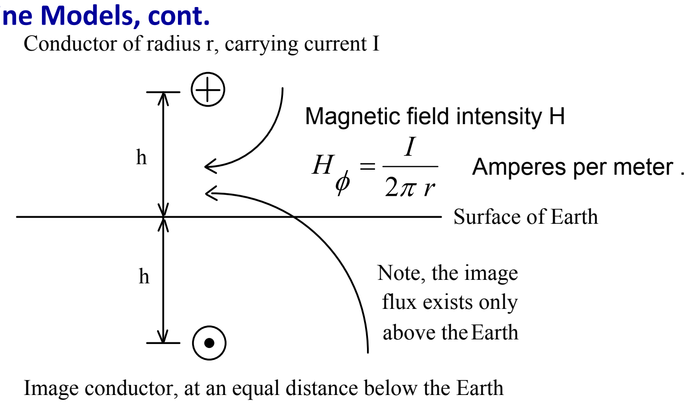
$$C = \frac{q_l}{V_{rg}} = \frac{2\pi\varepsilon_o}{\ln\left(\frac{2h}{r}\right)}$$

ln(10000) = 9.2, C = 6.0 pf/m
ln(1000) = 6.9, C = 8.1 pf/m
ln(100) = 4.6, C = 12.1 pf/m

Reasonable estimate is 10 pF/m

6. Transmission Line Models, cont.

Wire over Earth inductance



$$\Phi = \frac{\mu_o I}{2\pi}\left[\int_r^h \frac{dx}{x} + \int_h^{2h-r} \frac{dx}{x}\right] = \frac{\mu_o I}{2\pi}\ln\left[\frac{h(2h-r)}{rh}\right] = \frac{\mu_o I}{2\pi}\ln\left[\frac{(2h-r)}{r}\right]$$

$$\Phi \approx \frac{\mu_o I}{2\pi}\ln\left(\frac{2h}{r}\right)$$ for $h >> r$.

$(4\pi \bullet 10^{-7}) / 2\pi$ Henries per meter length = 0.2 μH/m

$$L_{ext} = \frac{N\Phi}{I} = \frac{\mu_o}{2\pi}\ln\left(\frac{2h}{r}\right)$$

ln(10000) = 9.2, L = 1.8 μH/m
ln(1000) = 6.9, L = 1.4 μH/m
ln(100) = 4.6, L = 0.92 μH/m

Reasonable estimate is 1 μH/m

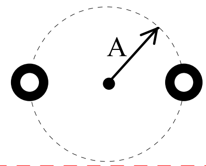
$$\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.0 \bullet 10^{-6})(10 \bullet 10^{-12})}} = \frac{1}{\sqrt{10 \bullet 10^{-18}}} = \frac{10^9}{\sqrt{10}} = \frac{10^9}{\sqrt{10}} \approx 3 \bullet 10^8$$

6. Transmission Line Models, cont.

Symmetric bundles have an equivalent radius

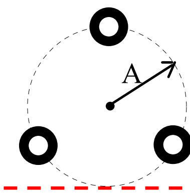
$$r_{eq} = \left[N r A^{N-1} \right]^{\frac{1}{N}}$$

Double Bundle, Each Conductor Has Radius r



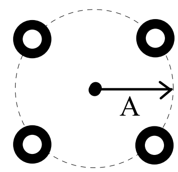
$$r_{eq} = \sqrt{2rA}$$

Triple Bundle, Each Conductor Has Radius r



$$r_{eq} = \sqrt[3]{3rA^2}$$

Quadruple Bundle, Each Conductor Has Radius r

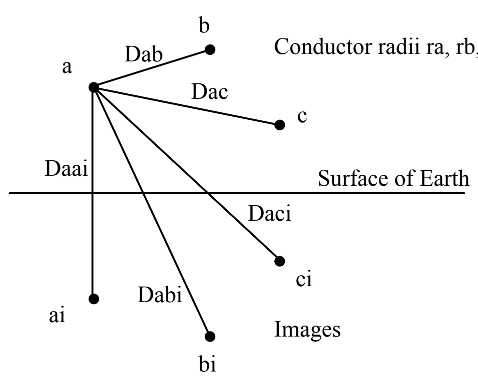


$$r_{eq} = \sqrt[4]{4rA^3}$$

6. Transmission Line Models, cont.

Three phases

Three Conductors Represented by Their Equivalent Line Charges



$$V_{ag} = \frac{1}{2\pi\varepsilon_o} \left[q_a \ln \frac{D_{aai}}{r_a} + q_b \ln \frac{D_{abi}}{D_{ab}} + q_c \ln \frac{D_{aci}}{D_{ac}} \right]$$

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \frac{1}{2\pi\varepsilon_o} \begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

If the transmission line is symmetric, then the "P matrix" has the equal diagonal, equal off-diagonal property that permits 0-1-2 analysis rather than a-b-c analysis

6. Transmission Line Models, cont.

$$\begin{bmatrix}\Phi_a\\ \Phi_b\\ \Phi_c\end{bmatrix} = \frac{\mu_o}{2\pi}\begin{bmatrix}\ln\frac{D_{aai}}{r_a} & \ln\frac{D_{abi}}{D_{ab}} & \ln\frac{D_{aci}}{D_{ac}}\\ \ln\frac{D_{bai}}{D_{ba}} & \ln\frac{D_{bbi}}{r_b} & \ln\frac{D_{bci}}{D_{bc}}\\ \ln\frac{D_{cai}}{D_{ca}} & \ln\frac{D_{cbi}}{D_{cb}} & \ln\frac{D_{cci}}{r_c}\end{bmatrix}\begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}$$

If the transmission line is symmetric, then the "L matrix" has the equal diagonal, equal off-diagonal property that permits 0-1-2 analysis rather than a-b-c analysis

$$L_{012}^{avg} = \begin{bmatrix}L_S + 2L_M & 0 & 0\\ 0 & L_S - L_M & 0\\ 0 & 0 & L_S - L_M\end{bmatrix}$$

6. Transmission Line Models, cont.

Summary of Positive/Negative Sequence Capacitance and Inductance Calculations

Computation of positive/negative sequence capacitance

$$C_{+/-} = \frac{2\pi\varepsilon_o}{\ln\frac{GMD_{+/-}}{GMR_{C+/-}}} \text{ farads per meter,}$$

where

$$GMD_{+/-} = \sqrt[3]{D_{ab} \bullet D_{ac} \bullet D_{bc}} \text{ meters,}$$

where D_{ab}, D_{ac}, D_{bc} are

- distances between phase conductors if the line has one conductor per phase, or
- distances between phase bundle centers if the line has symmetric phase bundles,

and where

- $GMR_{C+/-}$ is the actual conductor radius r (in meters) if the line has one conductor per phase, or
- $GMR_{C+/-} = \sqrt[N]{N \bullet r \bullet A^{N-1}}$ if the line has symmetric phase bundles.

6. Transmission Line Models, cont.

Summary of Positive/Negative Sequence Capacitance and Inductance Calculations

Computation of positive/negative sequence inductance

$$L_{+/-} = \frac{\mu_o}{2\pi} \ln \frac{GMD_{+/-}}{GMR_{L+/-}} \text{ henrys per meter,}$$

where $GMD_{+/-}$ is the same as for capacitance, and

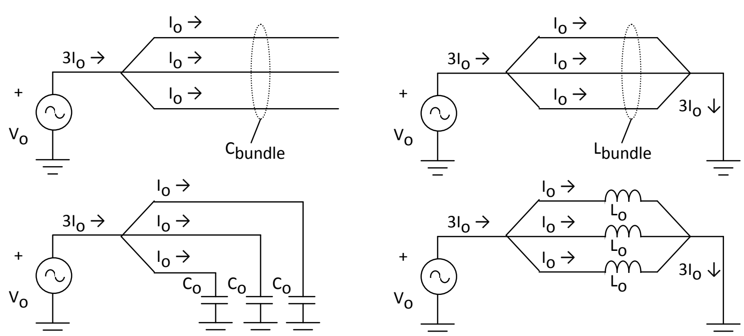
- for the single conductor case, $GMR_{L+/-}$ is the conductor r_{gmr} (in meters), which takes into account both stranding and the $e^{-1/4}$ adjustment for internal inductance. If r_{gmr} is not given, then assume $r_{gmr} = re^{-1/4}$, and
- for bundled conductors, $GMR_{L+/-} = \sqrt[N]{N \bullet r_{gmr} \bullet A^{N-1}}$ if the line has symmetric phase bundles.

Computation of positive/negative sequence resistance

R is the 60Hz resistance of one conductor if the line has one conductor per phase. If the line has symmetric phase bundles, then divide the one-conductor resistance by N.

6. Transmission Line Models, cont.

Summary of Zero Sequence Capacitance and Inductance Calculations



$$C_0 = \frac{1}{3} \bullet \frac{2\pi\varepsilon_o}{\ln \frac{GMD_{C0}}{GMR_{C0}}} \text{ farads per meter,}$$

where GMD_{C0} is the average height (with sag factored in) of the a-b-c bundle above perfect Earth. GMD_{C0} is computed using

$$GMD_{C0} = \sqrt[9]{D_{aa^i} \bullet D_{bb^i} \bullet D_{cc^i} \bullet D^2_{ab^i} \bullet D^2_{ac^i} \bullet D^2_{bc^i}} \text{ meters,}$$

$$GMR_{C0} = \sqrt[9]{GMR^3_{C+/-} \bullet D^2_{ab} \bullet D^2_{ac} \bullet D^2_{bc}} \text{ meters}$$

6. Transmission Line Models, cont.

Summary of Zero Sequence Capacitance and Inductance Calculations

Computation of zero sequence inductance

$$L_0 = 3 \bullet \frac{\mu_o}{2\pi} \ln \frac{\delta}{GMR_{L0}} \text{ Henrys per meter,}$$

where skin depth $\delta = \sqrt{\frac{\rho}{2\pi\mu_o f}}$ meters.

The geometric mean bundle radius is computed using

$$GMR_{L0} = \sqrt[9]{GMR_{L+/-}^3 \bullet D_{ab}^2 \bullet D_{ac}^2 \bullet D_{bc}^2} \text{ meters,}$$

where $GMR_{L+/-}$, D_{ab}, D_{ac}, and D_{bc} were shown previously.

Computation of zero sequence resistance

There are two components of zero sequence line resistance. First, the equivalent conductor resistance is the 60Hz resistance of one conductor if the line has one conductor per phase. If the line has symmetric phase bundles with N conductors per bundle, then divide the one-conductor resistance by N.

Second, the effect of resistive Earth is included by adding the following term to the conductor resistance:

$3 \bullet 9.869 \bullet 10^{-7} f$ ohms per meter (see Bergen),

6. Transmission Line Models, cont.

Summary of Zero Sequence Capacitance and Inductance Calculations

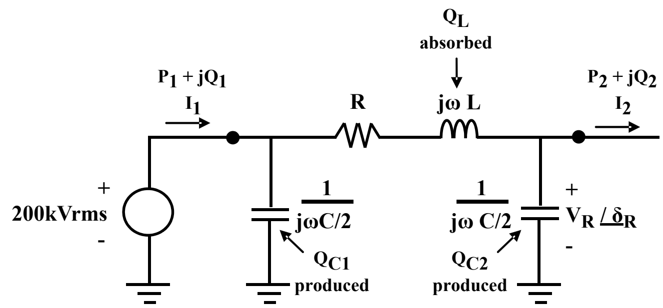
As a general rule,

- $C_{+/-}$ usually works out to be about 12 picoF per meter,
- $L_{+/-}$ works out to be about 1 microH per meter (including internal inductance).
- C_0 is usually about 6 picoF per meter.
- L_0 is usually about 2 microH per meter if the line has ground wires and typical Earth resistivity, or about 3 microH per meter for lines without ground wires or poor Earth resistivity.

The velocity of propagation, $\frac{1}{\sqrt{LC}}$, is approximately the speed of light (3 x 10^8 m/s) for positive and negative sequences, and about 0.8 times that for zero sequence.

6. Transmission Line Models, cont.

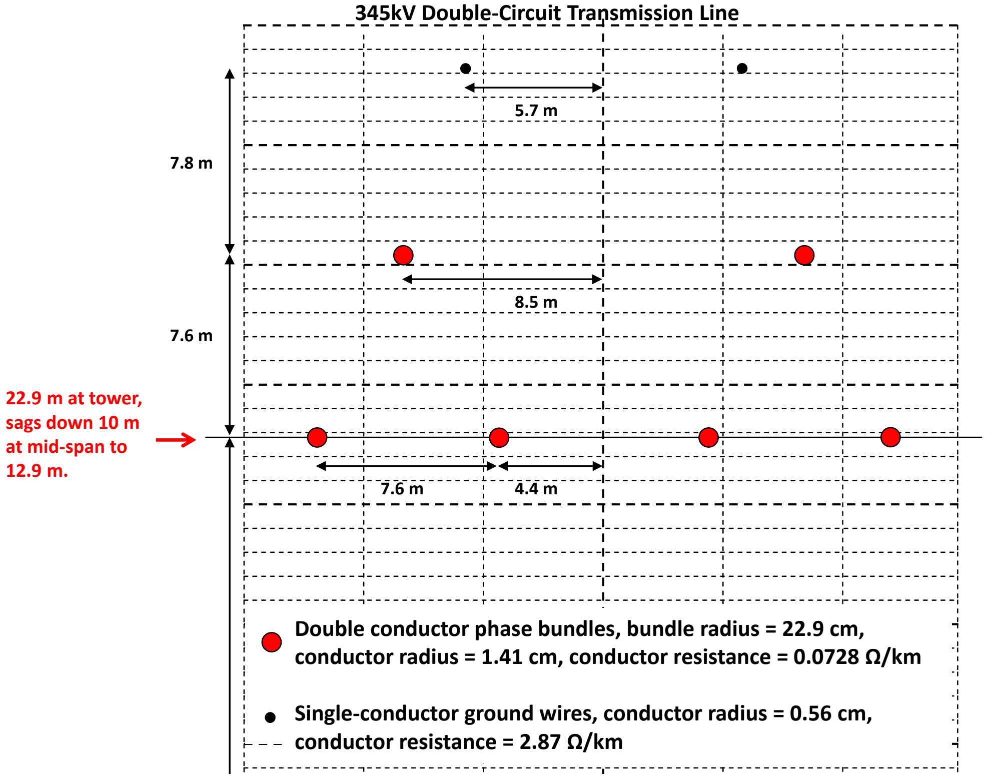
Ready for Use!



One circuit of the 345kV line geometry, 100km long

6. Transmission Line Models, cont.

345kV Double-Circuit Transmission Line



Short Circuits

1. Introduction

Voltage sags are due mostly to faults on either transmission systems or distribution feeders. Transmission faults affect customers over a wide area, possibly dozens of miles, but distribution faults usually affect only the customers on the faulted feeder or on adjacent feeders served by the same substation transformer.

Single-phase faults (i.e., line-to-ground) are the most common type of faults, followed by line-to-line, and three-phase. Since single-phase and line-to-line faults are unbalanced, their resulting sag voltages are computed using symmetrical components. Transformer connections affect the propagation of positive, negative, and zero sequence components differently. Thus, the characteristics of a voltage sag changes as it propagates through a network.

Typically, a transmission voltage sag passes through two levels of transformers before reaching a 480V load (e.g., 138kV:12.47kV at the entrance to the facility, and 12.47kV:480V at the load). 120V loads likely have a third transformer (e.g., 480V:120V). It is not intuitively obvious how the sag changes, but the changes can be computed using symmetrical components and are illustrated in this report.

2. Symmetrical Components

An unbalanced set of *N* related phasors can be resolved into *N* systems of phasors called the symmetrical components of the original phasors. For a three-phase system (i.e. *N* = 3), the three sets are:

1. Positive Sequence - three phasors, equal in magnitude, 120° apart, with the same sequence (a-b-c) as the original phasors.
2. Negative Sequence - three phasors, equal in magnitude, 120° apart, with the opposite sequence (a-c-b) of the original phasors.
3. Zero Sequence - three identical phasors (i.e. equal in magnitude, with no relative phase displacement).

The original set of phasors is written in terms of the symmetrical components as follows:

$$\tilde{V}_a = \tilde{V}_{a0} + \tilde{V}_{a1} + \tilde{V}_{a2} \ ,$$
$$\tilde{V}_b = \tilde{V}_{b0} + \tilde{V}_{b1} + \tilde{V}_{b2} \ ,$$
$$\tilde{V}_c = \tilde{V}_{c0} + \tilde{V}_{c1} + \tilde{V}_{c2} \ ,$$

where 0 indicates zero sequence, 1 indicates positive sequence, and 2 indicates negative sequence.

The relationships among the sequence components for a-b-c are

Positive Sequence	Negative Sequence	Zero Sequence
$\tilde{V}_{b1} = \tilde{V}_{a1} \bullet 1\angle -120^{\circ}$	$\tilde{V}_{b2} = \tilde{V}_{a2} \bullet 1\angle +120^{\circ}$	$\tilde{V}_{a0} = \tilde{V}_{b0} = \tilde{V}_{c0}$
$\tilde{V}_{c1} = \tilde{V}_{a1} \bullet 1\angle +120^{\circ}$	$\tilde{V}_{c2} = \tilde{V}_{a2} \bullet 1\angle -120^{\circ}$	

The symmetrical components of all a-b-c voltages are usually written in terms of the symmetrical components of phase a by defining

$$a = 1\angle +120^{\circ} \text{ , so that } a^2 = 1\angle +240^{\circ} = 1\angle -120^{\circ} \text{ , and } a^3 = 1\angle +360^{\circ} = 1\angle 0^{\circ} .$$

Substituting into the previous equations for $\tilde{V}_a, \tilde{V}_b, \tilde{V}_c$ yields

$$\tilde{V}_a = \tilde{V}_{a0} + \tilde{V}_{a1} + \tilde{V}_{a2} \ ,$$
$$\tilde{V}_b = \tilde{V}_{a0} + a^2\tilde{V}_{a1} + a\tilde{V}_{a2} \ ,$$
$$\tilde{V}_c = \tilde{V}_{a0} + a\tilde{V}_{a1} + a^2\tilde{V}_{a2} \ .$$

In matrix form, the above equations become

$$\begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} , \quad \begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \end{bmatrix} \qquad (1)$$

or in matrix form

$$\tilde{V}_{abc} = T \bullet \tilde{V}_{012} \text{ , and } \tilde{V}_{012} = T^{-1} \bullet \tilde{V}_{abc} \ , \qquad (2)$$

where transformation matrix *T* is

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{, and } T^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} . \qquad (3)$$

If $\tilde{V}_{abc}$ represents a balanced set (i.e. $\tilde{V}_b = \tilde{V}_a \bullet 1\angle -120^{\circ} = a^2\tilde{V}_a$, $\tilde{V}_c = \tilde{V}_a \bullet 1\angle +120^{\circ} = a\tilde{V}_a$), then substituting into $\tilde{V}_{012} = T^{-1} \bullet \tilde{V}_{abc}$ yields

$$\begin{bmatrix} \tilde{V}_{a0} \\ \tilde{V}_{a1} \\ \tilde{V}_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ a^2\tilde{V}_a \\ a\tilde{V}_a \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{V}_a \\ 0 \end{bmatrix} .$$

Hence, balanced voltages or currents have only positive sequence components, and the positive sequence components equal the corresponding phase a voltages or currents.

However, balanced voltages are rare during voltage sags. Most often, one phase is affected significantly, and the other two less significantly. Thus, all three sequence voltages $\tilde{V}_{a0}, \tilde{V}_{a1}, \tilde{V}_{a2}$ exist during most sags, and these sequence voltages are shifted differently by transformers when propagating through a system. When recombined to yield phase voltages $\tilde{V}_a, \tilde{V}_b, \tilde{V}_c$, it is clear that the form of phase voltages must also change as transformers are encountered.

3. Transformer Phase Shift

The conventional positive-sequence and negative-sequence model for a three-phase transformer is shown below. Admittance y is a series equivalent for resistance and leakage reactance, tap t is the tap (in per unit), and angle θ is the phase shift.

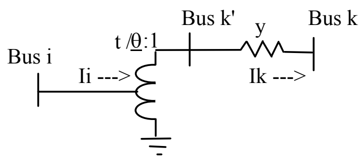


Figure 1. Positive- and Negative-Sequence Model of Three-Phase Transformer

For grounded-wye:grounded-wye and delta:delta transformers, θ is +0˚, and thus positive- and negative-sequence voltages and currents pass through unaltered (in per unit). However, for wye-delta and delta-wye transformers, θ is sequence-dependent and is defined as follows:

- For positive sequence, θ is +30˚ if bus i is the high-voltage side, or –30˚ if bus i is the low-voltage side

  and oppositely

- For negative sequence, θ is –30˚ if bus i is the high-voltage side, or +30˚ if bus i is the low-voltage side

In other words, positive sequence voltages and currents on the high-voltage side *lead* those on the low-voltage side by 30˚. Negative sequence voltages and currents on the high-voltage side *lag* those on the low-voltage side by 30˚.

For zero-sequence voltages and currents, transformers do not introduce a phase shift, but they may block zero-sequence propagation as shown in Figure 2.

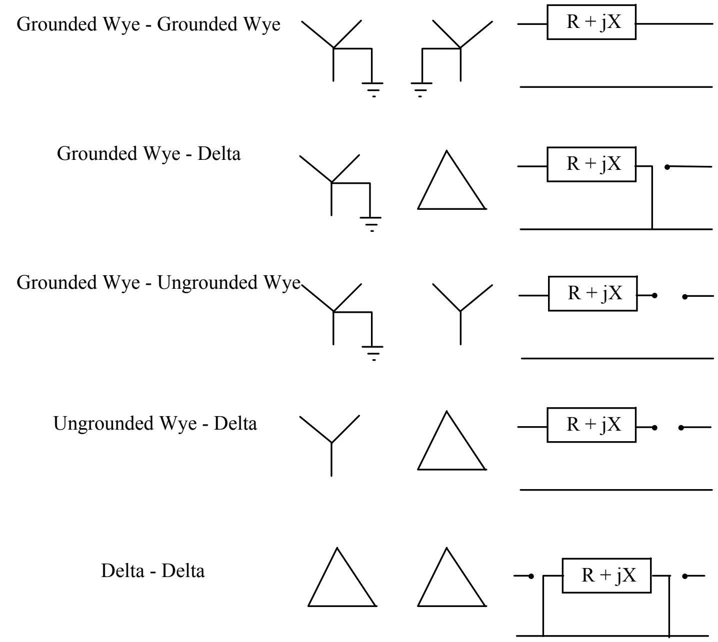


Figure 2. Zero-Sequence Model of Three-Phase Transformer

It can be seen in the above figure that only the grounded-wye:grounded-wye transformer connection permits the flow of zero-sequence from one side of a transformer to the other.

Thus, due to phase shift and the possible blocking of zero-sequence, transformers obviously play an important role in unbalanced voltage sag propagation.

4. System Impedance Matrices

Fault currents and voltage sags computations require elements of the impedance matrix Z for the study system. While each of the three sequences has its own impedance matrix, positive- and negative-sequence matrices are usually identical. Impedance elements are usually found by

- building the system admittance matrix Y, and then inverting it to obtain the entire Z,

or by

- using Gaussian elimination and backward substitution to obtain selected columns of Z.

The admittance matrix Y is easily built according to the following rules:

- The diagonal terms of Y contain the sum of all branch admittances connected directly to the corresponding bus.

- The off-diagonal elements of *Y* contain the negative sum of all branch admittances connected directly between the corresponding busses.

The procedure is illustrated by the three-bus example in Figure 3.

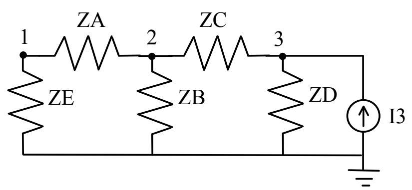


Figure 3. Three-Bus Admittance Matrix Example

Applying KCL at the three independent nodes yields the following equations for the bus voltages (with respect to ground):

At bus 1, $\frac{V_1}{Z_E} + \frac{V_1 - V_2}{Z_A} = 0$,

At bus 2, $\frac{V_2}{Z_B} + \frac{V_2 - V_1}{Z_A} + \frac{V_2 - V_3}{Z_C} = 0$,

At bus 3, $\frac{V_3}{Z_D} + \frac{V_3 - V_2}{Z_C} = I_3$.

Collecting terms and writing the equations in matrix form yields

$$\begin{bmatrix} \frac{1}{Z_E} + \frac{1}{Z_A} & -\frac{1}{Z_A} & 0 \\ -\frac{1}{Z_A} & \frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C} & -\frac{1}{Z_C} \\ 0 & -\frac{1}{Z_C} & \frac{1}{Z_C} + \frac{1}{Z_D} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_3 \end{bmatrix},$$

or in matrix form,

$$YV = I \ ,$$

Besides being the key for fault calculations, the impedance matrix, $Z = Y^{-1}$, is also physically significant. Consider Figure 4.

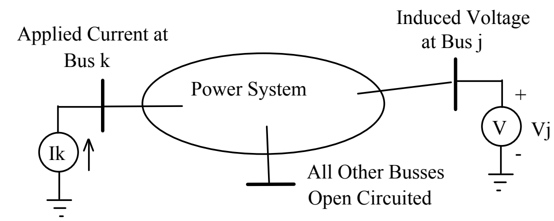


Figure 4. Physical Significance of the Impedance Matrix

Impedance matrix element $z_{j,k}$ is defined as

$$z_{j,k} = \left. \frac{V_j}{I_k} \right|_{I_m = 0, m = 1,2,\cdots,N, m \neq k} , \qquad (4)$$

where I_k is a current source attached to bus *k*, V_j is the resulting voltage at bus *j*, and all busses except *k* are open-circuited. The depth of a voltage sag at bus k is determined directly by multiplying the phase sequence components of the fault current at bus k by the matrix elements $z_{j,k}$ for the corresponding phase sequences.

5. Short Circuit Calculations

Short circuit calculations require positive, negative, and zero sequence impedance information, depending on whether or the fault is balanced or not. For example, the commonly-studied, but relatively rare, three-phase fault is balanced. Therefore, only positive sequence impedances are required for its study.

Consider the three-phase fault represented by the one-line diagram in Figure 5, where V_{TH} and Z_{TH} are the Thevenin equivalent circuit parameters for bus k.

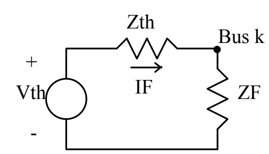


Figure 5. Three-Phase Fault at Bus *k*

The fault current and voltage are clearly

$$I_k^F = \frac{V_{TH}}{Z_{TH} + Z_F} \text{ , and } V_k^F = V_{TH} - Z_{TH} I_k^F = V_{TH} \left[\frac{Z_F}{Z_{TH} + Z_F} \right] .$$

In a large power system, the Thevenin equivalent impedance for a bus is the corresponding diagonal impedance matrix element, and the Thevenin equivalent voltage is usually assumed to be 1.0 /0 pu.

The type of machine models used when building impedance matrices affects the Thevenin equivalent impedances and fault calculations. Rotating machines actually have time-varying impedances when subjected to disturbances. However, for simplification purposes, their impedances are usually divided into three zones - subtransient (first few cycles), transient (5 cycles - 60 cycles), and steady-state (longer than 60 cycles). When performing fault studies, the time period of interest is usually a few cycles, so that machines are represented by their subtransient impedances when forming the impedance matrices.

Developing the equations for fault studies requires adept use of both a-b-c and 0-1-2 forms of the circuit equations. The use of sequence components implies that the system impedances (but not the system voltages and currents) are symmetric. In general, there are six equations and six unknowns to be solved, regardless of the type of fault studied.

It is common in fault studies to assume that the power system is initially unloaded and that all voltages are 1.0 per unit. When there are multiple sources, this assumption *requires* that there are no shunt elements connected, such as loads, capacitors, etc., *except* for rotating machines (whose Thevenin equivalent voltages are 1.0 pu.).

Since wye-delta transformers shift positive, negative, and zero sequence components differently, it is important to model transformers according to the rules given earlier. This means that the pre-fault voltages all have magnitude 1.0 pu., but that the pre-fault voltage angles can be $0^o, +30^o$, or -30^o, depending upon the net transformer phase shift between them and the chosen reference bus.

Balanced Three-Phase Fault

Consider the three-phase fault at bus *k*, as shown in Figure 6.

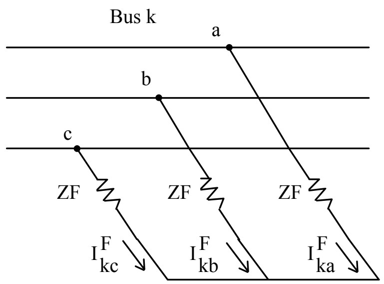


Figure 6: Three-Phase Fault at Bus *k*

The Thevenin equivalent circuit equation, assuming no other current injections in the system, is

$$\begin{bmatrix} V_{ka}^F \\ V_{kb}^F \\ V_{kc}^F \end{bmatrix} = \begin{bmatrix} V_{ka}^{Pre} \\ V_{kb}^{Pre} \\ V_{kc}^{Pre} \end{bmatrix} - \begin{bmatrix} z_{ka,ka} & z_{ka,kb} & z_{ka,kc} \\ z_{kb,ka} & z_{kb,kb} & z_{kb,kc} \\ z_{kc,ka} & z_{kc,kb} & z_{kc,kc} \end{bmatrix} \begin{bmatrix} I_{ka}^F \\ I_{kb}^F \\ I_{kc}^F \end{bmatrix},$$

or in sequence form,

$$\begin{bmatrix} V_{k0}^F \\ V_{k1}^F \\ V_{k2}^F \end{bmatrix} = \begin{bmatrix} V_{k0}^{Pre} \\ V_{k1}^{Pre} \\ V_{k2}^{Pre} \end{bmatrix} - \begin{bmatrix} z_{k0,k0} & 0 & 0 \\ 0 & z_{k1,k1} & 0 \\ 0 & 0 & z_{k2,k2} \end{bmatrix} \begin{bmatrix} I_{k0}^F \\ I_{k1}^F \\ I_{k2}^F \end{bmatrix}.$$

In abbreviated form, the above equations are

$$V_{kabc}^F = V_{kabc}^{Pre} - Z_{k-k,abc} I_{kabc}^F \text{ , and } V_{k012}^F = V_{k012}^{Pre} - Z_{k-k,012} I_{k012}^F \text{ ,} \qquad (5)$$

where V_k^F consists of the voltages at bus *k* during the fault, V_k^{Pre} consists of the pre-fault voltages, I_k^F gives the fault currents, and Z_{k-k} contains the individual impedance elements extracted from the impedance matrix.

The above matrix equations represents three equations (repeated in abc and 012 form), but there are six unknowns represented by V_k^F and I_k^F, so that three additional equations are required. The additional equations are found by observing that

$$V_{kabc}^F = Z_F I_{kabc}^F \text{ , or } V_{k012}^F = Z_F I_{k012}^F .$$

Substituting into the Thevenin equation, and recognizing that all zero- and negative-sequence voltages and currents are zero for a balanced fault yields

$$\begin{bmatrix} 0 \\ Z_F I_{k1}^F \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ V_{k1}^{Pre} \\ 0 \end{bmatrix} - \begin{bmatrix} z_{k0,k0} & 0 & 0 \\ 0 & z_{k1,k1} & 0 \\ 0 & 0 & z_{k2,k2} \end{bmatrix} \begin{bmatrix} 0 \\ I_{k1}^F \\ 0 \end{bmatrix},$$

so that the positive sequence fault current is found to be

$$I_{k1}^F = \frac{V_{k1}^{Pre}}{z_{k1,k1} + Z_F}, I_{k0}^F = 0, I_{k2}^F = 0 . \qquad (6)$$

Substituting into Thevenin equation

$$V_{k012}^F = V_{k012}^{Pre} - Z_{k-k,012} I_{k012}^F \qquad (7)$$

yields the fault voltage at Bus k. Similarly, because the impedance matrix relates the voltages at network busses to current injections at network busses, the voltage at any other bus j is found using

$$V_{j012}^{F} = V_{j012}^{Pre} - Z_{j-k,012} I_{k012}^{F} . \qquad (8)$$

Note that the minus sign is needed because the fault current has been drawn as positive outward.
Once the fault voltages are known at neighboring busses, the contribution currents through the connected branches can be easily found.

Single-Phase to Ground Fault

Consider the single-phase fault at bus k, as shown in Figure 7.

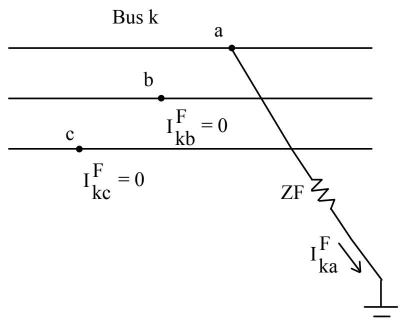


Figure 7: Single-Phase Fault at Bus k, Phase a

As before, the Thevenin equivalent circuit equations, assuming no other current injections in the system, is

$$V_{k012}^{F} = V_{k012}^{Pre} - Z_{k-k,012} I_{k012}^{F} .$$

Examining I_{k012}^{F} shows that in this case

$$I_{k012}^{F} = T^{-1} I_{kabc}^{F} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{ka}^{F} \\ I_{kb}^{F} = 0 \\ I_{kc}^{F} = 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_{ka}^{F} \\ I_{ka}^{F} \\ I_{ka}^{F} \end{bmatrix} . \qquad (9)$$

Substituting into the Thevenin equation yields

$$\begin{bmatrix} V_{k0}^{F} \\ V_{k1}^{F} \\ V_{k2}^{F} \end{bmatrix} = \begin{bmatrix} V_{k0}^{Pre} = 0 \\ V_{k1}^{Pre} = V_{ka}^{Pre} \\ V_{k2}^{Pre} = 0 \end{bmatrix} - \begin{bmatrix} z_{k0,k0} & 0 & 0 \\ 0 & z_{k1,k1} & 0 \\ 0 & 0 & z_{k2,k2} \end{bmatrix}\begin{bmatrix} I_{ka}^{F}/3 \\ I_{ka}^{F}/3 \\ I_{ka}^{F}/3 \end{bmatrix}$$

Add the three rows yields

$$V_{k0}^{F} + V_{k1}^{F} + V_{k2}^{F} = V_{ka}^{F} = V_{ka}^{Pre} - \frac{1}{3} I_{ka}^{F}\left(z_{k0,k0} + z_{k1,k1} + z_{k2,k2}\right).$$

From the circuit it is obvious that

$$V_{ka}^{F} = I_{ka}^{F} Z_F ,$$

so that

$$I_{ka}^{F} Z_F = V_{ka}^{Pre} - \frac{1}{3} I_{ka}^{F}\left(z_{k0,k0} + z_{k1,k1} + z_{k2,k2}\right) .$$

Solving for I_{ka}^{F} yields

$$I_{ka}^{F} = \frac{3V_{ka}^{Pre}}{z_{k0,k0} + z_{k1,k1} + z_{k2,k2} + 3Z_F} . \qquad (10)$$

Now, using

$$I_{k0}^{F} = I_{k1}^{F} = I_{k2}^{F} = \frac{I_{ka}^{F}}{3} ,$$

all network voltages can be found from

$$V_{j012}^{F} = V_{j012}^{Pre} - Z_{j-k,012} I_{k012}^{F} .$$

Note that if $z_{k0,k0} < z_{k1,k2}$, a single-phase fault will have a higher value than does a three-phase fault.

Line-to-Line Fault

Consider the line-to-line fault at bus k, as shown in Figure 8.

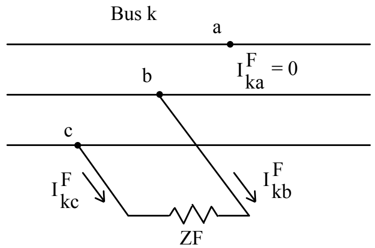


Figure 8. Line-to-Line Fault Between Phases b and c at Bus *k*

Examining I_{k012}^F shows that in this case

$$I_{k012}^F = T^{-1} I_{kabc}^F = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{ka}^F = 0 \\ I_{kb}^F \\ I_{kc}^F = -I_{kb}^F \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ I_{kb}^F\left(a - a^2\right) \\ I_{kb}^F\left(a^2 - a\right) \end{bmatrix} . \qquad (11)$$

Note that there is no zero sequence fault current.

Substituting into the Thevenin equation yields

$$\begin{bmatrix} V_{k0}^F \\ V_{k1}^F \\ V_{k2}^F \end{bmatrix} = \begin{bmatrix} 0 \\ V_{ka}^{\Pr e} \\ 0 \end{bmatrix} - \begin{bmatrix} z_{k0,k0} & 0 & 0 \\ 0 & z_{k1,k1} & 0 \\ 0 & 0 & z_{k2,k2} \end{bmatrix}\begin{bmatrix} 0 \\ I_{kb}^F\left(a - a^2\right)/3 \\ I_{kb}^F\left(a^2 - a\right)/3 \end{bmatrix} .$$

Subtracting the last two rows of the Thevenin equation yields

$$V_{k1}^F - V_{k2}^F = V_{ka}^{\Pr e} - z_{k1,k1}\frac{I_{kb}^F\left(a - a^2\right)}{3} + z_{k2,k2}\frac{I_{kb}^F\left(a^2 - a\right)}{3} ,$$

or

$$I_{kb}^F\left[\frac{z_{k2,k2}\left(a^2 - a\right)}{3} - \frac{z_{k1,k1}\left(a - a^2\right)}{3}\right] = V_{k1}^F - V_{k2}^F - V_{ka}^{\Pr e} .$$

From the circuit, we see that

$$I_{kb}^F = \frac{V_{kb}^F - V_{kc}^F}{Z_F} .$$

Using $V_{abc} = TV_{012}$, we find that

$$V_{kb}^F - V_{kc}^F = V_{k1}^F\left(a^2 - a\right) + V_{k2}^F\left(a - a^2\right),$$

so that

$$I_{kb}^F = \frac{V_{k1}^F\left(a^2 - a\right) + V_{k2}^F\left(a - a^2\right)}{Z_F}, \text{ or } I_{kb}^F = \frac{\left(V_{k1}^F - V_{k2}^F\right)\left(a^2 - a\right)}{Z_F} .$$

Combining equations yields

$$I_{kb}^F\left[\frac{z_{k2,k2}\left(a^2 - a\right)}{3} - \frac{z_{k1,k1}\left(a - a^2\right)}{3}\right] = \frac{I_{kb}^F Z_F}{\left(a^2 - a\right)} - V_{ka}^{\Pr e} .$$

Collecting terms yields

$$I_{kb}^F\left[\frac{z_{k2,k2}\left(a^2 - a\right)}{3} - \frac{z_{k1,k1}\left(a - a^2\right)}{3} - \frac{Z_F}{\left(a^2 - a\right)}\right] = -V_{ka}^{\Pr e} ,$$

or

$$I_{kb}^F\left[\frac{Z_F}{\left(a^2 - a\right)} + \frac{\left(a - a^2\right)}{3}\left(z_{k1,k1} + z_{k2,k2}\right)\right] = V_{ka}^{\Pr e} .$$

Simplifying yields

and where $I_{kc}^F = -I_{kb}^F, I_{ka}^F = 0$. All network voltages can now be found from

$$I_{kb}^F = \frac{-j\sqrt{3}V_{ka}^{\Pr e}}{z_{k1,k1} + z_{k2,k2} + Z_F} , \qquad (12)$$

$$V_{j012}^F = V_{j012}^{\Pr e} - Z_{j-k,012} I_{k012}^F .$$

6. Calculation Procedure

Step 1. Pick a system MVA base and a VLL base at one point in the network. The system MVA base will be the same everywhere. As you pass through transformers, vary the system VLL base according to the line-to-line transformer turns ratio.

Step 2. The system base phase angle changes by 30º each time you pass through a YΔ (or ΔY) transformer. ANSI rules state that transformers must be labeled so that high-side positive sequence voltages and currents lead low-side positive sequence voltages and currents by 30º. Negative sequence does the opposite (i.e., -30º shift). Zero sequence gets no shift. The "Net 30º " phase shift between a faulted bus k and a remote bus j is ignored until the last step in this procedure.

Step 3. Begin with the positive sequence network and balanced three-phase case. Assume that the system is "at rest" with no currents flowing. This assumption requires that the only shunt ties are machines which are represented as Thevenin equivalents with 1.0 pu voltage in series with subtransient impedances. Loads (except large machines), line capacitance, shunt capacitors, and shunt inductors are ignored. Convert all line/transformer/source impedances to the system base using $Z_{pu}^{new} = Z_{pu}^{old} \bullet \left[\frac{S_{base}^{new}}{S_{base}^{old}}\right] \bullet \left[\frac{V_{base}^{old}}{V_{base}^{new}}\right]^2$. S_{base} is three-phase MVA. V_{base} is line-to-line. If a transformer is comprised of three identical single-phase units, Z_{pu}^{old} is the impedance of any one transformer on its own base, and S_{base}^{old} is three times the rated power of one transformer. For a delta connection, V_{base} line-to-line is the rated coil voltage of one transformer. For a wye connection, V_{base} line-to-line is the rated coil voltage multiplied by $\sqrt{3}$.

Step 4. For small networks, you can find the fault current at any bus k "by hand" by turning off all voltage sources and computing the positive-sequence Thevenin equivalent impedance at the faulted bus, $Z_{kk,1}$. Ignore the Net 30º during this step because actual impedances are not shifted when reflected from one side to the other side of YΔ transformers. Once the Thevenin impedance is known, then use $I_{k1}^F = \frac{V_{k1}^{pre}}{Z_{kk,1} + Z_F}$, followed by $V_{k1}^F = V_{k1}^{pre} - Z_{kk,1} \bullet I_{k1}^F$.

Step 5. The key to finding 012 currents in a branch during the fault is to know the voltage on each end of the branch. For a branch with positive sequence impedance $\bar{z}_1$ between busses j and k, first find $V_{j1}^F = V_{j1}^{pre} - Z_{jk,1} \bullet I_{k1}^F$. Positive-sequence current flow through the branch during the fault is $I_{jk,1}^F = \frac{V_{j,1}^F - V_{k,1}^F}{\bar{z}_1}$. **Note that $\bar{z}_1$ is the physical positive sequence impedance of the branch - it is not an element of the Z matrix.** An accuracy check should be made by making sure that the sum of all the branch currents into the faulted bus equals I_{k1}^F.

Step 6. While continuing to ignore the Net 30º, voltage sag propagation at all other buses j can be computed with $V_{j1}^F = V_{j1}^{pre} - Z_{jk,1} \bullet I_{k1}^F$.

Step 7. For larger networks, "hand" methods are not practical, and the Z matrix should be built. Form the admittance matrix Y, and invert Y to obtain Z. Ignore the "Net 30º " when forming Y. Matrix inversion can be avoided if Gaussian elimination is used to find only the kth column of Z.

Step 8. Unbalanced faults also require negative sequence impedances. Faults with ground currents also require zero sequence impedances. Negative sequence impedances are usually the same as positive. Zero sequence impedances can be larger or smaller and are dramatically affected by grounding. YΔ transformers introduce broken zero sequence paths. Prefault negative sequence and zero sequence voltages are always zero.

Step 9. After the 012 fault currents are determined, continue to ignore the "Net 30º " and use $V_{j012}^F = V_{j012}^{pre} - Z_{jk,012} \bullet I_{k012}^F$ for each sequence to find 012 bus voltages.

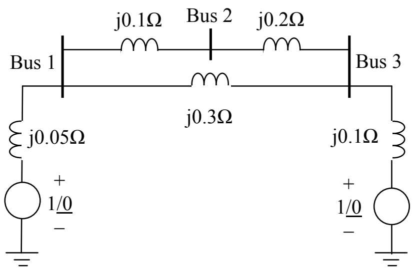
Step 10. Next, compute branch currents (ignoring the Net 30º) between buses j and k for each sequence using

$$I_{jk,0} = \frac{V_{j,0}^F - V_{k,0}^F}{\bar{z}_0},\ I_{jk,1} = \frac{V_{j,1}^F - V_{k,1}^F}{\bar{z}_1},\ I_{jk,2} = \frac{V_{j,2}^F - V_{k,2}^F}{\bar{z}_2}.$$

Step 11. As the last step, **include the Net 30º** between bus j and faulted bus k. Do this by adding the Net 30º to V_{j1}^F and $I_{jk,1}$ calculations, and subtracting the Net 30º from V_{j2}^F and $I_{jk,2}$ calculations. Then, use $V_{abc} = T \bullet V_{012}$, $I_{abc} = T \bullet I_{012}$ to find the abc bus voltages and branch currents.

Short Circuit Problem #1

The positive-sequence one-line diagram for a network is shown below. Prefault voltages are all 1.0pu.



a. Use the definition $z_{jk} = \frac{\partial V_j}{\partial I_k} = \left.\frac{V_j}{I_k}\right|_{I_m=0, m \neq k}$ to fill in column 1 of the Z matrix.

Now, a solidly-grounded three-phase fault occurs at bus 1.

b. Compute the fault current

c. Use the fault current and Z matrix terms to compute the voltages at busses 2 and 3.

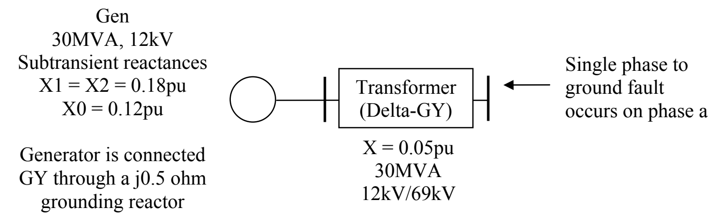
d. Find the magnitude of the current flowing in the line connecting busses 2 and 3.

	1	2	3
1			
2			
3			

Short Circuit Problem #2.

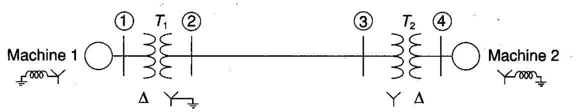
A 30MVA, 12kV generator is connected to a delta - grounded wye transformer. The generator and transformer are isolated and not connected to a "power grid." Impedances are given on equipment bases.

A single-phase to ground fault, with zero impedance, suddenly appears on phase a of the 69kV transformer terminal. Find the resulting a-b-c generator currents (magnitude **in amperes** and phase). Regarding reference angle, assume that the pre-fault phase a voltage on the transformer's 69kV bus has angle = 0.



Short Circuit Problem #3

A one-line diagram for a two-machine system is shown below.



All values in pu on equipment base

Machines 1 and 2:	100 MVA	20 kV	$X_1 = X_2 = 0.2$
	$X_0 = 0.04$	$X_n = 0.05$	
Transformers T_1 and T_2:	100 MVA	20Δ/345Y kV	
	$X_1 = X_2 = X_0 = 0.08$		

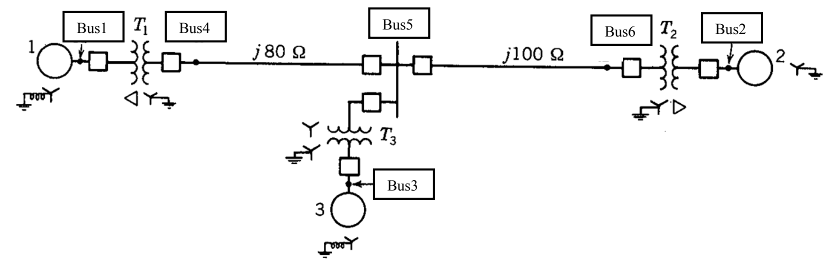
The transmission line between busses 2 and 3 has X1 = X2 = 0.12pu, X0 = 0.40pu on a 100MVA, 345kV base.

Using a base of 100MVA, 345kV in the transmission line, draw one line diagrams in per unit for positive, negative, and zero-sequences.

Then,

a. Compute the phase a fault current (in pu) for a three-phase bolted fault at bus 2.

b. Compute the phase a fault current (in pu) for a line-to-ground fault at bus 2, phase a.



Prob. 6.15.

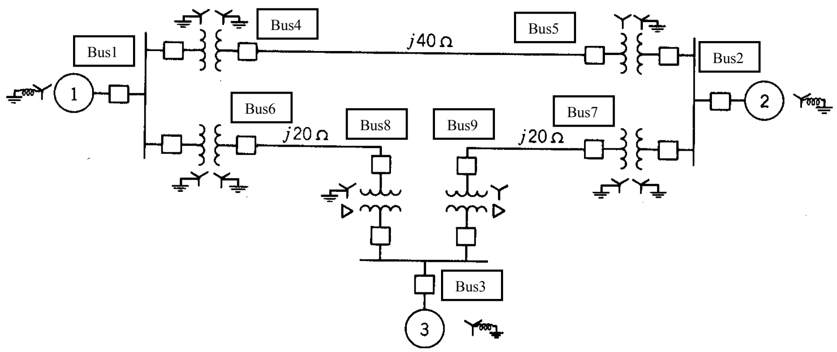
6.15 The one-line diagram of an unloaded power system is shown . Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are rated as follows:

Generator 1:	20 MVA, 13.8 kV, $X'' = 0.20$ per unit
Generator 2:	30 MVA, 18 kV, $X'' = 0.20$ per unit
Generator 3:	30 MVA, 20 kV, $X'' = 0.20$ per unit
Transformer T_1:	25 MVA, 220Y/13.8Δ kV, $X = 10\%$
Transformer T_2:	Single-phase units each rated 10 MVA, 127/18 kV, $X = 10\%$
Transformer T_3:	35 MVA, 220Y/22Y kV, $X = 10\%$

Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. **Use a 100 MVA, 220kv base in the transmission line.**

Draw the negative- and zero-sequence impedance networks for the power system

The neutrals of generators 1 and 3 are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Each generator has negative- and zero-sequence reactances of 20 and 5%, respectively, on its own rating as base. The zero-sequence reactance of the transmission line is 210 Ω from *B* to *C* and 250 Ω from *C* to *E*.



Prob. 6.16.

Draw the impedance diagram for the power system shown . Mark impedances in per unit. Neglect resistance, and use a base **100 MVA, 138kV in the** 40-Ω line. The ratings of the generators, motors, and transformers are:

Generator 1:	20 MVA, 18 kV, $X'' = 20\%$
Generator 2:	20 MVA, 18 kV, $X'' = 20\%$
Synchronous motor 3:	30 MVA, 13.8 kV, $X'' = 20\%$
Three-phase Y-Y transformers:	20 MVA, 138Y/20Y kV, $X = 10\%$
Three-phase Y-Δ transformers:	15 MVA, 138Y/13.8Δ kV, $X = 10\%$

Draw the negative- and zero-sequence impedance networks

The negative-sequence reactance of each synchronous machine is equal to its subtransient reactance. The zero-sequence reactance of each machine is 8% based on its own rating. The neutrals of the machines are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Assume that the zero-sequence reactances of the transmission lines are 300% of their positive-sequence reactances.

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Short Circuit Calculations

Short Circuit Problem #4.
Balanced Three-Phase Fault, Stevenson Prob. 6.15. A three-phase balanced fault, with $Z_F = 0$, occurs at Bus 4. Determine

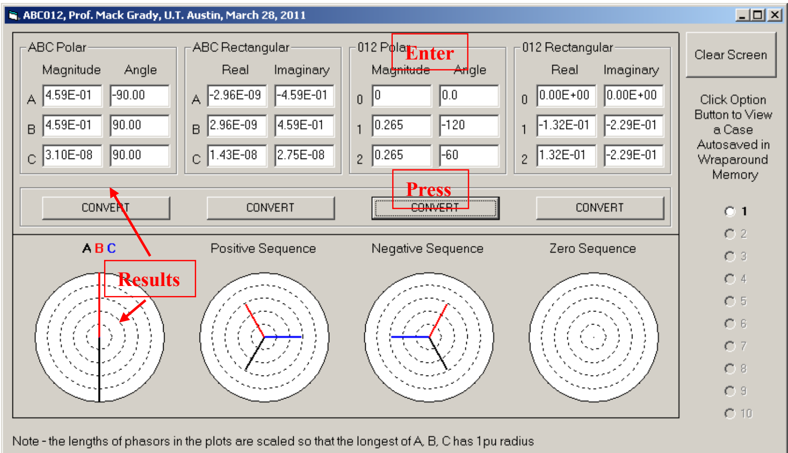
a. I_{4a}^F (in per unit and in amps)
b. Phasor abc line-to-neutral voltages at the terminals of Gen 1
c. Phasor abc currents flowing out of Gen 1 (in per unit and in amps)

Short Circuit Problem #5.
Line to Ground Fault, Stevenson Prob. 6.15. Repeat #4 for phase a-to-ground fault at Bus 4, again with $Z_F = 0$.

Short Circuit Problem #6.
Repeat #4, Using Stevenson Prob. 6.16.

Short Circuit Problem #7.
Repeat #5, Using Stevenson Prob. 6.16.

Stevenson Problem 6.15, Phase A to Ground Fault at Bus #4



Enter Polar Form 012 **Currents** at Gen #1, Compute the ABC Currents

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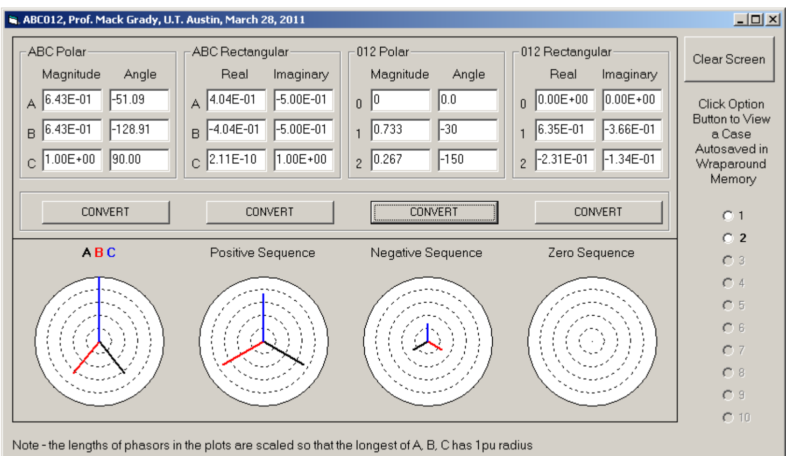

Enter Polar Form 012 **Voltages** at Gen #1, Compute the ABC Voltages

Voltage Sag Propagation Along Feeders

1. Introduction

Short circuit equations provide the theoretical framework for determining the voltage sag at a bus due to a fault anywhere in the system. However, the short circuit equations by themselves provide little insight. We now proceed with examples to provide this insight by showing how a sag propagates for various transformer situations.

2. Impact of Transmission System Faults on Customers

Consider the typical situation shown in Figure 1. A fault occurs at bus k of the transmission system, causing a voltage sag that affects a substation (bus j) and the customers connected to its feeders. There can be as many as three transformers between the customer's load and the transmission fault point, and each of these transformers can have a 30^{O} phase shift. Typically, all three of the transformers shown (i.e., T1, T2, and T3) are delta connected on the high side, and grounded-wye connected on the low side.

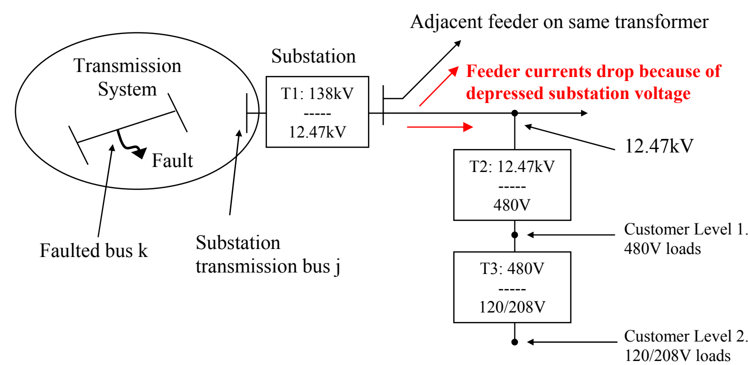

Figure 1. Example System for Analyzing the Propagation of Transmission Voltage Sags into Customer Low-Voltage Busses

The standard assumption for fault calculations is that

- the circuit is initially unloaded, or at least that the voltages are all close to 1.0 per unit.

Using this assumption, *and further assuming that there are no significant contributors of fault current on the feeders*, then the actual location of the customer is not important because all points on the three 12.47kV feeders shown (including the substation 12.47kV bus) will experience the same sag. Furthermore, the sag experienced on the substation 12.47kV bus will be the same as on substation 138kV bus j, except for possible zero-sequence component blocking and positive/negative-phase shifts.

The significance of the above paragraph is that for transmission faults, one monitor at either the substation 138kV bus or at the substation 12.47kV bus is adequate to predict voltage sag levels anywhere on the substation's feeders, provided there are no significant contributors of fault current on the feeders.

If the transmission fault is electrically far away, then the sag experienced at the substation and at the customer site will be small. Alternatively, if the fault is immediately at substation 138kV bus j, then the sag will be the most severe possible. Thus, it is reasonable to assume that an electrical "proximity" factor exists, where a **proximity factor** of zero (i.e., 0%) indicates that the fault is at substation 138kV bus j, and a **proximity factor** of unity (i.e., 100%) indicates that the transmission fault bus k is very far away. From knowledge of the physical significance of the impedance matrix, and from examining Thevenin equations

$$V_{k012}^{F} = V_{k012}^{\Pr e} - Z_{k-k,012} I_{k012}^{F} ,$$

$$V_{j012}^{F} = V_{j012}^{\Pr e} - Z_{j-k,012} I_{k012}^{F} ,$$

this **proximity factor P** is approximated using the ratio of positive-sequence impedances

$$P = 1 - \left| \frac{z_{j1,k1}}{z_{k1,k1}} \right| . \qquad (1)$$

By coding the short circuit equations into a Visual Basic program, and employing (1), voltage sag propagation for the situation described in Figure 1 can now be illustrated. Assuming that the transmission fault is relatively close to the substation (i.e., proximity factor = 25%), and that T1, T2, and T3 are all delta:grounded-wye transformers, the line-to-neutral voltages for single-phase, phase-to-phase, and three-phase transmission faults are shown in Figures 2 – 4, respectively. Both phasor plots and magnitude bar charts are given.

It is important to note that if a transformer is connected grounded-wye:grounded-wye or delta:delta, then the voltage sag on the low-voltage side of the transformer is the same as on the high-voltage side, as illustrated in Figure 5 for the single-phase fault.

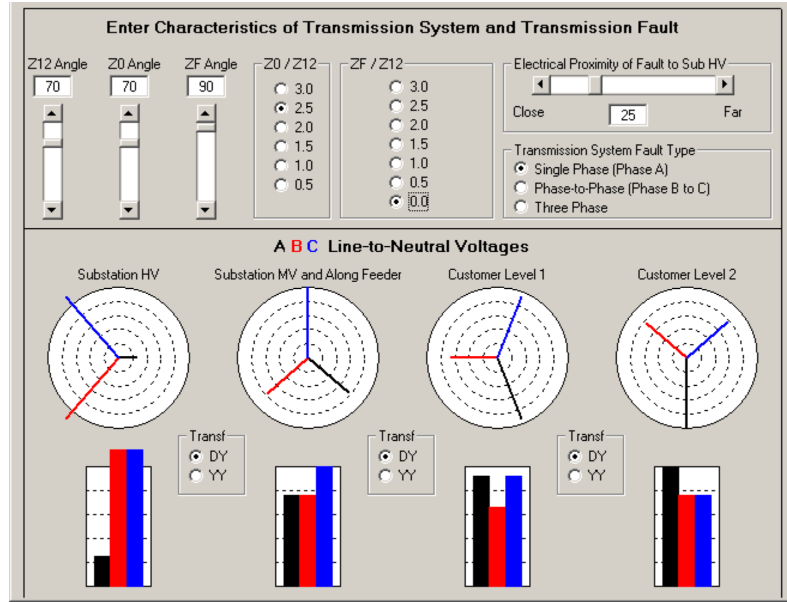

Figure 2. Propagation of Close-In Single Phase Fault on the Transmission System (all three transformers have delta:grounded-wye connections)

Note the voltage swell on phases b and c at the substation 138kV bus. Note also that two phases are affected after the first transformation, then one phase is affected after two transformations, and again two phases are affected after three transformations.

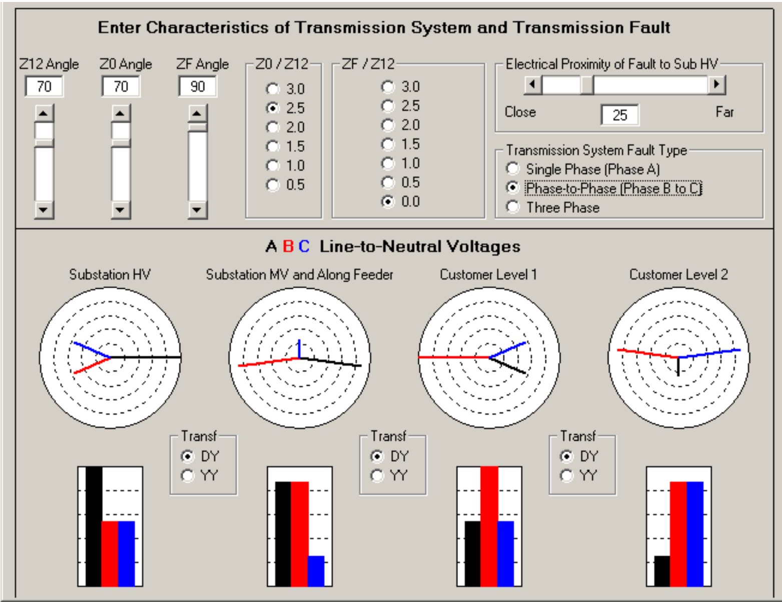

Figure 3. Propagation of Close-In Phase-to-Phase Fault on the Transmission System (all three transformers have delta:grounded-wye connections)

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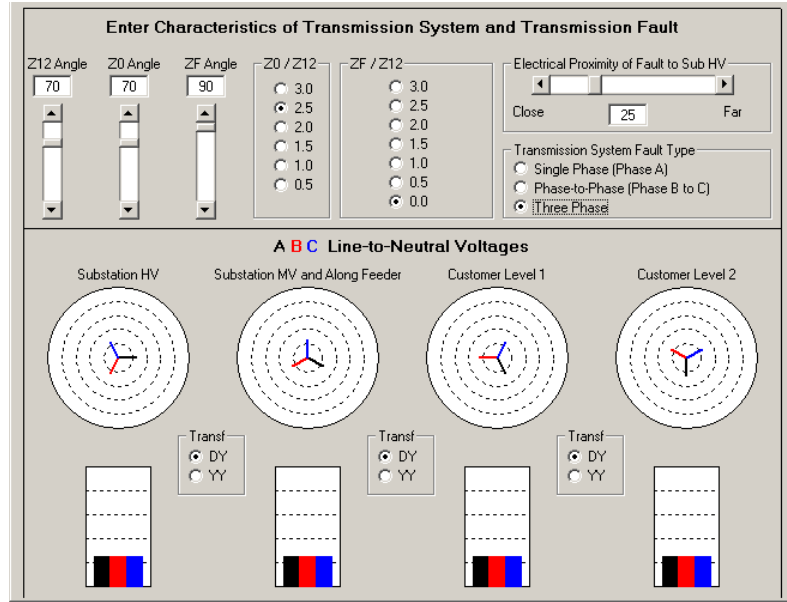

Figure 4. Propagation of Close-In Three-Phase Fault on the Transmission System (all three transformers have delta:grounded-wye connections)

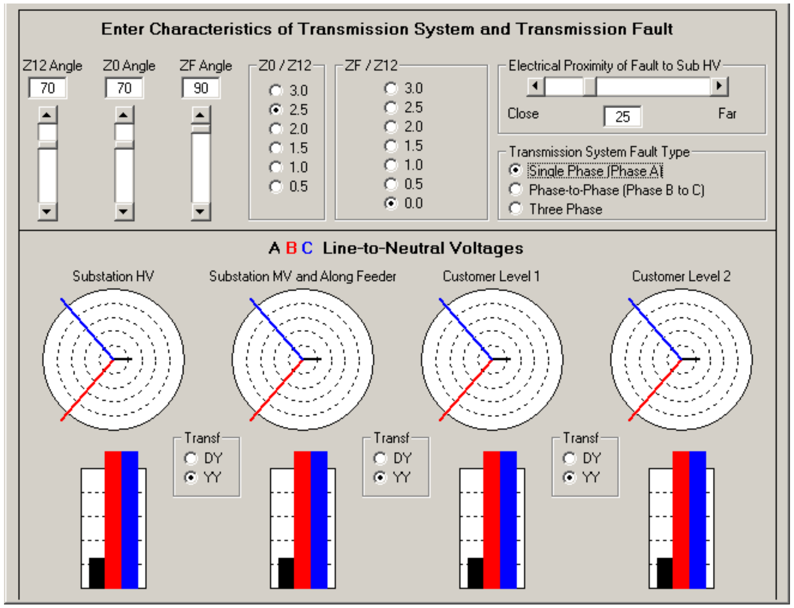

Figure 5. Situation in Figure 2 Repeated, but with all Three Transformers Having Grounded-Wye:Grounded-Wye Connections

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3. Impact of Distribution System Faults on Adjacent Feeders

Now, consider the situation in Figure 6 where a fault occurs on an adjacent feeder, and a monitor records the voltage waveform at the substation 12.47kV bus.

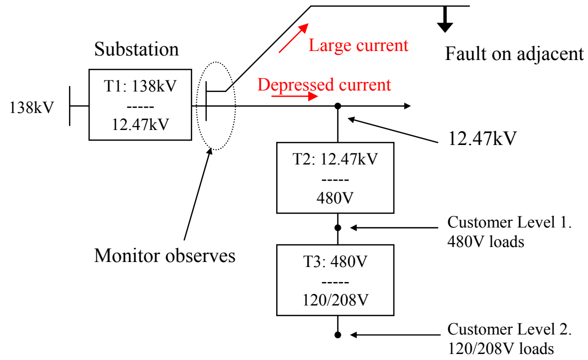

Figure 6. Substation Monitor Records Voltages when a Sag Occurs on an Adjacent Feeder.

As in Section 2, unless the customer's feeder has significant contributors to the fault current, the voltage sag at the substation 12.47kV bus will appear everywhere along the customer's 12.47kV feeder. However, to predict the voltage sag at Customer Levels 1 and 2, the a-b-c line-to-neutral **voltages at the substation 12.47kV bus must be**

- converted to positive/negative/zero-sequence components,
- shifted with the appropriate transformer phase shifts,
- converted back to a-b-c.