TOPICS

1. **Power Definitions and Equations. Why?**
   As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.

2. **Three-Phase Power. Why?**
   Losses are minimized because there are no ground or neutral return currents. Also, three-phase machines are smaller and much more efficient than single-phase machines of the same power rating.

3. **Transformer Models. Why?**
   To move power over long distances and minimize I^2R losses, transmission line voltages must be boosted to much higher levels than voltages produced by generators or required by end-users.

4. **Per Unit System. Why ?**
   To make modeling and simulation of circuits having transformers much easier. A by-product is that per-unitized equipment parameters such as ratings and impedances fall into narrow and predictable ranges.

5. **Symmetrical Components. Why?**
   Greatly simplifies analysis of normal power system operation, as well as abnormal events such as unbalanced faults.

6. **Transmission Line Models. Why?**
   Necessary for designing and simulating power systems for loadflow, short-circuit, and stability purposes.

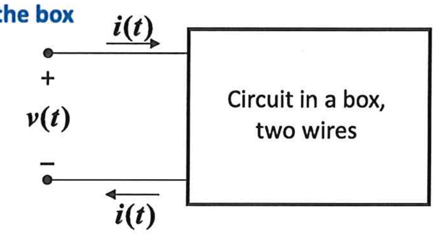
7. **Fault (Short-Circuit) Calculations and Voltage Sags. Why?**
   So that a power system can be properly protected by detecting and isolating faults within 0.1 second. 1 second is a very long time for a fault to exist - long enough to cause grid blackouts.

1. Power Definitions and Equations, cont.

As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.

Instantaneous power p(t) flowing into the box

$$v(t) = V\sin(\omega_o t + \delta),$$
$$i(t) = I\sin(\omega_o t + \theta)$$
$$p(t) = v(t) \bullet i(t)$$



$$p(t) = v(t) \bullet i(t) = V\sin(\omega_o t + \delta) \bullet I\sin(\omega_o t + \theta)$$

zero average

$$p(t) = VI\left[\frac{\cos(\delta - \theta) - \cos(2\omega_o t + \delta + \theta)}{2}\right]$$

peak

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o + T} p(t)dt \;= \frac{VI}{2}\cos(\delta - \theta) = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\delta - \theta)$$

rms

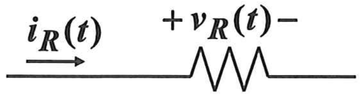
Average power P_{avg} flowing into the box

$$P_{avg} = V_{rms} I_{rms} \cos(\delta - \theta)$$

Power factor

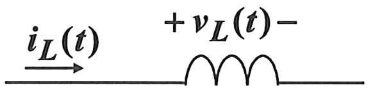
1. Power Definitions and Equations.

As with any other technology, the underlying physics must be understood so that equations and models can be developed that simulate actual behavior.



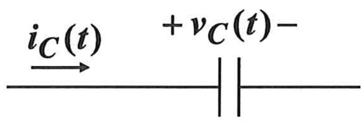
$$i_R(t) = \frac{v_R(t)}{R}$$

$i(t) = I\ sin(\omega t)$,
$v(t) = IR\ sin(\omega t)$,
$\therefore$ $i(t)$ in phase with $v(t)$



$$v_L(t) = L\frac{di(t)}{dt}$$

$i(t) = I\ sin(\omega t)$,
$v(t) = \omega L\ cos(\omega t)$,
$\therefore$ $i(t)$ lags $v(t)$ by 90°



$$i_C(t) = C\frac{dv(t)}{dt}$$

$v(t) = V\ sin(\omega t)$,
$i(t) = \omega C\ cos(\omega t)$,
$\therefore$ $i(t)$ leads $v(t)$ by 90°

1. Power Definitions and Equations, cont.

Thanks to Charles Steinmetz, Steady-State AC problems are greatly simplified with phasor analysis because differential equations are replaced by complex numbers

	Time Domain	Frequency Domain
Resistor	$i_R(t) = \frac{v_R(t)}{R}$	$Z_R = \frac{\tilde{V}_R}{\tilde{I}_R} = R$
Inductor	voltage leads current $v_L(t) = L\frac{di(t)}{dt}$	$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L} = j\omega L$
Capacitor	current leads voltage $i_C(t) = C\frac{dv(t)}{dt}$	$Z_C = \frac{\tilde{V}_C}{\tilde{I}_C} = \frac{1}{j\omega C}$

1. Power Definitions and Equations, cont.

Voltage and Current Phasors for R's, L's, C's

Resistor $Z_R = \dfrac{\tilde{V}_R}{\tilde{I}_R} = R, \;\; \tilde{V}_R = R\tilde{I}_R$

Voltage and Current in phase

Q = 0

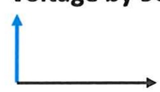
Inductor $Z_L = \dfrac{\tilde{V}_L}{\tilde{I}_L} = j\omega L, \;\; \tilde{V}_L = j\omega L\tilde{I}_L$

Voltage leads Current by 90°

Q > 0

Capacitor $Z_C = \dfrac{\tilde{V}_C}{\tilde{I}_C} = \dfrac{1}{j\omega C}, \;\; \tilde{V}_C = \dfrac{\tilde{I}_C}{j\omega C}$

Current leads Voltage by 90°



Q < 0

1. Power Definitions and Equations, cont.

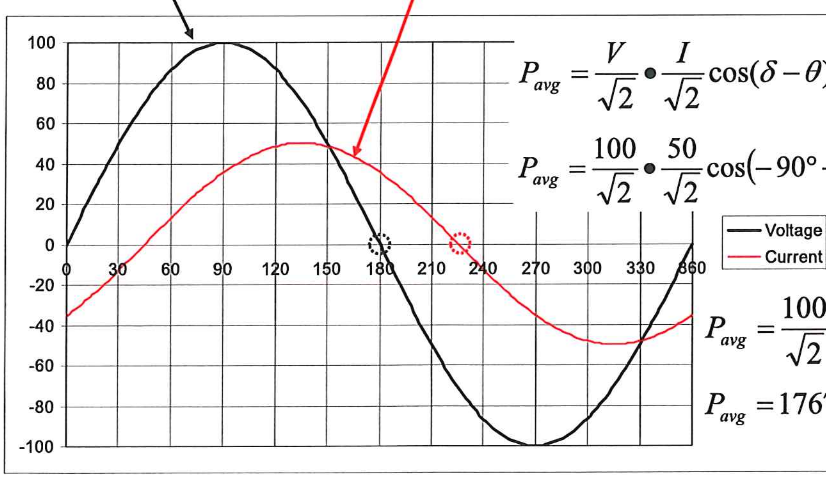
Converting Time Domain Waveforms to Phasor Domain

Using a cosine reference,

Voltage cosine has peak = 100V, phase angle = -90º

Current cosine has peak = 50A, phase angle = -135º

Phasors $\tilde{V} = \dfrac{100}{\sqrt{2}}\angle -90°\,V, \quad \tilde{I} = \dfrac{50}{\sqrt{2}}\angle -135°\,A$



$$P_{avg} = \frac{V}{\sqrt{2}} \bullet \frac{I}{\sqrt{2}} \cos(\delta - \theta)$$

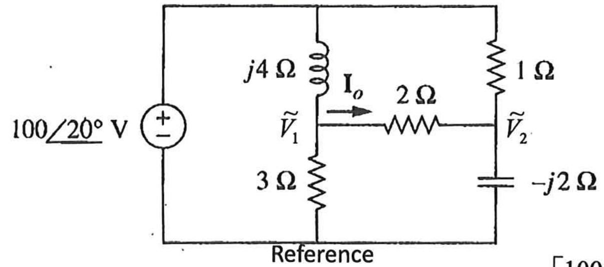
$$P_{avg} = \frac{100}{\sqrt{2}} \bullet \frac{50}{\sqrt{2}} \cos(-90° - (-135°))$$

$$P_{avg} = \frac{100}{\sqrt{2}} \bullet \frac{50}{\sqrt{2}} \cos(45°)$$

$$P_{avg} = 1767W$$

1. Power Definitions and Equations, cont.

Circuit analysis using the Nodal Method. Write KCL equations at major nodes 1 and 2, and solve for phasor voltages V_1 and V_2.



$$\begin{bmatrix} \frac{1}{j4}+\frac{1}{3}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+1+\frac{1}{-j2} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{100\angle 20^\circ}{j4} \\ \frac{100\angle 20^\circ}{1} \end{bmatrix}$$

$$\tilde{V}_1 = \frac{\begin{bmatrix} \frac{100\angle 20^\circ}{j4} & -\frac{1}{2} \\ \frac{100\angle 20^\circ}{1} & \frac{1}{2}+1+\frac{1}{-j2} \end{bmatrix}}{D}$$

$$D = \left[\frac{1}{j4}+\frac{1}{3}+\frac{1}{2}\right] \bullet \left[\frac{1}{2}+1+\frac{1}{-j2}\right] - \left[-\frac{1}{2}\right] \bullet \left[-\frac{1}{2}\right]$$

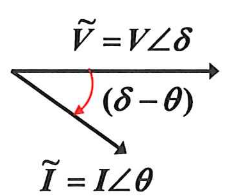
$$\tilde{V}_2 = \frac{\begin{bmatrix} -\frac{1}{2} & \frac{100\angle 20^\circ}{j4} \\ \frac{1}{2}+1+\frac{1}{-j2} & \frac{100\angle 20^\circ}{1} \end{bmatrix}}{D}$$

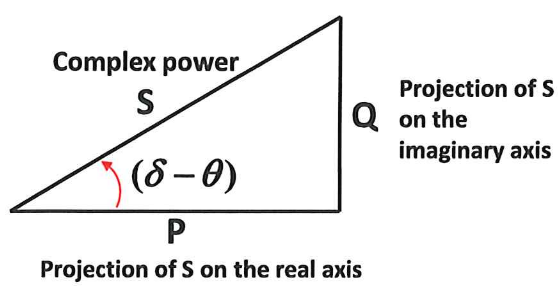
1. Power Definitions and Equations, cont.

Active power P_{avg} and reactive power Q form a power triangle

$$P_{avg} = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\delta-\theta), \quad Q = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\sin(\delta-\theta),$$

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\tilde{I}\right]^* = \left[V\angle\delta\right] \bullet \left[I\angle\theta\right]^* = VI\angle(\delta-\theta)$$





$\cos(\delta-\theta)$ is the power factor

1. Power Definitions and Equations, cont.

Resistor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^{*} = \frac{V^2}{Z^*} = \frac{V^2}{R}$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^{*} = I^2 Z = I^2 R$$

Thus ⇨ $P = \dfrac{V^2}{R} = I^2 R,\ Q = 0$

Inductor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^{*} = \frac{V^2}{Z^*} = \frac{V^2}{-j\omega L} = j\frac{V^2}{\omega L}$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^{*} = I^2 Z = j\omega L I^2$$

Thus ⇨ $P = 0,\ Q = \dfrac{V^2}{\omega L} = \omega L I^2$

Inductor consumes reactive power

Capacitor

$$S = P + jQ = \left[\tilde{V}\right] \bullet \left[\frac{\tilde{V}}{Z}\right]^{*} = \frac{V^2}{Z^*} = \frac{V^2}{\dfrac{1}{-j\omega C}} = -j\omega C V^2$$

Alternatively,

$$S = P + jQ = \left[\tilde{I}Z\right] \bullet \left[\tilde{I}\right]^{*} = I^2 \frac{1}{j\omega C} = -j\omega L I^2$$

Thus ⇨ $P = 0,\ Q = -\omega C V^2 = \dfrac{-I^2}{\omega C}$

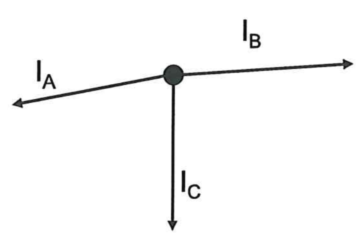
Capacitor produces reactive power

Always use rms values of voltage and current in the above equations

1. Power Definitions and Equations, cont.

Question: Why is the sum of power out of a node = 0?

Answer: KCL and conservation of power



Question: What about reactive power Q?

Answer: It depends.

Question: Can you be a bit more specific?

Answer: Unlike P, there is no physical for Q to be conserved.

When voltage and current are not sinusoidal, then cross products of voltage and current exist and Q is not conserved.

But power systems are mostly sinusoidal, so as shown on the right with phasors, both P and Q are conserved.

$$\tilde{I}_A + \tilde{I}_B + \tilde{I}_C = 0$$

$$\tilde{V}\left(\tilde{I}_A + \tilde{I}_B + \tilde{I}_C\right) = 0$$

$$\tilde{V}\left(\tilde{I}_A + \tilde{I}_B + \tilde{I}_C\right)^{*} = 0$$

$$P_A + jQ_A + P_B + jQ_B + P_C + jQ_C = 0$$

$$P_A + P_B + P_C = 0$$

$$Q_A + Q_B + Q_C = 0$$