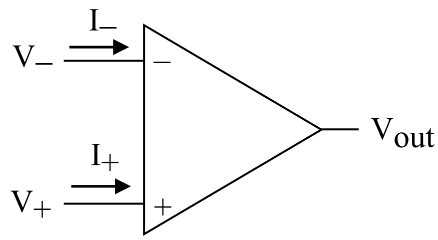
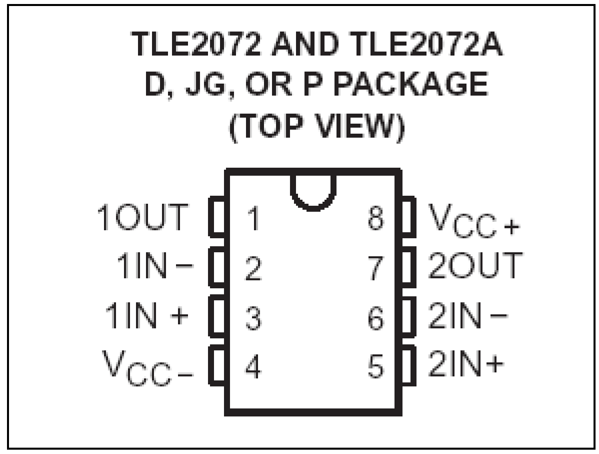
Op Amps

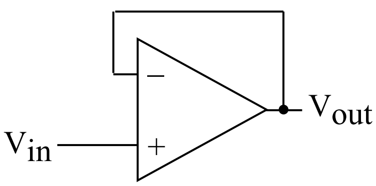




Assumptions for ideal op amp

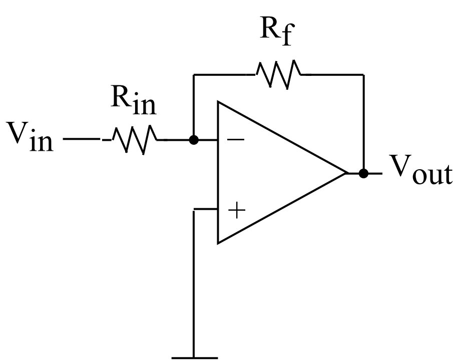
- $V_{out} = K(V_+ - V_-)$, K large (hundreds of thousands, or one million).
- $I_+ = I_- = 0$.
- Voltages are with respect to power supply ground.
- Output current is not limited.

Buffer Amplifier (converts high impedance signal to low impedance signal)



$V_{out} = K(V_{in} - V_{out})$, so $V_{out} + KV_{out} = KV_{in}$, so $V_{out}(1+K) = KV_{in}$, so $V_{out} = V_{in} \bullet \dfrac{K}{1+K}$. Since K is large, then $\boxed{V_{out} = V_{in}}$.

Inverting Amplifier (used for proportional control signal)



$V_{out} = K(0 - V_-) = -KV_-$, so $V_- = -\dfrac{V_{out}}{K}$.

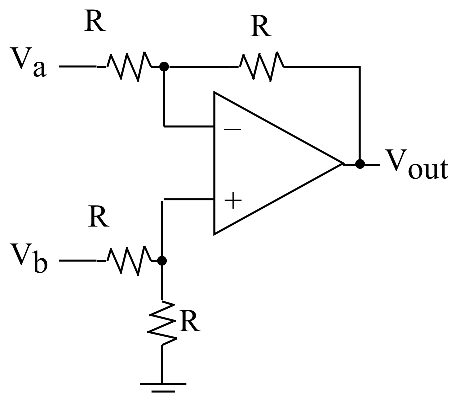
KCL at the – node is $\dfrac{V_- - V_{in}}{R_{in}} + \dfrac{V_- - V_{out}}{R_f} = 0$.

Eliminating V_- yields

$$\frac{-\dfrac{V_{out}}{K} - V_{in}}{R_{in}} + \frac{-\dfrac{V_{out}}{K} - V_{out}}{R_f} = 0, \text{ so}$$

$-V_{out}\left(\dfrac{1}{KR_{in}} + \dfrac{1}{KR_f} + \dfrac{1}{R_f}\right) = \dfrac{V_{in}}{R_{in}}$. For large K, then $\dfrac{-V_{out}}{R_f} = \dfrac{V_{in}}{R_{in}}$, so $\boxed{V_{out} = -V_{in}\dfrac{R_f}{R_{in}}}$.

Inverting Difference (used for error signal)



$V_{out} = K(V_+ - V_-) = K\left(\frac{V_b}{2} - V_-\right)$, so

$V_- = \frac{V_b}{2} - \frac{V_{out}}{K}$.

KCL at the – node is $\frac{V_- - V_a}{R} + \frac{V_- - V_{out}}{R} = 0$, so

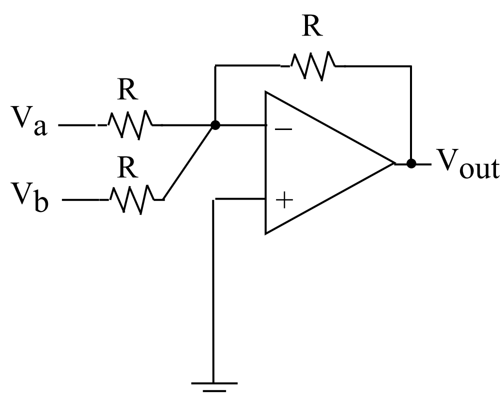
$V_- - V_a + V_- - V_{out} = 0$, yielding $V_- = \frac{V_a + V_{out}}{2}$.

Eliminating V_- yields

$V_{out} = K\left(\frac{V_b}{2} - \frac{V_a + V_{out}}{2}\right)$, so $V_{out} + K\frac{V_{out}}{2} = K\left(\frac{V_b - V_a}{2}\right)$, or $V_{out}\left(1 + \frac{K}{2}\right) = K\left(\frac{V_b - V_a}{2}\right)$.

For large K, then $\boxed{V_{out} = -(V_a - V_b)}$.

Inverting Sum (used to sum proportional and integral control signals)



$V_{out} = K(0 - V_-) = -KV_-$, so $V_- = \frac{-V_{out}}{K}$.

KCL at the – node is

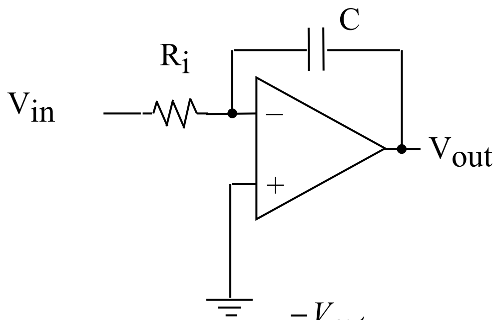
$\frac{V_- - V_a}{R} + \frac{V_- - V_b}{R} + \frac{V_- - V_{out}}{R} = 0$, so

$3V_- = V_a + V_b + V_{out}$.

Substituting for V_- yields $3\left(\frac{-V_{out}}{K}\right) = V_a + V_b + V_{out}$, so $V_{out}\left(\frac{-3}{K} - 1\right) = V_a + V_b$.

Thus, for large K, $\boxed{V_{out} = -(V_a + V_b)}$

Inverting Integrator (used for integral control signal)



$V_{out} = K(V_+ - V_-) = K(0 - V_-)$, so

$V_- = -\frac{V_{out}}{K}$. KCL at the − node is

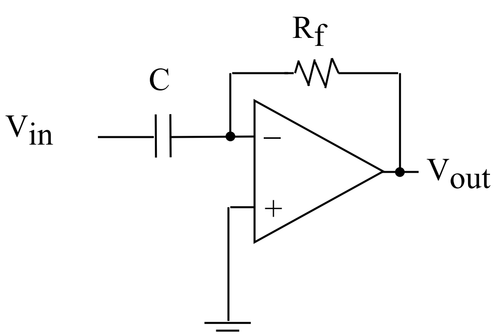
$\frac{V_- - V_{in}}{R_i} + C\frac{d}{dt}\left(V_- - V_{out}\right) = 0$.

Eliminating V_- yields $\frac{\frac{-V_{out}}{K} - V_{in}}{R_i} + C\frac{d}{dt}\left(\frac{-V_{out}}{K} - V_{out}\right) = 0$. For large K,

$-C\frac{d}{dt}V_{out} = \frac{V_{in}}{R_i}$, or $\frac{d}{dt}V_{out} = \frac{-V_{in}}{R_i C}$. So $\boxed{V_{out} = \frac{-1}{R_i C}\int V_{in}dt.}$

Lower R_i or C to increase integrator response.

Inverting Differentiator



$V_{out} = K(V_+ - V_-) = K(0 - V_-)$, so $V_- = -\frac{V_{out}}{K}$.

KCL at the – node is

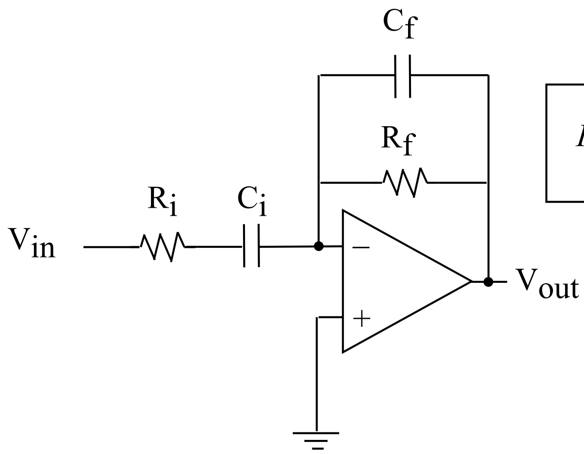
$\frac{V_- - V_{out}}{R_f} + C\frac{d}{dt}\left(V_- - V_{in}\right) = 0$. Eliminating V_-

yields

$\frac{\frac{-V_{out}}{K} - V_{out}}{R_f} + C\frac{d}{dt}\left(\frac{-V_{out}}{K} - V_{in}\right) = 0$. For large K, we have $\frac{-V_{out}}{R_f} + C\frac{d}{dt}\left(-V_{in}\right) = 0$, so

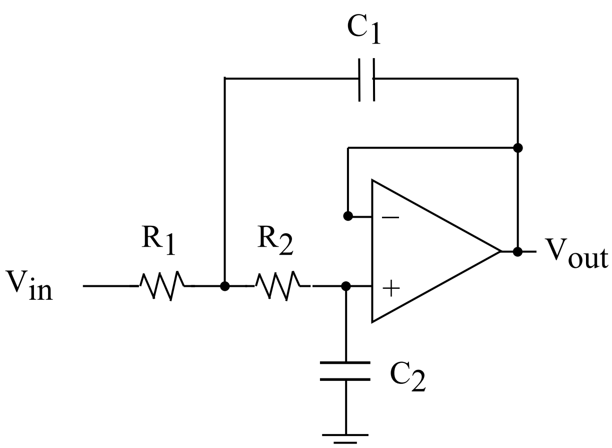
$\boxed{V_{out} = -R_f C\frac{dV_{in}}{dt}.}$ Raise R_f or C to increase differentiator response.

Bandpass Filter



$$\boxed{H(\omega) = -R_f C_i \frac{j\omega}{(1 + j\omega R_i C_i)(1 + j\omega R_f C_f)}}$$

Butterworth Low-Pass



$$\boxed{H(\omega) = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{\left(s^2 + 2\alpha s + \omega_o^2\right)}}$$

$$\boxed{\alpha = \frac{R_1 + R_2}{2 R_1 R_2 C_1}} \qquad \boxed{\omega_o^2 = \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\boxed{\zeta^2 = \frac{(R_1 + R_2)^2 C_2}{4 R_1 R_2 C_1}}$$

Place the demonimator's breakpoint at ω_o . Let $\boxed{R_1 = R_2}$, which leads to $\boxed{\zeta = \sqrt{\frac{C_2}{C_1}}}$. It is clear that underdamped response requires $C_2 < C_1$. Substituting $R_1 = R_2$ into the equation for ω_o yields $\boxed{R_1 = R_2 = \frac{1}{\omega_o \sqrt{C_1 C_2}}}$.

If we want $f_o = 10kHz$, then $\omega_o = 2\pi \bullet 10kHz$. Selecting C_1 = 0.01 μF and C_2 = 0.001 μF, then $R_1 = R_2$ should be approximately 5 kΩ (use parallel 10 kΩ resistors). The corresponding $\boxed{\zeta = \sqrt{\frac{C_2}{C_1}} = 0.316}$.