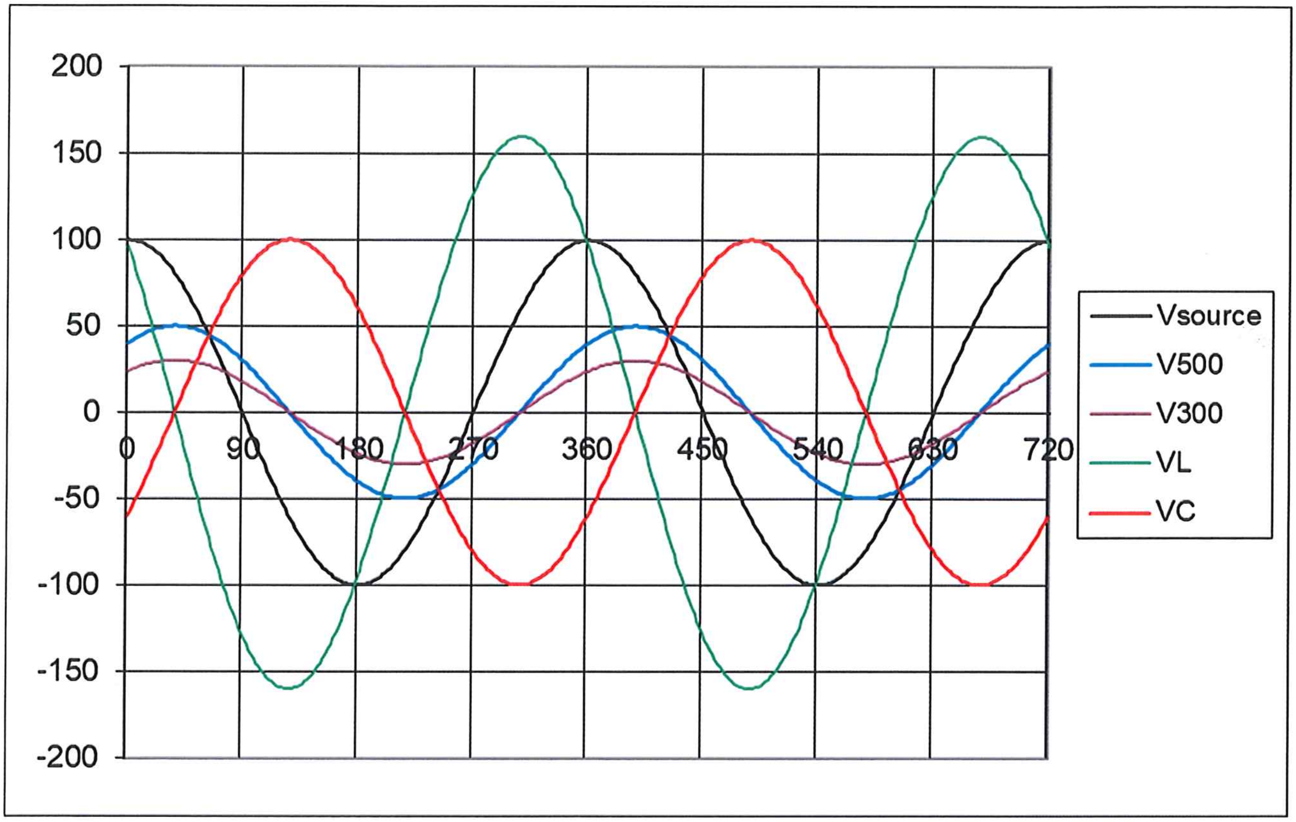
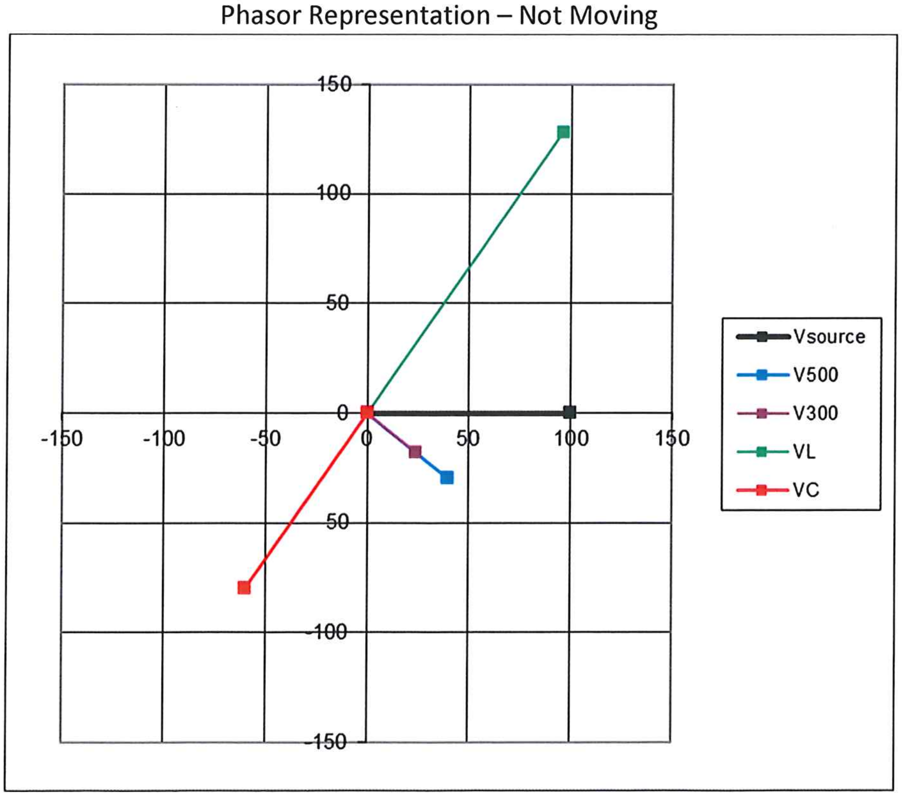
		Variable=	Vsource	V500	V300	VL	VC	KVL
wt	wt	Mag=	100	50	30	160	100	Sum
degrees	radians	Ang=	0	-36.9	-36.9	53.1	-126.9	Check

Time-Domain Waveforms



Phasor Representation – Not Moving



1. Consider KCL and KVL equations. For example, for three branch currents leaving a node, $i_1 + i_2 + i_3 = 0$.

2. For a linear circuit with sinusoidal excitation, with all sources having radian frequency ω radians per second, all voltages and currents in the circuit will also be sinusoidal with ω. The above KCL equation becomes
$I_1 \cos(\omega t + \theta_1) + I_2 \cos(\omega t + \theta_2) + I_3 \cos(\omega t + \theta_3) = 0$.

3. Using trig identity $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$, the above KCL equation becomes
$$I_1[\cos(\omega t)\cos(\theta_1) - \sin(\omega t)\sin(\theta_1)] + I_2[\cos(\omega t)\cos(\theta_2) - \sin(\omega t)\sin(\theta_2)]$$
$$+ I_3[\cos(\omega t)\cos(\theta_3) - \sin(\omega t)\sin(\theta_3)] = 0.$$

4. Factoring out the time varying terms
$$\cos(\omega t)[I_1 \cos(\theta_1) + I_2 \cos(\theta_2) + I_3 \cos(\theta_3)] + \sin(\omega t)[I_1 \sin(\theta_1) + I_2 \sin \theta_2) + I_3 \sin(\theta_3)] = 0.$$

5. The only way that the above expression can be zero for any arbitrary time t is if both bracketed terms are zero. Thus,
$$I_1 \cos(\theta_1) + I_2 \cos(\theta_2) + I_3 \cos(\theta_3) = 0,$$
$$I_1 \sin(\theta_1) + I_2 \sin(\theta_2) + I_3 \sin(\theta_3) = 0.$$

6. Multiplying the sin terms by complex $j = \sqrt{-1}$ and then adding the two equations yields
$$[I_1 \cos(\theta_1) + I_2 \cos(\theta_2) + I_3 \cos(\theta_3)] + j[I_1 \sin(\theta_1) + I_2 \sin \theta_2) + I_3 \sin(\theta_3)] = 0.$$
The above complex equation is actually two equations (i.e., real and imaginary).

7. Grouping by branch currents,
$$I_1[\cos(\theta_1) + j\sin(\theta_1)] + I_2[\cos(\theta_2) + j\sin(\theta_2)] + I_3[\cos(\theta_3) + j\sin(\theta_3)] = 0$$

8. Invoking Euler's Rule $e^{j\theta} = \cos(\theta) + j\sin(\theta)$ and substituting into the above,
$$I_1 e^{j\theta_1} + I_2 e^{j\theta_2} + I_3 e^{j\theta_3} = 0.$$

9. Define phasors $\tilde{I}_1 = I_1 e^{j\theta_1}$, etc., the above equation becomes
$$\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 0.$$
Phasors are complex numbers. Phasors are not time-varying.

10. Pre-multiplying (8) by $e^{j\omega t}$ yields
$$e^{j\omega t}\left[I_1 e^{j\theta_1} + I_2 e^{j\theta_2} + I_3 e^{j\theta_3}\right] = 0.$$
which can also be written as
$$I_1 e^{j(\omega t + \theta_1)} + I_2 e^{j(\omega t + \theta_2)} + I_3 e^{j(\omega t + \theta_3)} = 0.$$

11. If the above equation is true, then the following is also true,
$Real\left\{I_1 e^{j(\omega t+\theta_1)} + I_2 e^{j(\omega t+\theta_2)} + I_3 e^{j(\omega t+\theta_3)}\right\} = 0$.

12. which from Euler's rule is the same as the original KCL equation
$I_1 \cos(\omega t + \theta_1) + I_2 \cos(\omega t + \theta_2) + I_3 \cos(\omega t + \theta_3) = 0$.

13. So if we solve complex equation $\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 0$ in the phasor domain, we automatically satisfy KCL.

How do resistors, inductors, and capacitors fit into phasor analysis?

For R. Given $i(t) = I\cos(\omega t)$, then $v(t) = R \bullet i(t) = RI\cos(\omega t) = V\cos(\omega t)$.
Thus voltage and current are in phase.

Voltage leads by 90°

For L. Given $i(t) = I\cos(\omega t)$, then

$$v(t) = L \bullet \frac{di(t)}{dt} = -\omega LI\sin(\omega t) = -\omega LI\cos(\omega t - 90°) = \omega LI\cos(\omega t + 90°) = V\cos(\omega t + 90°).$$

Thus voltage leads current by 90°, so **for an inductor, current lags voltage by 90°.**

For C. Given $v(t) = V\cos(\omega t)$, then

$$i(t) = C \bullet \frac{dv(t)}{dt} = -\omega CV\sin(\omega t) = -\omega CV\cos(\omega t - 90°) = \omega CV\cos(\omega t + 90°) = I\cos(\omega t + 90°).$$

Thus **for a capacitor, current leads voltage by 90°.**

Current leads by 90°

Useful slogan: ELI the ICE man.
For inductor L, voltage E leads current I by 90° degrees.
For capacitor C, current I leads voltage E by 90° degrees.

So, in phasor notation,
For R, $\tilde{V} = R \bullet \tilde{I}$, define $Z = R\ \Omega$.
For L, $\tilde{V} = j\omega L \bullet \tilde{I}$, define $Z = j\omega L\ \Omega$.
For C, $\tilde{I} = j\omega C \bullet \tilde{V}$, so $\tilde{V} = \dfrac{\tilde{I}}{j\omega C}$, define $Z = \dfrac{1}{j\omega C}\ \Omega$.

Summarizing, with phasor analysis, all the rules you used for DC circuits now apply for steady-state, single-frequency AC circuits. Voltages, currents, and impedances are complex numbers. Once phasor voltages and currents are computed, then conversion to the time domain is done using

- Phasor $\tilde{I}_1 = I_1 e^{j\theta_1}$ is shorthand notation for time domain $i_1 = I_1\cos(\omega t + \theta_1)$.