5. Circuit Theorems

**P 5.4-1**

*Answer:* $R_i = 5 \Omega$ and $v_{oc} = 2$ V

![Figure P 5.4-1](image)

**P 5.4-2** The circuit shown in Figure P 5.4-2a is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-2a. Find the value of the open-circuit voltage $v_{oc}$ and Thévenin resistance $R_i$.

*Answer:* $v_{oc} = -12$ V and $R_i = 16$ $\Omega$

![Figure P 5.4-2](image)

**P 5.4-3** The circuit shown in Figure P 5.4-3a is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-3a. Find the value of the open-circuit voltage $v_{oc}$ and Thévenin resistance $R_i$.

*Answer:* $v_{oc} = 2$ V and $R_i = 4$ $\Omega$

![Figure P 5.4-3](image)

**P 5.4-4** Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-4.

![Figure P 5.4-4](image)

**P 5.4-5** Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-5.

![Figure P 5.4-5](image)

**P 5.4-6** Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-6.

![Figure P 5.4-6](image)

**P 5.4-7** The equivalent circuit in Figure P 5.4-7 is obtained by replacing part of the original circuit by its Thévenin equivalent circuit. The values of the parameters of the Thévenin equivalent circuit are

$v_{oc} = 15$ V and $R_i = 60$ $\Omega$

Determine the following:

(a) The values of $V_s$ and $R_{eq}$ (Four resistors in the original circuit have equal resistance, $R_a$).

(b) The value of $R_i$ required to cause $i = 0.2$ A.

(c) The value of $R_{eq}$ required to cause $v = 12$ V.

![Figure P 5.4-7](image)
P 5.4-8 A resistor, $R$, was connected to a circuit box as shown in Figure P 5.4-8. The voltage $v$ was measured. The resistance was changed, and the voltage was measured again. The results are shown in the table. Determine the Thévenin equivalent of the circuit within the box and predict the voltage $v$ when $R = 8 \, \text{k}\Omega$.

![Figure P 5.4-8](image)

<table>
<thead>
<tr>
<th>$R$ (k$\Omega$)</th>
<th>$v$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure P 5.4-8

P 5.4-9 A resistor, $R$, was connected to a circuit box as shown in Figure P 5.4-9. The current $i$ was measured. The resistance was changed, and the current was measured again. The results are shown in the table.

(a) Specify the value of $R$ required to cause $i = 2 \, \text{mA}$.
(b) Given that $R > 0$, determine the maximum possible value of the current $i$.

<table>
<thead>
<tr>
<th>$R$ (k$\Omega$)</th>
<th>$i$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Hint: Use the data in the table to represent the circuit by a Thévenin equivalent.

![Figure P 5.4-9](image)

Figure P 5.4-9

P 5.4-10 For the circuit of Figure P 5.4-10, specify the resistance $R$ that will cause current $i_a$ to be 2 mA. The current $i_a$ has units of amps.

Hint: Find the Thévenin equivalent circuit of the circuit connected to $R$.

![Figure P 5.4-10](image)

Figure P 5.4-10

P 5.4-11 For the circuit of Figure P 5.4-11, specify the value of the resistance $R_t$ that will cause current $i_a$ to be $-2 \, \text{A}$.

Answer: $R_t = 12 \, \text{k}\Omega$

![Figure P 5.4-11](image)

Figure P 5.4-11

P 5.4-12 The circuit shown in Figure P 5.4-12 contains an adjustable resistor. The resistance $R$ can be set to any value in the range $0 \leq R \leq 100 \, \text{k}\Omega$.

(a) Determine the maximum value of the current $i_a$ that can be obtained by adjusting $R$. Determine the corresponding value of $R$.
(b) Determine the maximum value of the voltage $v_a$ that can be obtained by adjusting $R$. Determine the corresponding value of $R$.
(c) Determine the maximum value of the power supplied to the adjustable resistor that can be obtained by adjusting $R$. Determine the corresponding value of $R$.

![Figure P 5.4-12](image)

Figure P 5.4-12

P 5.4-13 The circuit shown in Figure P 5.4-13 consists of two parts, the source (to the left of the terminals) and the load. The load consists of a single adjustable resistor having resistance $0 \leq R_L \leq 20 \, \Omega$. The resistance $R$ is fixed but unspecified. When $R_L = 4 \, \Omega$, the load current is measured to be $i_a = 0.375 \, \text{A}$. When $R_L = 8 \, \Omega$, the value of the load current is $i_a = 0.300 \, \text{A}$.

(a) Determine the value of the load current when $R_L = 10 \, \Omega$.
(b) Determine the value of $R$.

![Figure P 5.4-13](image)

Figure P 5.4-13

P 5.4-14 The circuit shown in Figure P 5.4-14 contains an unspecified resistance, $R$. Determine the value of $R$ in each of the following two ways.

(a) Write and solve mesh equations.
(b) Replace the part of the circuit connected to the resistor $R$ by a Thévenin equivalent circuit. Analyze the resulting circuit.

![Figure P 5.4-14](image)

Figure P 5.4-14
5. Circuit Theorems

P 5.4-15 Consider the circuit shown in Figure P 5.4-15. Replace the part of the circuit to the left of terminals a-b by its Thévenin equivalent circuit. Determine the value of the current \( i_0 \).

---

P 5.4-16 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 5.4-16a shows a circuit with a voltmeter that measures the voltage \( v_{\text{m}} \). In Figure P 5.4-16b, the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. The voltmeter measures \( v_{\text{m}} \), the ideal value of \( v_{\text{m}} \).

---

P 5.4-17 Given that \( 0 \leq R \leq \infty \) in the circuit shown in Figure P 5.4-17, consider these two observations:

Observation 1: When \( R = 2 \, \Omega \) then \( v_R = 4 \, V \) and \( i_R = 2 \, A \).
Observation 2: When \( R = 6 \, \Omega \) then \( v_R = 6 \, V \) and \( i_R = 1 \, A \).

Determine the following:
(a) The maximum value of \( i_R \) and the value of \( R \) that causes \( i_R \) to be maximal.
(b) The maximum value of \( v_R \) and the value of \( R \) that causes \( v_R \) to be maximal.
(c) The maximum value of \( p_R = i_R v_R \) and the value of \( R \) that causes \( p_R \) to be maximal.

---

P 5.4-18 Consider the circuit shown in Figure P 5.4-18. Determine

(a) The value of \( v_R \) that occurs when \( R = 9 \, \Omega \).
(b) The value of \( R \) that causes \( v_R = 5.4 \, V \).
(c) The value of \( R \) that causes \( i_R = 300 \, mA \).

---

P 5.4-19 The circuit shown in Figure P 5.4-19a can be reduced to the circuit shown in Figure P 5.4-19b using source transformations and equivalent resistances. Determine the values of the source voltage \( v_{\text{oc}} \) and the resistance \( R \).

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As \( R_m \to 0 \), the ammeter becomes an ideal ammeter and \( i_{am} \to i_{ai} \). When \( R_m > 0 \), the ammeter is not ideal and \( i_{am} < i_{ai} \). The difference between \( i_{am} \) and \( i_{ai} \) is a measurement error caused by the fact that the ammeter is not ideal.

(a) Determine the value of \( i_{ai} \).

(b) Express the measurement error that occurs when \( R_m = 20 \Omega \) as a percentage of \( i_{ai} \).

(c) Determine the maximum value of \( R_m \) required to ensure that the measurement error is smaller than 2 percent of \( i_{ai} \).

P 5.5-11 Determine values of \( R_L \) and \( i_s \) that cause the circuit shown in Figure P 5.5-11b to be the Norton equivalent circuit of the circuit in Figure P 5.5-11a.

\[ R_L = 3 \Omega \] and \( i_s = -2 \text{ A} \)

![Figure P 5.5-11](image)

P 5.5-12 Use Norton’s theorem to formulate a general expression for the current \( i \) in terms of the variable resistance \( R \) shown in Figure P 5.5-12.

\[ i = \frac{20}{8 + R} \text{ A} \]

![Figure P 5.5-12](image)

Section 5.6 Maximum Power Transfer

P 5.6-1 The circuit shown in Figure P 5.6-1 consists of two parts separated by a pair of terminals. Consider the part of the circuit to the left of the terminals. The open circuit voltage is \( v_{oc} = 8 \text{ V} \), and short-circuit current is \( i_{sc} = 2 \text{ A} \). Determine the values of (a) the voltage source voltage \( v_s \) and the resistance \( R_s \), and (b) the resistance \( R \) that maximizes the power delivered to the resistor to the right of the terminals, and the corresponding maximum power.

![Figure P 5.6-1](image)

P 5.6-2 The circuit model for a photovoltaic cell is given in Figure P 5.6-2 (Edelson, 1992). The current \( i_s \) is proportional to the solar insolation (kW/m²).

(a) Find the load resistance, \( R_L \), for maximum power transfer.

(b) Find the maximum power transferred when \( i_s = 1 \text{ A} \).

![Figure P 5.6-2](image)

P 5.6-3 For the circuit in Figure P 5.6-3, (a) find \( R \) such that maximum power is dissipated in \( R \), and (b) calculate the value of maximum power.

\[ R = 60 \Omega \] and \( P_{max} = 54 \text{ mW} \)

![Figure P 5.6-3](image)

P 5.6-4 For the circuit in Figure P 5.6-4, prove that for \( R_s \) variable and \( R_L \) fixed, the power dissipated in \( R_L \) is maximum when \( R_s = 0 \).

![Figure P 5.6-4](image)

P 5.6-5 Determine the maximum power that can be absorbed by a resistor, \( R \), connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of \( R \).

![Figure P 5.6-5](image)
Section 5.8 Using PSpice to Determine the Thévenin Equivalent Circuit

P 5.8-1 The circuit shown in Figure P 5.8-1 is separated into two parts by a pair of terminals. Call the part of the circuit to the left of the terminals circuit A and the part of the circuit to the right of the terminal circuit B. Use PSpice to do the following:

(a) Determine the node voltages for the entire circuit.
(b) Determine the Thévenin equivalent circuit of circuit A.
(c) Replace circuit A by its Thévenin equivalent and determine the node voltages of the modified circuit.
(d) Compare the node voltages of circuit B before and after replacing circuit A by its Thévenin equivalent.

Section 5.9 How Can We Check

P 5.9-1 For the circuit of Figure P 5.9-1, the current has been measured for three different values of $R$ and is listed in the table. Are the data consistent?

<table>
<thead>
<tr>
<th>$R(\Omega)$</th>
<th>$i(\text{mA})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>16.5</td>
</tr>
<tr>
<td>500</td>
<td>43.8</td>
</tr>
<tr>
<td>0</td>
<td>97.2</td>
</tr>
</tbody>
</table>

P 5.9-2 Your lab partner built the circuit shown in Figure P 5.9-2 and measured the current $i$ and voltage $v$ corresponding to several values of the resistance $R$. The results are shown in the table in Figure P 5.9-2. Your lab partner says that $R_L = 8000 \Omega$ is required to cause $i = 1 \text{ mA}$. Do you agree? Justify your answer.

P 5.9-3 In preparation for lab, your lab partner determined the Thévenin equivalent of the circuit connected to $R_1$ in Figure P 5.9-3. She says that the Thévenin resistance is $R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$ and the open-circuit voltage is $v_{oc} = \frac{R_2}{R_1 + R_2} V$. In lab, you built the circuit using $R = 110 \Omega$ and $R_L = 40 \Omega$ and measured that $i = 54.5 \text{ mA}$. Is this measurement consistent with the prelab calculations? Justify your answers.

P 5.9-4 Your lab partner claims that the current $i$ in Figure P 5.9-4 will be no greater than 12.0 mA, regardless of the value of the resistance $R$. Do you agree?