HW#6 Write the equations only

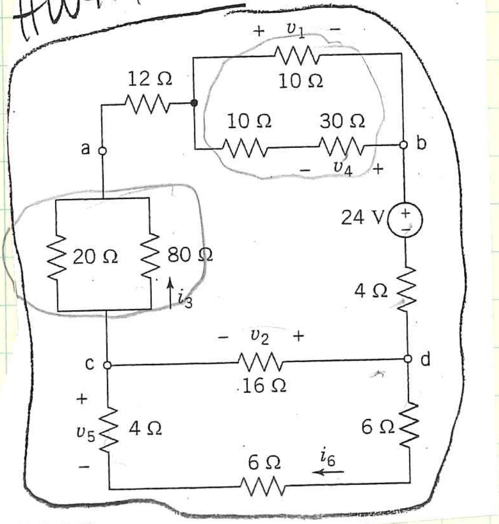

Figure P 3.6-19

Methods of Analysis of Resistive Circuits

4.1 Introduction

To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. The constitutive equations of the circuit elements, such as Ohm's law, provide the remaining equations. The unknown variables are element currents and voltages. Solving the equations provides the values of the element current and voltages.

This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits. A circuit with only 6 elements has 6 element currents and 6 element voltages. We could have 12 equations in 12 unknowns. In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method.

The node voltage method uses a new type of variable called the node voltage. The "node voltage equations" or, more simply, the "node equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the node voltage equations are the node voltages. After solving the node voltage equations, we determine the values of the element currents and voltages from the values of the node voltages.

It's easier to write node voltage equations for ~~some~~ MOST types of circuit than for others. Starting with the easiest case, we will learn how to write node voltage equations for circuits that consist of:

- Resistors and independent current sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

The mesh current method uses a new type of variable called the mesh current. The "mesh current equations" or, more simply, the "mesh equations," are a set of simultaneous equations that represent a

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given electric circuit. The unknown variables of the mesh current equations are the mesh currents. After solving the mesh current equations, we determine the values of the element currents and voltages from the values of the mesh currents.

It's easier to write mesh current equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write mesh current equations for circuits that consist of:

- Resistors and independent voltage sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

Major (More than 2 branches

To write a set of node equations, we do two things:

1. Express element currents as functions of the node voltages.
2. Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit except for the reference node.

EXAMPLE 4.2-2 Node Equations

Obtain the node equations for the circuit in Figure 4.2-6.

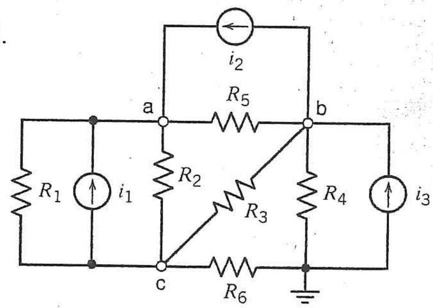


FIGURE 4.2-6 The circuit for Example 4.2-2.

Solution

Let v_a denote the node voltage at node a, v_b denote the node voltage at node b, and v_c denote the node voltage at node c. Apply KCL at node a to obtain

$$-\left(\frac{v_a - v_c}{R_1}\right) + i_1 - \left(\frac{v_a - v_c}{R_2}\right) + i_2 - \left(\frac{v_a - v_b}{R_5}\right) = 0$$

Separate the terms of this equation that involve v_a from the terms that involve v_b and the terms that involve v_c to obtain.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right) v_a - \left(\frac{1}{R_5}\right) v_b - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_c = i_1 + i_2$$

Determine the node voltages for the circuit in Figure 4.2-6 when $i_1 = 1$ A, $i_2 = 2$ A, $i_3 = 3$ A, $R_2 = 2V$, $R_3 = 10\ \Omega$, $R_4 = 4\ \Omega$, $R_5 = 5\ \Omega$, and $R_6 = 2\ \Omega$.

Solution

The node equations are

Form the Admittance Matrix

$$\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{5}\right) v_a - \left(\frac{1}{5}\right) v_b - \left(\frac{1}{5} + \frac{1}{2}\right) v_c = 1 + 2$$

$$-\left(\frac{1}{5}\right) v_a + \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right) v_b - \left(\frac{1}{10}\right) v_c = -2 + 3$$

$$-\left(\frac{1}{5} + \frac{1}{2}\right) v_a - \left(\frac{1}{10}\right) v_b + \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{10} + \frac{1}{2}\right) v_c = -1$$

$$0.9v_a - 0.2v_b - 0.7v_c = 3$$

$$-0.2v_a + 0.55v_b - 0.1v_c = 1$$

$$-0.7v_a - 0.1v_b + 1.3v_c = -1$$

The node equations can be written using matrices as

$$A\,v = b$$

where

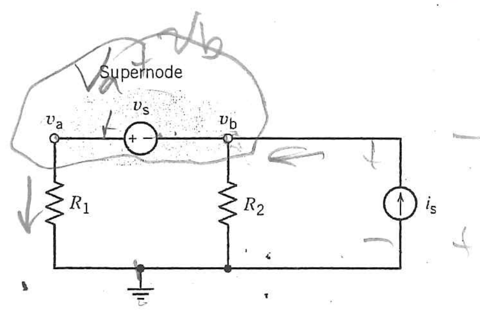
The Admittance Matrix "Y" Matrix

$$A = \begin{bmatrix} 0.9 & -0.2 & -0.7 \\ -0.2 & 0.55 & -0.1 \\ -0.7 & 0.1 & 1.3 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \text{ and, } v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

This matrix equation is solved using MATLAB in Figure 4.2-7.

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 7.1579 \\ 5.0526 \\ 3.4737 \end{bmatrix}$$

Consequently, $v_a = 7.1579$ V, $v_b = 5.0526$ V, and $v_c = 3.4737$ V



IGURE 4.3-2 Circuit with a supernode at incorporates v_a and v_b.

A **supernode** consists of two nodes connected by an independent or a dependent voltage source.

EXAMPLE 4.3-1 Node Equations

e the values node voltages, v_1 and v_2, in the circuit shown in Figure 4.3-3a.

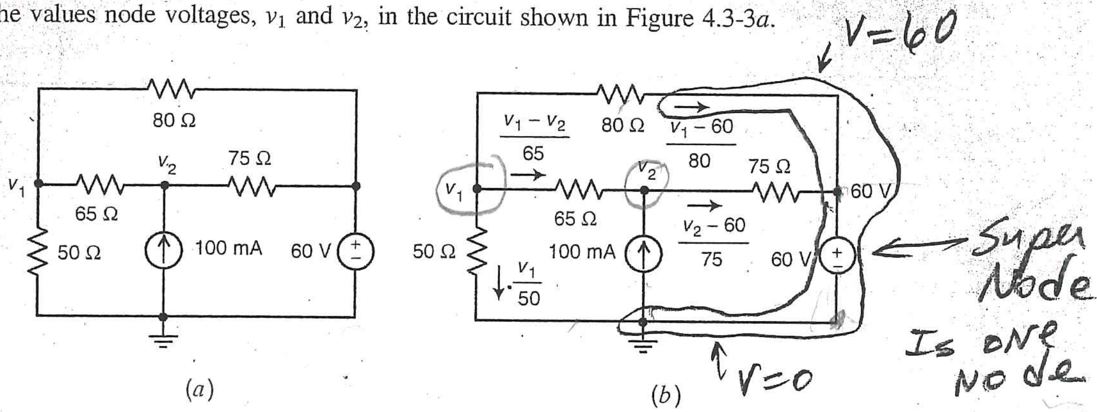
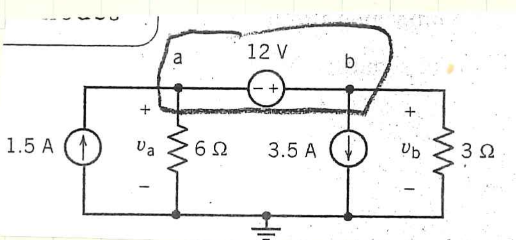


FIGURE 4.3-3 The circuit considered in Example 4.3-1.



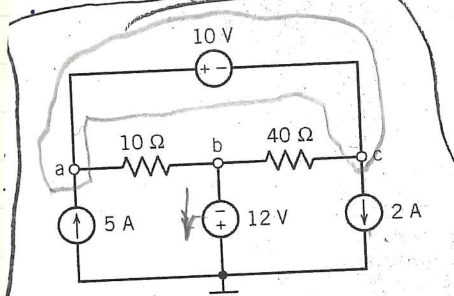


FIGURE 4.3-7 The circuit for Example 4.3-3.

4.5 Mesh Current Analysis with Independent Voltage Sources

In this and succeeding sections, we consider the analysis of circuits using Kirchhoff's voltage law (KVL) around a closed path. A *closed path* or a *loop* is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.

A mesh is a special case of a loop.

A **mesh** is a loop that does not contain any other loops within it.

Uh-oh

Mesh current analysis is applicable only to planar networks. A planar circuit is one that can be drawn on a plane, without crossovers. An example of a nonplanar circuit is shown in Figure 4.5-1, in which the crossover is identified and cannot be removed by redrawing the circuit. For planar networks, the meshes in the network look like windows. There are four meshes in the circuit shown in Figure 4.5-2.

If you can mash your current down ON PAPER so there are no crossovers, you can use mesh.

V_1

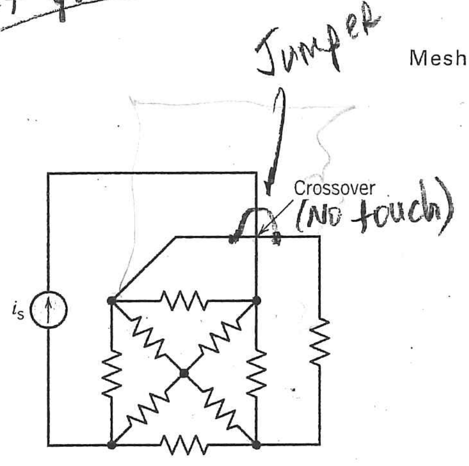


FIGURE 4.5-1 Nonplanar circuit with a crossover.

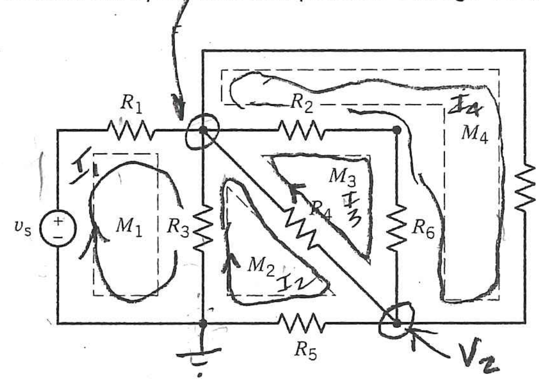


FIGURE 4.5-2 Circuit with four meshes. Each mesh is identified by dashed lines.

4 Mesh Equations OR 2 Nodal Equations to solve it

To write a set of mesh equations, we do two things:

1. Express element voltages as functions of the mesh currents.
2. Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.

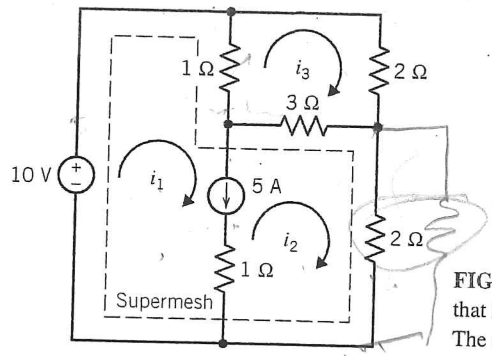


FIGURE 4.6-4 Circuit with a supermesh that incorporates mesh 1 and mesh 2. The supermesh is indicated by the dashed line.

A **supermesh** is one larger mesh created from two meshes that have an independent or dependent current source in common.