Methods of Analysis of Resistive Circuits

4.1 Introduction

To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. The constitutive equations of the circuit elements, such as Ohm's law, provide the remaining equations. The unknown variables are element currents and voltages.

Solving the equations provides the values of the element current and voltages.

This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits. A circuit with only 6 elements has 6 element currents and 6 element voltages.

We could have 12 equations in 12 unknowns. In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method.

The node voltage method uses a new type of variable called the node voltage. The "node voltage equations" or, more simply, the "node equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the node voltage equations are the node voltages. After solving the node voltage equations, we determine the values of the element currents and voltages from the values of the node voltages.

It's easier to write node voltage equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write node voltage equations for circuits that consist of:

- Resistors and independent current sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

The mesh current method uses a new type of variable called the mesh current. The "mesh current equations" or, more simply, the "mesh equations," are a set of simultaneous equations that represent a...
given electric circuit. The unknown variables of the mesh current equations are the mesh currents. After solving the mesh current equations, we determine the values of the element currents and voltages from the values of the mesh currents.

It’s easier to write mesh current equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write mesh current equations for circuits that consist of:

- Resistors and independent voltage sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

To write a set of node equations, we do two things:

1. Express element currents as functions of the node voltages.
2. Apply Kirchhoff’s current law (KCL) at each of the nodes of the circuit except for the reference node.

**Example 4.2-2 Node Equations**

Obtain the node equations for the circuit in Figure 4.2-6.

**Solution**

Let \( v_a \) denote the node voltage at node a, \( v_b \) denote the node voltage at node b, and \( v_c \) denote the node voltage at node c. Apply KCL at node a to obtain

\[
- \left( \frac{v_a - v_c}{R_1} \right) + i_1 - \left( \frac{v_a - v_b}{R_3} \right) + i_2 - \left( \frac{v_a - v_b}{R_5} \right) = 0
\]

Separate the terms of this equation that involve \( v_a \) from the terms that involve \( v_b \), and the terms that involve \( v_c \) to obtain.

\[
\left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) v_a - \left( \frac{1}{R_3} \right) v_b - \left( \frac{1}{R_1} + \frac{1}{R_5} \right) v_c = i_1 + i_2
\]

Determine the node voltages for the circuit in Figure 4.2-6 when \( i_1 = 1 \text{ A}, i_2 = 2 \text{ A}, i_3 = 3 \text{ A}, i_4 = 2 \text{ A}, R_2 = 2 \Omega, R_3 = 10 \Omega, R_4 = 4 \Omega, R_5 = 5 \Omega, \) and \( R_6 = 2 \Omega. \)

**Solution**

The node equations are

\[
\begin{align*}
\frac{1}{5} + \frac{1}{2} + \frac{1}{5} v_a - \frac{1}{5} v_b - \frac{1}{5} + \frac{1}{2} v_c = 1 + 2 \\
- \frac{1}{5} + \frac{1}{10} + \frac{1}{2} + \frac{1}{4} v_b - \frac{1}{10} v_c = -2 + 3 \\
- \frac{1}{5} + \frac{1}{2} v_b - \frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{10} + \frac{1}{2} v_c = -1
\end{align*}
\]

The node equations can be written using matrices as

\[
A v = b
\]
where

\[
A = \begin{bmatrix}
0.9 & -0.2 & -0.7 \\
-0.2 & 0.55 & -0.1 \\
-0.7 & 0.1 & 1.3
\end{bmatrix}, \quad b = \begin{bmatrix}
3 \\
1 \\
-1
\end{bmatrix}
\]

and, \( v = \begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} \)

This matrix equation is solved using MATLAB in Figure 4.2-7.

\[
v = \begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} = \begin{bmatrix}
7.1579 \\
5.0526 \\
3.4737
\end{bmatrix}
\]

Consequently, \( v_a = 7.1579 \) V, \( v_b = 5.0526 \) V, and \( v_c = 3.4737 \) V.

A supernode consists of two nodes connected by an independent or a dependent voltage source.

**Example 4.3-1 Node Equations**

Find the values node voltages, \( v_1 \) and \( v_2 \), in the circuit shown in Figure 4.3-3a.

\[ V = 60 \]

\[ V = 0 \]

![FIGURE 4.3-3 The circuit considered in Example 4.3-1.]

![FIGURE 4.3-7 The circuit for Example 4.3-3.]

\[ \text{Is one node} \]

\[ \text{Super Node} \]
4.5 **Mesh Current Analysis with Independent Voltage Sources**

In this and succeeding sections, we consider the analysis of circuits using Kirchhoff's voltage law (KVL) around a closed path. A **closed path** or a **loop** is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.

A mesh is a special case of a loop.

A mesh is a loop that does not contain any other loops within it.

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**Mesh current analysis is applicable only to planar networks.** A planar circuit is one that can be drawn on a plane, without crossovers. An example of a nonplanar circuit is shown in Figure 4.5-1, in which the crossover is identified and cannot be removed by redrawing the circuit. For planar networks, the meshes in the network look like windows. There are four meshes in the circuit shown in Figure 4.5-2.

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**FIGURE 4.5-1** Nonplanar circuit with a crossover.

**FIGURE 4.5-2** Circuit with four meshes. Each mesh is identified by dashed lines.

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To write a set of mesh equations, we do two things:
1. Express element voltages as functions of the mesh currents.
2. Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.
A supermesh is one larger mesh created from two meshes that have an independent or dependent current source in common.