7.2 Capacitors

A capacitor is a circuit element that stores energy in an electric field. A capacitor can be constructed using two parallel conducting plates separated by distance d as shown in Figure 7.2-1. Electric charge is stored on the plates, and a uniform electric field exists between the conducting plates whenever there is a voltage across the capacitor. The space between the plates is filled with a dielectric material. Some capacitors use impregnated paper for a dielectric, whereas others use mica sheets, ceramics, metal films, or just air. A property of the dielectric material, called the dielectric constant, describes the relationship between the electric field strength and the capacitor voltage. Capacitors are represented by a parameter called the *capacitance*. The capacitance of a capacitor is proportional to the dielectric constant and surface area of the plates and is inversely proportional to the distance between the plates. In other words, the capacitance C of a capacitor is given by

$$C = \frac{\epsilon A}{d}$$

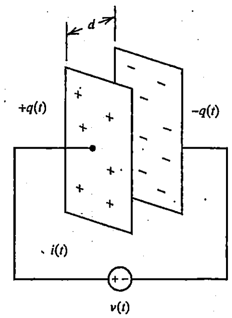

FIGURE 7.2-1 A capacitor connected to a voltage source.

where ϵ is the dielectric constant, A the area of the plates, and d the distance between plates. ~~The unit of capacitance is~~ coulomb per volt and is called farad (F) in honor of Michael Faraday.

A capacitor voltage $v(t)$ deposits a charge $+q(t)$ on one plate and a charge $-q(t)$ on the other plate. We say that the charge $q(t)$ is stored by the capacitor. The charge stored by a capacitor is proportional to the capacitor voltage $v(t)$. Thus, we write

$$q(t) = Cv(t) \tag{7.2-1}$$

where the constant of proportionality C is the capacitance of the capacitor.

Capacitance is a measure of the ability of a device to store energy in the form of a separated charge or an electric field.

In general, the capacitor voltage $v(t)$ varies as a function of time. Consequently, $q(t)$, the charge stored by the capacitor, also varies as a function of time. The variation of the capacitor charge with respect to time implies a capacitor current $i(t)$, given by

$$i(t) = \frac{d}{dt}q(t)$$

We differentiate Eq. 7.2-1 to obtain

$$i(t) = C\frac{d}{dt}v(t) \tag{7.2-2}$$

Equation 7.2-2 is the current–voltage relationship of a capacitor. The current and voltage in Eq. 7.7-2 adhere to the passive convention. Figure 7.2-2 shows two alternative symbols to represent capacitors in circuit diagrams. In both Figure 7.2-2a and b, the capacitor current and voltage adhere to the passive sign convention and are related by Eq. 7.2-2.

Now consider the waveform shown in Figure 7.2-3, in which the voltage changes from a constant voltage of zero to another constant voltage of 1 over an increment of time, Δt. Using Eq. 7.2-2, we obtain

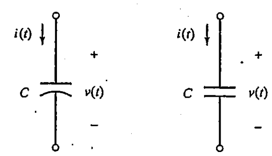

FIGURE 7.2-2 Circuit symbols of a capacitor.

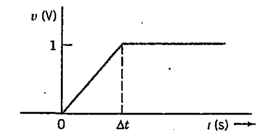

FIGURE 7.2-3 Voltage waveform in which the change in voltage occurs over an increment of time, Δt.

$$i(t) = \begin{cases} 0 & t < 0 \\ \dfrac{C}{Dt} & 0 < t < Dt \\ 0 & t > Dt \end{cases}$$

Thus, we obtain a pulse of height equal to $C/\Delta t$. As Δt decreases, the current will increase. Clearly, Δt cannot decline to zero or we would experience an infinite current. An infinite current is an impossibility because it would require infinite power. Thus, an instantaneous ($Dt = 0$) change of voltage across the capacitor is not possible. In other words, we cannot have a discontinuity in $v(t)$.

The voltage across a **capacitor** cannot change instantaneously.

Now, let us find the voltage $v(t)$ in terms of the current $i(t)$ by integrating both sides of Eq. 7.2-2. We obtain

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \tag{7.2-3}$$

This equation says that the capacitor voltage $v(t)$ can be found by integrating the capacitor current from time $-\infty$ until time t. To do so requires that we know the value of the capacitor current from time $\tau = -\infty$ until time $\tau = t$. Often, we don't know the value of the current all the way back to $\tau = -\infty$. Instead, we break the integral up into two parts:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + \frac{1}{C}\int_{-\infty}^{t_0} i(\tau)d\tau = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0) \tag{7.2-4}$$

This equation says that the capacitor voltage $v(t)$ can be found by integrating the capacitor current from some convenient time $\tau = t_0$ until time $\tau = t$, provided that we also know the capacitor voltage at time t_0. Now we are required to know only the capacitor current from time $\tau = t_0$ until time $\tau = t$. The time t_0 is called the **initial time**, and the capacitor voltage $v(t_0)$ is called the **initial condition**. Frequently, it is convenient to select $t_0 = 0$ as the initial time.

Capacitors are commercially available in a variety of types and capacitance values. Capacitor types are described in terms of the dielectric material and the construction technique. Miniature metal film capacitors are shown in Figure 7.2-4. Miniature hermetically sealed polycarbonate capacitors are shown in Figure 7.2-5. Capacitance values typically range from picofarads (pF) to microfarads (μF).

EXERCISE 7.2-1 Determine the current $i(t)$ for $t > 0$ for the circuit of Figure E 7.2-1b when $v_s(t)$ is the voltage shown in Figure E 7.2-1a.

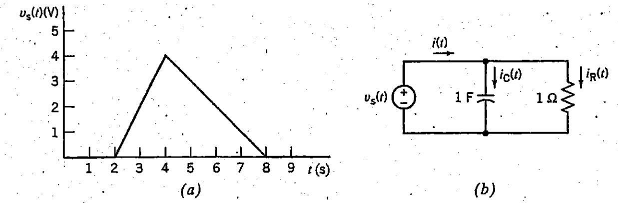

FIGURE E 7.2-1 (*a*) The voltage source voltage. (*b*) The circuit.

Hint: Determine $i_C(t)$ and $i_R(t)$ separately, then use KCL.

Answer: $v(t) = \begin{cases} 2t - 2 & 2 < t < 4 \\ 7 - t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$

7.3 Energy Storage in a Capacitor

Consider a capacitor that has been connected to a battery of voltage v. A current flows and a charge is stored on the plates of the capacitor, as shown in Figure 7.3-1. Eventually, the voltage across the capacitor is a constant, and the current through the capacitor is zero. The capacitor has stored energy by virtue of the separation of charges between the capacitor plates. These charges have an electrical force acting on them.

The forces acting on the charges stored in a capacitor are said to result from an electric field. An *electric field* is defined as the force acting on a unit positive charge in a specified region. Because the charges have a force acting on them along a direction x, we recognize that the energy required originally to separate the charges is now stored by the capacitor in the electric field.

The energy stored in a capacitor is

$$w_c(t) = \int_{-\infty}^{t} vi\, d\tau$$

Remember that v and i are both functions of time and could be written as $v(t)$ and $i(t)$. Because

$$i = C\frac{dv}{dt}$$

we have

$$w_c = \int_{-\infty}^{t} vC\frac{dv}{d\tau}\, d\tau = C\int_{v(-\infty)}^{v(t)} v\, dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

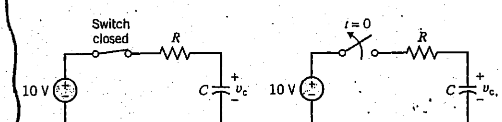

FIGURE 7.3-1 A circuit (*a*) where the capacitor is charged and $v_c = 10$ V and (*b*) the switch is opened at $t = 0$.

Because the capacitor was uncharged at $t = -\infty$, set $v(-\infty) = 0$. Therefore,

$$w_c(t) = \frac{1}{2}Cv^2(t)\ \text{J} \tag{7.3-1}$$

Therefore, as a capacitor is being charged and $v(t)$ is changing, the energy stored, w_c, is changing. Note that $w_c(t) > 0$ for all $v(t)$, so the element is said to be passive.

Because $q = Cv$, we may rewrite Eq. 7.3-1 as

$$w_c = \frac{1}{2C}q^2(t)\ \text{J} \tag{7.3-2}$$

The capacitor is a storage element that stores but does not dissipate energy. For example, consider a 100-mF capacitor that has a voltage of 100 V across it. The energy stored is

$$w_c = \frac{1}{2}Cv^2 = \frac{1}{2}(0.1)(100)^2 = 500\ \text{J}$$

As long as the capacitor is not connected to any other element, the energy of 500 J remains stored. Now if we connect the capacitor to the terminals of a resistor, we expect a current to flow until all the energy is dissipated as heat by the resistor. After all the energy dissipates, the current is zero and the voltage across the capacitor is zero.

As noted in the previous section, the requirement of conservation of charge implies that the voltage on a capacitor is continuous. Thus, the *voltage and charge on a capacitor cannot change instantaneously*. This statement is summarized by the equation

$$v(0^+) = v(0^-)$$

where the time just prior to $t = 0$ is called $t = 0^-$ and the time immediately after $t = 0$ is called $t = 0^+$. The time between $t = 0^-$ and $t = 0^+$ is infinitely small. Nevertheless, the voltage will not change abruptly.

To illustrate the continuity of voltage for a capacitor, consider the circuit shown in Figure 7.3-1. For the circuit shown in Figure 7.3-1a, the switch has been closed for a long time and the capacitor voltage has become $v_c = 10$ V. At time $t = 0$, we open the switch, as shown in Figure 7.3-1b. Because the voltage on the capacitor is continuous,

$$v_c(0^+) = v_c(0^-) = 10\ \text{V}$$

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EXAMPLE 7.3-1 Energy Stored by a Capacitor

A 10-mF capacitor is charged to 100 V, as shown in the circuit of Figure 7.3-2. Find the energy stored by the capacitor and the voltage of the capacitor at $t = 0^+$ after the switch is opened.

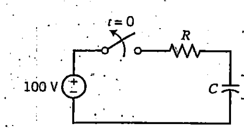

FIGURE 7.3-2 Circuit of Example 7.3-1 with $C = 10$ mF.

Solution

The voltage of the capacitor is $v = 100$ V at $t = 0^-$. Because the voltage at $t = 0^+$ cannot change from the voltage at $t = 0^-$, we have

$$v(0^+) = v(0^-) = 100\ \text{V}$$

The energy stored by the capacitor at $t = 0^+$ is

$$w_c = \frac{1}{2}Cv^2 = \frac{1}{2}(10^{-2})(100)^2 = 50\ \text{J}$$

EXERCISE 7.3-1. A 200-μF capacitor has been charged to 100 V. Find the energy stored by the capacitor. Find the capacitor voltage at $t = 0^+$ if $v(0^-) = 100$ V.

Answer: $w(1) = 1$ J and $v(0^+) = 100$ V

EXERCISE 7.3-2 A constant current $i = 2$ A flows into a capacitor of 100μF after a switch is closed at $t = 0$. The voltage of the capacitor was equal to zero at $t = 0^-$. Find the energy stored at (a) $t = 1$ s and (b) $t = 100$ s.

Answer: $w(1) = 20$ kJ and $w(100) = 200$ MJ

7.4 Series and Parallel Capacitors

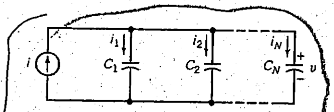

FIGURE 7.4-1 Parallel connection of N capacitors.

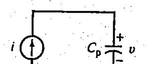

FIGURE 7.4-2 Equivalent circuit for N parallel capacitors.

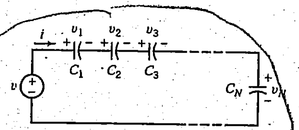

FIGURE 7.4-3 Series connection of N capacitors.

Series

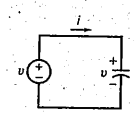

FIGURE 7.4-4 Equivalent circuit for N series capacitors.

First, let us consider the parallel connection of N capacitors as shown in Figure 7.4-1. We wish to determine the equivalent circuit for the N parallel capacitors as shown in Figure 7.4-2.

Using KCL, we have

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

Because

$$i_n = C_n \frac{dv}{dt}$$

and v appears across each capacitor, we obtain

$$\begin{aligned} i &= C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt} \\ &= (C_1 + C_2 + C_3 + \cdots + C_N)\frac{dv}{dt} \\ &= \left(\sum_{n=1}^{N} C_n\right)\frac{dv}{dt} \end{aligned} \tag{7.4-1}$$

For the equivalent circuit shown in Figure 7.4-2,

$$i = C_p \frac{dv}{dt} \tag{7.4-2}$$

Comparing Eqs. 7.4-1 and 7.4-2, it is clear that

$$C_p = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{n=1}^{N} C_n$$

Thus, the equivalent capacitance of a set of N parallel capacitors is simply the sum of the individual capacitances. It must be noted that all the parallel capacitors will have the same initial condition $v(0)$.

Now let us determine the equivalent capacitance C_s of a set of N series-connected capacitances, as shown in Figure 7.4-3. The equivalent circuit for the series of capacitors is shown in Figure 7.4-4.

Using KVL for the loop of Figure 7.4-3, we have

$$v = v_1 + v_2 + v_3 + \cdots + v_N \tag{7.4-3}$$

Because, in general,

$$v_n(t) = \frac{1}{C_n}\int_{t_0}^{t} i\,d\tau + v_n(t_0)$$

where i is common to all capacitors, we obtain

$$\begin{aligned} v &= \frac{1}{C_1}\int_{t_0}^{t} i\,d\tau + v_1(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i\,d\tau + v_N(t_0) \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i\,d\tau + \sum_{n=1}^{N} v_n(t_0) \\ &= \left(\sum_{n=1}^{N}\frac{1}{C_n}\right)\int_{t_0}^{t} i\,d\tau + \sum_{n=1}^{N} v_n(t_0) \end{aligned} \tag{7.4-4}$$

From Eq. 7.4-3, we note that at $t = t_0$,

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0) = \sum_{n=1}^{N} v_n(t_0) \tag{7.4-5}$$

Substituting Eq. 7.4-5 into Eq. 7.4-4, we obtain

$$v = \left(\sum_{n=1}^{N}\frac{1}{C_n}\right)\int_{t_0}^{t} i\,d\tau + v(t_0) \tag{7.4-6}$$

Using KVL for the loop of the equivalent circuit of Figure 7.4-4 yields

$$v = \frac{1}{C_s}\int_{t_0}^{t} i\,d\tau + v(t_0) \tag{7.4-7}$$

Comparing Eqs. 7.4-6 and 7.4-7, we find that

$$\frac{1}{C_s} = \sum_{n=1}^{N}\frac{1}{C_n} \tag{7.4-8}$$

Series Caps

For the case of two series capacitors, Eq. 7.4-8 becomes

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \tag{7.4-9}$$

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EXAMPLE 7.4-1 Parallel and Series Capacitors

Find the equivalent capacitance for the circuit of Figure 7.4-5 when $C_1 = C_2 = C_3 = 2$ mF, $v_1(0) = 10$ V, and $v_2(0) = v_3(0) = 20$ V.

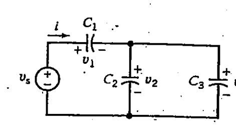

FIGURE 7.4-5 Circuit for Example 7.4-1.

Solution

Because C_2 and C_3 are in parallel, we replace them with C_p, where

$$C_p = C_2 + C_3 = 4 \text{ mF}$$

The voltage at $t = 0$ across the equivalent capacitance C_p is equal to the voltage across C_2 or C_3, which is $v_2(0) = v_3(0) = 20$ V. As a result of replacing C_2 and C_3 with C_p we obtain the circuit shown in Figure 7.4-6.

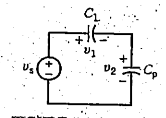

FIGURE 7.4-6 Circuit resulting from Figure 7.4-5 by replacing C_2 and C_3 with C_p.

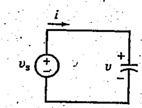

FIGURE 7.4-7. Equivalent circuit for the circuit of Example 7.4-1.

We now want to replace the series of two capacitors C_1 and C_p with one equivalent capacitor. Using the relationship of Eq. 7.4-9, we obtain

$$C_s = \frac{C_1 C_p}{C_1 + C_p} = \frac{(2 \times 10^{-3})(4 \times 10^{-3})}{(2 \times 10^{-3}) + (4 \times 10^{-3})} = \frac{8}{6}\ \text{mF}$$

The voltage at $t = 0$ across C_s is

$$v(0) = v_1(0) + v_p(0)$$

where $v_p(0) = 20$ V, the voltage across the capacitance C_p at $t = 0$. Therefore, we obtain

$$v(0) = 10 + 20 = 30\ \text{V}$$

Thus, we obtain the equivalent circuit shown in Figure 7.4-7.

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EXERCISE 7.4-1 Find the equivalent capacitance for the circuit of Figure E 7.4-1

Answer: $C_{eq} = 4$ mF

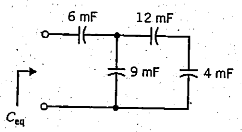


FIGURE E 7.4-1

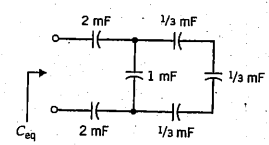


FIGURE E 7.4-2

EXERCISE 7.4-2 Determine the equivalent capacitance C_{eq} for the circuit shown in Figure E 7.4-2.

Answer: 10/19 mF

7.5 Inductors

An inductor is a circuit element that stores energy in a magnetic field. An inductor can be constructed by winding a coil of wire around a magnetic core as shown in Figure 7.5-1. Inductors are represented by a parameter called the *inductance*. The inductance of an inductor depends on its size, materials, and method of construction. For example, the inductance of the inductor shown in Figure 7.5-1 is given by

$$L = \frac{\mu N^2 A}{l}$$ HeNRY H

where N is the number of turns—that is, the number of times that the wire is wound around the core—A is the cross-sectional area ~~of the core in square~~ meters; l the length of the winding in meters; and μ is a property of the magnetic core known as the permeability. The unit of inductance is called henry (H) in honor of the American physicist Joseph Henry. Practical inductors have inductances ranging from 1 μH to 10 H. Inductors are wound in various forms, as shown in Figure 7.5-2.

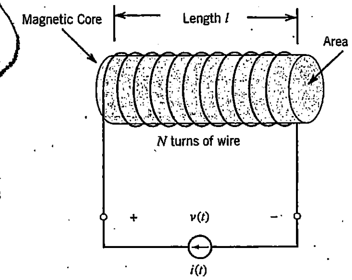


FIGURE 7.5-1 An inductor connected to a current source.

Inductance is a measure of the ability of a device to store energy in the form of a magnetic field.

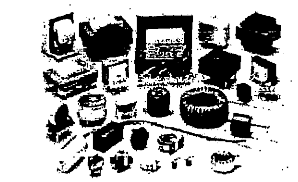
In Figure 7.5-1, a current source is used to cause a coil current $i(t)$. We find that the voltage $v(t)$ across the coil is proportional to the rate of change of the coil current. That is,

$$v(t) = L\frac{d}{dt}i(t) \tag{7.5-1}$$

where the constant of proportionality is L, the inductance of the inductor.

Integrating both sides of Eq. 7.5-1, we obtain

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau \tag{7.5-2}$$



Courtesy of Vishay Intertechnology, Inc.

FIGURE 7.5-2 Elements with inductances arranged in various forms of coils.

This equation says that the inductor current $i(t)$ can be found by integrating the inductor voltage from time $-\infty$ until time t. To do so requires that we know the value of the inductor voltage from time $\tau = -\infty$ until time $\tau = t$. Often, we don't know the value of the voltage all the way back to $\tau = -\infty$. Instead, we break the integral up into two parts:

$$i(t) = \frac{1}{L}\int_{-\infty}^{t_0} v(\tau)d\tau + \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau \tag{7.5-3}$$

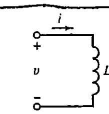


FIGURE 7.5-3 Circuit symbol for an inductor.

This equation says that the inductor current $i(t)$ can be found by integrating the inductor voltage from some convenient time $\tau = t_0$ until time $\tau = t$, provided that we also know the inductor current at time t_0. Now we are required to know only the inductor voltage from time $\tau = t_0$ until time $\tau = t$. The time t_0 is called the **initial time**, and the inductor current $i(t_0)$ is called the **initial condition**. Frequently, it is convenient to select $t_0 = 0$ as the initial time.

Equations 7.5-1 and 7.5-3 describe the current–voltage relationship of an inductor. The current and voltage in these equations adhere to the passive convention. The circuit symbol for an inductor is shown in Figure 7.5-3. The inductor current and voltage in Figure 7.5-3 adhere to the passive sign convention and are related by Eqs. 7.5-1 and 7.5-3.

Consider the voltage of an inductor when the current changes at $t = 0$ from zero to a constantly increasing current and eventually levels off as shown in Figure 7.5-4. Let us determine the voltage of the inductor. We may describe the current (in amperes) by

$$i(t) = \begin{cases} 0 & t \le 0 \\ \frac{10t}{t_1} & 0 \le t \le t_1 \\ 10 & t \ge t_1 \end{cases}$$

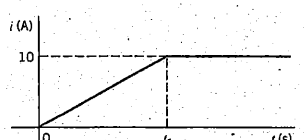


FIGURE 7.5-4 A current waveform. The current is in amperes.

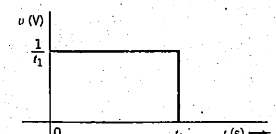


FIGURE 7.5-5 Voltage response for the current waveform of Figure 7.5-4 when $L = 0.1$ H.

Let us consider a 0.1-H inductor and find the voltage waveform. Because $v = L(di/dt)$, we have (in volts)

$$v(t) = \begin{cases} 0 & t < 0 \\ \dfrac{1}{t_1} & 0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$

The resulting voltage pulse waveform is shown in Figure 7.5-5. Note that as t_1 decreases, the magnitude of the voltage increases. Clearly, we cannot let $t_1 = 0$ because the voltage required would then become infinite, and we would require infinite power at the terminals of the inductor. Thus, instantaneous changes in the current through an inductor are not possible.

The current in an inductance cannot change instantaneously.

$\frac{1}{2} L I^2$

An ideal inductor is a coil wound with resistanceless wire. Practical inductors include the actual resistance of the copper wire used in the coil. For this reason, practical inductors are far from ideal elements and are typically modeled by an ideal inductance in series with a small resistance.

EXAMPLE 7.5-1 Inductor Current and Voltage

Find the voltage across an inductor, $L = 0.1$ H, when the current in the inductor is

$$i(t) = 20te^{-2t} \text{ A}$$

for $t > 0$ and $i(0) = 0$.

Solution

The voltage for $t < 0$ is

$$v(t) = L\frac{di}{dt} = (0.1)\frac{d}{dt}\left(20te^{-2t}\right) = 2\left(-2te^{-2t} + e^{-2t}\right) = 2e^{-2t}(1 - 2t) \text{ V}$$

The voltage is equal to 2 V when $t = 0$, as shown in Figure 7.5-6*b*. The current waveform is shown in Figure 7.5-6*a*.

7.6 *Energy Storage in an Inductor*

The power in an inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i \tag{7.6-1}$$

The energy stored in the inductor is stored in its magnetic field. The energy stored in the inductor during the interval t_0 to t is given by

$$w = \int_{t_0}^{t} p\, d\tau = L\int_{i(t_0)}^{i(t)} i\, di$$

Integrating the current between $i(t_0)$ and $i(t)$, we obtain

$$w = \frac{L}{2}\left[i^2(t)\right]_{i(t_0)}^{i(t)} = \frac{L}{2}i^2(t) - \frac{L}{2}i^2(t_0) \tag{7.6-2}$$

Usually, we select $t_0 = -\infty$ for the inductor and then the current $i(-\infty) = 0$. Then we have

$$\boxed{w = \frac{1}{2}Li^2} \tag{7.6-3}$$

Note that $w(t) \geq 0$ for all $i(t)$, so the inductor is a passive element. The inductor does not generate or dissipate energy but only stores energy. It is important to note that inductors and capacitors are fundamentally different from other devices considered in earlier chapters in that they have memory.

EXAMPLE 7.6-1 Inductor Voltage and Current

Find the current in an inductor, $L = 0.1$ H, when the voltage across the inductor is

$$v = 10te^{-5t} \text{ V}$$

Assume that the current is zero for $t \leq 0$.

Solution

The voltage as a function of time is shown in Figure 7.6-1*a*. Note that the voltage reaches a maximum at $t = 0.2$ s. The current is

$$i = \frac{1}{L}\int_0^t v\, d\tau + i(t_0)$$

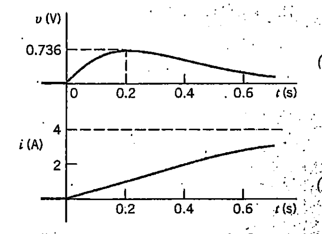


FIGURE 7.6-1 Voltage and current for Example 7.6-1.

Because the voltage is zero for $t < 0$, the current in the inductor at $t = 0$ is $i(0) = 0$. Then we have

$$i = 10\int_0^t 10\,\tau e^{-5\tau}\, d\tau = 100\left[\frac{-e^{-5\tau}}{25}(1 + 5\tau)\right]_0^t = 4\left(1 - e^{-5t}(1 + 5t)\right) \text{ A}$$

The current as a function of time is shown in Figure 7.6-1*b*.

7.7 Series and Parallel Inductors

A series and parallel connection of inductors can be reduced to an equivalent simple inductor. Consider a series connection of N inductors as shown in Figure 7.7-1. The voltage across the series connection is

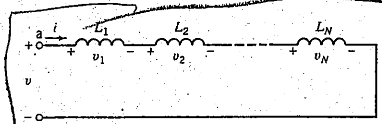


FIGURE 7.7-1 Series of N inductors.

$$\begin{aligned} v &= v_1 + v_2 + \cdots + v_N \\ &= L_1\frac{di}{dt} + L_2\frac{di}{dt} + \cdots + L_N\frac{di}{dt} \\ &= \left(\sum_{n=1}^{N} L_N\right)\frac{di}{dt} \end{aligned}$$

Because the equivalent series inductor L_s, as shown in Figure 7.7-2, is represented by

$$v = L_s\frac{di}{dt}$$

we require that

$$\boxed{L_s = \sum_{n=1}^{N} L_n} \quad (7.7\text{-}1)$$

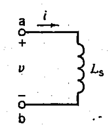


FIGURE 7.7-2 Equivalent inductor L_s for N series inductors.

Thus, an equivalent inductor for a series of inductors is the sum of the N inductors.

Now, consider the set of N inductors in parallel, as shown in Figure 7.7-3. The current i is equal to the sum of the currents in the N inductors:

$$i = \sum_{n=1}^{N} i_n$$

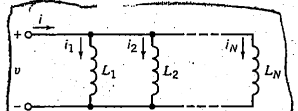


FIGURE 7.7-3 Connection of N parallel inductors.

However, because

$$i_n = \frac{1}{L_n}\int_{t_0}^{t} v\,d\tau + i_n(t_0)$$

we may obtain the expression

$$i = \left(\sum_{n=1}^{N}\frac{1}{L_n}\right)\int_{t_0}^{t} v\,d\tau + \sum_{n=1}^{N} i_n(t_0) \quad (7.7\text{-}2)$$

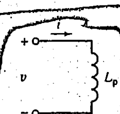


FIGURE 7.7-4 Equivalent inductor L_p for the connection of N parallel inductors.

The equivalent inductor L_p, as shown in Figure 7.7-4, is represented by the equation

$$i = \frac{1}{L_p}\int_{t_0}^{t} v\,d\tau + i(t_0) \quad (7.7\text{-}3)$$

When Eqs. 7.7-2 and 7.7-3 are set equal to each other, we have

$$\boxed{\frac{1}{L_p} = \sum_{n=1}^{N}\frac{1}{L_n}} \quad (7.7\text{-}4)$$

2. The circuit includes one or more switches that open or close at time t_0. We denote the time immediately before the switch opens or closes as t_0^- and the time immediately after the switch opens or closes as t_0^+. Often, we will assume that $t_0 = 0$.
3. The circuit includes at least one capacitor or inductor.
4. We will assume that the switches in a circuit have been in position for a long time at $t = t_0$, the switching time. We will say that such a circuit is at *steady state* immediately before the time of switching. A circuit that contains only constant sources and is at steady state is called a *dc circuit*. All the element currents and voltages in a dc circuit are constant functions of time.

We are particularly interested in the current and voltage of energy storage elements after the switch opens or closes. (Recall from Section 2.9 that open switches act like open circuits and closed switches act like short circuits.) In Table 7.8-1, we summarize the important characteristics of the behavior of an inductor and a capacitor. In particular, notice that neither a capacitor voltage nor an inductor current can change instantaneously. (Recall from Sections 7.2 and 7.5 that such changes would require infinite power, something that is not physically possible.) However, instantaneous changes to an inductor voltage or a capacitor current are quite possible.

Suppose that a dc circuit contains an inductor. The inductor current, like every other voltage and current in the dc circuit, will be a constant function of time. The inductor voltage is proportional to the derivative of the inductor current, $v = L(di/dt)$, so the inductor voltage is zero. Consequently, the inductor acts like a short circuit.

An **inductor in a dc circuit** behaves as a **short circuit**.

Similarly, the voltage of a capacitor in a dc circuit will be a constant function of time. The capacitor current is proportional to the derivative of the capacitor voltage $i = C\,(dv/dt)$, so the capacitor current is zero. Consequently, the capacitor acts like a open circuit.

IMPORTANT

Table 7.8-1 Characteristics of Energy Storage Elements

VARIABLE	INDUCTORS	CAPACITORS
Passive sign convention	i, L, $+\ v\ -$	i, C, $+\ v\ -$
Voltage	$v = L\dfrac{di}{dt}$	$v = \dfrac{1}{C}\int_0^t i\,d\tau + v(0)$
Current	$i = \dfrac{1}{L}\int_0^t v\,d\tau + i(0)$	$i = C\dfrac{dv}{dt}$
Power	$i = Li\dfrac{di}{dt}$	$p = Cv\dfrac{dv}{dt}$
Energy	$w = \dfrac{1}{2}Li^2$	$w = \dfrac{1}{2}Cv^2$
An instantaneous change is not permitted for the element's	Current	Voltage
Will permit an instantaneous change in the element's	Voltage	Current
This element acts as a (see note below)	Short circuit to a constant current into its terminals	Open circuit to a constant voltage across its terminals

Note: Assumes that the element is in a circuit with steady-state condition.

A **capacitor** in a **dc circuit** behaves as an **open circuit**.

Our plan to analyze switched circuits has two steps:

1. Analyze the dc circuit that exists before time t_0 to determine the capacitor voltages and inductor currents. In doing this analysis, we will take advantage of the fact that capacitors behave as open circuits and inductors behave as short circuits when they are in dc circuits.
2. Recognize that capacitor voltages and inductor currents cannot change instantaneously, so the capacitor voltages and inductor currents at time t_0^+ have the same values that they had at time t_0^-.

The following examples illustrate this plan.

EXAMPLE 7.8-1 Initial Conditions in a Switched Circuit

Consider the circuit Figure 7.8-1. Prior to $t = 0$, the switch has been closed for a long time. Determine the values of the capacitor voltage and inductor current immediately after the switch opens at time $t = 0$.

Solution

1. To find $v_c(0^-)$ and $i_L(0^-)$, we consider the circuit before the switch opens, that is for $t < 0$. The circuit input, the voltage source voltage, is constant. Also, before the switch opens, the circuit is at steady state. Because the circuit is a dc circuit, the capacitor will act like an open circuit, and the inductor will act like a short circuit. In Figure 7.8-2, we replace the capacitor by an open circuit having voltage $v_c(0^-)$ and the inductor by a short circuit having current $i_L(0^-)$. First, we notice that

$$i_L(0^-) = \frac{10}{5} = 2\text{ A}$$

Next, using the voltage divider principle, we see that

$$v_c(0^-) = \left(\frac{3}{5}\right)10 = 6\text{ V}$$

2. The capacitor voltage and inductor current cannot change instantaneously, so

$$v_c(0^+) = v_c(0^-) = 6\text{ V}$$

and

$$i_L(0^+) = i_L(0^-) = 2\text{ A}$$

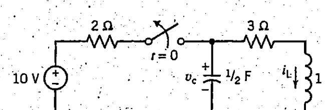


FIGURE 7.8-1 Circuit with an inductor and a capacitor. The switch is closed for a long time prior to opening at $t = 0$.

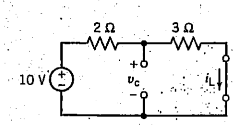


FIGURE 7.8-2 Circuit of Figure 7.8-1 for $t < 0$.

EXAMPLE 7.8-2 Initial Conditions in a Switched Circuit

Find $i_L(0^+)$, $v_c(0^+)$, $dv_c(0^+)/dt$, and $di_L(0^+)/dt$ for the circuit of Figure 7.8-3. We will use $dv_c(0^+)/dt$ to denote $dv_c(t)/dt|_{t=0^+}$.

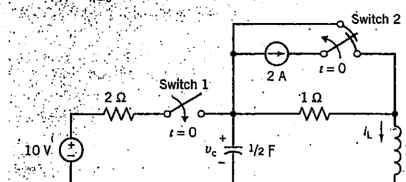


FIGURE 7.8-3 Circuit for Example 7.8-2. Switch 1 closes at $t = 0$ and switch 2 opens at $t = 0$.

Assume that switch 1 has been open and switch 2 has been closed for a long time and steady-state conditions prevail at $t = 0^-$.

Solution

First, we redraw the circuit for $t = 0^-$ by replacing the inductor with a short circuit and the capacitor with an open circuit, as shown in Figure 7.8-4. Then we note that

$$i_L(0^-) = 0$$

and

$$v_c(0^-) = -2\text{ V}$$

Therefore, we have

$$i_L(0^+) = i_L(0^-) = 0$$

and

$$v_c(0^+) = v_c(0^-) = -2\text{ V}$$

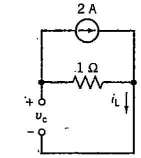


FIGURE 7.8-4 Circuit of Figure 7.8-3 at $t = 0^-$.

To find $dv_c(0^+)/dt$ and $di_L(0^+)/dt$, we throw the switch at $t = 0$ and redraw the circuit of Figure 7.8-3, as shown in Figure 7.8-5. (We did not draw the current source because its switch is open.)

Because we wish to find $dv_c(0^+)/dt$, we recall that

$$i_c = C\frac{dv_c}{dt}$$

so

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

Similarly, because for the inductor

$$v_L = L\frac{di_L}{dt}$$

we may obtain $di_L(0^+)/dt$ as

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

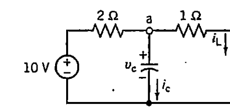


FIGURE 7.8-5 Circuit of Figure 7.8-3 at $t = 0^+$ with the switch closed and the current source disconnected.

Using KVL for the right-hand mesh of Figure 7.8-5, we obtain

$$v_L - v_c + 1i_L = 0$$

Therefore, at $t = 0^+$,

$$v_L(0^+) = v_c(0^+) - i_L(0^+) = -2 - 0 = -2\text{ V}$$

Hence, we obtain

$$\frac{di_L(0^+)}{dt} = -2 \text{ A/s}$$

Similarly, to find i_c, we write KCL at node a to obtain

$$i_c + i_L + \frac{v_c - 10}{2} = 0$$

Consequently, at $t = 0^+$,

$$i_c(0^+) = \frac{10 - v_c(0^+)}{2} - i_L(0^+) = 6 - 0 = 6 \text{ A}$$

Accordingly,

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{6}{1/2} = 12 \text{ V/s}$$

Thus, we found that at the switching time $t = 0$, the current in the inductor and the voltage of the capacitor remained constant. However, the inductor voltage did change instantaneously from $v_L(0^-) = 0$ to $v_L(0^+) = -2$ V, and we determined that $di_L(0^+)/dt = -2$ A/s. Also, the capacitor current changed instantaneously from $i_c(0^-) = 0$ to $i_c(0^+) = 6$ A, and we found that $dv_c(0^+)/dt = 12$ V/s.

7.9 Operational Amplifier Circuits and Linear Differential Equations

This section describes a procedure for designing operational amplifier circuits that implement linear differential equations such as

$$2\frac{d^3}{dt^3}y(t) + 5\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = 6x(t) \tag{7.9-1}$$

The solution of this equation is a function $y(t)$ that depends both on the function $x(t)$ and on a set of initial conditions. It is convenient to use the initial conditions:

$$\frac{d^2}{dt^2}y(t) = 0, \quad \frac{d}{dt}y(t) = 0, \quad \text{and} \quad y(t) = 0 \tag{7.9-2}$$

Having specified these initial conditions, we expect a unique function $y(t)$ to correspond to any given function $x(t)$. Consequently, we will treat $x(t)$ as the input to the differential equation and $y(t)$ as the output.

Section 6.6 introduced the notion of diagramming operations as blocks and equations as block diagrams. Section 6.6 also introduced blocks to represent addition and multiplication by a constant. Figure 7.9-1 illustrates two additional blocks, representing integration and differentiation.

Suppose that we were somehow to obtain $\frac{d^3}{dt^3}y(t)$. We could then integrate three times to obtain $\frac{d^2}{dt^2}y(t)$, $\frac{d}{dt}y(t)$, and $y(t)$, as illustrated in Figure 7.9-2.

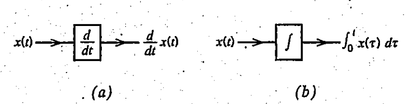


FIGURE 7.9-1 Block diagram representations of (a) differentiation and (b) integration.

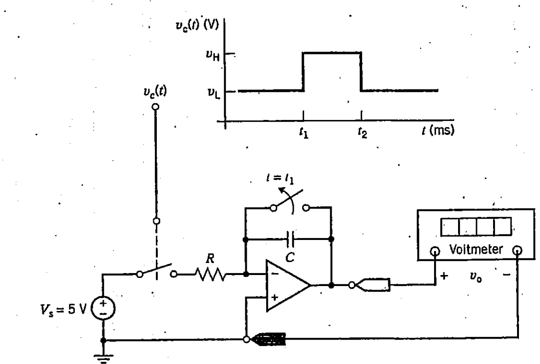


FIGURE 7.12-4 Using an operational amplifier integrator to measure an interval of time.

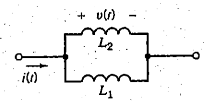
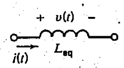
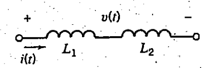
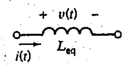
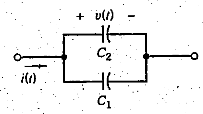
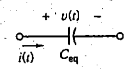
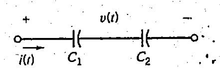
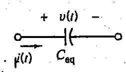
7.13 SUMMARY

- Table 7.13-1 summarizes the element equations for capacitors and inductors. (Notice that the voltage and current referred to in these equations adhere to the passive convention.) Unlike the circuit elements we encountered in previous chapters, the element equations for capacitors and inductors involve derivatives and integrals.
- Circuits that contain capacitors and/or inductors are able to store energy. The energy stored in the electric field of a capacitor is equal to $\frac{1}{2}Cv^2(t)$, where $v(t)$ is the voltage across the capacitor. The energy stored in the magnetic field of a inductor is equal to $\frac{1}{2}Li^2(t)$, where $i(t)$ is the current in the inductor.
- Circuits that contain capacitors and/or inductors have memory. The voltages and currents in that circuit at a particular time depend not only on other voltages and currents at that same instant of time but also on previous values of those currents and voltages. For example, the voltage across a capacitor at time t_1 depends on the voltage across that capacitor at an earlier time t_0 and on the value of the capacitor current between t_0 and t_1.
- A set of series or parallel capacitors can be reduced to an equivalent capacitor. A set of series or parallel inductors can readily be reduced to an equivalent inductor. Table 7.13-2 summarizes the equations required to do so.

Table 7.13-1 Element Equations for Capacitors and Inductors

CAPACITOR	INDUCTOR
$+\ v(t)\ -$, $i(t)$, C	$+\ v(t)\ -$, $i(t)$, L
$i(t) = C\frac{d}{dt}v(t)$	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v(t) = L\frac{d}{dt}i(t)$

Table 7.13-2 Parallel and Series Capacitors and Inductors

SERIES OR PARALLEL CIRCUIT	EQUIVALENT CIRCUIT	EQUATION
		$L_{eq} = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}$
		$L_{eq} = L_1 + L_2$
		$C_{eq} = C_1 + C_2$
		$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$

- In the absence of unbounded currents, the voltage across a capacitor cannot change instantaneously. Similarly, in the absence of unbounded voltages, the current in an inductor cannot change instantaneously. In contrast, the current in a capacitor and voltage across an inductor are both able to change instantaneously.
- We sometimes consider circuits that contain capacitors and inductors and have only constant inputs. (The voltages of the independent voltage sources and currents of the independent current sources are all constant.) When such a circuit is at steady state, all the currents and voltages in that circuit will be constant. In particular, the voltage across any capacitor will be constant. The current in that capacitor will be zero due to the derivative in the equation for the capacitor current. Similarly, the current through any inductor will be constant and the voltage across any inductor will be zero. Consequently, the capacitors will act like open circuits and the inductors will act like short circuits. Notice that this situation occurs only when all of the inputs to the circuit are constant.
- An op amp and a capacitor can be used to make circuits that perform the mathematical operations of integration and differentiation. Appropriately, these important circuits are called the integrator and the differentiator.
- The element voltages and currents in a circuit containing capacitors and inductors can be complicated functions of time. MATLAB is useful for plotting these functions.

PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

Section 7.2 Capacitors

P 7.2-1 ⊕ A 15-μF capacitor has a voltage of 5 V across it at $t = 0$. If a constant current of 25 mA flows through the capacitor, how long will it take for the capacitor to charge up to 150 μC?

Answer: $t = 3$ ms

P 7.2-2 The voltage $v(t)$ across a capacitor and current $i(t)$ in that capacitor adhere to the passive convention. Determine the current $i(t)$ when the capacitance is $C = 0.125$ F, and the voltage is $v(t) = 12\cos(2t + 30°)$ V.

Hint:

$$\frac{d}{dt}A\cos(\omega t+\theta) = -A\sin(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta) = -A\omega\sin(\omega t+\theta) = A\omega\cos\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$$

Answer: $i(t) = 3\cos(2t + 120°)$ A

P 7.2-3 ⊕ The voltage $v(t)$ across a capacitor and current $i(t)$ in that capacitor adhere to the passive convention. Determine the capacitance when the voltage is $v(t) = 12\cos(500t - 45°)$ V and the current is $i(t) = 3\cos(500t + 45°)$ mA.

Answer: $C = 0.5\ \mu$F

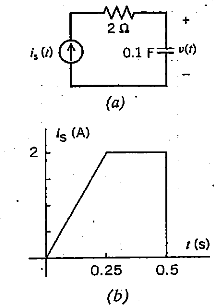


Figure P 7.2-10

P 7.2-11 Determine $i(t)$ for $t \geq 0$ for the circuit of Figure P 7.2-11*a* when $v_s(t)$ is the voltage shown in Figure P 7.2-11*b*.

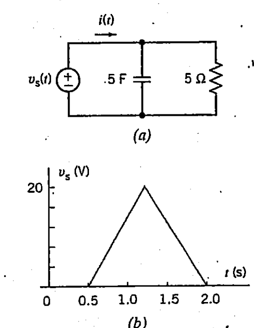


Figure P 7.2-11

HW #13

P 7.2-12 The capacitor voltage in the circuit shown in Figure P 7.2-12 is given by

$$v(t) = 12 - 10e^{-2t}\text{ V} \quad \text{for } t \geq 0$$

Determine $i(t)$ for $t > 0$.

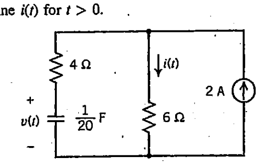


Figure P 7.2-12

HW#13

P 7.2-13 ⊕ The capacitor voltage in the circuit shown in Figure P 7.2-13 is given by

$$v(t) = 2.4 + 5.6e^{-5t}\text{ V} \quad \text{for } t \geq 0$$

Determine $i(t)$ for $t > 0$.

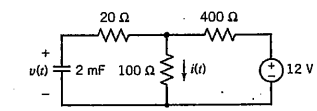


Figure P 7.2-13

P 7.2-14 ⊕ The capacitor voltage in the circuit shown in Figure P 7.2-14 is given by

$$v(t) = 10 - 8e^{-5t}\text{ V} \quad \text{for } t \geq 0$$

Determine $i(t)$ for $t > 0$.

HW#13

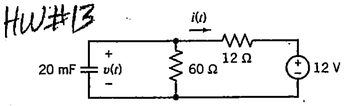


Figure P 7.2-14

P 7.2-15 ⊕ Determine the voltage $v(t)$ for $t > 0$ for the circuit of Figure P 7.2-15*b* when $i_s(t)$ is the current shown in Figure P 7.2-15*a*. The capacitor voltage at time $t = 0$ is $v(0) = -12$ V.

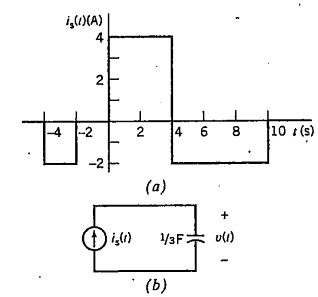


Figure P 7.2-15 (*a*) The voltage source voltage. (*b*) The circuit.

P 7.2-16 ⊕ The input to the circuit shown in Figure P 7.2-16 is the current

$$i(t) = 3.75e^{-1.2t}\text{ A} \quad \text{for } t > 0$$

The output is the capacitor voltage

$$v(t) = 4 - 1.25e^{-1.2t}\text{ V} \quad \text{for } t > 0$$

Find the value of the capacitance C.

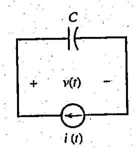


Figure P 7.2-16

HW 13

P 7.2-17 The input to the circuit shown in Figure P 7.2-17 is the current

$$i(t) = 3e^{-25t} \text{ A} \quad \text{for } t > 0$$

The initial capacitor voltage is $v_C(0) = -2$ V. Determine the current source voltage $v(t)$ for $t > 0$.

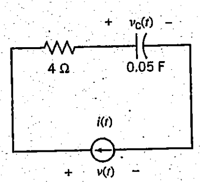


Figure P 7.2-17

P 7.2-18 The input to the circuit shown in Figure P 7.2-18 is the current

$$i(t) = 3e^{-25t} \text{ A} \quad \text{for } t > 0$$

The output is the voltage

$$v(t) = 9.6e^{-25t} + 0.4 \text{ V} \quad \text{for } t > 0$$

The initial capacitor voltage is $v_C(0) = -2$ V. Determine the values of the capacitance C and resistance R.

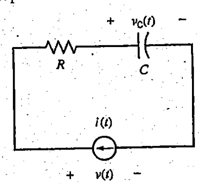


Figure P 7.2-18

P 7.2-19 The input to the circuit shown in Figure P 7.2-19 is the voltage

$$v(t) = 8 + 5e^{-10t} \text{ V} \quad \text{for } t > 0$$

Determine the current $i(t)$ for $t > 0$.

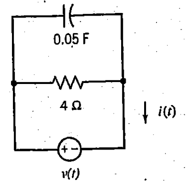


Figure P 7.2-19

P 7.2-20 The input to the circuit shown in Figure P 7.2-20 is the voltage:

$$v(t) = 3 + 4e^{-2t} \text{ A} \quad \text{for } t > 0$$

The output is the current $i(t) = 0.3 - 1.6e^{-2t}$ V for $t > 0$. Determine the values of the resistance and capacitance.

Answers: $R = 10\,\Omega$ and $C = 0.25$ F

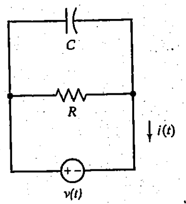


Figure P 7.2-20

P 7.2-21 Consider the capacitor shown in Figure P 7.2-21. The current and voltage are given by

$$i(t) = \begin{cases} 0.5 & 0 < t < 0.5 \\ 2 & 0.5 < t < 1.5 \\ 0 & t > 1.5 \end{cases}$$

and

$$v(t) = \begin{cases} 2t + 8.6 & 0 \le t \le 0.5 \\ at + b & 0.5 \le t \le 1.5 \\ c & t \ge 1.5 \end{cases}$$

where a, b, and c are real constants. (The current is given in amps, the voltage in volts, and the time in seconds.) Determine the values of a, b, and c.

Answers: $a = 8$ V/s, $b = 5.6$ V, and $c = 17.6$ V

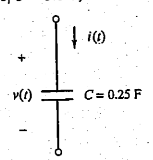


Figure P 7.2-21

P 7.2-22 At time $t = 0$, the voltage across the capacitor shown in Figure P 7.2-22 is $v(0) = -20$ V. Determine the values of the capacitor voltage at times 1 ms, 3 ms, and 7 ms.

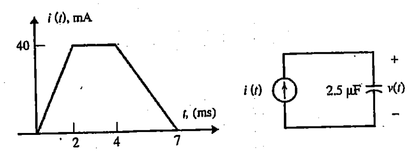


Figure P 7.2-22

Section 7.3 Energy Storage in a Capacitor

P 7.3-1 The current i through a capacitor is shown in Figure P 7.3-1. When $v(0) = 0$ and $C = 0.5$ F, determine and plot $v(t)$, $p(t)$, and $w(t)$ for $0 \text{ s} < t < 6$ s.

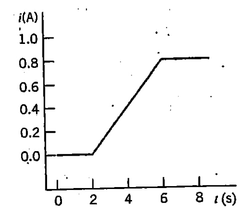


Figure P 7.3-1

P 7.3-2 In a pulse power circuit, the voltage of a 10-μF capacitor is zero for $t < 0$ and

$$v = 5\left(1 - e^{-4000t}\right) \text{ V} \quad t \ge 0$$

Determine the capacitor current and the energy stored in the capacitor at $t = 0$ ms and $t = 10$ ms.

P 7.3-3 If $v_c(t)$ is given by the waveform shown in Figure P 7.3-3, sketch the capacitor current for $-1 \text{ s} < t < 2$ s. Sketch the power and the energy for the capacitor over the same time interval when $C = 1$ mF.

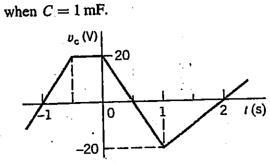


Figure P 7.3-3

P 7.3-4 The current through a 2-μF capacitor is $50\cos(10t + \pi/6)$ μA for all time. The average voltage across the capacitor is zero. What is the maximum value of the energy stored in the capacitor? What is the first nonnegative value of t at which the maximum energy is stored?

P 7.3-5 A capacitor is used in the electronic flash unit of a camera. A small battery with a constant voltage of 6 V is used to charge a capacitor with a constant current of 10 μA. How long does it take to charge the capacitor when $C = 10$ μF? What is the stored energy?

P 7.3-6 The initial capacitor voltage of the circuit shown in Figure P 7.3-6 is $v_c(0^-) = 3$ V. Determine (a) the voltage $v(t)$ and (b) the energy stored in the capacitor at $t = 0.2$ s and $t = 0.8$ s when

$$i(t) = \begin{cases} 3e^{5t} \text{ A} & 0 < t < 1 \\ 0 & t \ge 1 \text{ s} \end{cases}$$

Answers:
(a) $18e^{5t}$ V, $0 \le t < 1$
(b) $w(0.2) = 6.65$ J and $w(0.8) = 2.68$ kJ

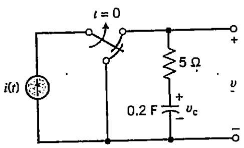


Figure P 7.3-6

HW #13

P 7.3-7 (a) Determine the energy stored in the capacitor in the circuit shown in Figure P 7.3-7 when the switch is closed and the circuit is at steady state. (b) Determine the energy stored in the capacitor when the switch is open and the circuit is at steady state.

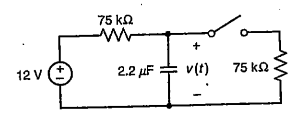


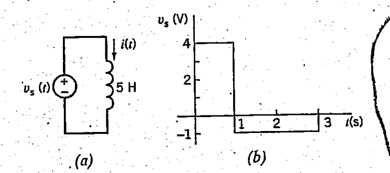
Figure P 7.3-7

Section 7.4 Series and Parallel Capacitors

P 7.4-1 Find the current $i(t)$ for the circuit of Figure P 7.4-1.

Answer: $i(t) = -1.2 \sin 100t$ mA

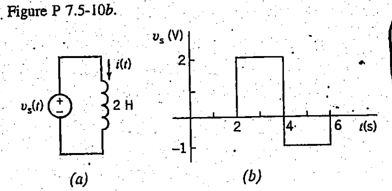
HW #14



(a) (b)

Figure P 7.5-9

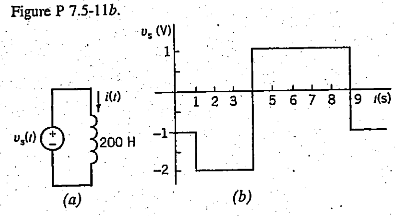
P 7.5-10 Determine $i(t)$ for $t \geq 0$ for the current of Figure P 7.5-10*a* when $i(0) = 1$ A and $v_s(t)$ is the voltage shown in Figure P 7.5-10*b*.



(a) (b)

Figure P 7.5-10

P 7.5-11 ⊕ Determine $i(t)$ for $t \geq 0$ for the circuit of Figure P 7.5-11*a* when $i(0) = 25$ mA and $v_s(t)$ is the voltage shown in Figure P 7.5-11*b*.



(a) (b)

Figure P 7.5-11

P 7.5-12 ⊕ The inductor current in the circuit shown in Figure P 7.5-12 is given by

$$i(t) = 6 + 4e^{-8t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

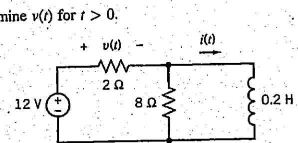


Figure P 7.5-12

P 7.5-13 ⊕ The inductor current in the circuit shown in Figure P 7.5-13 is given by

$$i(t) = 5 - 3e^{-4t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

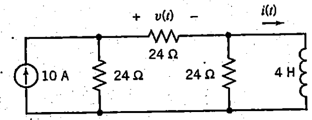


Figure P 7.5-13

P 7.5-14 ⊕ The inductor current in the circuit shown in Figure P 7.5-14 is given by

$$i(t) = 3 + 2e^{-3t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

HW #14

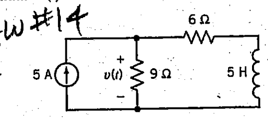
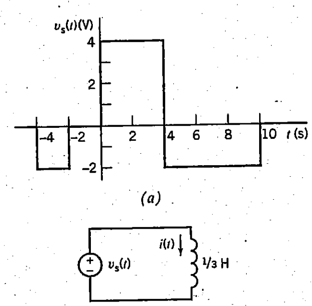


Figure P 7.5-14

P 7.5-15 Determine the current $i(t)$ for $t > 0$ for the circuit of Figure P 7.5-15*b* when $v_s(t)$ is the voltage shown in Figure P 7.5-15*a*. The inductor current at time $t = 0$ is $i(0) = -12$ A.



(a)

(b)

Figure P 7.5-15 (*a*) The voltage source voltage. (*b*) The circuit.

P 7.5-16 ⊕ The input to the circuit shown in Figure P 7.5-16 is the voltage

$$v(t) = 15e^{-4t} \text{ V} \quad \text{for } t > 0$$

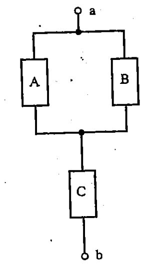


Figure P 7.7-12

Section 7.8 Initial Conditions of Switched Circuits

P 7.8-1 ⊕ The switch in Figure P 7.8-1 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answers: $v_c(0^+) = 12$ V, $i_L(0^+) = 0$, $v_c(\infty) = 4$ V, and $i_L(\infty) = 1$ mA

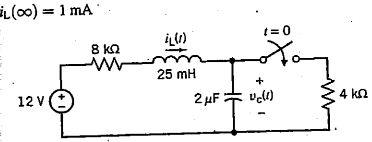


Figure P 7.8-1

P 7.8-2 The switch in Figure P 7.8-2 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 6$ V, $i_L(0^+) = 1$ mA, $v_c(\infty) = 3$ V, and $i_L(\infty) = 1.5$ mA

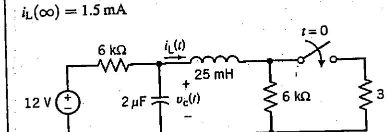


Figure P 7.8-2

HW #14

P 7.8-3 ⊕ The switch in Figure P 7.8-3 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

HW #14

Answers: $v_c(0^+) = 0$ V, $i_L(0^+) = 0$, $v_c(\infty) = 8$ V, and $i_L(\infty) = 0.5$ mA

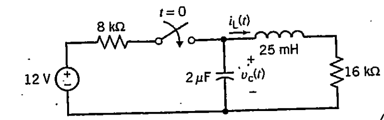


Figure P 7.8-3

P 7.8-4 The switch in the circuit shown in Figure P 7.8-4 has been closed for a long time before it opens at time $t=0$. Determine the values of $v_R(0\text{-})$ and $v_L(0\text{-})$, the voltage across the 4-Ω resistor and the inductor immediately before the switch opens, and the values of $v_R(0\text{+})$ and $v_L(0\text{+})$, the voltage across the 4-Ω resistor and the inductor immediately after the switch opens.

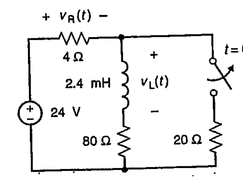


Figure P 7.8-4

P 7.8-5 The switch in the circuit shown in Figure P 7.8-5 has been open for a long time before it closes at time $t=0$. Determine the values of $i_R(0\text{-})$ and $i_C(0\text{-})$, the current in one of the 20-Ω resistors and in the capacitor immediately before the switch closes, and the values of $i_R(0\text{+})$ and $i_C(0\text{+})$, the current in that 20-Ω resistor and in the capacitor immediately after the switch closes.

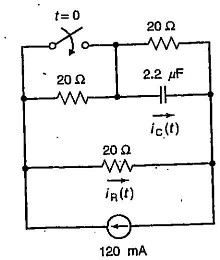


Figure P 7.8-5

(b) Determine the value of $v_R(t)$ immediately after the switch closes.

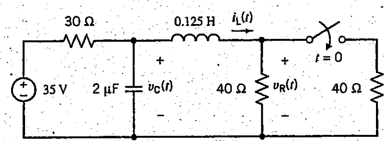


Figure P 7.8-12

HW 14

P 7.8-13 The circuit shown in Figure P 7.8-13 has reached steady state before the switch opens at time $t = 0$. Determine the values of $i_L(t)$, $v_C(t)$, and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

Answers: $i_L(0-) = 0.4$ A, $v_C(0-) = 16$ V, $v_R(0-) = 0$ V, and $v_R(0+) = -12$ V

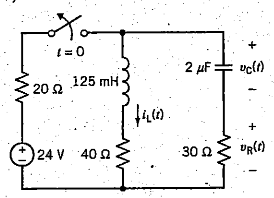


Figure P 7.8-13

Section 7.9 Operational Amplifier Circuits and Linear Differential Equations

P 7.9-1 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{1}{2}\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + y(t) = \frac{5}{2}x(t)$$

P 7.9-2 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{1}{2}\frac{d^2}{dt^2}y(t) + y(t) = -\frac{5}{2}x(t)$$

P 7.9-3 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$2\frac{d^3}{dt^3}y(t) + 16\frac{d^2}{dt^2}y(t) + 8\frac{d}{dt}y(t) + 10y(t) = -4x(t)$$

P 7.9-4 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{d^3}{dt^3}y(t) + 16\frac{d^2}{dt^2}y(t) + 8\frac{d}{dt}y(t) + 10y(t) = 4x(t)$$

Section 7.11 How Can We Check . . . ?

P 7.11-1 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t) = \begin{cases} 0.025 & t < 1 \\ -\frac{t}{25} + 0.065 & 1 < t < 3 \\ \frac{t}{50} - 0.115 & 3 < t < 9 \\ 0.065 & t < 9 \end{cases}$$

and

$$v(t) = \begin{cases} 0 & t < 1 \\ -4 & 1 < t < 3 \\ 2 & 3 < t < 9 \\ 0 & t > 9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Verify that the inductor current does not change instantaneously.

P 7.11-2 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t) = \begin{cases} -\frac{t}{200} + 0.025 & t < 1 \\ -\frac{t}{100} + 0.03 & 1 < t < 4 \\ \frac{t}{100} - 0.03 & 4 < t < 9 \\ 0.015 & t < 9 \end{cases}$$

and

$$v(t) = \begin{cases} -1 & t < 1 \\ -2 & 1 < t < 4 \\ 1 & 4 < t < 9 \\ 0 & t > 9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Is this homework solution correct? Justify your answer.