Hw #1

2.5 :

(a) : $v(t) = 227 \times \sqrt{2} \cos(\omega t + 30^\circ) = 391.7 \cos(\omega t + 30^\circ)$ V

(b) : $\tilde{I} = \frac{\tilde{V}}{20} = 13.89 \angle 30^\circ$ A

$i(t) = 19.58 \cos(\omega t + 30^\circ)$ A

(c) : $Z = j\omega L = j(2\pi \times 60)(10 \times 10^{-3}) = 3.771 \angle 90^\circ$ Ω

$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{227 \angle 30^\circ}{3.771 \angle 90^\circ} = 73.46 \angle -60^\circ$ A

$i(t) = 73.46 \times \sqrt{2} \cos(\omega t - 60^\circ) = 103.9 \cos(\omega t - 60^\circ)$ A

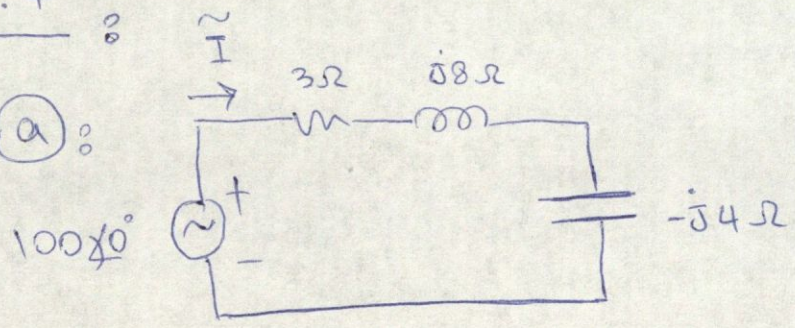
(d) $Z = -j25$ Ω

$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{277 \angle 30^\circ}{25 \angle -90^\circ} = 11.08 \angle 120^\circ$ A

$i(t) = 11.08 \times \sqrt{2} \cos(\omega t + 120^\circ) = 15.67 \cos(\omega t + 120^\circ)$ A

2.7 :

(a) :



(b) : $Z = 3 + j8 - j4 = 3 + 4j = 5 \angle 53.1^\circ\ \Omega$

(c) : $\tilde{I} = \dfrac{100 \angle 0^\circ}{5 \angle 53.1^\circ} = 20 \angle -53.1^\circ\ A$

The current lags by 53.1°.

$P.F = \cos(53.1^\circ) = 0.6$

2.11 :

(a): $P(t) = v(t) \cdot i(t) = 391.7 \times 19.58 \cos^2(\omega t + 30^\circ)$

$= 0.7669 \times 10^4 \times \left(\frac{1}{2}\right)\left[1 + \cos(2\omega t + 60^\circ)\right]$

$= 3.834 \times 10^3 + 3.834 \times 10^3 \cos(2\omega t + 60^\circ)$ Watt

$P = VI \cos(\theta_V - \theta_I) = 277 \times 13.85 \cos(0^\circ) = 3.836$ kW

$Q = VI \sin(\theta_V - \theta_I) = 0$ VAR, $\quad P.F = \cos(0^\circ) = 1$

(b): $P(t) = v(t) \cdot i(t) = 391.7 \times 103.9 \cos(\omega t + 30^\circ)\cos(\omega t - 60^\circ)$

$= 4.07 \times 10^4 \times \frac{1}{2} \times \left[\cos(90^\circ) + \cos(2\omega t - 30^\circ)\right]$

$= 2.035 \times 10^4 \cos(2\omega t - 30^\circ)$ W

$P = VI \cos(\theta_V - \theta_I) = 277 \times 73.46 \cos(30^\circ + 60^\circ) = 0$ W

$Q = VI \sin(\theta_V - \theta_I) = 277 \times 73.46 \sin(90^\circ) = 20.35$ KVAR

$P.F = \cos(90^\circ) = 0$ Lagging.

(c): $P(t) = V(t) \cdot i(t) = 391.7 \times 15.67 \cos(\omega t + 30^\circ) \cos(\omega t + 120^\circ)$

$= 6.138 \times 10^3 \times \frac{1}{2} [\cos(-90^\circ) + \cos(2\omega t + 150^\circ)]$

$= 3.069 \times 10^3 \cos(2\omega t + 150^\circ)$ W

$P = VI \cos(\theta_V - \theta_I) = 277 \times 11.08 \cos(30^\circ - 120^\circ) = 0$ watt

$Q = VI \sin(\theta_V - \theta_I) = 277 \times 11.08 \sin(-90^\circ) = -3.069$ KVAR

$P.F = \cos(\theta_V - \theta_I) = \cos(-90^\circ) = 0$ Leading.

2.12 :

(a): $P_R(t) = 678.8 \times 67.88 \cos^2(\omega t + 45^\circ)$

$= 4.608 \times 10^4 \times \frac{1}{2}\left[1 + \cos(2\omega t + 90^\circ)\right]$

$= 2.304 \times 10^4 + 2.304 \times 10^4 \cos(2\omega t + 90^\circ)$ ω

(b): $P(t) = \left[678.8 \cos(\omega t + 45^\circ)\right]\left[27.15 \cos(\omega t + 45^\circ + 90^\circ)\right]$

$= 1.843 \times 10^4 \cos(\omega t + 45^\circ) \cos(\omega t + 135^\circ)$

$= 1.843 \times 10^4 \times \frac{1}{2}\left[\cos(-90^\circ) + \cos(2\omega t + 180^\circ)\right]$

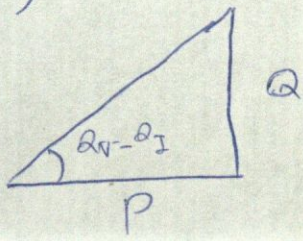
$= 9.215 \times 10^3 \cos(2\omega t + 180^\circ) = -9.215 \cos(2\omega t)$ ω

(c): $P = \frac{\tilde{V}_{rms}^2}{R} = \frac{\left(\frac{678.8}{\sqrt{2}}\right)^2}{10} = 2.304 \times 10^4$ ω

(d): $Q = \frac{V_{rms}^2}{X} = \frac{\left(\frac{678.8}{\sqrt{2}}\right)^2}{25} = 9.215 \times 10^3$ VAR

(e): $\theta_V - \theta_I = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{9.215 \times 10^3}{2.304 \times 10^4}\right) = 21.8^\circ$

P.F $= \cos(\theta_V - \theta_I) = 0.928$ Leading



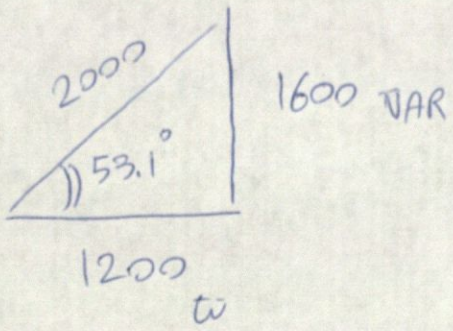
2.25

$$S_R = \tilde{V}_R \tilde{I}_R^* = R\tilde{I}\tilde{I}^* = R|I|^2 = 3 \times 20^2 = 1200 + j0$$

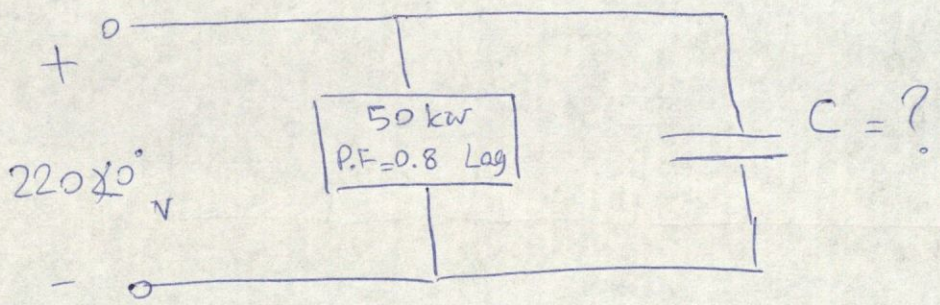
$$S_L = \tilde{V}_L \tilde{I}^* = z_L \tilde{I}\tilde{I}^* = jX_L|I|^2 = j8 \times 20^2 = 0 + j3200$$

$$S_C = \tilde{V}_C \tilde{I}^* = z_C \tilde{I}\tilde{I}^* = -jX_C|I|^2 = -j4 \times 20^2 = 0 - j1600$$

$$S_T = S_R + S_L + S_C = 2000 \angle 53.1^\circ$$



2.27 :



$P_{old} = 50\,kw$, $\cos^{-1} 0.8 = 36.87^\circ \Rightarrow Q_{old} = 36.87^\circ$

$$Q_{old} = V_{old} \times I_{old} \times \sin(Q_{old}) = \underbrace{V_{old} \times I_{old} \times \cos(Q_{old})}_{P_{old}} \times \frac{\sin(Q_{old})}{\cos(Q_{old})}$$

$$= P_{old} \times \tan(Q_{old}) = 37.5 \quad KVAR$$

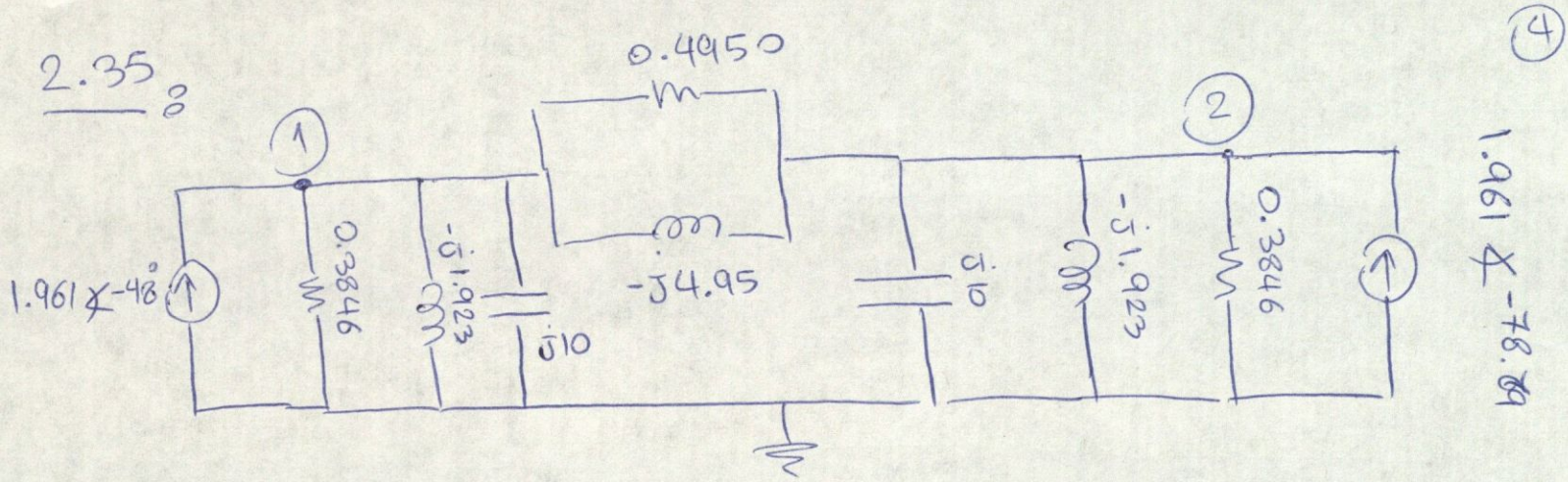
$S_{old} = 50000 + j\,37500$, $Q_{New} = \cos^{-1} 0.95 = 18.19^\circ$

$S_{New} = 50000 + j\,50000 \tan(18.19) = 50000 + j\,16430$

$S_{cap} = S_{old} - S_{New} = -j\,21070\ VA$

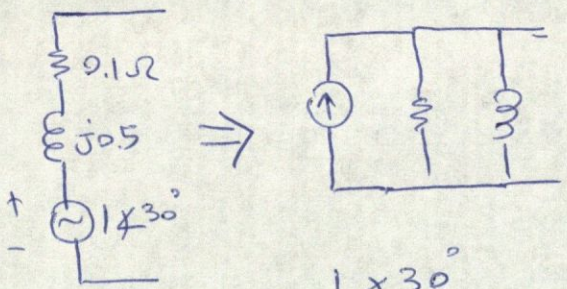
$$C = \frac{21070}{(377)(220)^2} = 1155\ \mu F$$

2.35 :



$$\begin{bmatrix} 0.3846+0.4950+j(10-1.923-4.95) & -(0.495-j4.95) \\ -(0.495-j4.950) & 0.3846+0.495+j(10-1.923-4.95 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.961\angle -48.69 \\ 1.961\angle -78.69 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.8796+j3.127 & -0.495+j4.95 \\ -0.495+j4.95 & 0.8796+j3.127 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.961\angle -48.69 \\ 1.961\angle -78.69 \end{bmatrix}$$



$$\frac{1}{0.1+j0.5} = \underbrace{0.3846 - j1.923}_{Y}$$

$$\underbrace{\frac{1}{0.1+j0.5}}_{Z}$$

$$\frac{1\angle 30^\circ}{0.1+j0.5} = 1.961\angle -48^\circ$$

$$\frac{1}{0.02+j0.2} = 0.495 + (-j4.95)$$