6.1.3 RELATION BETWEEN RADIANS AND DEGREES

The angle π radians corresponds to 180 degrees. Therefore,

$$\begin{aligned} \text{one radian} &= \frac{180}{\pi} = 57.30\,\text{degrees}, \\ \text{one degree} &= \frac{\pi}{180} = 0.01745\,\text{radians}. \end{aligned} \tag{6.1.1}$$

6.1.5 PERIODICITY RELATIONSHIPS

When n is any integer,

$$\begin{aligned} \sin(\alpha + n2\pi) &= \sin\alpha, \\ \cos(\alpha + n2\pi) &= \cos\alpha, \\ \tan(\alpha + n\pi) &= \tan\alpha. \end{aligned} \tag{6.}$$

6.1.6 SYMMETRY RELATIONSHIPS

$$\sin(-\alpha) = -\sin\alpha, \qquad \cos(-\alpha) = +\cos\alpha, \qquad \tan(-\alpha) = -\tan\alpha.$$

6.1.11 DEFINITIONS IN TERMS OF EXPONENTIALS

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}. \qquad e^{iz} = \cos z + i\sin z.$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}. \qquad e^{-iz} = \cos z - i\sin z.$$

$$\tan z = \frac{\sin z}{\cos z} = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}.$$

6.1.13 ANGLE SUM AND DIFFERENCE RELATIONSHIPS

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta. \\ \cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta. \\ \tan(\alpha \pm \beta) &= \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}. \\ \cot(\alpha \pm \beta) &= \frac{\cot\alpha\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}. \end{aligned}$$

6.1.14 DOUBLE ANGLE FORMULAE

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1+2\tan^2\alpha}.$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}.$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}.$$

$$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}.$$

6.1.18 PRODUCTS OF SINE AND COSINE

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta).$$

$$\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta).$$

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta).$$

6.1.19 SUMS OF CIRCULAR FUNCTIONS

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2}.$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$