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CRC

STANDARD MATHEMATICAL TABLES AND FORMULAE

6.5.2 DEFINITION OF e^z

$$\exp(z) = e^z = \lim_{m\to\infty}\left(1+\frac{z}{m}\right)^m$$
$$= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \dots .$$

If $z = x + iy$, then $e^z = e^x e^{iy} = e^x(\cos y + i \sin y)$

6.5.3 DERIVATIVE AND INTEGRAL OF e^x

The derivative of e^x is e^x. The integral of e^x is e^x.

6.5.4 CIRCULAR FUNCTIONS IN TERMS OF EXPONENTIALS

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \qquad e^{iz} = \cos z + i \sin z.$$
$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \qquad e^{-iz} = \cos z - i \sin z.$$

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CHAPTER 10 Sinusoidal Steady-State Analysis

2021_1001

#2

IN THIS CHAPTER

When all of the inputs to the circuit are sinusoids having the same frequency, the forced response $v_f(t)$ is also a sinusoid having the same frequency as the inputs. As time goes on, the transient part of the response dies out. The part of the response that is left is called the steady-state response. Once the transient part of the response has died out, we say that the circuit is "at steady state." In the case of sinusoidal inputs having the same frequency, the steady-state response is equal to the forced response, a sinusoid at the input frequency.

We can choose the output of our circuit to be any voltage or current that is of interest to us. We conclude that when a circuit satisfies the two conditions that (1) all of the inputs are sinusoidal and have the same frequency and (2) the circuit is at steady state, then all of the currents and voltages are sinusoidal and have the same frequency as the inputs. Traditionally, sinusoidal currents have been called *alternating currents* (*ac*) and circuits that satisfy the above conditions are called *ac circuits*.

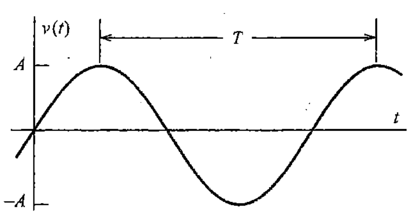


FIGURE 10.2-1 A sinusoidal function.

To summarize, an ac circuit is a steady-state circuit in which all of the inputs are sinusoidal and have the same frequency. All of the currents and voltages of an ac circuit are sinusoidal at the input frequency.

Consider the sinusoidal function

$$v(t) = A\sin(\omega t) \text{ V} \quad (10.2\text{-}1)$$

by the property

$$v(t+T) = v(t) \quad (10.2\text{-}2)$$

for all time. The constant T is called the "period of oscillation" or just the "period." The reciprocal of T defines the frequency or number of cycles per second, denoted by f, where

$$f = \frac{1}{T} \quad (10.2\text{-}3)$$

The units of frequency are hertz (Hz) in honor of the scientist Heinrich Hertz, shown in Figure 10.2-2. The angular frequency of the sinusoidal function is

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (10.2\text{-}4)$$

The units of angular frequency are radians per second.

Courtesy of the Institution of Electrical Engineers

FIGURE 10.2-2 Heinrich R. Hertz (1857–1894).

Next, consider the effect of replacing t by $t+t_a$ where t_a is some arbitrary constant time. As shown in Figure 10.2-3, $v(t+t_a)$ is a sinusoid that is identical to $v(t)$ except that $v(t+t_a)$ is advanced from $v(t)$ by time t_a. We have

$$v(t+t_a) = A\sin(\omega(t+t_a)) = A\sin(\omega t + \omega t_a) = A\sin(\omega t + \theta) \text{ V}$$

where θ is in radians and is called the phase angle of the sinusoid $A\sin(\omega t+\theta)$. The phase angle in radians is related to the time t_a by

$$\theta = \omega t_a = \frac{2\pi}{T} t_a = 2\pi \frac{t_a}{T} \quad (10.2\text{-}5)$$

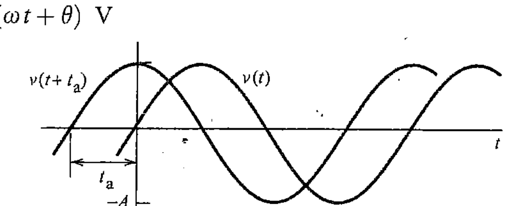


FIGURE 10.2-3 Advancing a sinusoid in time.

Similarly, replacing t by $t - t_d$ produces a sinusoid that is identical to $v(t)$ except that $v(t-t_d)$ is delayed from $v(t)$ by time t_d. We have

#3

10.16 SUMMARY

- With the pervasive use of ac electric power in the home and industry, it is important for engineers to analyze circuits with sinusoidal independent sources.
- The steady-state response of a linear circuit to a sinusoidal input is itself a sinusoid having the same frequency as the input signal.
- Circuits that contain inductors and capacitors are represented by differential equations. When the input to the circuit is sinusoidal, the phasors and impedances can be used to represent the circuit in the frequency domain. In the frequency domain, the circuit is represented by algebraic equations. The original circuit, represented by a differential equation, is called the time-domain representation of the circuit.
- The steady-state response of a linear circuit with a sinusoidal input is obtained as follows:

  1. Transform the circuit into the frequency domain, using phasors and impedances.
  2. Represent the frequency-domain circuit by algebraic equations, for example, mesh or node equations.
  3. Solve the algebraic equations to obtain the response of the circuit.
  4. Transform the response into the time domain, using phasors.

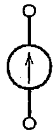
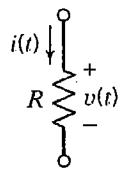
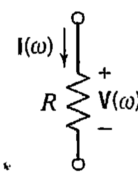
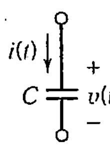
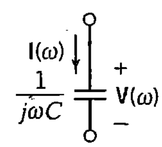
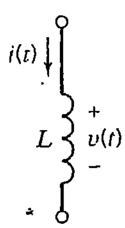
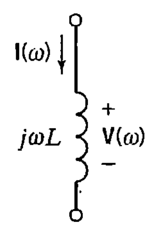
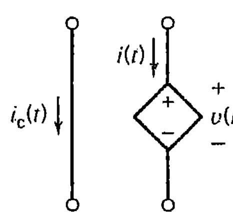
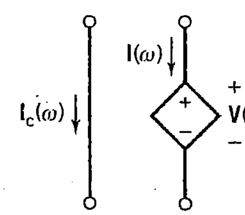
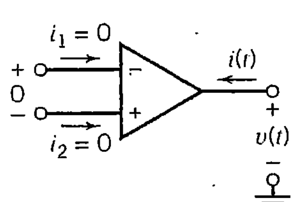
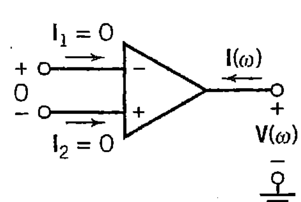
- Table 10.16-1 summarizes the relationships used to transform a circuit from the time domain to the frequency domain or vice versa.
- When a circuit contains several sinusoidal sources, we distinguish two cases.

  1. When all of the sinusoidal sources have the same frequency, the response will be a sinusoid with that frequency, and the problem can be solved in the same way that it would be if there was only one source.
  2. When the sinusoidal sources have different frequencies, superposition is used to break the time-domain circuit up into several circuits, each with sinusoidal inputs all at the same frequency. Each of the separate circuits is analyzed separately and the responses are summed *in the time domain.*

- MATLAB greatly reduces the computational burden associated with solving mesh or node equations having complex coefficients.

#4

Table 10.16-1 Time-Domain and Frequency-Domain Relationships

ELEMENT	TIME DOMAIN	FREQUENCY DOMAIN
Current Source	 $i(t) = A\cos(\omega t + \theta)$	 $\mathbf{I}(\omega) = Ae^{j\theta}$
Voltage source	 $v(t) = B\cos(\omega t + \phi)$	 $\mathbf{V}(\omega) = Be^{j\phi}$
Resistor	 $v(t) = R\, i(t)$	 $\mathbf{V}(\omega) = R\,\mathbf{I}(\omega)$
Capacitor	 $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau$	 $\mathbf{V}(\omega) = \frac{1}{j\omega C}\,\mathbf{I}(\omega)$
Inductor	 $v(t) = L\frac{d}{dt} i(t)$	 $\mathbf{V}(\omega) = j\omega L\,\mathbf{I}(\omega)$
CCVS	 $v(t) = K\, i_c(t)$	 $\mathbf{V}(\omega) = K\,\mathbf{I}_c(\omega)$
Ideal op amp	 $i_1 = 0$, $i_2 = 0$	 $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$

In summary, if a set of sinusoidal voltages $v_i(t)$ satisfy KVL for an ac circuit, the corresponding phasor voltages $\mathbf{V}_i(\omega)$ satisfy the same KVL equation. Similarly, if a set of sinusoidal currents $i_i(t)$ satisfy KCL for an ac circuit, the corresponding phasor currents $\mathbf{I}_i(\omega)$ satisfy the same KCL equation.

SAME #5

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EXAMPLE 10.3-4 Kirchhoff's Laws for AC Circuits

The input to the circuit shown in Figure 10.3-3 is the voltage source voltage,

$$v_s(t) = 25\cos(100t + 15°)\ \text{V}$$

The output is the voltage across the capacitor,

$$v_C(t) = 20\cos(100t - 22°)\ \text{V}$$

Determine the resistor voltage $v_R(t)$.

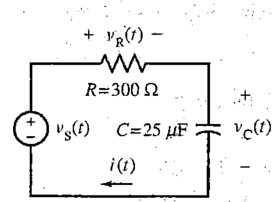

FIGURE 10.3-3 The circuit in Example 10.3-4

Solution

Apply KVL to get

$$v_R(t) = v_s(t) - v_C(t) = 25\cos(100t + 15°) - 20\cos(100t - 22°)$$

Writing the KVL equation using phasors, we have

$$\begin{aligned}\mathbf{V}_R(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_C(\omega) &= 25\,\underline{/15°} - 20\,\underline{/-22°}\\ &= (24.15 + j6.47) - (18.54 - j7.49)\\ &= 5.61 + j13.96\\ &= 15\,\underline{/68.1°}\ \text{V}\end{aligned}$$

Converting the phasor $\mathbf{V}_R(\omega)$ to the corresponding sinusoid, we have

$$\mathbf{V}_R(\omega) = 15\,\underline{/68.1°}\ \text{V} \quad \Leftrightarrow \quad v_R(t) = 15\cos(100t + 68.1°)\ \text{V}$$

10.4 Impedances

NEW

We've seen that all of the currents and voltages of an ac circuit are sinusoids at the input frequency. Figure 10.4-1a shows an element of an ac circuit. The element voltage and element current are labeled as $v(t)$ and $i(t)$. We can write

$$v(t) = V_m\cos(\omega t + \theta)\ \text{V} \quad \text{and} \quad i(t) = I_m\cos(\omega t + \phi)\ \text{A} \qquad (10.4\text{-}1)$$

where V_m and I_m are the amplitudes of the sinusoidal voltage and current, θ and ϕ are the phase angles of the voltage and current, and ω is the input frequency. The corresponding phasors are

$$\mathbf{V}(\omega) = V_m\,\underline{/\theta}\ \text{V} \quad \text{and} \quad \mathbf{I}(\omega) = I_m\,\underline{/\phi}\ \text{A}$$

Figure 10.4-1b shows the circuit element again, now labeled with the phasor voltage and current $\mathbf{V}(\omega)$ and $\mathbf{I}(\omega)$. Notice that the voltage and current adhere to the passive convention in both Figure 10.4-1a and Figure 10.4-1b.

The impedance of an element of an ac circuit is defined to be the ratio of the voltage phasor to the current phasor. The impedance is denoted as $\mathbf{Z}(\omega)$ so

$$\mathbf{Z}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)} = \frac{V_m\,\underline{/\theta}}{I_m\,\underline{/\phi}} = \frac{V_m}{I_m}\,\underline{/(\theta - \phi)}\ \Omega \qquad (10.4\text{-}2)$$

Consequently,

$$\mathbf{V}(\omega) = \mathbf{Z}(\omega)\,\mathbf{I}(\omega) \qquad (10.4\text{-}3)$$

A resistor from an ac circuit is shown in Figure 10.4-4*a*. We know that the resistor voltage is a sinusoid at the input frequency so we can write

$$v_R(t) = A\cos(\omega t + \theta)$$

The resistor current is

$$i_R(t) = \frac{v_R(t)}{R} = \frac{A}{R}\cos(\omega t + \theta)$$

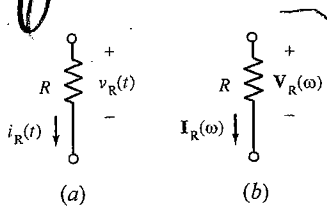

FIGURE 10.4-4 A resistor in an ac circuit represented (*a*) in the time domain and (*b*) in the frequency domain.

The impedance of the resistor is the ratio of the voltage phasor to the current phasor:

$$\mathbf{Z}_R(\omega) = \frac{\mathbf{V}_R(\omega)}{\mathbf{I}_R(\omega)} = \frac{A\angle\theta}{\frac{A}{R}\angle\theta} = R\ \Omega \tag{10.4-8}$$

The impedance of a resistor is numerically equal to the resistance. Using Eq. 10.4-3, we write

$$\mathbf{V}_R(\omega) = R\,\mathbf{I}_R(\omega) \tag{10.4-9}$$

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EXAMPLE 10.4-1 Impedances

The input to the ac circuit shown in Figure 10.4-5 is the source voltage

$$v_S(t) = 12\cos(1000t + 15^\circ)\ \text{V}$$

Determine (a) the impedances of the capacitor, inductor, and resistance and (b) the current $i(t)$.

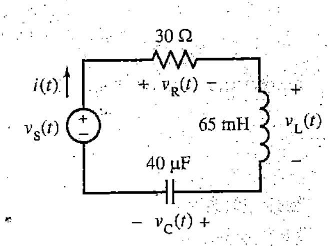

FIGURE 10.4-5 The AC circuit in Eample 10.4-1.

Solution

(a) The input frequency is $\omega = 1000$ rad/s. Using Eq. 10.4-4 shows that the impedance of the capacitor is

$$\mathbf{Z}_C(\omega) = \frac{1}{j\omega C} = \frac{1}{j1000\left(40\times 10^{-6}\right)} = \frac{25}{j} = -j25\ \Omega$$

Using Eq. 10.4-6 shows that the impedance of the inductor is

$$\mathbf{Z}_L(\omega) = j\omega L = j1000\,(0.065) = j65\ \Omega$$

Using Eq. 10.4-8, the impedance of the resistor is

$$\mathbf{Z}_R(\omega) = R = 30\ \Omega$$

(b) Apply KVL to write

$$12\cos(1000t + 15^\circ) = v_R(t) + v_L(t) + v_C(t)$$

Using phasors, we get

$$12\angle 15^\circ = \mathbf{V}_R(\omega) + \mathbf{V}_L(\omega) + \mathbf{V}_C(\omega) \tag{10.4-10}$$

Using Eqs. 10.4-5, 10.4-7, and 10.4-9, we get

$$12\angle 15^\circ = 30\,\mathbf{I}(\omega) + j65\,\mathbf{I}(\omega) - j25\,\mathbf{I}(\omega) = (30 + j40)\,\mathbf{I}(\omega) \tag{10.4-11}$$

Solving for $\mathbf{I}(\omega)$ gives

$$\mathbf{I}(\omega) = \frac{12\angle 15^\circ}{30 + j40} = \frac{12\angle 15^\circ}{50\angle 53.13^\circ} = 0.24\angle 38.13^\circ\ \text{A}$$

The corresponding sinusoid is

$$i(t) = 0.24\cos(1000t - 38.13^\circ)\ \text{A}$$

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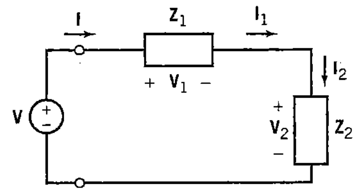
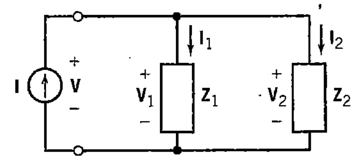
Table 10.5-1 Voltage and Current Division in the Frequency Domain

SAME

	CIRCUIT	EQUATIONS
Voltage division		$I_1 = I_2 = I$ $V_1 = \frac{Z_1}{Z_1 + Z_2} V$ $V_2 = \frac{Z_2}{Z_1 + Z_2} V$
Current division		$V_1 = V_2 = V$ $I_1 = \frac{Z_2}{Z_1 + Z_2} I$ $I_2 = \frac{Z_1}{Z_1 + Z_2} I$

SAME

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EXAMPLE 10.5-2 Voltage Division Using Impedances

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 10.5-4*a*. The input to the circuit is the voltage of the voltage source,

$$v_s(t) = 7.28 \cos(4t + 77°) \text{ V}$$

The output is the voltage across the inductor $v_o(t)$. Determine the steady-state output voltage $v_o(t)$.

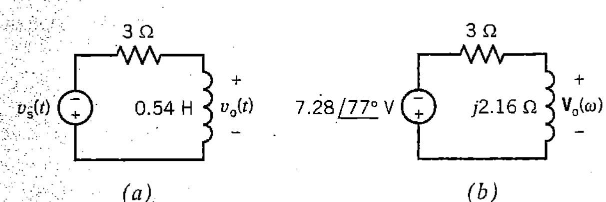


FIGURE 10.5-4 The circuit considered in Example 10.5-2 represented (*a*) in the time domain and (*b*) in the frequency domain.

Solution

The input voltage is sinusoid. The output voltage is also sinusoid and has the same frequency as the input voltage. The circuit has reached steady state. Consequently, the circuit in Figure 10.5-4*a* can be represented in the frequency domain, using phasors and impedances. Figure 10.5-4*b* shows the frequency-domain representation of the circuit from Figure 10.5-4*a*. The impedance of the inductor is $j\omega L = j(4)(0.54) = j2.16\,\Omega$, as shown in Figure 10.5-4*b*.

SAME

10.5 Series and Parallel Impedances

#8

Figure 10.5-1a shows a circuit called "Circuit A" connected to two series impedances. Using KCL Figure 10.5-1 shows that

$$\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I} \qquad (10.5\text{-}$$

Using Ohm's law in Figure 10.5-1a shows that

$$\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I}_1 = \mathbf{Z}_1\mathbf{I} \text{ and } \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2 = \mathbf{Z}_2\mathbf{I}$$

Using KVL in Figure 10.5-1a shows that

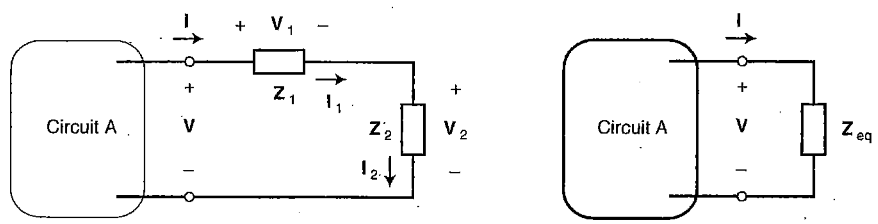
$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = (\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I} \qquad (10.5\text{-}$$

The impedance of the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$ is given by

$$\frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2$$

We say that the impedance $\mathbf{Z}_{eq}$ is equivalent to the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$ because replacing $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure 10.5-1a) by $\mathbf{Z}_{eq}$ in Figure 10.5-1b will not change the current or voltage of any element of Circuit A. Equation 10.5-3 generalizes to the case of n series impedances

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_n \qquad (10.5\text{-}4)$$



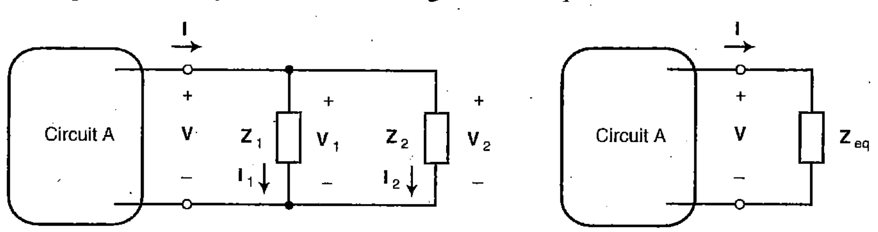
(a) (b)

FIGURE 10.5-1 Series impedances (a) and an equivalent impedance (b).

The voltages across the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure 10.5-1a are given by

$$\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I} = \mathbf{Z}_1\frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V} \text{ and } \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V} \qquad (10.5\text{-}5)$$

These equations show how $\mathbf{V}$, the voltage across the series impedances, is divided between the individual impedances. They are called the voltage division equations.



(a) (b)

FIGURE 10.5-2 Parallel impedances (a) and an equivalent impedance (b).

Figure 10.5-2a shows a circuit called "Circuit A" connected to two parallel impedances. Using KVL in Figure 10.5-2a shows that

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V} \qquad (10.5\text{-}6)$$

Using Ohm's law in Figure 10.5-2a shows that

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} = \frac{\mathbf{V}}{\mathbf{Z}_1} \text{ and } \mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_2} = \frac{\mathbf{V}}{\mathbf{Z}_2}$$

Using KCL in Figure 10.5-1a shows that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right)\mathbf{V} \qquad (10.5\text{-}7)$$

The impedance of the parallel combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$ is given by

$$\frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}}$$

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#9

EXAMPLE 10.5-5 Equivalent Impedance

Determine the equivalent impedance of the circuit shown in Figure 10.5-7a at the frequency $\omega = 1000$ rad/s.

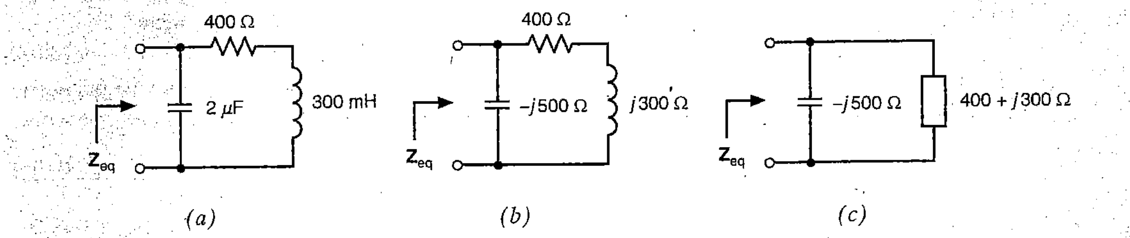

FIGURE 10.5-7 The circuit considered in Example 10.5-5 (a) in the time domain, (b) in the frequency domain, and (c) after replacing series impedances by an equivalent impedance.

Solution

Represent the circuit in the frequency domain as shown in Figure 10.5-7b. After replacing series impedances by an equivalent impedance, we have the circuit shown in Figure 10.5-7c. $\mathbf{Z}_{eq}$ is now seen to be the equivalent impedance of the parallel impedances in Figure 10.5-7c.

$$\mathbf{Z}_{eq} = \frac{-j500(400+j300)}{-j500+400+j300} = \frac{150,000-j200,000}{400-j200} = \frac{250,000\angle{-53.1^\circ}}{447.2\angle{-26.6^\circ}} = 599.0\angle{-26.5^\circ}\,\Omega$$

10.6 *Mesh and Node Equations*

SAME

We can analyze an ac circuit by writing and solving a set of simultaneous equations. Two methods, the node equations and the mesh equations, are quite popular. Before writing either the node equations or mesh equations, we represent the ac circuit in the frequency domain using phasors and impedances.

The node equations are a set of simultaneous equations in which the unknowns are the node voltages. We write the node equations by

1. Expressing the element voltages and currents (for example, the current and voltage of an impedance) in terms of the node voltages.
2. Applying KCL at the nodes of the ac circuit.

After writing and solving the node equations, we can determine all of the voltages and currents of the ac circuit using Ohm's and Kirchhoff's laws.

$\mathbf{I} = \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}}$

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EXAMPLE 10.6-2 Mesh Equation for AC Circuits

#10

Determine the mesh currents for the circuit shown in Figure 10.6-7.

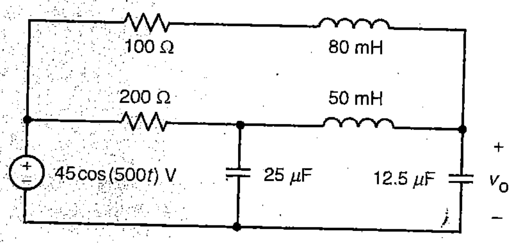

FIGURE 10.6-7 The AC circuit considered in Example 10.6-2.

Solution

First, we will represent the circuit in the frequency domain using phasors and impedances. The impedances of the capacitors and inductors are

$$-j\frac{1}{500(25\times 10^{-6})} = -j80\ \Omega,\ -j\frac{1}{500(12.5\times 10^{-6})} = -j160\ \Omega,\ j500(80\times 10^{-3}) = j40\ \Omega$$

and

$$j500(50\times 10^{-3}) = j25\ \Omega$$

The frequency domain representation of the circuit is shown in Figure 10.6-8. Also, the mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ are identified in Figure 10.6-8. Next, express the currents in the impedances as shown in Figure 10.6-9. The voltages across the impedances are labeled as $\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$, $\mathbf{V}_d$, $\mathbf{V}_e$, and Vo in Figure 10.6-9. Each of these voltages is expressed in terms of the mesh currents by multiplying an impedance by the currents in that impedance. For example,

$$\mathbf{V}_b = j40\mathbf{I}_1, \mathbf{V}_d = j25(\mathbf{I}_2 - \mathbf{I}_1) \text{ and } \mathbf{V}_e = -j80(\mathbf{I}_2 - \mathbf{I}_3)$$

Having expressed the impedance currents and voltages in terms of the mesh current, we next apply KVL to each of the meshes to obtain the following equations:

$$100\mathbf{I}_1 + j40\mathbf{I}_1 - j25(\mathbf{I}_2 - \mathbf{I}_1) - 100(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$200(\mathbf{I}_2 - \mathbf{I}_1) - j80(\mathbf{I}_2 - \mathbf{I}_3) - 45\angle 0^\circ = 0$$

and

$$-j25(\mathbf{I}_2 - \mathbf{I}_1) + (-j160)\mathbf{I}_3 - (-j80)(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

These simultaneous equations can be organized into a single matrix equation:

$$\begin{bmatrix} 300+j65 & -200 & -j25 \\ -200 & 200-j80 & j80 \\ -j25 & j80 & -j215 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 45 \\ 0 \end{bmatrix}$$

Solving, for example, using MATLAB, gives

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0.374\angle 115^\circ \\ 0.575\angle 25^\circ \\ 0.171\angle 28^\circ \end{bmatrix}$$

In the time domain, the mesh currents are

$$i_1(t) = 374\cos(500t + 15^\circ)\ \text{mA},\ i_2(t) = 575\cos(500t + 25^\circ)\ \text{mA}$$

and

$$i_3(t) = 171\cos(500t + 28^\circ)\ \text{mA}$$

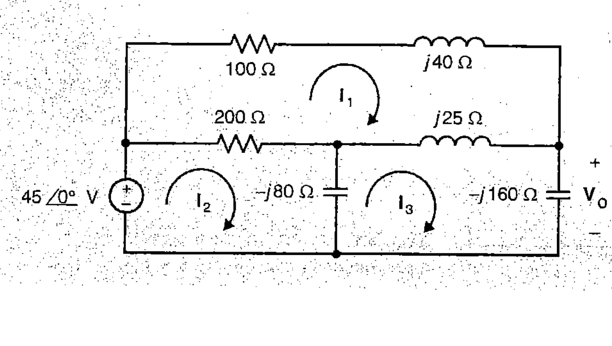

FIGURE 10.6-8 The AC circuit from Figure 10.6-7 represented in the frequency domain using phasors and impedances.

#11

10.7 SAME Thévenin and Norton Equivalent Circuits

In this section, we will determine the Thévenin and Norton equivalent circuits of an ac circuit.

Figure 10.7-1 illustrates the use of Thévenin and Norton equivalent circuits. In Figure 10.7-1*a*, an ac circuit is partitioned into two parts—circuit A and circuit B—that are connected at a single pair of terminals. (This is the only connection between circuits A and B. In particular, if the overall circuit contains a dependent source, then either both parts of that dependent source must be in circuit A or both parts must be in circuit B.) In Figure 10.7-1*b*, circuit A is replaced by its Thévenin equivalent circuit, which consists of a voltage source in series with an impedance. In Figure 10.7-1*c*, circuit A is replaced by its Norton equivalent circuit, which consists of a current source in parallel with an impedance. Replacing circuit A by its Thévenin or Norton equivalent circuit does not change the voltage or current of any element in circuit B. This means that if you looked at a list of the values of the currents and voltages of all the circuit elements in circuit B, you could not tell whether circuit B was connected to circuit A or connected to its Thévenin equivalent or connected to its Norton equivalent circuit.

Finding the Thévenin or Norton equivalent circuit of circuit A involves three parameters: the open-circuit voltage $\mathbf{V}_{oc}$, the short-circuit current $\mathbf{I}_{sc}$, and the Thévenin impedance $\mathbf{Z}_t$. Figure 10.7-2

EXAMPLE 10.7-1 Thévenin Equivalent Circuit

Find the Thévenin equivalent circuit of the ac circuit in Figure shown in Figure 10.7-3.

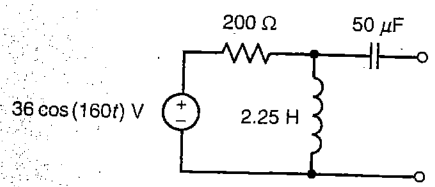

FIGURE 10.7-3 The circuit considered in Example 10.7-1.

Solution

We begin by representing the circuit from Figure 10.7-3 in the frequency domain, using phasors and impedance. The result, shown in Figure 10.7-4, corresponds to circuit A in Figures 10.7-1 and 10.7-2.

Next, we determine the open-circuit voltage using the circuit shown in Figure 10.7-5*a*. In Figure 10.7-5*a*, an open circuit is connected across the terminals of the circuit from Figure 10.7-3. The voltage across that open circuit

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10.8 Superposition

Suppose we encounter a circuit that is at steady state and all of its inputs are sinusoidal but not all of the input sinusoid have the same frequency. Such a circuit is not an ac circuit and the currents and voltages will not be sinusoidal. We can analyze this circuit using the principle of superposition.

The principle of superposition says that the output of a linear circuit due to several inputs working together is equal to the sum of the outputs working separately. The inputs to the circuit are the voltages of the independent voltage sources and the currents of the independent current sources.

When we set all but one input to zero, the other inputs become 0-V voltage sources and 0-A current sources. Because 0-V voltage sources are equivalent to short circuits and 0-A current sources are equivalent to open circuits, we replace the sources corresponding to the other inputs by open or short circuits. We are left with a steady-state circuit having a single sinusoidal input. Such a circuit is an ac circuit and we analyze it using phasors and impedances.

Thus, we use superposition to replace a circuit involving several sinusoidal inputs at different frequencies by several circuits each having a single sinusoidal input. We analyze each of the several ac circuits using phasors and impedances to obtain its sinusoidal output. The sum of those several sinusoidal outputs will be identical to the output of the original circuit. The following example illustrates this procedure.

EXAMPLE 10.8-1 Superposition

Determine the voltage $v_o(t)$ across the 8-Ω resistor in the circuit shown in Figure 10.8-1.

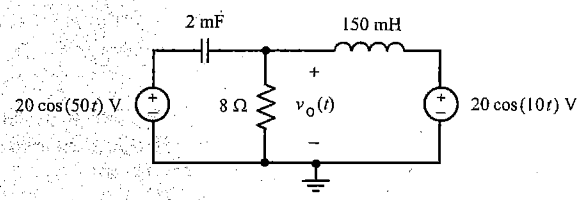

FIGURE 10.8-1 The circuit considered in Exampl

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HW6 Due Wed Oct6
HW7 Due Fri Oct 7

$$v(t) = A\cos(\omega t + \theta)\ \text{V}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

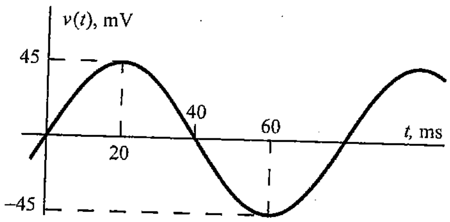

Figure P 10.2-4

P 10.2-5 Figure P 10.2-5 shows a sinusoidal voltage $v(t)$, plotted as a function of time t. Represent $v(t)$ by a function of the form $A\cos(\omega t + \theta)$.

Answer: $v(t) = 18\cos(393t - 27°)$

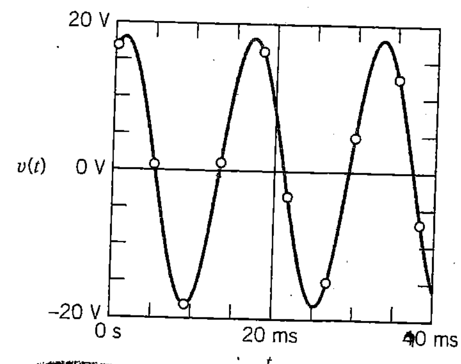

Figure P 10.2-5

HW 6

P 10.2-6 Figure P 10.2-6 shows a sinusoidal voltage $v(t)$, plotted as a function of time, t. Represent $v(t)$ by a function of the form $A\cos(\omega t + \theta)$.

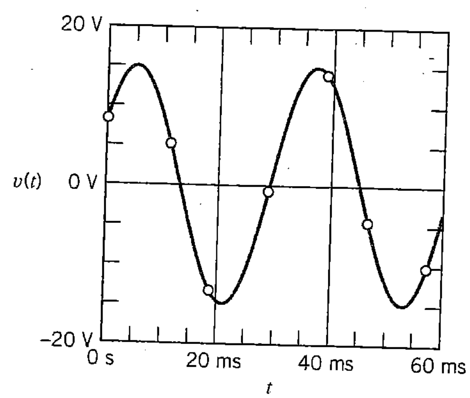

Figure P 10.2-6

Section 10.3 Phasors and Sinusoids

P 10.3-1 Express the current

$$i(t) = 2\cos(6t + 120°) + 2\sin(6t - 60°)\ \text{mA}$$

in the general form

$$i(t) = A\cos(\omega t + \theta)\ \text{mA}$$

HW6

where $A \geq 0$ and $-180° < \theta \leq 180°$.

P 10.3-2 Express the voltage

$$v(t) = 5\sqrt{2}\cos(8t) + 2\sin(8t + 45°)\ \text{V}$$

in the general form

$$v(t) = A\cos(\omega t + \theta)\ \text{V}$$

HW6

where $A \geq 0$ and $-180° < \theta \leq 180°$.

P 10.3-3 Determine the polar form of the quantity

$$\frac{\left(25\angle 36.9°\right)\left(80\angle -53.1°\right)}{(4 + j8) + (6 - j8)}$$

HW6

Answer: $200\angle -16.2°$

P 10.3-4 Determine the polar and rectangular form of the expression

$$5\angle +81.87°\left(4 - j3 + \frac{3\sqrt{2}\angle -45°}{7 - j1}\right)$$

HW7

Answer: $88.162\angle 30.127° = 76.2520 + j44.2506$

P 10.3-5 Determine the polar and rectangular form of the expression

$$\frac{\left(60\angle 120°\right)\left(-16 + j12 + 20\angle 15°\right)}{5\angle -75°}$$

HW7

P 10.3-6 The circuit shown in Figure 10.3-6 is at steady state. The current source currents are

$i_1(t) = 10\cos(25t)$ mA and $i_3(t) = 10\cos(25t + 135°)$ mA

Determine the voltage $v_2(t)$.

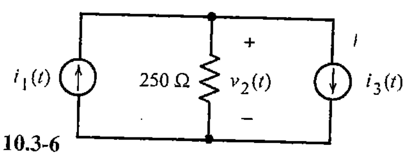

Figure P 10.3-6

P 10.3-7 The circuit shown in Figure 10.3-7 is at steady state. The inputs to this circuit are the current source current

HW7

$$i_1(t) = 0.12\cos(100t + 45°)\ \text{A}$$

and the voltage source voltage

$$v_2(t) = 24\cos(100t - 60°)\ \text{V}$$

Determine the current $i_2(t)$.

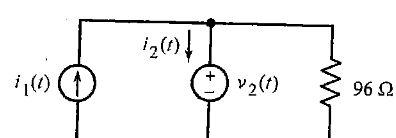

Figure P 10.3-7

P 10.3-8 ⊕ Given that

$$i_1(t) = 30\cos(4t + 45°)\ \text{mA}$$

and

$$i_2(t) = -40\cos(4t)\ \text{mA}$$